Quantum Entanglement:

The discovery, nature, and implications of a peculiar phenomenon

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When we study the world of the very small, observing events that take place in a billionth of a second over distances so small as to be inconceivable by the human mind, we come across phenomena that are completely impossible to explain using classical physics. Particles disappear and reappear in other places, at a whim. They travel straight through seemingly impenetrable barriers. They even seem to exist in two locations at once. Physicists in the early part of the 20th century were quite successful in developing a comprehensive theory that could explain this strange behavior and correctly predicted what particles would do in experimental settings. This theory, called *quantum mechanics*, is generally regarded to be the most successful scientific theory ever conceived. It is able to predict the behavior of particles to stunning degrees of accuracy, and it is verified again and again in laboratories all over the world. In this paper, my aim is to introduce the reader to the deepest and most tantalizing of these behaviors, which Erwin Schrödinger called "not one, but rather *the* characteristic trait of quantum mechanics," - quantum entanglement (Aczel 70). I will show how the interpretation of Schrödinger's famous wave function allows for an entangled quantum "state" to exist, and the trouble that this discovery caused in the physics community of the 1930s. I will then present several examples and real experiments to solidify the idea of entanglement and briefly describe some of its profound physical implications.

We will begin with a discussion of what the famous theoretical physicist Richard Feynman once called "the only mystery" in quantum mechanics; that is, the strange phenomena observed in a more thorough analysis of Thomas Young's famous double slit experiment. Young performed this experiment with visible light in the early part of the 19th century, and his results affirmed that light exhibits wave-like behavior. He obtained this result by shining a beam of light through two slits separated by a very small distance so that the light was incident on a viewing screen. Young observed that the light exhibited an interference pattern, a visual effect caused by the wave going through the slits and splitting up into two separate light waves, which then produce a bright band on the screen when their respective peaks meet, and a dark band when a peak meets a trough (Fig. 1). The experiment confirmed for physicists that light travels as a wave. However, in the early 20th century, the development of quantum mechanics was accompanied by the revelation that light must also be considered to travel as a particle, which we call a *photon* (Aczel 20). As a result of this "wave-particle duality," a very strange problem arises.

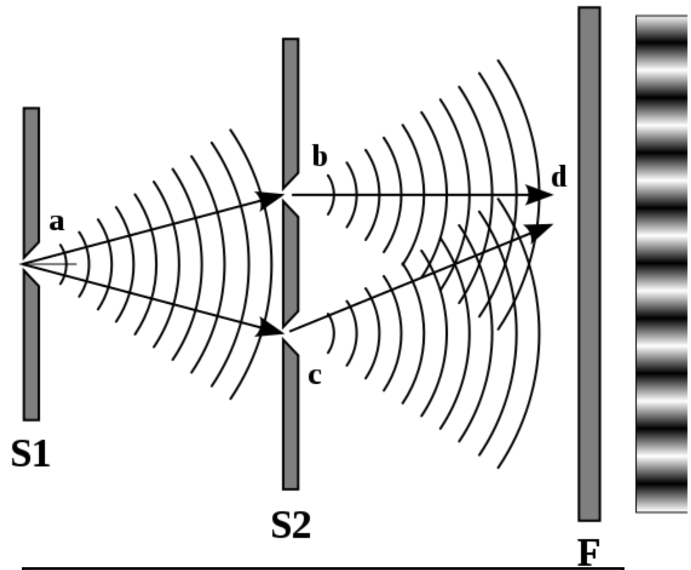


Fig. 1: The double-slit experiment. Light is emitted from a source at "a", travels through two slits at "b" and "c" and exhibits an interference pattern at "d." <http://en.wikipedia.org/wiki/File:Ebohr1.svg>

If we imagine the light beam as a beam of photons, then it is natural to ask the question, "How can particles interact to produce an interference pattern?" We have no way of describing a particle as having peaks and troughs, so we could stop here and conclude that in this experiment light must behave only as a wave. However, the experiment has been performed with light of such weak intensity that *only one photon at a time* travels through the apparatus and arrives at the screen (Aczel 21). The photon is expected to travel through one of the slits (or through neither) and arrive at the viewing screen so that we might see a collection of blips of light as the experiment progresses. This is not the case, however. We observe the same interference pattern as before – it is as if the photon goes through both slits at once and interferes with itself. Even stranger,if we set up the experiment so that we can observe through which slit the particle travels , *the interference pattern disappears*. How can the particle have gone through both slits, and why does it behave differently when we observe[1] it? The first question is addressed in quantum mechanics by the *principle of superposition of states*. If the particle can go through slit A or slit B, then the superposition principle says that "the particle is in state A when it passes through slit A and in state B when it passes through slit B. The superposition of states is a combination of 'particle goes through slit A' with 'particle goes through slit B.'…in a sense, then, the particle has gone through both slits, and as it arrived at the end of the experimental setup, it interfered with itself" (Aczel 25). The superposition principle says that in quantum mechanics, our particle must be described as being in a "state" that includes both slits. To make sense of this, and to find out why an observation changed the behavior of the particle, it is necessary to become acquainted with the work of Erwin Schrödinger, one of the sculptors of the quantum theory. The principle of superposition of states can be explained by Schrödinger's wave function. Schrödinger was able to formulate an explanation of the wave nature of particles using a differential equation that when solved, would yield a solution Ψ, called the wave function. This wave function gives us a mathematical way of analyzing the behavior of quantum mechanical particles. The magnitude of this solution squared, $|\Psi|^2$, gives the probability that a particle can be found in a certain region of space. In this way, it is natural in quantum mechanics to think of particles not as being located in a specific place, but instead as being described by a *wave of probabilities*. If one solution of Schrödinger's equation describes the particle as being in one particular state, then a superposition of two states is still a solution of Schrödinger's equation for that particle, since the solutions of the equation are waves and a sum of these waves must also be a solution (Aczel 68). Therefore, an explanation for the double slit experiment is that "the single photon does not choose one slit *or* the other to go through. It chooses both slits, that is, one slit *and* the other. The particle goes through both slits, and then it interferes with itself, as two waves do by superposition" (Aczel 69). However, when we impose a path for the photon to follow by observing its motion, we destroy the superposition and therefore the interference pattern. In this sense, the wave function describing the superposition "collapses" into one state (Fig. 2)[2]. Thus, the observation of this system, and any quantum system in general, inexorably changes that system. The power of the observer in quantum mechanics, and the problem of observing

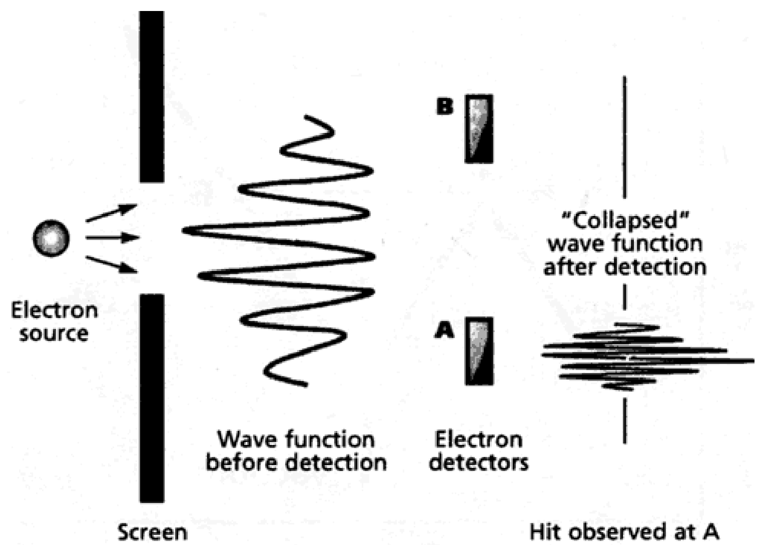


Figure 2: wave function collapse. <http://www.mukto-mona.com/Special_Event_/rationalist_day/2005/stenger2.gif>

and measuring a system, is the topic of a different paper, and we will not go into much more detail here. However, we will come back to the idea of the collapse of the wave function shortly.

In 1926, when Schrödinger published his paper on wave mechanics of quantum systems, he anticipated that a natural consequence of his wave function's interpretation of quantum behavior was that in a system of multiple particles, it is possible to observe not just interference of one particle with itself, but interference of *the state of two particles with itself*. That is, when two particles interact with each other, their wave functions become related in such a way that we must now describe the state of the two particles with absolutely no reference to their individual properties. This superposition of two-particle states gives rise to what Schrödinger later called *entanglement*. It was this entanglement of particles that Albert Einstein, along with colleagues Boris Podolsky and Nathan Rosen, sought to use to show that the theory of quantum mechanics was incomplete, in a joint paper published in *Physical Review* in 1935. For the reader, a discussion of the contents of this paper as well as its implications for the quantum theory, including the conflict it spurred with the proponents of quantum theory at the time of its publication, is instrumental in developing an understanding of entanglement. It is for this reason that we now proceed to a discussion of the "EPR incompleteness argument" (Steward 156).

Prior to the publication of the EPR (Einstein-Podolsky-Rosen) paper in 1935, Einstein had been involved in a series of heated arguments with the developers of the quantum theory, including Niels Bohr and Werner Heisenberg. Einstein agreed that the quantum theory was adequate in its ability to correctly predict the outcome of experiments, but he disagreed with the idea that the reality we experience is determined by probability and chance, as can be inferred from our earlier discussion of Schrödinger's wave equation. His main argument concerned a fundamental aspect of quantum mechanics: the Heisenberg uncertainty principle. This principle was proposed by Werner Heisenberg in 1927 and included relations between *observables* of a particle, which can be measured or predicted by an experimenter. The uncertainty principle states that certain observables of a particle are related in such a way that if we know one of them to a good degree of accuracy, then we know much less about the other (the most well-known are position and momentum). It implies, for example, that *if we know more about where the particle is, then we know less about what it is doing*.

Now, although Einstein was in agreement that the uncertainty principle correctly accounted for observed experimental phenomena, he had a problem with the quantum mechanical description of reality itself. His arguments with Bohr and Heisenberg in the years leading up to the publication of the EPR paper centered around the idea that "quantum theory was perhaps the correct theory of statistical laws but it did not provide an adequate treatment of individual elementary processes…[Einstein] felt there should be a deeper, independent, theoretical framework – what he called 'objective reality' – for dealing with the latter" (Steward 154). For example, Einstein thought that although we might not be able to measure both the position and momentum of a particle exactly, this does not mean that the particle does not have both of these properties before we gain knowledge of one of them by a measurement. Thus, "Einstein believed that there was something missing from the quantum theory, some variables, perhaps, such that if we could find the values of these variables, the uncertainty…would be gone" (Aczel 108). He attempted to prove the incompleteness of quantum theory in the EPR paper of 1935, but we will see that the very behavior that Einstein wanted to prove paradoxical, later called entanglement, would be vindicated years later.

In the EPR paper, entitled, "Can Quantum Mechanical Description of Physical Reality Be Considered Complete?" the authors use clear logic and simple mathematics to expose an apparent paradox, based on the concept of "elements of physical reality." According to EPR, "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" (Einstein 777). Here it is prudent to introduce an example that illustrates the above principle as well as the paradox EPR wanted to show. I will use the same example present in Steward's *Quantum Mechanics*, p.155. Consider two particles, A and B, which have collided and separated to an arbitrary distance. The system has a total momentum that is conserved throughout their interaction, and a total position that incorporates each of their individual locations. If we perform a measurement of particle A's momentum, then the momentum of particle B is immediately deduced without disturbing particle B whatsoever. In the same manner if we measure A's position, then we deduce B's position. Thus both the position and momentum of particle B must be elements of physical reality, possessed by the particle simultaneously. This contradicts the uncertainty principle and thus suggests, according to EPR, "the quantum mechanical description of physical reality given by wave functions is not complete" (Einstein 780). The argument seems perfectly reasonable – either quantum mechanics is incomplete, or there exists some kind of instant communication between two particles when one of them is measured – something Einstein referred to as "spooky action at a distance," and which we now refer to as *non-local communication*. EPR sought to supplement the existing quantum theory with a theory of "hidden variables," which could describe the properties of a quantum state of two particles without this non-local communication. This hidden variable theory was opposed by a different interpretation of the EPR phenomenon, commonly referred to as the Copenhagen Interpretation. This interpretation's main argument is as follows, beautifully worded in Ohanian's *Principles of Quantum Mechanics*:

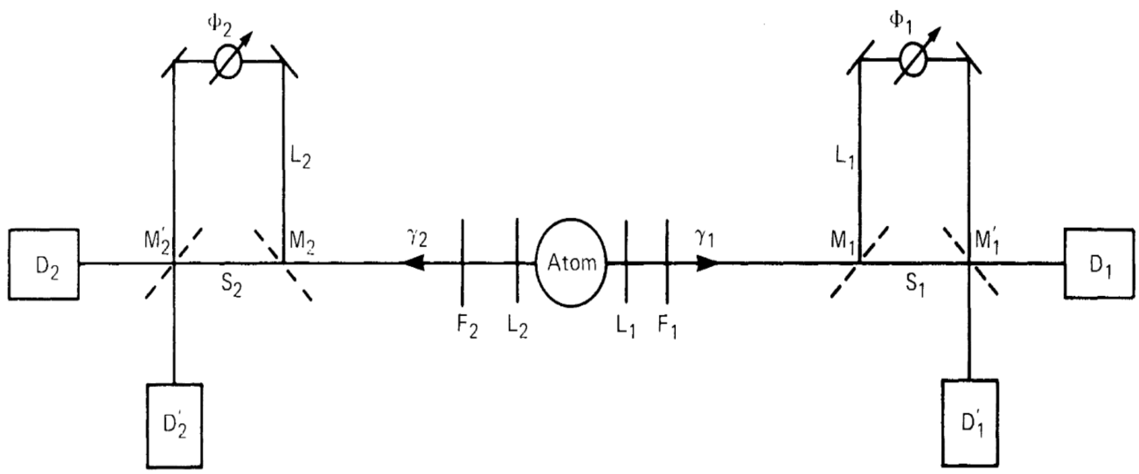
> We cannot measure one portion of the quantum-mechanical wave function and leave the rest undisturbed. When we measure any portion of the wave function, the *whole* wave function collapses. The strange simultaneous collapse of the states of both particles in the EPR *Gedankenexperiment* (German for "thought experiment") is no more remarkable than the simultaneous collapse of all parts of the wave function of a single particle…the system of the two particles has a single wave function, which happens to depend on two variables. The wave function cannot be regarded as consisting of separate, disjoint pieces" (373).

I want to point out an important distinction to the reader here. The non-local communication between the two particles, with which Einstein was uncomfortable, is a consequence of the Copenhagen interpretation and must be considered incompatible with Einstein's theory of hidden variables. Erwin Schrödinger, in a written reaction to the EPR paper, called this non-local communication "entanglement," and defined an entangled state of two particles: "When two systems, of which we know the states by their respective representations, enter into a temporary physical interaction due to known forces between them and when after a time of mutual influence the systems separate again, then they can no longer be described…by endowing each of them with a representative of its own" (Aczel 70). What Schrödinger means here is that two particles in an entangled state share a common wave function. A measurement on one of the particles initiates the non-local communication between the two particles.

Now we see that the nature of entanglement, as mentioned before, is essentially a superposition of multiple states of a system of multiple particles. In an entangled state, *the particles themselves do not have individual properties until measurement of the system results in a collapse of their shared wave function.* The reader may be left with a question – why should entanglement be right, and Einstein's hidden variable theory wrong? A theory to distinguish between the two alternatives was not developed until 1964 when physicist John Bell derived an inequality, now commonly referred to as Bell's Inequality, which could use the results of experiments to determine whether a quantum system exhibits behavior in support of entanglement or a hidden variable theory (Aczel 145). Bell's inequality tests the number of possible results of an experiment – if the inequality is violated, then quantum mechanics must exhibit non-local communication, and if the inequality holds, than the results are in support of a hidden variable theory. Alain Aspect, a French physicist, was the first to experimentally violate Bell's inequality in 1982. The results of his experiment, which utilized a complicated system of interferometers, were in strong support of entanglement (Scarani 82). To solidify the reader's understanding of entanglement, I will now provide several examples of EPR-type experiments, which exhibit the bizarre implications of the entanglement phenomenon and which will help to clarify the concepts involved.

First, we will examine a relatively simple example involving photon polarizations. A photon's polarization describes the direction in which its corresponding light wave moves through space. The scenario is attributed to Steward's *Quantum Mechanics*, p. 166. A source emits entangled pairs of photons with correlated polarizations so that the state of the two photons has a polarization along a particular axis – that is, if one photon has a polarization along some axis, then the other one must as well. The photons, A and B, travel in opposite directions and arrive at detectors D_A and D_B. Each of the detectors is covered with a piece of polaroid material with its axis aligned so that there is a 50% chance of the photon being transmitted and detected, and a 50% chance of the photon being blocked. Photon A takes a slightly shorter path than Photon B. Now, imagine that there are two physicists, one at each detector, monitoring the number of hits & misses. Each experimenter registers a random sequence of hits and misses, depending on whether the photon gets through the polaroid. Now, if the physicists have aligned their respective polaroids along the same axis, then they observe an interesting result. Their seemingly random sequences of hits and misses are *exactly the same*. When photon A registered a hit, so did photon B, and when photon A missed the detector, photon B missed as well. Now, the reader can likely intuit by now that this correlation is exhibited regardless of the distance between the detectors, and is directly attributed to the entangled state. When photon A meets the polaroid, the system of the two photons interferes with itself in such a way as to collapse to either "both photons are transmitted" or "both photons are blocked."

The second example is a bit more complicated, but includes an important result. The setup is attributed to J.D. Franson, in his article published in 1989 in *Physical Review Letters* entitled "Bell Inequality for Position and Time." The setup, depicted below, is called a Franson interferometer.



As in the prior example, two photons are emitted on opposite sides of the source, but this time there are 4 detectors. Set in each respective photon's path are half-silvered mirrors, represented by dashed lines in the figure, which allow each photon to take either the long path (photon is reflected and takes L_1, L_2) or the short path (photon is transmitted and takes S_1, S_2). Now, there are 4 possible configurations that can result: both photons take the short path (SS), both photons take the long path (LL) or each photon takes a different path (SL or LS).

This example is pertinent because we can observe the "interference between states" that was mentioned in our definition of entanglement. Thinking back to our discussion of the double slit experiment, we observed that the particle only exhibits an interference pattern if we do not observe which slit the particle goes through. In the context of this experiment, this effect is known as the "indistinguishability principle," and serves as the criteria for observing interference between states in this experiment as well. In the double slit experiment, there were two paths which were indistinguishable from each other: we had no way of knowing which path the particle had taken without observing it, and this produced an interference pattern. In the case of the Franson inteferometer, let's examine our four possible configurations. Remember, we are looking for two indistinguishable alternatives. If we compare hits at two detectors and find that the photons arrived at different times, we obviously know that we have either the SL or LS configuration. But are these outcomes distinguishable? The answer is most certainly 'yes,' because we know which photon took the short path by noting which detector registers a hit first. Therefore, since SL and LS are distinguishable, we observe no correlation between the results of the two detectors since no interference between states has occurred. Now, if two detectors register at the same time, we know that the photons have taken paths of the same length. We have two possible configurations that are indistinguishable from each other: SS and LL. The reader might object to this, noting that the time to take SS should be shorter than LL. However, the experiment performed by Franson used a source that emitted pairs of photons at various times, *allowing an inherent uncertainty in when the photons are emitted* (Franson 2205). Therefore, since we don't know when the pair enters the apparatus, we have absolutely no way of distinguishing between the states SS and LL. Interference between states occurs, and a direct correlation in the results is observed when detections are compared – the photons arrive at corresponding detectors whenever simultaneous hits are detected. The experiment is in strong support of entanglement.

The third and final example exhibits a fascinating phenomenon that is the subject of much current research in quantum physics[3]. It is called "quantum teleportation," and it is indeed teleportation in the sense that information travels between two particles and is not available at any point between them. However, it is important to distinguish that what is teleported here is not an actual particle, but rather the quantum state of one particle on to another. The experiment described here was performed at the Universität Innsbruck in Austria in 1997, and its results published in *Nature* by D. Bouwmeester, J. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger. It is the first experimental verification of quantum teleportation.

In the experiment, three photons were involved. Photon 1 was produced and passed through polaroid so as to acquire an initial state of polarization 45 degrees. Photon 2 and 3 were produced in an entangled state using the method of "parametric down conversion." This technique involves passing a photon through a crystal so that it splits into two photons of opposite polarization – the polarizations of the individual photons are not known, but they must be opposite. This indistinguishability produces the desired entanglement effect. A "Bell-state measurement" was then performed on photons 1 and 2, wherein photon 1 lost its polarization state, becoming entangled with photon 2. The Bell-state measurement is essentially a way of "asking" photons 1 and 2 about their polarization state by passing them through a beam splitter and into two detectors. If the two detectors register a coincidence, then the polarization of photon 2 is projected as opposite that of photon 1, and since photon 2 and photon 3 were entangled, *photon 3 acquires the polarization state that photon 1 had originally*. In the experiment, "the polarization of photon 3 is analyzed by passing it through a polarizing beam splitter selecting +45 degrees and -45 degrees polarization" (577). Teleportation is successful when photon 3 is detected with a polarization of +45 degrees. The quantum state of photon 1 was teleported instantaneously to photon 3. The reader will recall that in an entangled state, communication between particles happens instantaneously regardless of the distance separating the two particles, as affirmed in this experiment: "the transfer of quantum information from particle 1 to particle 3 can happen over arbitrary distances, hence the name teleportation" (576). This implies that there is no limit on the speed of the information transmitted. However, it would seem that this idea violates Einstein's theory of special relativity, which states that information cannot be transmitted faster than the speed of light. It is important to note here, and in the general idea of quantum teleportation using 3 particles, that the result of the Bell-state measurement on particles 1 and 2 must be communicated to an experimenter at particle 3 by classical means (shouting, a telephone call, an email, etc.). Therefore, although information is transferred to particle 3 with no limit on its speed, this information *is not useful* until realized by a classical information channel. Therefore, communication between entangled particles does not violate the theory of special relativity, because the information transferred is completely random and useless until discovered by classical means.

By now, the reader is familiar with several fascinating examples of the entanglement phenomenon. At this point, as we near the conclusion, it is worth exploring briefly one of the promising implications of communication between entangled particles. This is called quantum information theory. The computers of today use the binary system, in which the basic unit of information is a "bit" that takes on the value of 1 or 0." Therefore one bit can represent either 1 or 0, two bits can represent one of four numbers, and *N* bits can represent one of 2^Nnumbers (Steward 175). However, a quantum particle carrying information can represent both 1 and 0 simultaneously, since it can be represented by a superposition of these states. This unit of information is called the *qubit*, and has immense implications for the field of modern computing. Imagine a "quantum computer," which processes information in qubit form by way of multiple entangled particles. In this way, information processing speeds could increase exponentially, allowing quantum computers to perform tasks that would take a modern computer thousands of years to complete, such as factoring extremely large numbers. The field of quantum computing is growing rapidly as techniques to produce and sustain entangled states advance in sophistication[4].

Quantum entanglement is the embodiment of the deep mysteries inherent in quantum theory. It effectively contradicts all notions of causality; that one event follows another has no representation in the behavior of entangled particles. If one considers entanglement in the cosmic sense, the implications are quite astounding – imagine that at the time of the Big Bang, all matter in the universe interacted so intimately that today the entire universe consists of a vast web of entangled particles. I will leave it up to the reader to consider the possibilities arising from one tug on a strand of this web.

Notes

1. That is, when the experimenter by one method or another measures the particle's position at some point between the source and the screen. For a more detailed description, see the discussion of the Heisenberg Uncertainty Principle on page 5.
2. The figure depicts an electron traveling through the slit instead of the photons that have been discussed. Unlike the photon, the electron is a massive particle, and is considered "matter," while the photon does not possess mass and is essentially energy. It is important to note here that massive particles, in addition to photons, behave as waves in quantum mechanics.
3. For examples, the reader can explore any number of science websites. Popsci.com does a good job in a couple of articles:
   a. http://www.popsci.com/scitech/article/2004-10/atoms-beam
   b. http://www.popsci.com/scitech/article/2009-06/quantum-entaglement

4. See "Google Demonstrates Quantum Algorithm Promising Superfast Search," available at http://www.popsci.com/technology/article/2009-12/google-algorithm-uses-quantum-computing-sort-images-faster-ever

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