HONORS THESIS

"Does the Use of Manipulatives Really Make a Difference in Teaching a Unit on Quadrilaterals?"

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ABSTRACT

Does the use of manipulatives really make a difference in teaching a unit on quadrilaterals in a geometry class? Specifically, is there a notable improvement in students' achievement or attitudes toward geometry? Based on the theories of cognitive styles, does a particular type of learner respond best to the use of manipulatives? These were the questions considered in the study.

In order to investigate these questions, the experimenting student teacher conducted her research on her two geometry classes of tenth through twelfth graders. Due to a difference in class size and student ability, twelve students in the experimental group were paired with twelve students in the control group for the purposes of validity and consistency. The experimenter then administered two pretests, one achievement and one attitude/learning style, to both groups. Since these instruments were designed by the experimenter and the thesis advisor, they must be considered an informal means of assessing these qualities. With these scores documented, the experimenter began teaching a unit on quadrilaterals to both groups. The experimental group's lessons were enriched with manipulatives intended to add meaning by linking the concrete to the abstract mathematical concepts. The control group was taught in a traditional manner, void of the use of manipulatives. The entire fourteen-day unit was taught in this manner. At the end of

the unit, the chapter test, the achievement posttest, and the attitude posttest were administered, and the scores on each were documented.

Analyzing these scores, significant t values (at the 95% confidence level) were obtained by the experimental group in the areas of achievement and attitude when the pretest and posttest scores on each were compared. Their results on the t tests were clearly more impressive than those of the control group. Thus, the advantages of using manipulatives to add meaning by bridging the gap between the concrete realm and the abstract realm were supported by this testing. Specifically, t tests also revealed that visual learners followed by kinesthetic learners in the experimental group acceled the most of any group in achievement due to the use of manipulatives. In summary, substantial improvements in achievement and attitude were experienced by the experimental group who learned through manipulative activities.

THESIS STATEMENT

Does the use of manipulatives really make a difference in teaching a unit on quadrilaterals in a geometry class? Specifically, is there a notable improvement in students' achievement or attitudes toward geometry? Based on the theories of cognitive styles, does a particular type of learner respond best to the use of manipulatives?

INTRODUCTION

As a means of introduction, manipulatives are defined to be "concrete models that incorporate mathematical concepts, appeal to several senses, and can be touched and moved around by students" (Hynes 11). Used primarily in mathematics teaching, manipulatives can be used to teach a wide range of topics including algebra, geometry, and probability and statistics, not to mention fundamental elementary school mathematical concepts. Schultz (54) has further categorized manipulatives as active, passive, and nonmanipulative. Active manipulatives are those that students can touch and move. Passive manipulatives are those that the students observe the teacher touching and moving. Nonmanipulatives such as pictures on worksheets and bulletin boards cannot be moved or manipulated. Thus defined, manipulatives can be implemented by the classroom teacher to supplement a wide variety of lessons and classroom activities.

The premise underlying the use of manipulatives in the classroom is that students learn best by doing (Dessart 81). Manipulatives are "devices that allow the students to do geometry rather than to watch geometry" (Prevost 412). In a study by Corwin (Suydam 82) manipulative aids were found to help "the students visualize and understand the geometric concepts." Too often students look for patterns to solving problems instead of truly understanding the solutions. Erlwanger states, "students often proceed by manipulating meaningless symbols with no attempt to ask what the symbols mean" (Davis 1157). In further support Carpenter and his colleagues stated that "students appear to be learning many mathematical skills at a rote manipulation level and do not understand the concepts underlying the computation" (Dessart 48). Thus, by having students actively involved with manipulatives during the learning process, they are more likely to grasp the fundamental concepts being studied. Also, if actively involved, students are kept on task a greater majority of the time. Time on task is vitally important in two respects. First, "time on task has been directly related to achievement" (Dessart 4). Second, if the students are busily on task, fewer discipline problems are likely to result. Thus, the use of manipulatives in the classroom is supported by a variety of rationales.

Much research has been done on the ages and ability levels of students helped most by the use of manipulatives. Suydam stated that "achievement is enhanced across a variety of topics at every grade, achievement, and ability level" (Kennedy 7). According to Thornton and Wilmot (38), manipulatives can benefit the learning handicapped as well as the mathematically gifted if they are properly used. Commenting on ability levels, Shoecraft stated that "students who are low achievers in mathematics might have more need for concrete materials and would therefore find a manipulative approach to mathematics more conducive to learning than a more abstract, symbolic approach" (Threadgill 367). Further, Shoecraft commented that "high-achieving students would likely be less affected by instructional methods and be able to process information from either approach" (Threadgill 367). Thus, even for high achievers the results of using manipulatives in the classroom are not detrimental.

Another area explored in this research was the cognitive styles of the students. As defined in a project done by the East Lansing (Michigan) school personnel,

> "a child's cognitive style is the way he takes meaning from the world around him, how he comes to know what he knows. The technique used in determining a child's cognitive style is called `mapping.' By the use of tests and observations, and in interviews, the teacher seeks answers to the question of how a child derives meaning his own unique way. How does the child note his surroundings, seek meaning, and become informed? Is he a listener or a reader? Does he make up his own mind or seek consensus with his peer groups?" (Gartner 13)

According to Kagan and Kogan, a child's cognitive style "develops early in life and remains relatively fixed" (Ryan 1874). From Messick's (1976) theory that "cognitive style has a direct relationship to the way a person behaves," Hunt's (1977) theory that "the way teachers learn can significantly influence the way in which they teach" is supported. Teaching to just one type of learner can be very detrimental as algebra teachers are warned to "guard against an overemphasis on verbal kinds of instruction" (Dessart 21). According to Gartner and Riessman, "some children learn more readily by reading, others by hearing, and some learn faster when they can be physically involved in the process, doing things with their hands and bodies" (9). These authors are referring to the classification of students as visual, auditory, and tactile-kinesthetic learners. When preparing lesson plans, teachers should remember to adapt their teaching to the learning styles of the students (Dunn and Dunn). Due to the average classroom size in the public school system, individualized instruction has not been feasible. However, by practicing the maxim "Vary Thy Teaching" (Dessart 9) teachers can better meet the needs of individual learners. In other words, teachers should "provide a varied set of experiences for their students so that if they cannot learn by one approach, they will learn by another" (Dessart 22). Thus, since manipulatives are a method by which teachers can vary the presentation of topics, their use would be supported by the above sources. Specifically, in this research an attempt will be made to discover which, if any, type of learner (visual, auditory, or kinesthetic) is helped most by the implementation of manipulatives in the classroom.

The use of manipulatives in the mathematics classroom has been supported by many professional educational organizations. In particular, the National Council of Teachers of Mathematics (NCTM) has devoted several publications to this topic. In 1946,

> "NCTM's Eighteenth Yearbook was entitled Multi-Sensory Aids in the Teaching of Mathematics. In the Twenty-fifth Yearbook, the basic role of sensory learning continued to be emphasized, in

> particular in Hardgrove and Sueltz's (1960) chapter on 'Instructional Materials.' In 1963 the Cambridge Conference (1963, 35) made an even stronger case for the use of manipulative materials, stating that every student should have ample opportunity to manipulate physical objects. . .in 1973 the NCTM published the Thirty-fourth Yearbook, Instructional Aids in Mathematics. In 1980, An Agenda for Action (NCTM 1980, 12) continued this call for the use of manipulatives: Teachers should use diverse instructional strategies, materials, and resources, such as. . .the use of manipulatives, where suited, to illustrate or develop a concept or skill." (Worth 2)

Putting all of this into practice,

> "a middle school teacher Herbert (1985, 4) wrote that manipulatives allow teachers to create situations that draw mathematical responses from children. Such situations result 'in improvements in motivation, involvement, understanding, and achievement--overwhelming reasons to believe that manipulatives are good mathematics." (Kennedy 7)

Thus, from the findings of the NCTM and practicing teachers, the benefits of using manipulatives in the teaching of mathematics are well documented.

In addition to the support from teachers, the use of manipulatives has quite a foundation in learning theory. The basic link between the two is that manipulatives are intended to give students a clearer meaning of mathematical concepts.

> "The mental-discipline and stimulus-response theories of the nineteenth and early twentieth centuries gave way to meaning theory, espoused by William Brownell in the 1930's. This theory is based on the belief that children must understand the basic concepts that underlie what they are learning if learning is to be permanent. Brownell's discussion of learning generated interest in having children use

> manipulative materials to form the concepts necessary in learning mathematics." (Kennedy 6)

According to Jean Piaget (1952) and Richard Skemp (1982),

> "manipulative materials are significant learning aids in all four stages [of cognitive development]. Students' mental images and abstract ideas are based on their experiences. Hence, students who see and manipulate a variety of objects have clearer mental images and can represent abstract ideas more completely than those whose experiences are meager." (Kennedy 6)

Jerome Bruner's theory of learning and plea to educators is summarized below:

> "any subject can be taught effectively in some intellectually honest form to any child at any stage of development. . .[There is a need to rewrite] the basic subjects and their teaching materials in such a way that the pervading ideas and attitudes relating to them are given a central role." (Bruner 33, 18)

Dienes (1960) also advocates the use of manipulatives since they provide "'multiple embodiments' rather than a single representation of a concept" (Kennedy 6). Each manipulative device "supplies a proper concrete representation of a concept" (Kennedy 6). In conclusion, Kennedy offers this synopsis:

> "Learning theories suggest that children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics. The manipulatives help children understand both the meanings of mathematical ideas and the applications of these ideas to real-world situations." (6)

Regardless of the existing support, manipulatives are not being used extensively in the classroom. "One survey [Fey] reported that nine percent of elementary school classes (K-6) never used manipulatives and that thirty-seven percent used them less than once a week" (Worth 3). This fact suggests that the drawbacks to the use of manipulatives should be researched. "One reason may be the financial constraints on education today" (Worth 2). School budgets may not provide funds for teachers to purchase commercial manipulatives. "Post (1980) speculates that another reason has to do with teachers' concerns about management and control" (Worth 2). Unless carefully monitored, manipulatives may be transformed into play toys for the students. Post also mentions that "when achievement in mathematics is largely determined by students' ability to compute on standardized tests, 'widespread use of manipulatives seems almost counterproductive'" (Worth 2). When using manipulatives, teachers must remember the importance of helping students bridge the gap by "connecting the world of manipulatives and the world of [mathematical] symbols" (Bright 4). Of a similar philosophy Trueblood states that

> "prospective teachers resist using manipulatives in the classroom for two reasons: a lack of confidence in their own ability to use manipulative materials correctly and the general belief that children will become too dependent on these materials and, as a result, will not master basic computational algorithms and related concepts. This general belief seems related to a lack of confidence in helping

> children make the transition from the concrete to the abstract." (51)

Thus, for a variety of financial and pedagogical reasons manipulatives have not been implemented extensively in the public school system.

When choosing manipulative materials for classroom use, teachers should consider the following pedagogical and physical criteria. One pedagogical criterion is "the adaptability of materials in different contexts" (Moser 9). A certain manipulative device is much more valuable if it is versatile, useful in teaching many different concepts. Also, "an added advantage of using a familiar set of materials repeatedly is that valuable instructional time is not lost on 'play,' which is needed any time some new and unfamiliar materials are introduced" (Moser 9). Another pedagogical criterion is whether the manipulative provides a "clear representation of mathematical ideas" (Hynes 11). In other words, will the students clearly grasp the underlying concepts from using the manipulative device? Also, teachers should ensure that the chosen manipulatives are "appropriate for the students' developmental level" (Hynes 11). Manipulatives should not be too complex for younger students nor too childish for older students. Finally, Fennema suggests that teachers should choose "manipulative aids [that] do arouse students' interest and improve their motivation to learn mathematics" (Hynes 11). Teachers should also consider the physical criteria of manipulatives.

Before implementation, each manipulative device should be checked for its "durability, simplicity, attractiveness, reasonableness of cost, and manageability and ease of storage" (Hynes 12). Thus, a variety of selection criteria exists and should be considered by classroom teachers when choosing which manipulative devices to implement.

One type of manipulative device which is not as tangible as those previously mentioned is computer assisted instruction. In one study, Knerr (Dessart 108) found significant improvements in instructional effectiveness and learning competency when computer software was used to instruct students. For this reason, the experimenter used a computer assisted review activity with the experimental group in this research. The Geometric Supposer: Quadrilaterals software by Sunburst Communications, Inc. allows the student to choose any type of quadrilateral, and then "make any geometric construction that Euclid know how to make" (Schwartz 58). Thus, the students can construct parallel lines, perpendiculars, and angle bisectors; label points of intersection and midpoints; and measure angles and lengths of line segments (Yerushalmy 418). Therefore, based on the adaptability of this software, the experimenter chose the Geometric Supposer: Quadrilaterals to motivate the experimental students to apply the theorems they had learned as a review of the unit on quadrilaterals.

In too many classrooms mathematical activities are intended to keep students content and quiet (Dessart 19). Instead, these activities should challenge and stimulate discussion by the students. As described in the following procedure section, the geometry students in the experimental group were actively involved and on task in the learning process. The experimenter attempted to follow the guidelines for "the sound use of manipulatives in geometry and geometric-measurement instruction" suggested by Clements and Battista (29). According to them, first of all "students should be involved with four representations of a new idea" (29). The four representations are as follows: "use classroom manipulatives, examine the physical world, examine and draw diagrams, and learn names, definitions, or symbolizations" (Clements 29). This order represents a progression from the concrete to the abstract, bridging the gap between the world of manipulatives and the world of symbols. A second guideline is that "the use of manipulatives should promote the development of spatial visualization" (Clements 29). This spatial visualization is vital to the students' comprehension of geometrical concepts. Finally, Clements and Battista state that "activities with manipulatives should be oriented toward problem solving" (29). In the experimenter's classroom students practiced this by conjecturing conclusions, after using manipulatives, to theorems given their hypotheses. Thus, following these guidelines the experimenter began the research supported by this concluding statement:

> "Learning theories and evidence from research and classroom practice support the use of manipulative materials to help children learn and understand mathematics. Well-chosen and properly used manipulative materials enhance children's learning, generate interest, relieve boredom, and promote problem solving and computational skills." (Kennedy 7)

PROCEDURE

After visiting the school that the experimenter had been assigned for student teaching, plans were begun for the unit in which manipulatives would be utilized. The unit was on quadrilaterals with a slight mention of locus points. The students were tenth through twelfth graders. In the control group, the second period class, there were twenty-four students. This class was taught in a traditional manner without the use of manipulatives. In the experimental group, the third period class, there were fourteen students. This class was enriched with a wide variety of manipulative materials. Obviously, the difference in class numbers presented a problem. In terms of the procedure, the size difference had no effect. However, when analyzing the data and drawing the conclusions, the experimenter did pair the students between the two groups. The rationale and process by which this was completed will be described later in the discussion of data section of this thesis.

To begin the unit on quadrilaterals, two pretests were administered to both of the geometry classes. First, an achievement pretest (Sample Test 1) was given before instruction on the unit was begun. The first five items were to test recall of the previous unit and readiness for the one to be taught by the experimenter. Items six through eleven were objective items to determine how much the students already knew about parallelograms. Questions twelve through fifteen asked the students to define and list all of the properties they knew about a parallelogram, a rhombus, a trapezoid, and a rectangle. Questions sixteen and seventeen were intended to measure the student's ability to analyze a problem. In each, stated was the "given" for a proof, and students were asked to write down all of the facts they knew from the "given." In several of the above questions, the students were asked to list all that they knew about a particular question. For this reason the achievement pretests were graded on a point scale. For every correct fact, the student received a point. Helping to familiarize the experimenter with the new students, this pretest provided information about each student as well as a measure with which to compare the posttest when the unit was completed.

Also given to both classes, the second pretest was an attitude/learning style inventory. To measure the students' original attitudes toward geometry, a semantic differential adapted from the work of Osgood (Reisman 120) was used. Whereas his semantic differential used ten bipolar pairs, the experimenter included seven bipolar pairs to simplify the students' questionnaire. The differential is scored on a scale depending upon where each student places a check. The scale ranges from -3 to +3 and includes all of the inclusive integers. A sum total is obtained by adding the integer equivalent of the student's check for each of the seven bipolar pairs. Then, to find the student's attitude rating, this sum total is divided by seven to find the mean. See the sample semantic differential (Sample Test 2) for an example of the scoring. Also on this pretest were straightforward questions intended to determine the student's learning style, whether visual, auditory, or kinesthetic (Peterson 845). Questions 1, 2, 4, 5, 7, and 9 were aimed strictly at determining these sensory modality differences in students' learning styles. If in three out of these six questions a student gave a visual (auditory or kinesthetic) response, then the student was classified as a visual (auditory or kinesthetic) learner. Questions 3 and 6 were designed to classify a student as an introvert or an extrovert. Since many of the manipulatives were studied in groups, the experimenter desired a prediction of how well each student worked with others. In the initial testing, the experimenter decided to ask extra questions such as this to leave several areas open for possible analysis later in the research. Finally, question 8 was to determine left and right brain hemisphere learners. Again, this question was an extra direction that the experimenter could have explored later. Actually, in some cases the answer to this question was used as a tie breaker since a three in six score determined a student's learning style in this research system. Since right hemisphere learners tend to be visual learners (Davidson), students who preferred a chapter overview in question 8 were classified as visual learners in the event of a tie. Thus, much of the classification done in this study was premised on the results of the attitude/learning style and achievement pretests.

At this point in the research the experimenter began implementing manipulatives in the third period class. The second period class was taught the same material in a traditional manner, void of manipulatives. Henceforth, only the procedure with the third period class, the experimental group, will be described. The cooperating teacher had introduced the experimenter's unit on quadrilaterals by teaching the first section. The experimenter began teaching the second section of the unit with a lesson on the properties of parallelograms. In this lesson each student had his own geoboard, protractor/ruler, and rubber bands. Of course, the students needed a few moments of play as an orientation to the geoboards after receiving instruction about their use. The approach in this discovery lesson was for the students to analyze the parallelograms they had constructed (from definitions given on the previous day) on their geoboards (Photograph 1) and formulate the theorems in section two without having seen them first. These activities helped to build students' analysis and problem solving skills. The students were not required to formally prove the theorems since the cooperating teacher had never stressed this. After their discovery time and a few prompts from the experimenter, the students were able to formulate the following theorems:

1. "Opposite sides of a parallelogram are congruent." (Hirsch 223)

2. "Opposite angles of a parallelogram are congruent." (Hirsch 224)

3. "The diagonals of a parallelgram bisect each other." (Hirsch 224)

4. "The distance between two parallel lines is a constant." (Hirsch 225)

5. "The diagonal and the sides of a parallelogram form two congruent triangles." (Hirsch 223)

As an example of a passive manipulative (Schultz 54), the experimenter again emphasized the final theorem above with a paper parallelogram (Photographs 2 & 3). The experimenter drew the diagonal of the parallelogram and then folded it on the diagonal in order for the students to see the two congruent triangles as they overlapped. Overall, the students' participation was good. Especially, they were very attentive, and no discipline problems were experienced. Although they used rubber bands, the experimenter warned them that they had a quota, and all of them had to be returned. This discouraged any mischievous activity. Most importantly, many students seemed shocked that they could discover geometry theorems on their own. This really impressed and motivated the students.

The second day on section two was spent in a bulletin board game activity (please see Bulletin Board). To begin the activity students were paired, and each team was given a geoboard, protractor/ruler, and rubber bands. As a review each team was to read a property from the bulletin board and construct a parallelogram, a rectangle, a rhombus, and a square, checking each to see if the property applied to it (Photographs 4-7). This way, all of the teams were actively involved. Then, experimenter called on one team (per property) to come to the bulletin board. One teammate constructed the different quadrilaterals and demonstrated or disproved the particular property for each. The other teammate placed a check on the bulletin board where appropriate to match the quadrilaterals and their properties. The students were actively involved in the learning process and seemed to genuinely enjoy this type of review.

In the lesson on section three the students used geostrips and brads to prove that certain types of quadrilaterals are parallelograms. Each student was given six geostrips, with at least two congruent pairs coded by color (Photograph 8). Again, after some orientation and play time, the students settled down to work. Without having seen the theorems for this lesson, the students completed the conclusions to the following theorems given their hypotheses:

1. "If two sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram." (Hirsch 229)

2. "If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram." (Hirsch 230)

3. "If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram." (Hirsch 230)

Given the hypotheses above, the students were able to construct the conditions for each (Photographs 9-12) and conclude that the quadrilateral was a parallelogram in each case. Once finalized, the experimenter referred to the textbook (Hirsch 229-30), showing the students that they had formulated the theorems for this lesson. Several commented that this was more fun than the teacher just standing in front of the class reciting facts.

Parallelograms and parallel lines were studied in section four. For the manipulative portion of the lesson, each student was given a geoboard, a protractor/ruler, and five rubber bands. By this time the students were familiar with the properties of the geoboards. Since the pegs are arranged along parallel lines on a geoboard, the students could visualize the hypothesis of the following theorem: "If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal" (Hirsch 235). Also familiar with transversals, the students then placed another rubber band across the three parallel rubber bands to represent the transversal in the hypothesis and noted its division into three congruent segments. Prompted by the experimenter's question of what happens to a second transversal of the same three lines, the students constructed a second rubber band transversal on their geoboards (Photograph 13). Using their intuition they volunteered observations and suggested possible answers. Asked to back up their speculations with proof, the students used their rulers to measure the segments of the second transversal. After doing this, the students were able to formulate the conclusion to the above theorem. A second theorem covered in the lesson pertained to triangles and parallel lines: "If a segment joins the midpoints of two sides of a triangle, then it is parallel to the third side, and its length is one-half the length of the third side" (Hirsch 234). Each student constructed a triangle on his geoboard and then measured to find the midpoints of two of the legs, connecting them with a rubber band (Photograph 14). Analyzing this, the students first noticed that the segment was parallel to the third side. After a prompt by the experimenter, the students began testing their estimates of the segment's length. Completing this process for several triangles, the students eventually formulated the last part of the theorem's conclusion. The students were more challenged today to test their educated guesses before stating their conclusions. Nonetheless, they still seemed to prefer this active learning technique over the lecture method.

In section five the diagonals of rectangles and rhombuses were the focus of study. Each student was given three geostrips, a brad, and a protractor/ruler (Photograph 15).

For this activity, the experimenter stressed that the geostrips now represented the diagonals of a quadrilateral, not the sides as they had done previously. In Photographs 15 - 18 the cut out circles at the ends of each geostrip can be observed. These circles represented the vertices of the quadrilateral formed by the two diagonals. Either by sight or by filling in these circles on paper and connecting the dots, the students had to state what type of quadrilateral was formed in each case. The students investigated cases of congruent diagonals, perpendicular diagonals, and various other arrangements. From this activity the students were able to formulate the following theorems once the experimenter had stated the hypotheses in each case:

1. "A parallelogram is a rectangle if and only if its diagonals are congruent." (Hirsch 240)

2. "A parallelogram is a rhombus if and only if its diagonals are perpendicular." (Hirsch 241)

3. "A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles of the parallelogram." (Hirsch 241)

This activity was more successful than the experimenter had anticipated because the students discovered several connections. For example, when working with perpendicular diagonals, many of the students thought the parallelogram had to be a square. However, by constructing another figure with perpendicular diagonals, the students understood that perpendicular diagonals produce a square in a particular case (four right angles), but in general perpendicular diagonals always produce a rhombus. From this the students were able to transfer their new knowledge to conclude that a square is a particular case of a rhombus, an accurate conclusion. They also learned that theorems are always true in every case, not stated just to satisfy one particular case because a counterexample could be found. Similarly, many students thought the parallelogram in theorem one above had to be a square, but later understood that the general case of a rectangle had to be concluded by the theorem. The students, as well as the teacher, seemed very pleased by the mental excursion this activity launched. To close this day's lesson, the bulletin board activity was used again to review the new properties learned. Called upon at random, each student had to use their geostrips to construct supporting evidence for placing new checks on the bulletin board. In their seats the other students also followed along, completing the same geostrip constructions and verifying the accuracy of the checks placed on the bulletin board. Thus, the students were able to help each other learn as well as discovering information on their own.

Since these properties of diagonals were so vital to the students' classification of quadrilaterals, a second day was devoted to this topic. Also, at this point in the unit, the students had been exposed to a great deal of information necessary for them to distinguish among the different types of quadrilaterals. Thus, in effect today's activity was a review of all of the material covered thus far in the unit. First of all, the class was divided into four groups. Each group was given a different quadrilateral constructed from "orbit materials" (Photographs 19 - 22). The diagonals were also constructed in each manipulative figure. Thus, there was a parallelogram group, a rectangle group, a rhombus group, and a square group. Without the aid of notes or the textbook (Hirsch), the students in each group were to compile a complete list of the properties of their quadrilateral. Once formulated, each list was written on chalkboard and checked for accuracy and completeness. From the lists on the board, the students were able to revise and condense the information through class discussion led by the experimenter's questions. They summarized that:

1. A rectangle has all of the properties of parallelogram, four right angles, and congruent diagonals.

2. A rhombus has all of the properties of a parallelogram, four congruent sides, and diagonals that are perpendicular and that bisect opposite angles.

3. A square has all of the properties of a parallelogram, a rectangle, and a rhombus.

From their ability to easily formulate this synopsis, the experimenter concluded that the manipulatives helped this class to see the similarities and transfer the information more readily than the control group based on observation alone.

In section six of this unit, the topic changed from parallelograms to trapezoids. To supplement the instruction each student was given a geoboard, a protractor/ruler, and rubber bands. After reviewing the definition and parts of an isosceles trapezoid, each student constructed one on his geoboard and used rubber bands to represent the diagonals (Photographs 23 & 24). Measuring with the protractor/rulers and focusing on the angles and the diagonals, the students were able to formulate the following theorems:

1. "Each pair of base angles of an isosceles trapezoid is congruent." (Hirsch 246)

2. "The diagonals of an isosceles trapezoid are congruent." (Hirsch 247)

Similar to the triangle theorem from section four, the students then constructed the median of a general trapezoid on their geoboards (Photograph 25) and then made observations. Once again, they had to prove their educated guesses by measuring several examples. Through this familiar process the students were able to state the following theorem without having seen it first: "the median of a trapezoid is parallel to the bases, and its length is one-half the sum of the lengths of the bases. Although this activity required more knowledge of terminology and concentration, the students still seemed to enjoy being actively involved. As their teacher, the experimenter observed that actually constructing and studying isosceles trapezoids then general trapezoids helped the experimental students differentiate between the properties of each, avoiding some of the confusion experienced by the control group.

In the final section of the unit, quadrilaterals were abandoned for a study of locus points and locus theorems. This topic was so complex and difficult to understand that instruction without the aid of manipulatives would be next to impossible for the students to comprehend. First of all, a locus was defined to be "the set of all points, and only those points, that satisfy a given condition" (Hirsch 250). Basically, this section was intended to sharpen the students' problem solving abilities. However, on the sophomore level students are often discouraged by activities such as these. After the students seemed to understand the concept of locus points, the experimenter began to present the locus theorems. For each theorem one or two three-dimensional models were presented to illustrate the theorem for the students (Photographs 26 - 29). Reading the theorems meant nothing to the students; they were much too wordy. However, seeing and touching the models allowed the students to study the given for each theorem and then follow through the model to discover where the locus points had to be. In order to make the same visual connection the students did, please refer to Photographs 26 - 29 again as the following theorems are read:

> 1. "The locus of points in a plane equidistant from two given points is the perpendicular bisector of the segment joining the two points." (Hirsch 254)

2. "The locus of points equidistant from two given points is the perpendicular bisecting plane of the segment joining the given points." (Hirsch 255)

3. "In a plane, the locus of points equidistant from the sides of an angle is the bisecting ray of the angle, excluding its endpoint." (Hirsch 255)

In this case especially, manipulatives have a very important role in making wordy mathematical jargon real to the students. To support this, the experimenter's cooperating teacher and the school assistant principal observed the lesson and commented on how vital the manipulative models were to the students' comprehension. Proven by this lesson, manipulatives serve to simplify and tangibly represent difficult mathematical concepts if properly selected and used. The lesson on locus points concluded the presentation of new material in the unit on quadrilaterals.

The next two days were spent in review, followed by the chapter test on the third day. The first day of the review was planned as a computer activity. The software package utilized was Sunburst Communication's <u>Geometric Supposer: Quadrilaterals</u> (Photograph 30). This software allows the user to draw quadrilaterals and diagonals, label vertices and intersection points, and measure angles and segments. After reviewing the software, the experimenter created a worksheet which covered the main points from the unit. A copy of this worksheet has been included in the following pages (Geometry Worksheet). The worksheet was lengthy and intended to occupy the students for the entire period. Amazingly, some of the slower, failing students were the first to complete the worksheet. When finished, they willingly walked around helping others. Later, they commented on how much the computer/worksheet activity helped them review the material. When questioned as to whether they really enjoyed just having an opportunity to use the computers, they admitted that the computers were fun, but they convincingly stated that they had reviewed and learned material that they would not have by studying on their own or by doing assigned problems. The second review day was filled with assigned problems and factual reviews, identical to the lesson for the control group. On the following day both classes took the same chapter test(Sample Test 3), and the unit on quadrilaterals was completed.

Approximately one week after the graded chapter tests were returned to the students, two posttests were administered to both classes. One posttest (Sample Test 4) was an attitude inventory identical to the attitude pretest. The purpose of this was to measure any improvements or declines in students' attitudes toward geometry during the course of the unit on quadrilaterals. The second posttest (Sample Test 5) was an achievement posttest. Half of it was identical to twelve questions from the pretest. This was planned so as to determine the amount of improvement between the raw scores on these sections of the two tests. The second half of the achievement posttest was to test the students' recall of specifics from the unit. This section was planned for a comparison of raw scores on recall between the two classes, in an attempt to determine if manipulatives improve students' retention of information. Finally, the students in the experimental class were asked to fill out the questionnaire included in the following pages (Questionnaire). The questions were designed to measure how the students felt about using manipulatives, which activities they preferred, and what benefits the students experienced from their use. Further results from the posttests as well as the other tests mentioned in this procedure section have been thoroughly documented and analyzed in the following discussion of data section.

SAMPLE TESTS

Sample Test 1

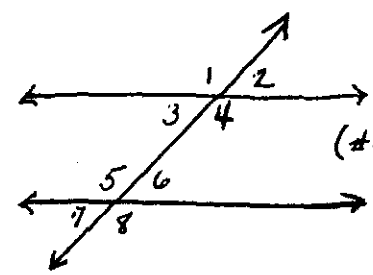
Name ____________

1. List methods of proving 2 triangles are congruent.

2. List methods of proving 2 <u>right</u> triangles are congruent.

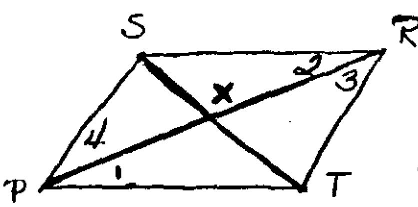
3. List the corresponding congruent parts of $\triangle ABC \cong \triangle DEF$

4. List 10 pairs of $\cong$ $\angle$s.



(#4+

5. Name 4 supplements of $\angle 4$



▱SR

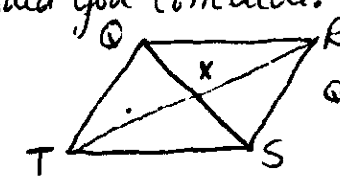
(#6–#1

6. $\overline{PS} \parallel$ ______.
7. $\triangle PTS \cong$ ______.
8. If $m\angle P = 70$ and $m\angle S = 110$, then $m\angle R =$ ______.
9. $\angle 1 \cong$ ______.
10. $\overline{PX} \cong$ ______.
11. If $PT = 10x$ and $SR = 8x + 6$, $x =$ ______.

■. Define and list properties of the following:

12. parallelogram
13. rhombus
14. trapezoid
15. rectangle

If the following were "GIVEN" for a proof, what could you conclude?



QUADRILATERAL QRST

16. $\overline{ST} \parallel \overline{QR}$, $\overline{TX} \cong \overline{RX}$
17. $\angle TQR$ and $\angle QRS$ are supplementary
$\angle QRS$ and $\angle RST$ are supplementary

Sample Test 2 Name ___________

Please place a check in the appropriate blank.

.29

Geometry

	strongly agree	agree	leaning toward	neutral	leaning toward	agree	strongly agree	
Changeless				✓				Changing
Strange	✓							Familiar
Unexplored					✓			Explored
Unpleasant			✓					Pleasant
Bad					✓			Good
Harmful							✓	Helpful
Worthless					✓			Valuable
	-3 +	0 +	-1 +	0 +	3 + $(3\times1)=3$	0 +	3	$= 2 \div 7$ 0.29

Please circle the appropriate answer.

1. When learning something new, do you prefer to listen to a lecture, to read books, or to apply the information in a new situation?

2. Which do you understand better: information presented orally in lecture or information arranged in a chart?

3. Faced with an important project to complete, do you prefer to work by yourself or with a group?

4. In the laboratory, do you learn better by performing the experiment or by watching others do it?

5. In your free time, are you most likely playing sports ⟹

reading a book or a magazine, or listening to music?

6. Are you a "people person" or are you more of an individualist?

7. In math class, which describes when you *fully* comprehend the material:
   - as the teacher verbally explains it,
   - as the teacher writes it on the chalkboard,
   - as you complete classwork or homework related to the topic.

8. When you start a new unit in geometry do you prefer that the teacher
   a) gives an overview of all the topics in the chapter or
   b) tells the steps to solve problems in the first section (jumps right in)?

9. When asking for directions to a party, do you prefer that someone draws you a map or that someone orally explains the directions?

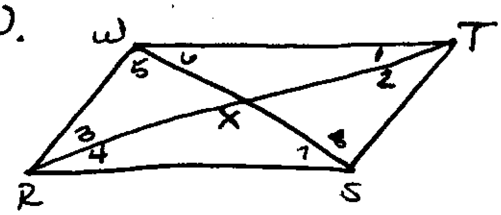
Sample Test 3   Name

Chapter 5 Test

Fill in the blank.

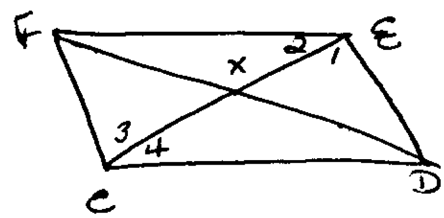
1. A quadrilateral with exactly one pair of opposite sides parallel is a ______________.
2. A quadrilateral in which both pairs of opposite sides are parallel is a ______________.
3. A parallelogram with four right angles is a ______________.
4. A parallelogram with four congruent sides is a ______________.
5. A quadrilateral which is both a rectangle and a rhombus is a ______________.

Exercises 6 – 12 refer to ▱ RSTW.



6. $\overline{RS} \parallel$ ____.
7. $\overline{TS} \cong$ ____.
8. $\overline{WX} \cong$ ____.
9. $\angle WRS \cong$ ____.
10. $\triangle RST \cong$ ____.
11. $\angle 1 \cong$ ____.
12. $\angle$ ____ and $\angle$ ____ are supplements of $\angle WRS$.

Exercises 13 – 19 refer to ▱ CDEF



13. $CE = 12$, $CX =$ ____.
14. $m\angle CDE = 72$, $m\angle EFC =$ ____.
15. $m\angle 1 + m\angle 2 = 103$; $m\angle FCD =$ ____.
16. $m\angle 4 = 87$, $m\angle 2 =$ ____.
17. $m\angle 3 = 4x + 4$; $m\angle 4 = 6x$; $m\angle FED = 104$; $x =$ ____.
18. $XE = 2y + 2$; $CE = 12$; $y =$ ____.
19. $m\angle FCD = 70$; $m\angle CFE = 110$; $m\angle FED =$ ____.

Please write whether the statement is SOMETIMES, ALWAYS, or NEVER true.

20. If each diagonal bisects a pair of opposite angles, then the parallelogram is a rhombus.

21. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a rectangle.

22. The median of a trapezoid is parallel to the bases.

23. In $\triangle CDE$, if X and Y are the midpoints of $\overline{CE}$ and $\overline{DE}$, respectively, then $XY = CD$.

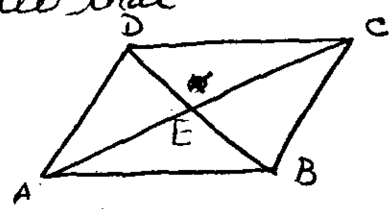
24. In $\triangle ABC$, if X and Y are the midpoints of $\overline{AC}$ and $\overline{BC}$, then $\overline{XY}$ is parallel to $\overline{AB}$.

25. In quadrilateral ABCD, if $\overline{AC} \cong \overline{BD}$ then ABCD is a rectangle.

26. A rectangle is a square.

27. In a parallelogram ABCD, if $\overline{AC}$ is perpendicular to $\overline{BD}$ then parallelogram ABCD is a square.

Does the following given information guarantee that ABCD is a parallelogram? (yes or no)

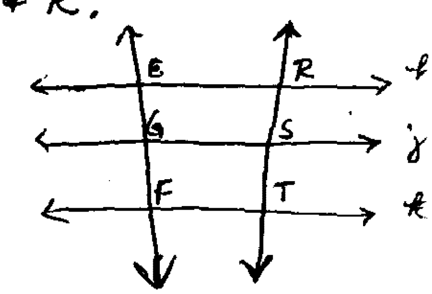


28. $\overline{AE} \cong \overline{CE}$; $\overline{BE} \cong \overline{DE}$

29. $\overline{AC} \perp \overline{BD}$

30. $\overline{AB} \parallel \overline{CD}$; $\overline{AB} \cong \overline{CD}$

Exercises 31 – 33 refer to the 3 parallel lines h, j, + K.



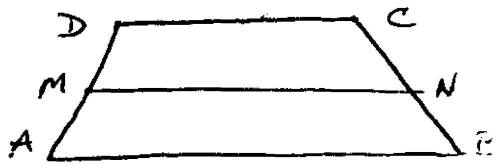
31. If $EG = 7$, $GF = 7$, $RS = 8$, then $ST =$ ______.

32. If $GF = \frac{1}{2} EF$ and $RS = 5.6$, then $RT =$ ______.

33. If $\overline{EG} \cong \overline{RS}$, does quadrilateral GSRE have to be a parallelogram?

Exercises 34 – 38 Name

Given trapezoid ABCD. $\overline{MN}$ is the median



34. $AB = 16$, $DC = 10$, $MN =$ ______.

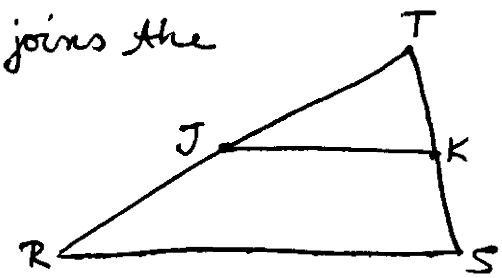
35. $DC = 12$, $MN = 15$, $AB =$ ______.

36. $\angle DMN \cong$ ______ because segment $\overline{MN}$ is ____________ to base $\overline{AB}$.

37. If $\overline{AD} \cong \overline{BC}$, then $\angle DAB \cong$ ______.

38. If $\overline{CB} \cong \overline{AD}$, then $\overline{DB} \cong$ ______.

Exercises 39 – 42 refer to $\triangle RST$. $\overline{JK}$ joins the midpoints of $\overline{RT}$ and $\overline{ST}$.



39. $\overline{JK}$ is ____________ to $\overline{RS}$.

40. $JK = 7.2$, $RS =$ ______.

41. If $RS = 6c + 10$, then $JK =$ ______.

42. $\angle TKJ \cong$ ______.

43. The set of all points, and only those points, that satisfy a given condition is a ____________.

Draw a sketch and describe the solution.

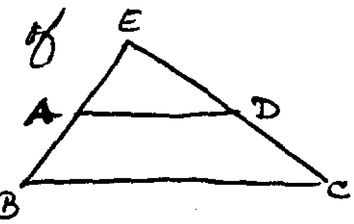
44. In a plane, the locus of points equidistant from the sides of an angle.

45. The locus of points in a plane equidistant from 2 given points.

#6. Do one of these 2 proofs.

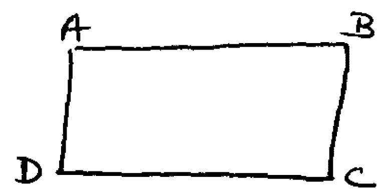
a. Given: points A and D are the midpoints of $\overline{BE}$ and $\overline{CE}$.

Prove: Quadrilateral ABCD is a trapezoid.



b. Given: ABCD is a rectangle ; $\overline{AB} \cong \overline{BC}$

Prove: ABCD is a square.



Sample Test 4

Name ______________

Place one check per line in the blank most representative of your feelings or attitudes when you think of geometry.

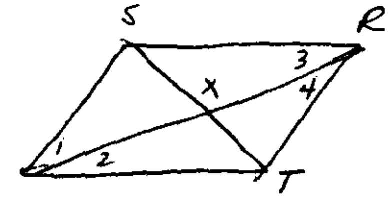
"GEOMETRY"

	STRONGLY AGREE	AGREE	LEANING TOWARD	NEUTRAL	LEANING TOWARD	AGREE	STRONGLY AGREE	
Changeless	____	____	____	____	____	____	____	Changing
Strange	____	____	____	____	____	____	____	Familiar
Unexplored	____	____	____	____	____	____	____	Explored
Unpleasant	____	____	____	____	____	____	____	Pleasant
Bad	____	____	____	____	____	____	____	Good
Harmful	____	____	____	____	____	____	____	Helpful
Worthless	____	____	____	____	____	____	____	Valuable

Posttest
Sample Test 5

Name ________

1. Given ▱ SRTP



▱SRTP
\# 1

a) $\overline{PS} \parallel$ ____.

B) $\triangle PTS \cong$ ____

C) If $m\angle P = 70$ and $m\angle S = 110$, then $m\angle R =$ ____.

D) $\angle 1 \cong$ ____

E) $\overline{PX} \cong$ ____

F) If $PT = 10x$ and $SR = 8x + 6$, $x =$ ____.

2. Define and list properties of the following: (as many " you can

A) parallelogram

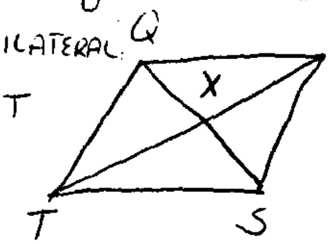
B) rhombus

C) trapezoid

D) rectangle

3. If the following were "given" for a proof, what could you conclude?

QUADRILATERAL QRST



A) $\overline{ST} \parallel \overline{QR}$, $\overline{TX} \cong \overline{RX}$

B) $\angle TQR$ and $\angle QRS$ are supplementary;
$\angle QRS$ and $\angle RST$ are supplementary.

(over)

Posttest
(con't.)

4. What facts can you recall about the DIAGONALS of the following?

A) Parallelogram

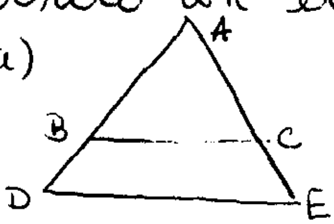
B) Rectangle

C) Rhombus

D) Square

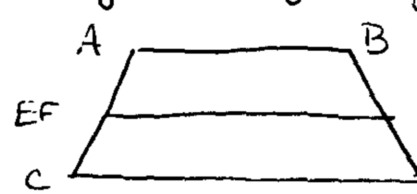
5. Write an equation to find the following lengths:

a)



$\overline{BC} \parallel \overline{DE}$

b)



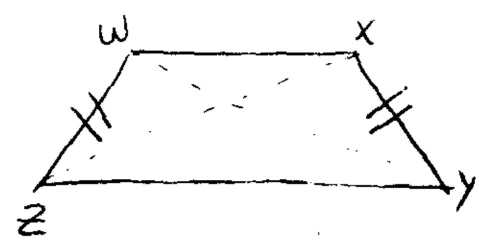
$\overline{EF} \parallel \overline{CD}$

a) BC = ________

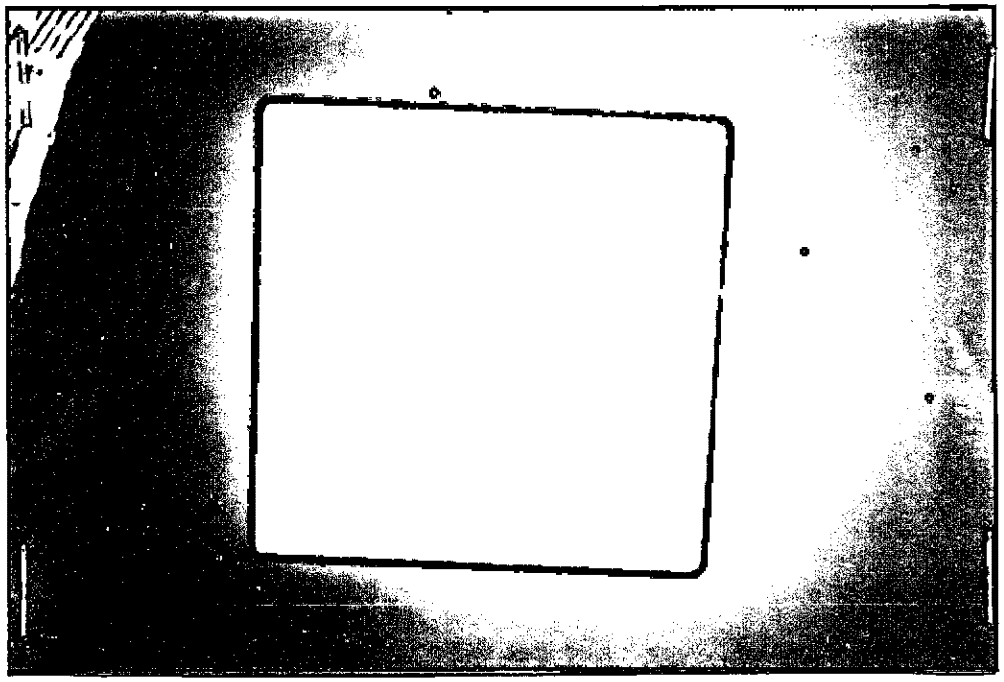
b) EF = ________

6. State all that you know (other than $\overline{WZ} \cong \overline{XY}$) from the following figure:

trapezoid
WXYZ



PHOTOGRAPHS



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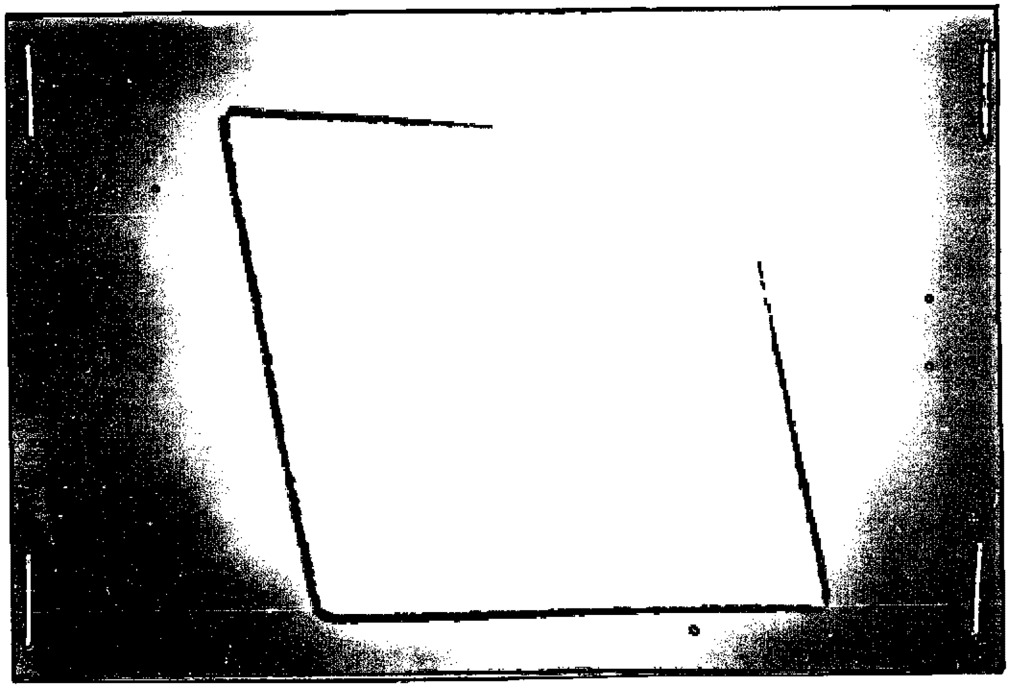

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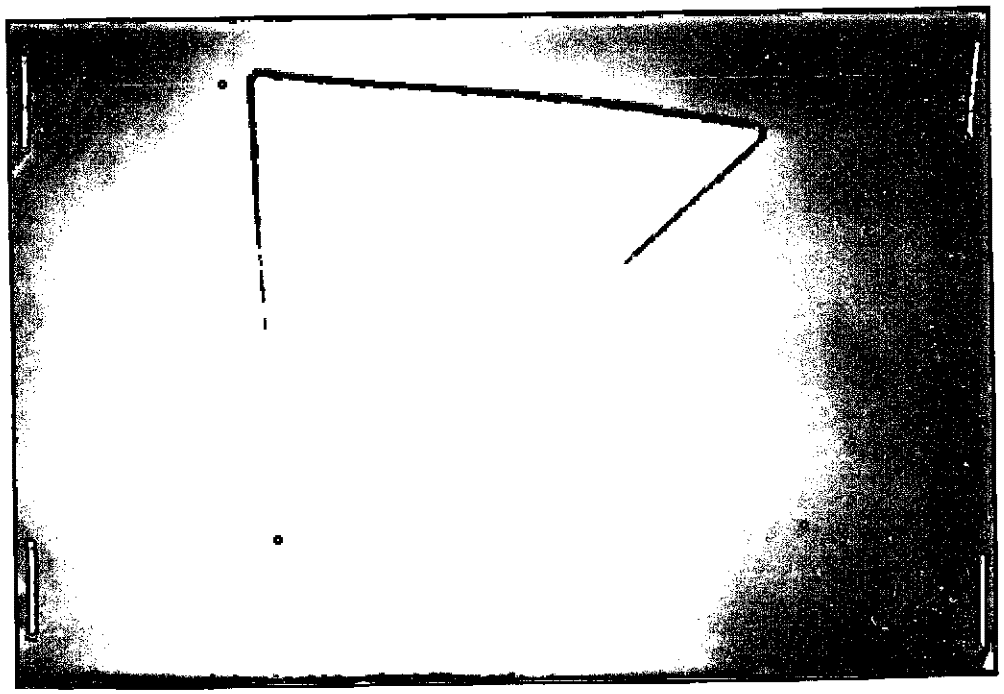

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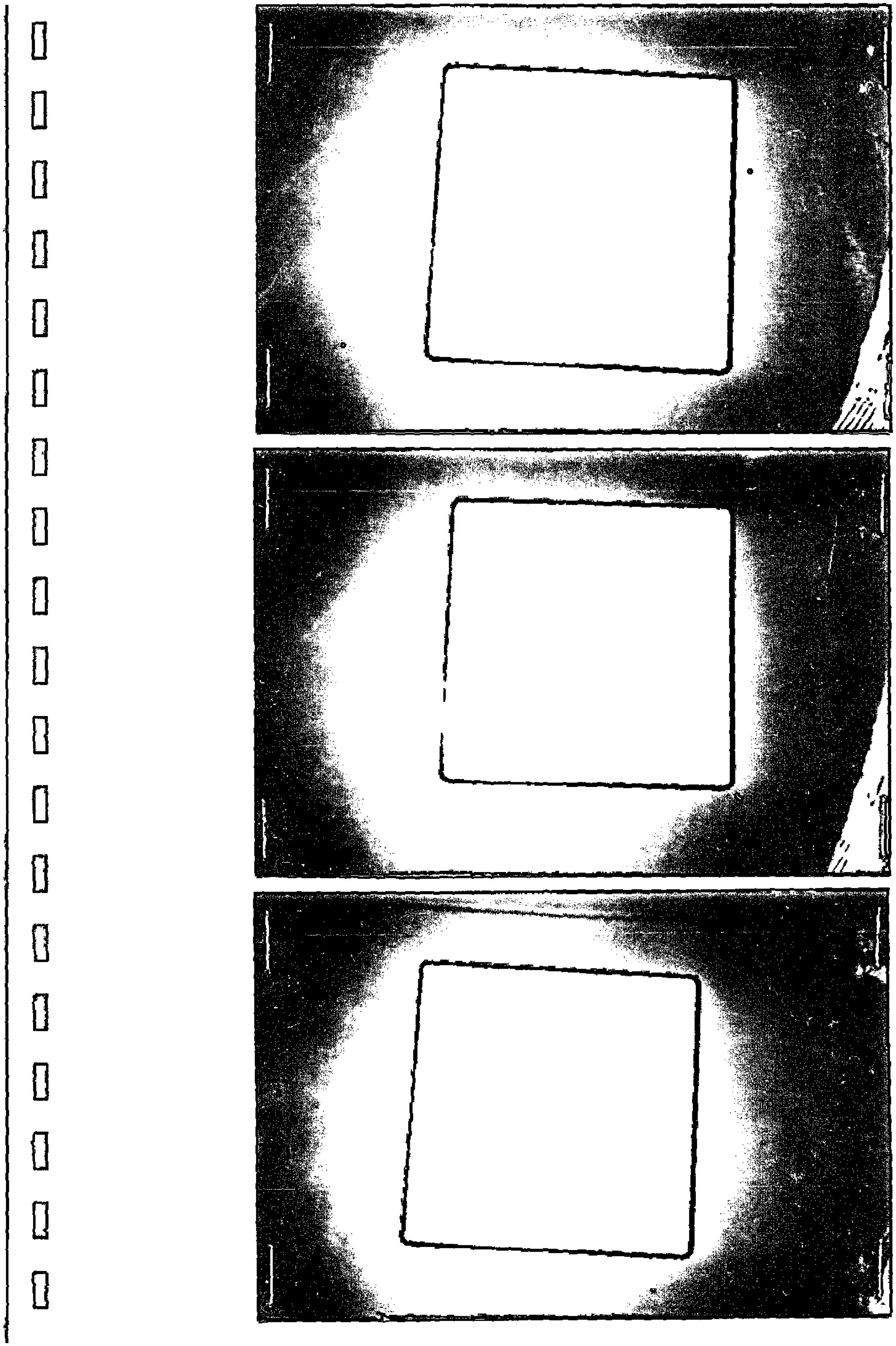

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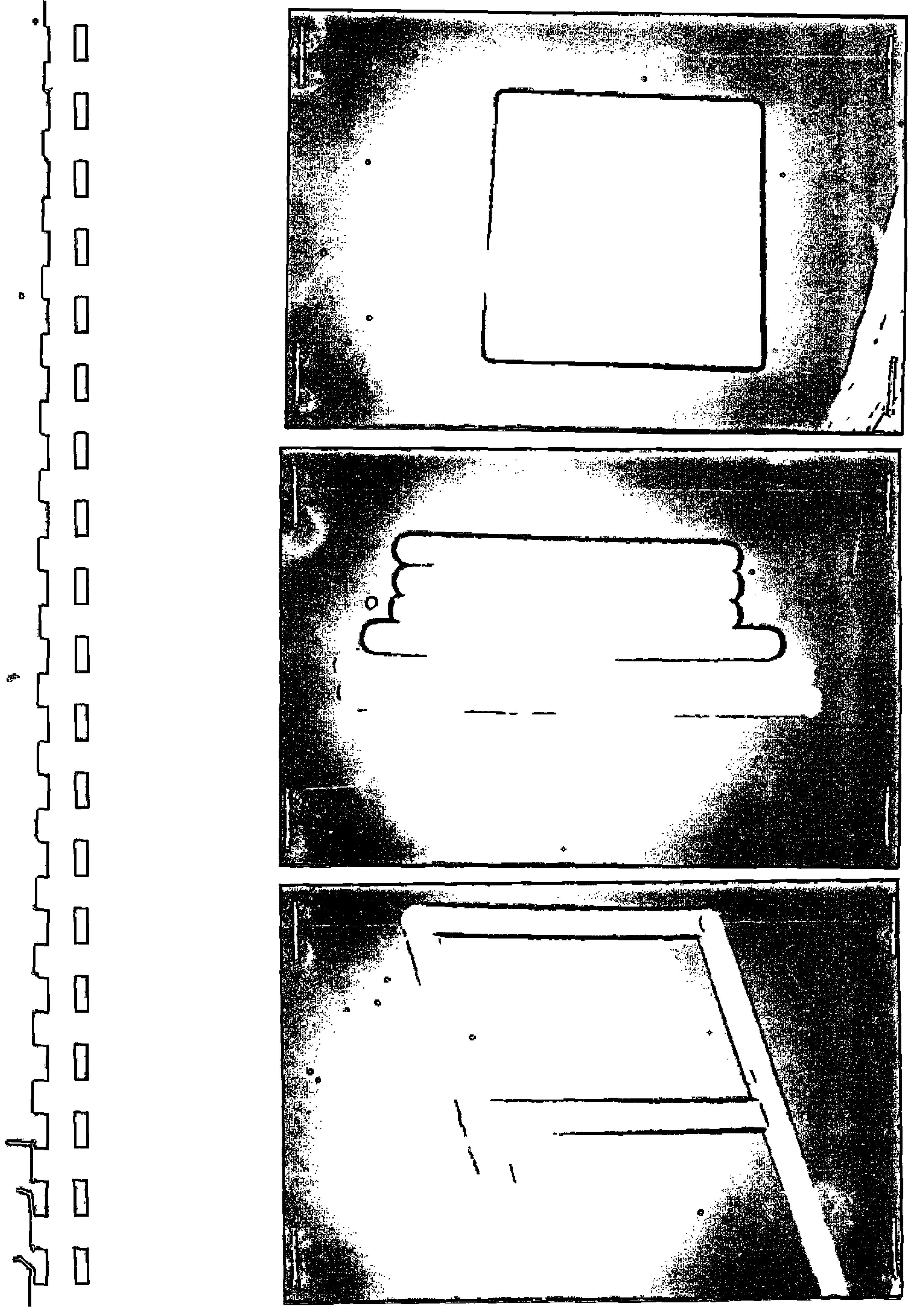

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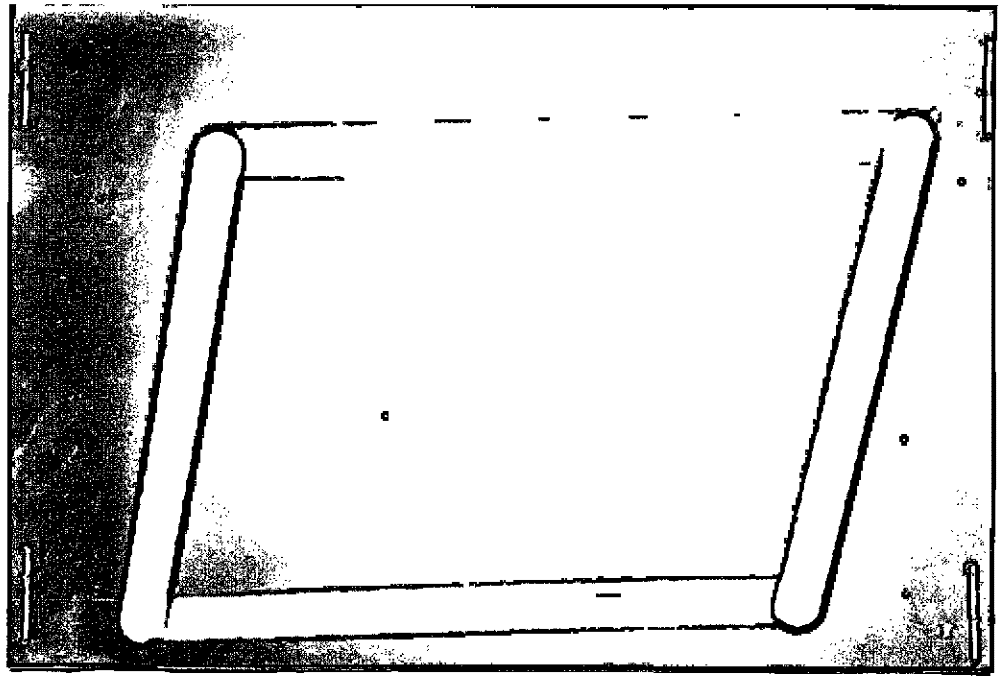

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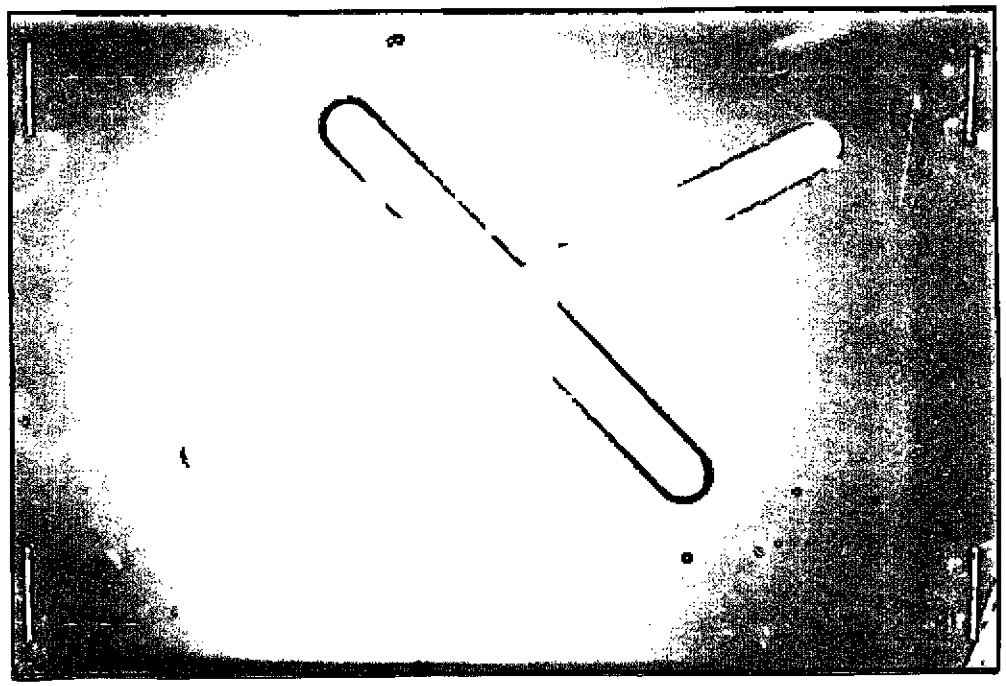

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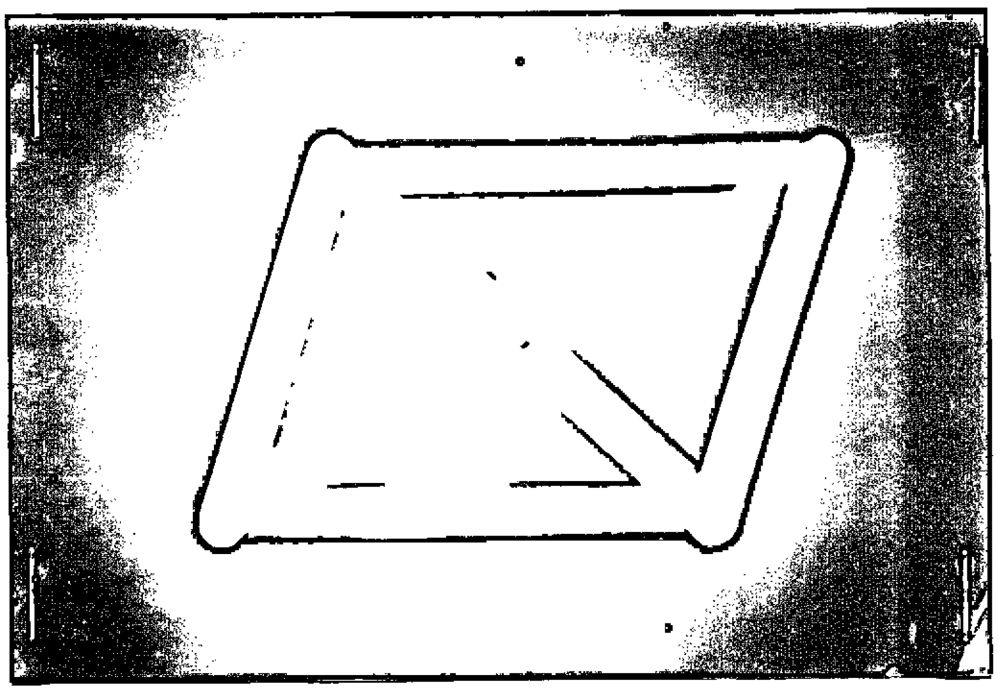

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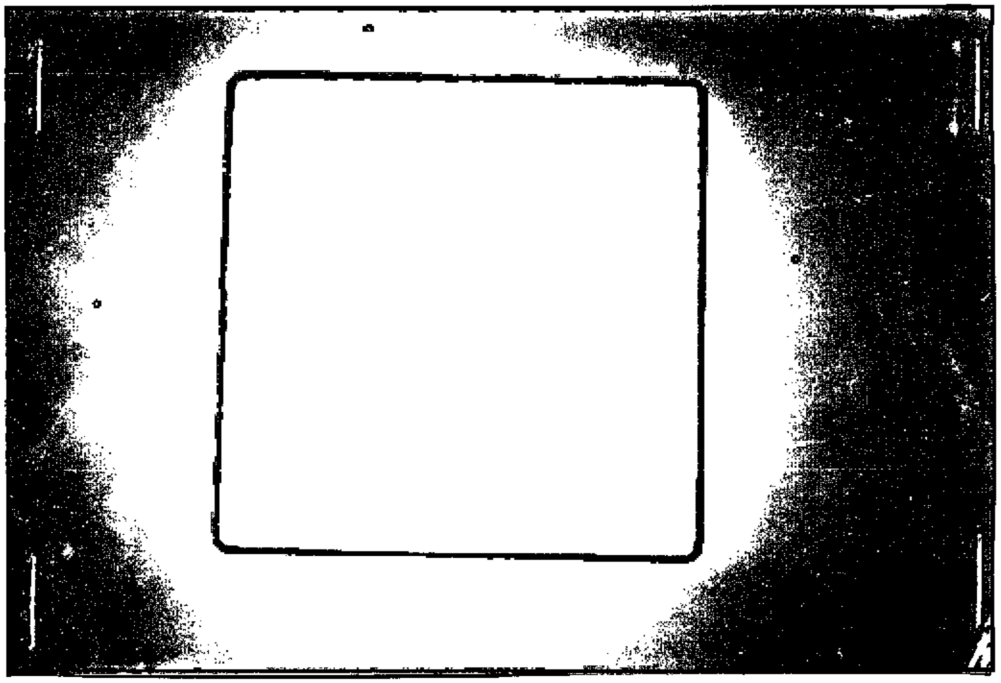

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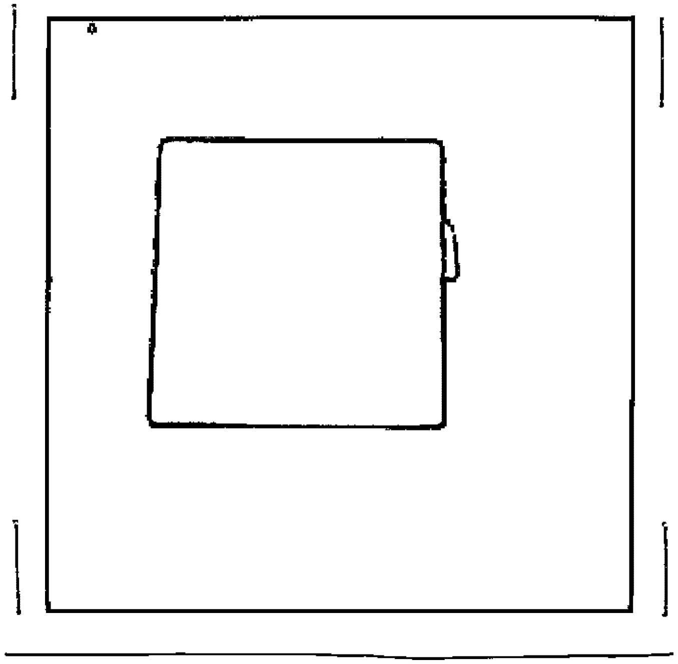

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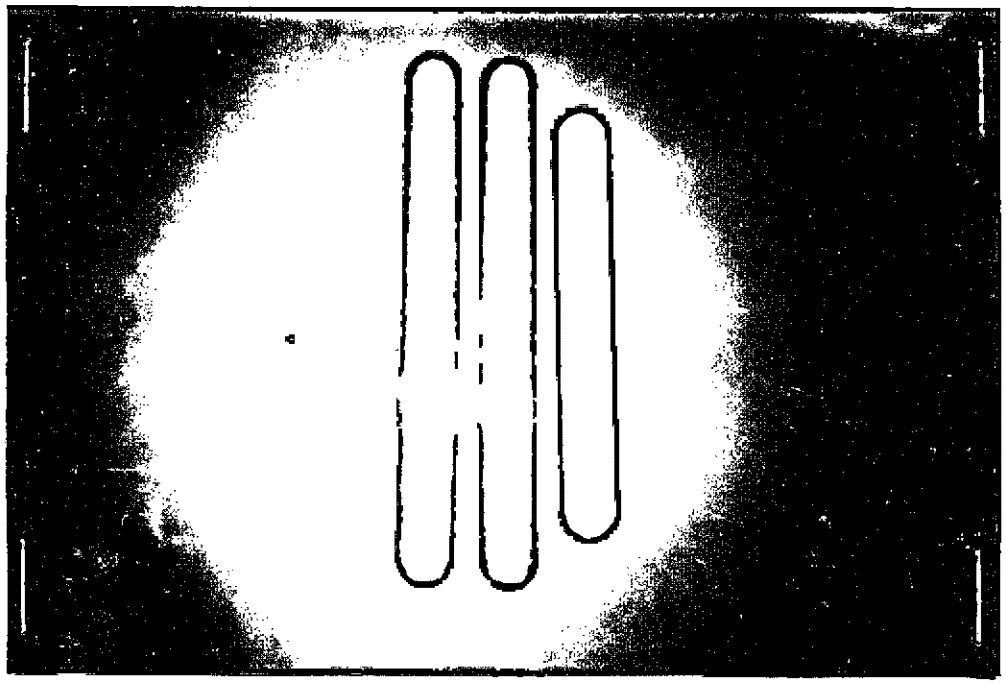

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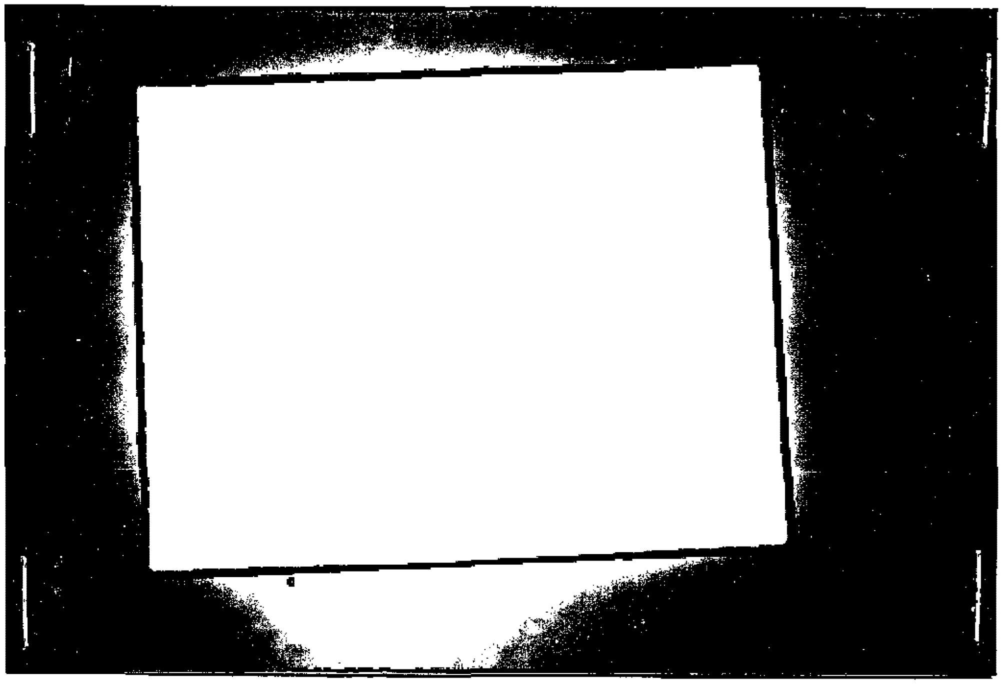

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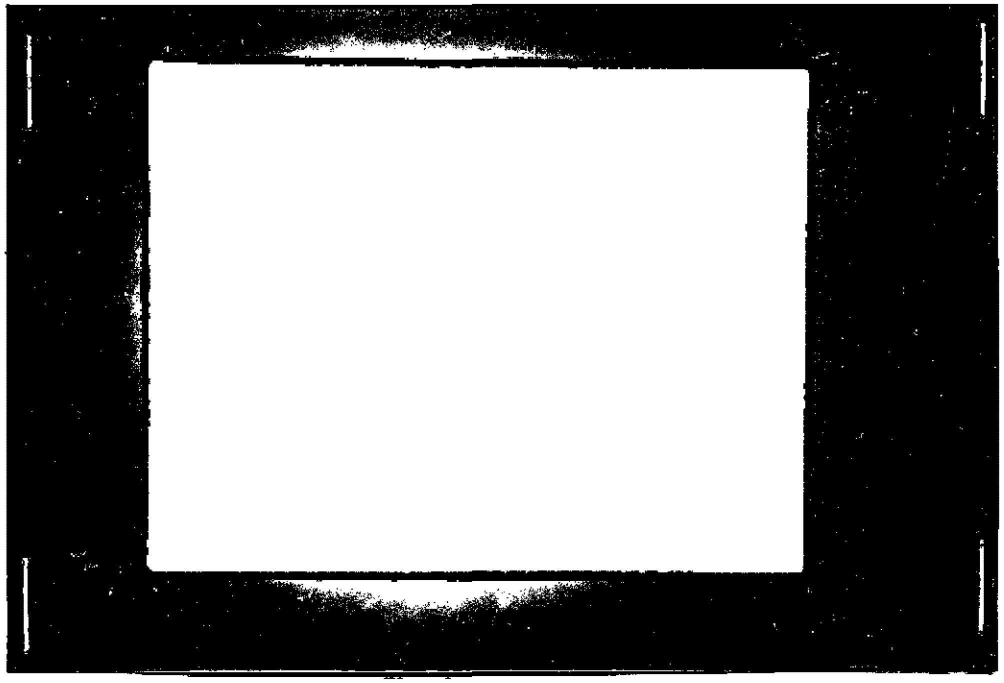

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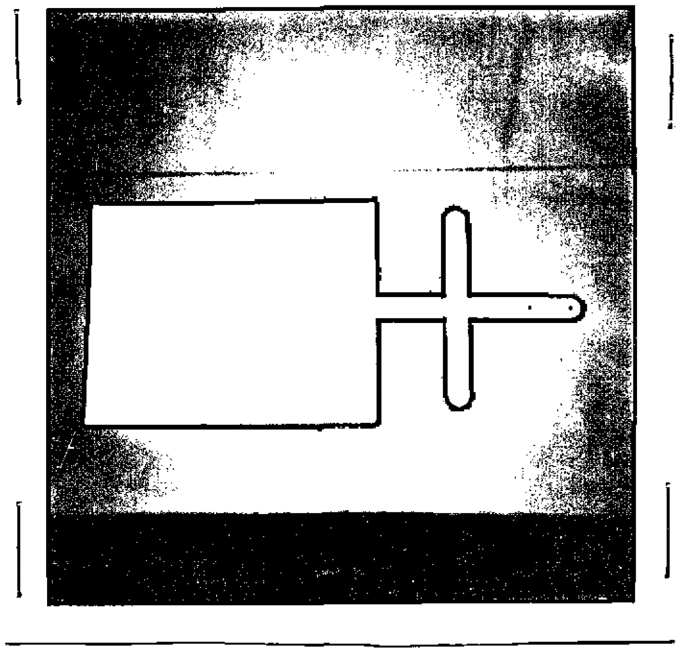

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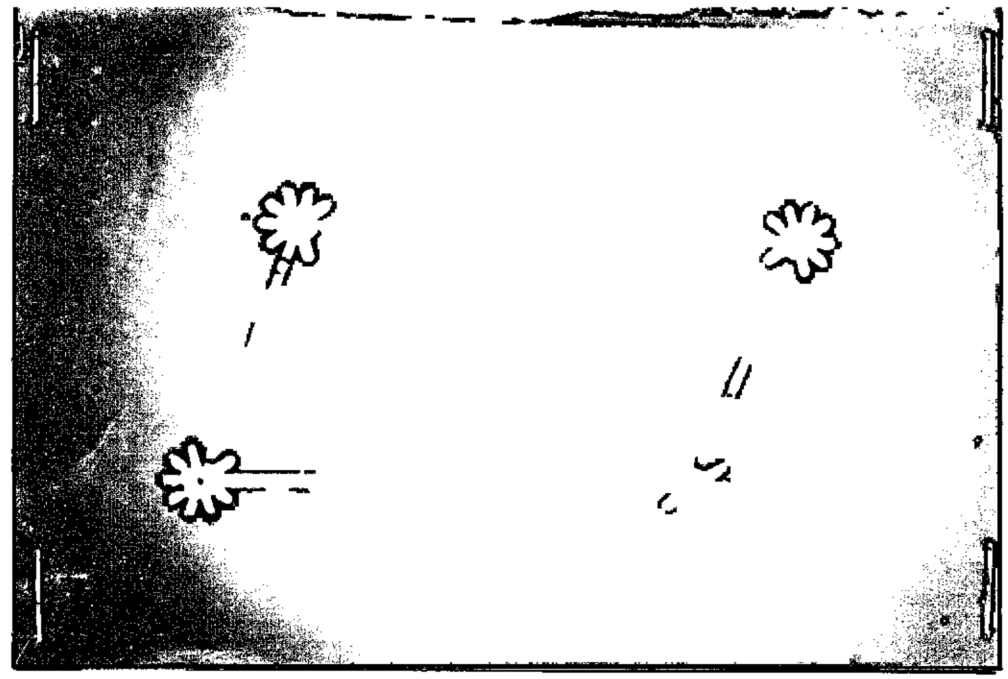

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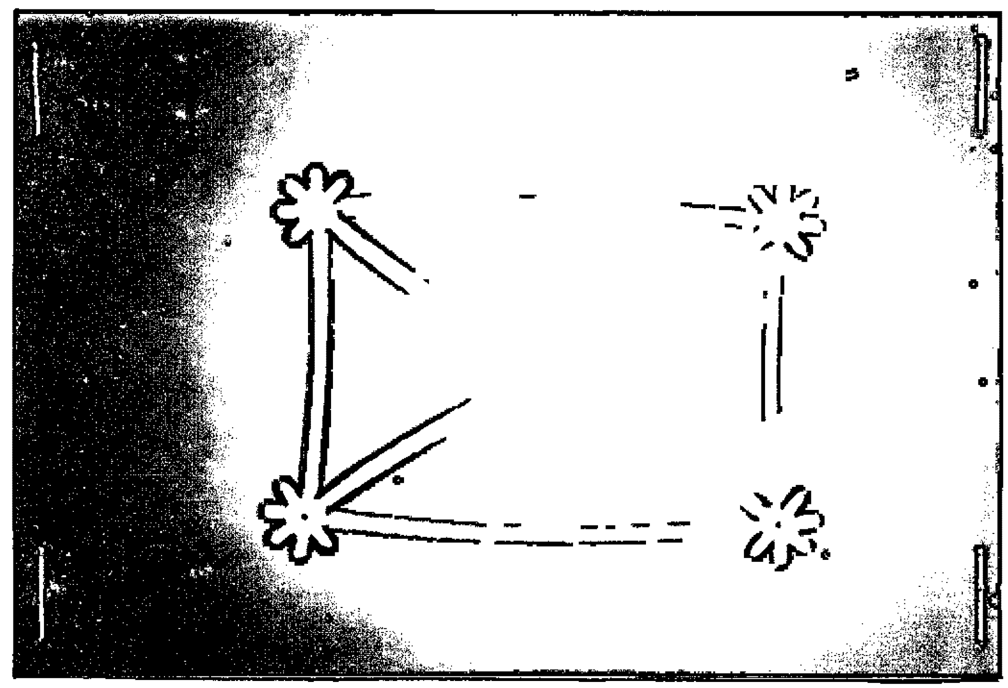

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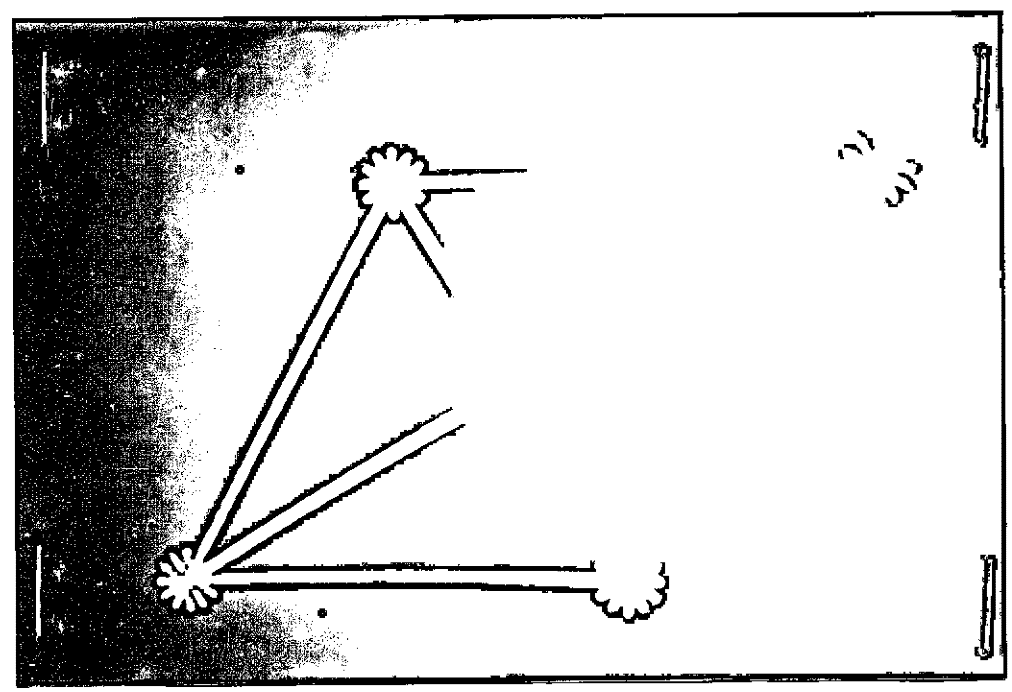

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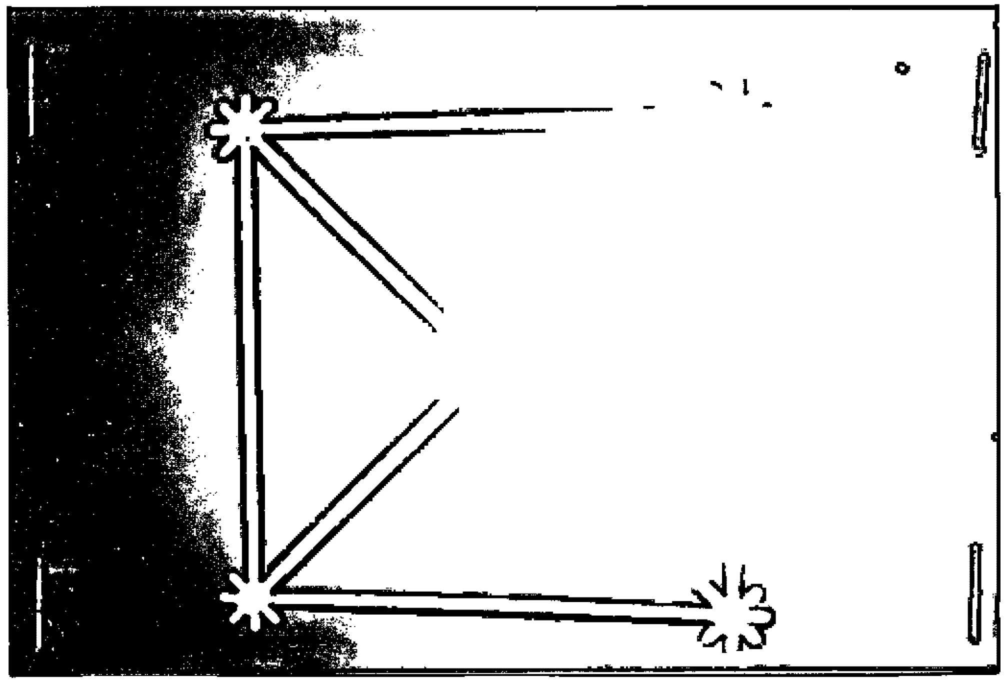

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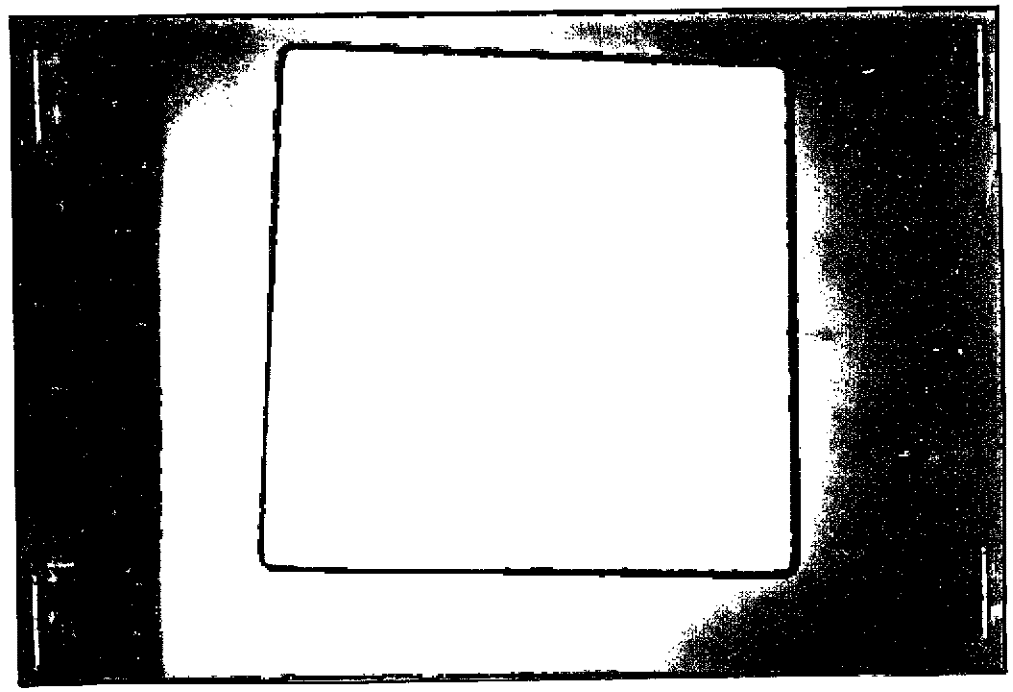

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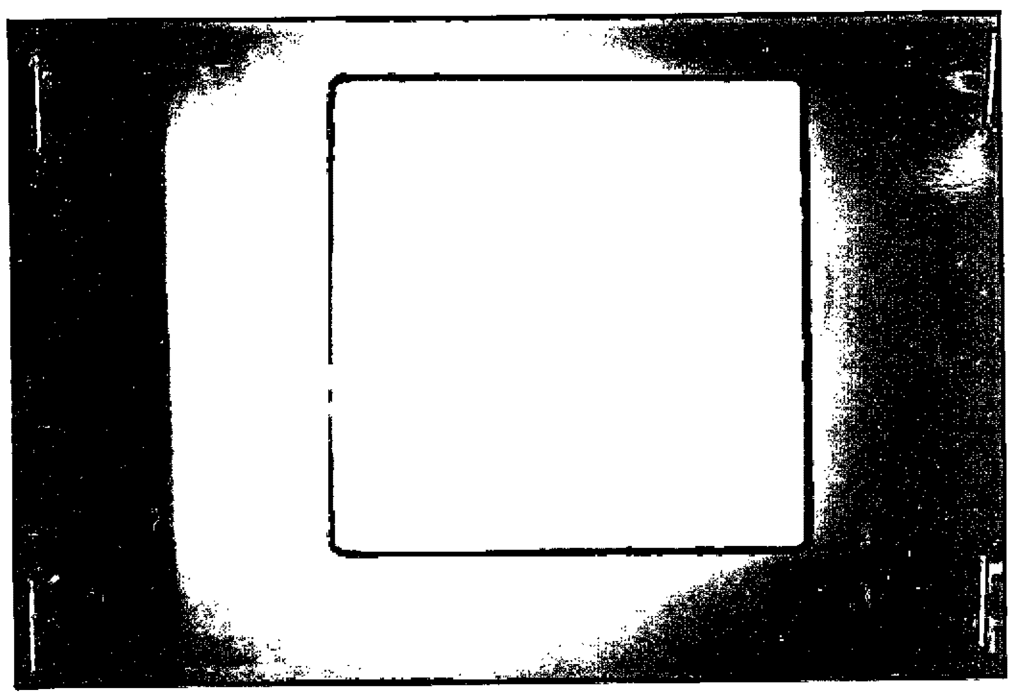

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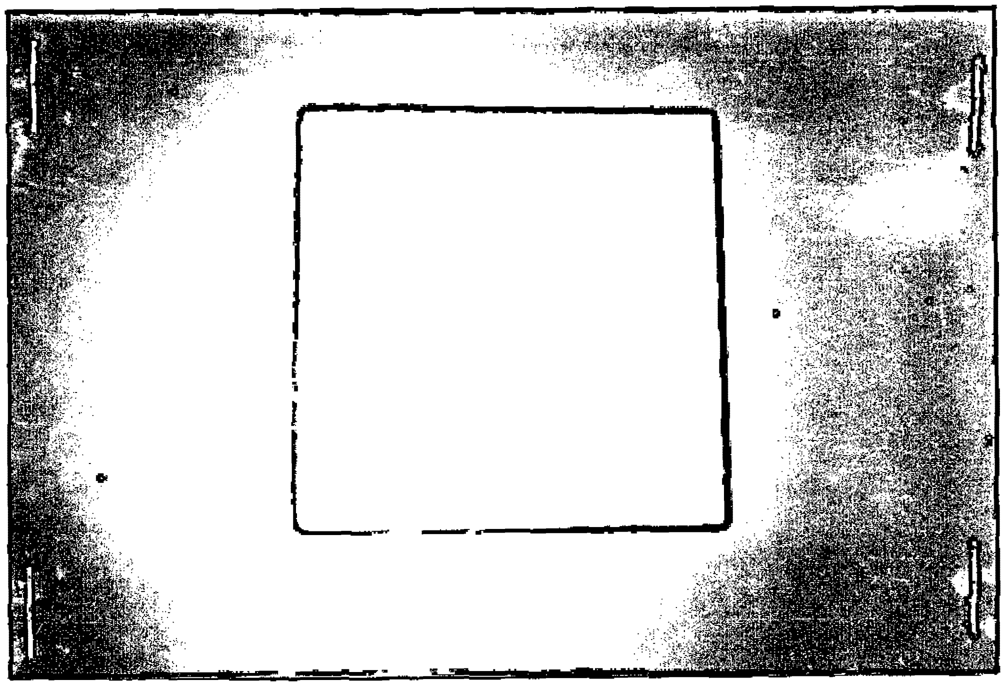

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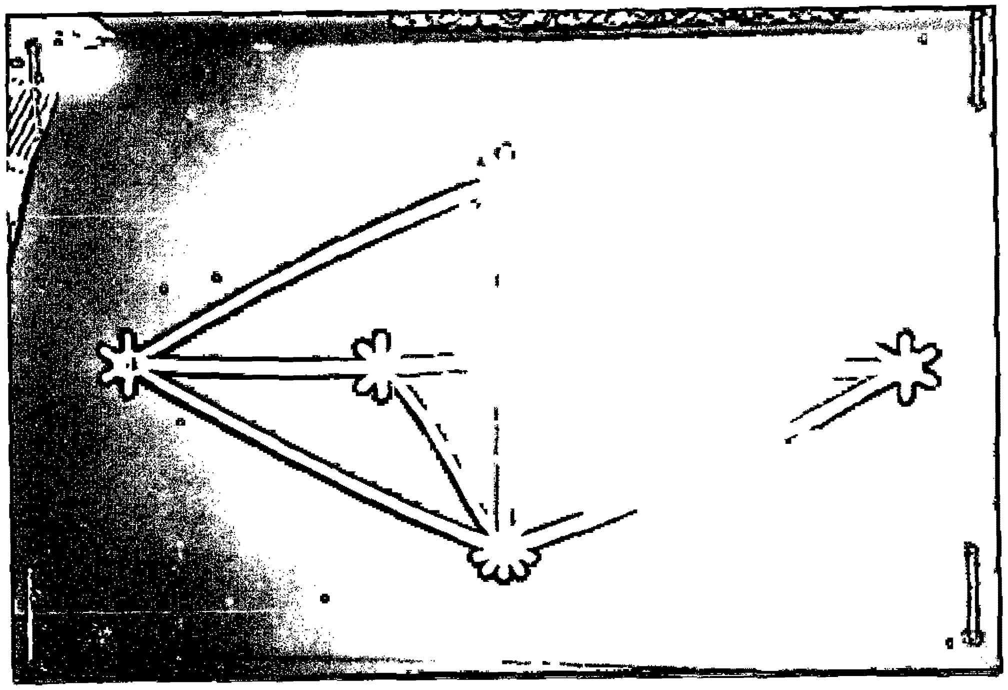

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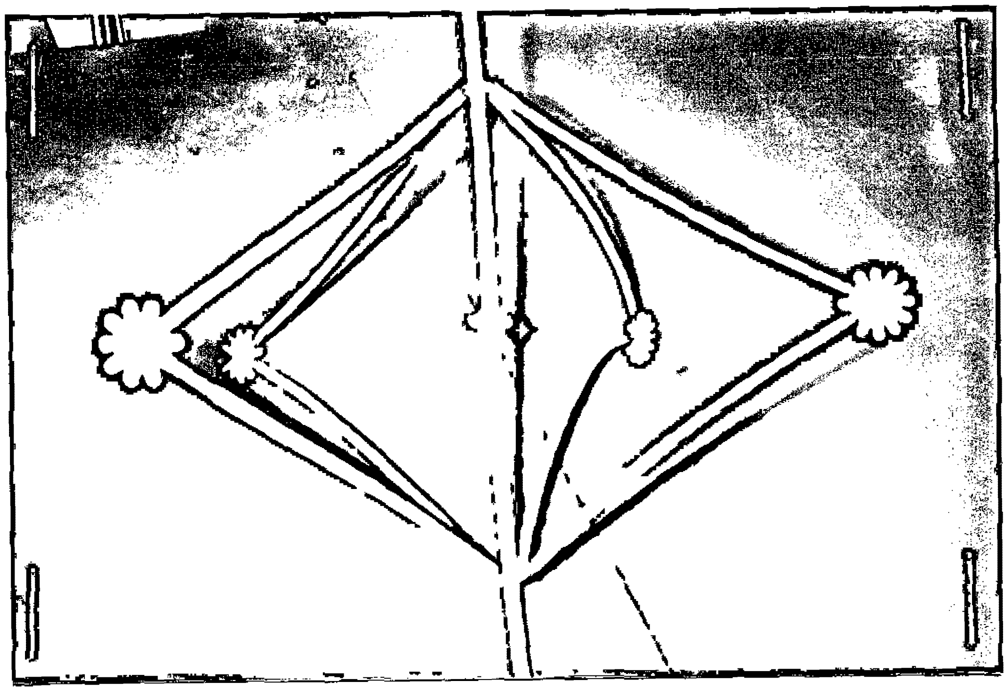

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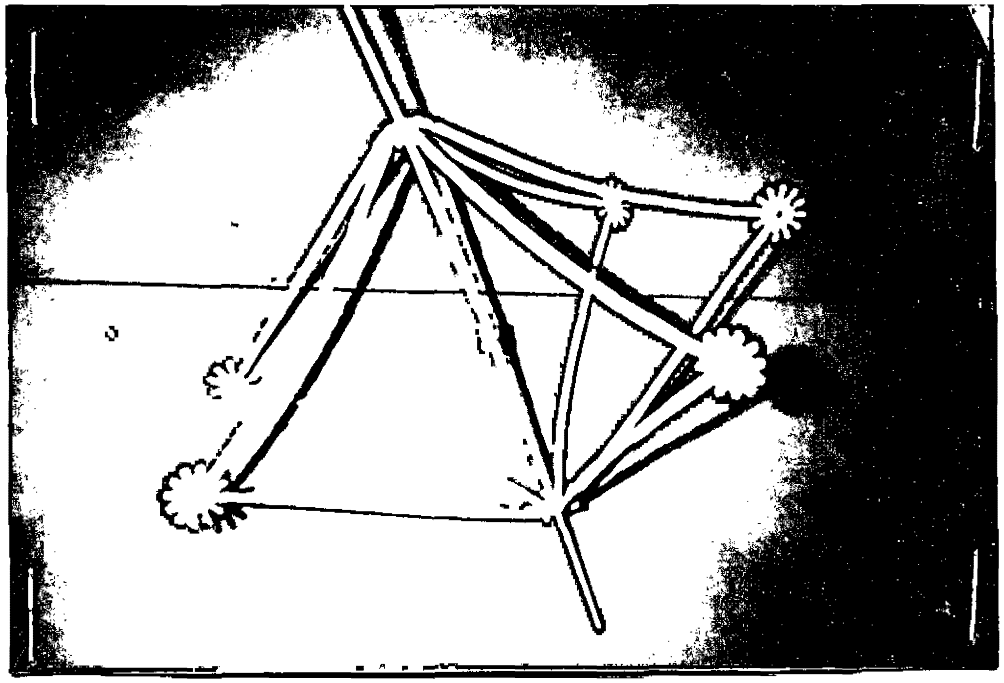

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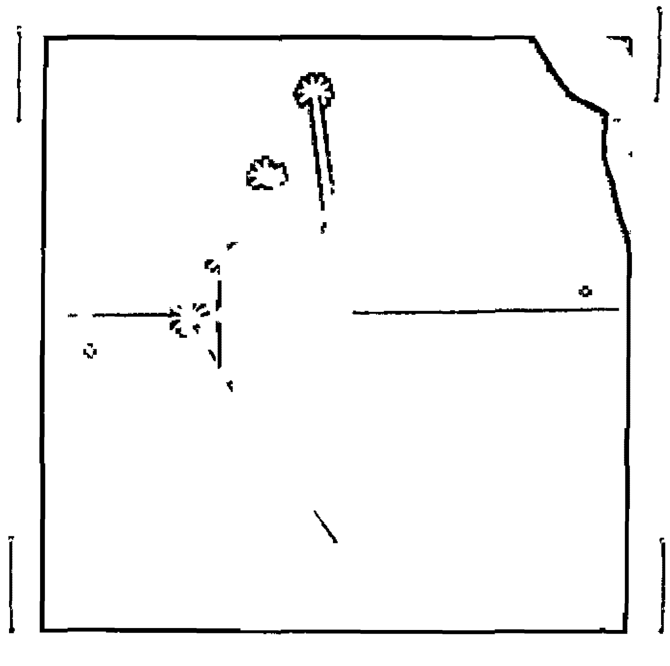

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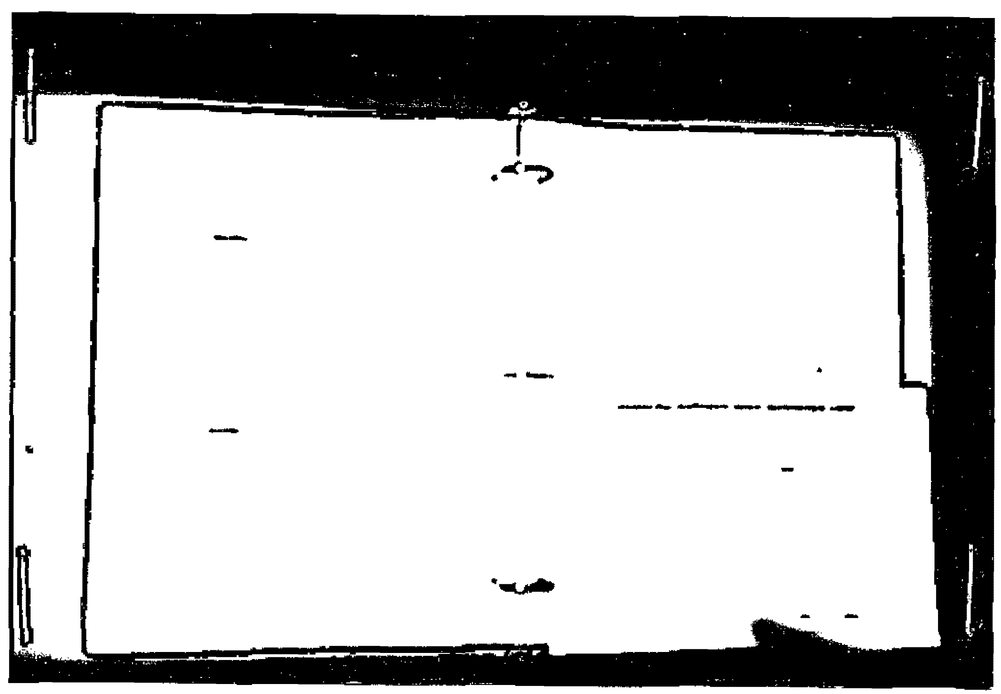

30

BULLETIN BOARD

BULLETIN BOARD

PROPERTY	PARALLELOGRAM	RECTANGLE	RHOMBUS	SQUARE
OPPOSITE SIDES ARE PARALLEL	*	*	*	*
OPPOSITE SIDES ARE CONGRUENT	*	*	*	*
OPPOSITE ANGLES ARE CONGRUENT	*	*	*	*
DIAGONAL FORMS 2 CONGRUENT TRIANGLES	*	*	*	*
DIAGONALS BISECT EACH OTHER	*	*	*	*
DIAGONALS ARE CONGRUENT		*		*
DIAGONALS ARE PERPENDICULAR			*	*
DIAGONALS BISECT OPPOSITE ANGLES			*	*
ALL ANGLES ARE RIGHT ANGLES		*		*
ALL SIDES ARE CONGRUENT			*	*

GEOMETRY WORKSHEET

Name ______

Geometry - Chapter 5
Review Worksheet

Draw a parallelogram (Random).

Choose an angle ____; its measure is ____.

Its opposite angle ____ has measure ____.

Choose a 3rd angle ____; its measure is ____.

Its opposite angle ____ has measure ____.

Supporting theorem:

$\angle$ ____ is a supplement of $\angle$ ____ and $\angle$ ____.

Prove by the measures above:

Supporting theorem:

Choose a side ____; its measure is ____.

Its opposite side ____ has measure ____.

Choose a 3rd side ____; its measure is ____.

Its opposite side ____ has measure ____.

Supporting theorem:

Draw 2 diagonals and label their intersection.

Diagonal ____ has measure ____. Diagonal ____ has measure ____

Measure the segment from a vertex ____ to the point of intersection of the 2 diagonals; its measure is ____. Measure from the same intersecti point ____ to the opposite vertex ____; its measure is ____.

So, intersection point ____ is the ________ of diagonal ____.

Measure the segment from a 3rd vertex ____ to the point of intersection of the diagonals; its measure is ____. Measure from the same intersection point ____ to the opposite vertex ____; its measure is ____. So, intersecti

point ___ is the ______ of diagonal ____.
Supporting theorem:

New Shape (N). Parallelogram. Rectangle.
∠ ___ has measure ____. ∠ ___ has measure ____.
∠ ___ has measure ____. ∠ ___ has measure ____.
Side ___ has measure ____; its opposite side ___ has measure ____.
Side ___ has measure ____; its opposite side ___ has measure ____.
2) Supporting theorems:

Draw 2 diagonals.
Diagonal ____ has measure ____. Diagonal ____ has measure ____
So, diagonal ___ is ______ to diagonal ____.
Supporting theorem:

All of the above are also characteristics of a ________
which has the additional property that ________

Name ________

New Shape. Parallelogram. Rhombus.

Side ___ has measure ___. Side ___ has measure ___.
Side ___ has measure ___. Side ___ has measure ___.
Draw 2 diagonals.
Diagonal ___ has measure ___. Diagonal ___ has measure ___
Label the intersection of the 2 diagonals ___.
Measure the 4 angles formed by the 2 diagonals.
∠ ___ has measure ___. ∠ ___ has measure ___.
∠ ___ has measure ___. ∠ ___ has measure ___.
So, the diagonals of a rhombus are ________.
Choose a vertex ∠ ___; its measure is ___.
The diagonal ___ cuts through the vertex ∠ ___ (above). The 2 angles formed, ∠ ___ and ∠ ___ have measures ___ and ___.
The vertex angle's opposite ∠ in the rhombus is ∠ ___; it is cut by the same diagonal ___. The 2 angles formed, ∠ ___ and ∠ ___, have measures ___ and ___.
Supporting theorem:

Using the small angles from the 2 problems above:
∠ ___ and ∠ ___ are alternate interior angles.
This means that side ___ and side ___ are ________
∠ ___ and ∠ ___ are alternate interior angles.
This means that side ___ and side ___ are ________.
This redefines the fact that a rhombus is a ________
based upon the supporting theorem:

New Shape. (N) Trapezoid. Random.
Label. Subdivide segment. Segment (choose a leg). Sections 2
Repeat this process to get the midpoint of the other leg).
Draw the segment connecting the 2 points above.
This segment ___ has measure ___ and is called the ___
Base ___ has measure ___.
Base ___ has measure ___.
Set up a relationship equating these 3 measures:

Supporting theorem:

Choose an angle ___ formed by the median and the leg ___; the angle's measure is ___.
Angle ___ is formed by the base and the same leg; its measure is ___
So, ∠ ___ is ___ to ∠ ___.
These 2 angles are ___ angles.
Now we can say that segment ___ is parallel to segment ___ due to the supporting theorem:

This proves that in a trapezoid, the median is ___ to the ___.

New Shape (N). Your own. Sides + angles. ∠DAB = 60. AB = 2. ∠ABC = 120. BC = !
∠BCD = 60. Draw.
The measure of side $\overline{AB}$ is ___; its opposite side ___ has measure ___
Draw $\overline{AC}$.
m∠DCA = ___. m∠CAB = ___.
So, side ___ is ___ to side ___ since alt. interior ∠s are
Based on these facts (*), we can say that quadrilateral ABCD is a ___ because of the supporting theorem:

QUESTIONNAIRE

Name ____________

Questionnaire

MANIPULATIVES = geo-boards, geo-strips, locus models, etc.

1. Did you enjoy using manipulatives as we studied quadrilaterals? Explain.

2. Do you feel the manipulatives helped you learn and/or retain the facts about quadrilaterals better than just the textbook? Explain.

3. Was there a particular type of manipulative activity you liked best?

4. Did you learn anything from going to the computer room? Do you feel that the worksheet and computer program helped you review for the test?

Did the manipulatives make geometry more fun and interesting? Explain.

(over)

6. Did you ever feel as if the activities were too childish for you? Explain.

7. Please use this space to give me any additional feedback about manipulatives that you wish. Any comments you make (positive or negative) will be a great help.

Thank You!!

DISCUSSION OF DATA

In the thesis statement one of the goals was to determine if the use of manipulatives produced any differences in the students' achievement or attitudes toward geometry. Further, analyzing the effects on visual, auditory, and kinesthetic learners specifically in the areas of achievement and attitude was another goal. In this section of the thesis data is presented, and the process by which it was analyzed is described. To draw the conclusions, t tests for correlated means (Linton 212-215) were performed to analyze the significance of the data.

When compiling the data, it was necessary to pair students between the experimental group and the control group for several reasons. First of all, there was a difference in class size. In the statistical comparisons, the number of students needed to stay constant for the purpose of continuity. Also, the ability levels differed between the two groups. These conditions were beyond the experimenter's control. Pairing was an option to compensate for this. The fact that pairing did occur must be a constraint on the research because of possible errors. When pairing the students, the decision was based on the students' achievement and ability, as revealed by the students' previous chapter test averages and pretest scores. Out of the fourteen students in the experimental group, twelve of them were matched with similar students in the control group. Data from these twelve pairs of students was gathered throughout the research and used to draw the conclusions on achievement and attitudes.

ACHIEVEMENT

In the area of achievement, two means of comparison were used. First of all, the differences between the students' pretest and posttest (recall) scores in both the control group and the experimental group were identified. The second comparison was between the students' quadrilateral chapter test scores and the averages of their previous chapter test scores. As a student teacher the experimenter did not administer these previous chapter tests; however, the cooperating teacher assisted in constructing the test and validated that it was similar to the previous chapter tests. Therefore, this comparison was a valid measure of achievement.

Considering the pretest and posttest scores specifically, the difference in t values between the experimental group and the control group was conclusive (Exhibits 1 & 2). Exhibit 1 presents the pretest and posttest scores for the twelve students in the experimental group who were successfully paired with twelve students in the control group. Also presented in Exhibit 1 are the results of the t tests on this data. For the research the 95% confidence level was used to find the critical value for t (Hoel 402). Similarly, the same information for the control group is presented in Exhibit 2. Comparing the obtained value for t with the critical value for t in each exhibit, there is a significant difference between the pretest and posttest scores in both cases. This is to be expected because an entire unit of material was taught in the elapsed time. However, the t value of 4.180 for the experimental group is more significant than the control group's t value of 3.015. This difference can be attributed to the experimental group's use of manipulatives. These results are most impressive considering that the students in the experimental group may have had the same ability as their partners in the control group, but they were not high achievers. Whereas the control group was more academic in nature, the students in the experimental group were not likely to study for tests or quizzes. Examining Exhibits 1 and 2 more closely, the total raw score for the experimental group on the pretest was 70 compared to the total raw score of 120 for the control group. Looking to the posttest (recall) scores, the experimental group's total score of 151 was a 116% increase as compared to a 38% increase in the control group's score. Considering the natures of the experimental students and the t test results, the use of manipulatives produced a definite increase in achievement.

As another measure of achievement, the quadrilateral chapter test score was compared to the average of the previous chapter tests for both groups. The data and t test results for the experimental group are presented in Exhibit 3; the same data for the control group is presented in Exhibit 4. Since neither t value is significant (above the critical value), the null hypothesis (Linton 213) is supported. For both groups, no significant differences in achievement were found between the students' quadrilateral chapter test scores and their average scores of the previous chapter tests. Although this proved to be an inconclusive measure of achievement, several comments can be made about this comparison. First of all, the experimental group was not one for studying for tests. Any possible improvements in performance would have been surpassed by the control group due to their fine studying habits. Since neither t value was significant, further support is added for the similarity of this test to the previous tests. Also, the general meaning nature of manipulatives may not transfer over to the specifics demanded by a chapter test. This may explain why the experimental group performed so well on the general recall posttest. Another final reason is that teachers must make a trade-off when using manipulatives. The experimental group missed a review day that the control group had due to the computer room activity. Although the control group students worked individually on the same assignment that the experimental group had for homework during this one review day, time still had to be swapped in order for the computer activity to take place. Thus, teachers must make sure that the manipulative activities are just as worth-while as the missed class time. In other words, missing the one in-class review day may have hurt the experimental group's performance on the chapter test. This trade-off which must take place is a choice which teachers must ponder. In summary, in spite of the inconclusive results from the chapter test comparison, the pretest and posttest (recall) comparison provided strong support for the improvement in students' achievement brought about by the use of manipulatives in the mathematics classroom.

While teaching the unit, the experimenter observed that the low achieving students seemed to participate more in class if manipulatives were used. This observation brought about the question of whether manipulatives help to improve the achievement of slower students in particular. For this reason the student pairs were again analyzed, and more t tests were run on just the average to below average (C or below) students. Thus, in Exhibits 5 and 6 data for only nine pairs of average or below average students is presented. In these exhibits student performance on the pretest and posttest is compared. As expected, the t values for both the experimental group and the control group are significant. Further, the t value for the experimental group is still more significant than the t value for the control group. Thus, the slower students also showed improvement due to the use of manipulatives. Unfortunately, due to the limited number of pairs in this study, an accurate comparison of t values between the low and high achievers in just the experimental group could not be justifiably performed. Due to the experimenter's observations, this is an area well worthy of further study. However, if manipulatives keep slower students on task and out of trouble, they also have numerous other benefits for the classroom teacher.

Examining the same hypothesis as in Exhibits 5 and 6, t tests were performed on the differences between the quadrilateral chapter test scores and the average scores of the previous chapter tests for the same nine pairs of students. The data and t test results for these average or below average students are presented in Exhibits 7 and 8. In Exhibit 7 the obtained value for t in the experimental group is not significant, and the null hypothesis is supported. Oddly, the t value for the control group is significant in Exhibit 8. One possible reason for this is the nature of the control group; they were more grade conscious. Perhaps certain students noticed their deficiency and determined to improve their performance, especially for a new teacher. Otherwise, the comparisons in Exhibits 7 and 8 proved to be inconclusive. Thus, the benefits of using manipulatives with slower and underachieving students remains a topic for further study (Thornton 38).

ATTITUDES

Another area of interest was whether using manipulatives causes a positive improvement in students' attitudes toward geometry. A student's attitude toward a subject has a great influence on his performance. According to Jacobs (1974), "attitude is positively correlated with achievement" (Dessart 23). Further, an unfavorable attitude can produce anxiety over math, evidenced by stress and tension (Dessart 24). Therefore, to ensure student success teachers must attempt to implant proper, positive attitudes toward mathematics in the students.

In order to measure changes in the students' attitudes, an attitude test was administered before the unit was begun and again after the unit was completed. In Exhibits 9 and 10, the students' scores on the pretest and the posttest for both groups are presented. The t test results for both groups are also given in Exhibits 9 and 10. The obtained value for t for the control group (Exhibit 10) is not significant. In fact, there was no difference between the total raw scores on the pretest and the posttest. This is important to disprove any attitude improvements due to the new teacher situation. However, Exhibit 9 does show a significant t value for the experimental group. Thus, the use of manipulatives did produce a positive improvement in the students' attitudes toward geometry. In fact, the total raw score more than doubled in the positive direction between the pretest and the posttest. Further, the number of experimental students with overall negative attitudes toward geometry decreased from five to two. The two negative scores were only slightly negative, -0.29 and -0.14. Thus, the improvement in attitudes of the students who used manipulatives is very well supported. The students seemed interested and occupied when using the manipulatives, and this was well documented by the test results.

LEARNING STYLES

As a link between attitudes and learning styles, the comments by experimental students on a questionnaire given after the unit was completed have been compiled. Their responses were sorted by learning style (visual, auditory, and kinesthetic). Some selected responses have been chosen for inclusion in this thesis. Surprisingly, even though manipulatives are not envisioned to help auditory learners, one auditory learner commented, "The manipulatives made the theorems easier to understand." One above average, kinesthetic learner commented that manipulatives helped, "Especially the locus models, if I didn't have them I wouldn't have known it." One below average, kinesthetic learner commented, "In the textbooks it just shows you one side, but the manipulatives give you a three-dimensional figure." A below average, visual learner said, "Everybody enjoys having time out from always writing and trying to learn from what the teacher is doing." Another below average, visual learner admitted, "Manipulatives made me pay attention, more than just looking at the book." Finally, another below average, visual learner declared, "I think my grade in this class went up because of the manipulatives. They made me understand it completely." From these comments, manipulatives clearly improve the achievement and the attitudes of all types of learners in geometry.

In the next stage of research, the twelve students in each group were sorted by learning style (visual, auditory, or kinesthetic). In Exhibit 11, the posttest scores for the five visual, one auditory, and six kinesthetic learners in both the control and experimental groups are presented. The average score in each category has also been calculated. Evaluating these scores, the visual experimental group scored the highest, followed by the kinesthetic control group. Although the visual experimental learners' result was in accordance with expectations, the kinesthetic control learners made a surprising showing. For this reason, t tests were performed on this data to analyze the results more closely. In Exhibit 12 the data for the visual experimental group is presented. The pretest and posttest scores are compared to determine the degree of improvement. For this group a very significant t value of 4.079 was calculated as to be expected. Exhibit 13 presents the same data for the visual control group. Here, the t value is not significant in accordance with the hypothesis of this research. The t tests were not performed for the auditory learners since only one student was in this category in both the control group and the experimental group. However, in Exhibit 14 very interesting results are found for the kinesthetic experimental group. The obtained value for t is very significant. In contrast, the kinesthetic control group results in Exhibit 15 reveal a t value which is not significant. This is surprising considering their first place showing in Exhibit 11. One reason for this is that these students are high achievers and scored well on their pretests. Thus, there was not much room for improvement. Returning to Exhibit 14, the kinesthetic experimental group is not a strong group of learners, as evidenced by their pretest scores. In fact, the total post score in Exhibit 14 is less than the total pre score of the control group in Exhibit 15. Thus, the comparison made in Exhibit 11 is too general to be accurate and informative. However, the significant t value for this group lends support for the use of manipulatives with kinesthetic learners. The manipulatives were designed for this type of learner and obviously proved to be effective. To summarize the results of the t tests, no significant improvements resulted in either of the control groups (Exhibits 13 & 15). The manipulatives produced a significant improvement in the scores of the kinesthetic experimental learners (Exhibit 14). The most impressive results occurred with the visual experimental group (Exhibit 12). This group had the lowest total pre score (26) of any group analyzed, well below the kinesthetic experimental group's score of 45. The low ability, visual experimental group showed a phenomenal improvement. Their post score of 82 was only two points below that of the high ability, kinesthetic control group.

Obviously, this group responded most dramatically to the use manipulatives in terms of pretest and posttest scores.

In Exhibit 16 the chapter test scores of both the control group and the experimental group have been categorized by the students' learning styles. The mean score in each category was also calculated. Once again, the strong, kinesthetic control learners scored highest on the chapter test. Due to their ability and tendency to study, this result was not surprising. Not much variation in scores was evident among the other classifications. For this reason, t tests were completed to check for progress between the quadrilateral chapter test and the average of the previous chapter tests. Exhibits 17 through 20 present the data and t test results for the visual and kinesthetic learners in both the control group and the experimental group. Once again, t tests could not be performed to find the improvement of the auditory learners since only one student was classified in this manner. Although the greatly improved, visual experimental learners had the greatest obtained value for t, none of the t tests in Exhibits 17 through 20 were conclusive since the obtained t values were less than the critical values for t. Thus, no improvements in achievement due to the use of manipulatives were evident from the analysis of the chapter test scores.

Another method of comparison chosen was examining the number of correct responses to open ended questions on the posttest. The motivation for this comparison was to discover if the experimental students would recall more facts from having worked with the manipulatives. In Exhibit 21 posttest question four is stated, and the students' scores are presented. For every correct fact answered, the students received one point. In Exhibit 21 their raw scores are classified according to the learning styles of the students, and the average score for each category has been calculated. Once again, the slower, visual experimental students made a surprising showing, having the highest mean score of all of the categories. This group was closely followed by the academic, kinesthetic control group. The mean scores of the auditory and kinesthetic experimental learners were third and fourth respectively. Thus, allowing for the exceptional study habits of the kinesthetic control group, the experimental students achieved the highest scores on recalling facts about diagonals. Hopefully, this is due to the extensive use of geostrips to teach the section of diagonals to the experimental group. If so, manipulatives also help to improve the recall of the students who use them in the learning process. The raw scores from question six on the same posttest are presented in Exhibit 22. Analyzing the mean scores in each category, very little difference in scores was found among the different categories of learners. Perhaps the nature of this question was too general or open for the students to give detailed responses.

Whatever the reason, the analysis of question six was inconclusive. In addition to Exhibit 21, further tests of this type would be necessary to yield supporting evidence for the claim that manipulatives increase students' retention of information.

The data presented in Exhibits 1 through 22 was compiled from attitude and achievement pretests and posttests administered by the experimenter during her student teaching experience. The scores on these instruments have been analyzed to determine if the use of manipulatives effected an improvement in students' achievement or attitudes toward geometry. Another dimension of the research was to determine the cognitive styles which benefited most from the use of manipulatives. Having described the results of the t tests on this data, the experimenter must now present the limitations of this research and draw the final conclusions.

EXHIBITS

EXHIBIT 1

CUMULATIVE SCORES ON EQUAL SECTIONS
EXPERIMENTAL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	7	7	0	0
STUDENT 2	6	15	-9	81
STUDENT 3	6	10	-4	16
STUDENT 4	6	22	-16	256
STUDENT 5	11	13	-2	4
STUDENT 6	2	8	-6	36
STUDENT 7	5	24	-19	361
STUDENT 8	8	12	-4	16
STUDENT 9	9	14	-5	25
STUDENT 10	3	11	-8	64
STUDENT 11	4	8	-4	16
STUDENT 12	3	7	-4	16
TOTALS	70	151	-81	891

THE TOTAL NUMBER OF SUBJECTS (N)

N = 12

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 5.594

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -6.75

THE OBTAINED VALUE FOR t

t = 4.180

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.796

EXHIBIT 2

CUMULATIVE SCORES ON EQUAL SECTIONS
CONTROL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	9	19	-10	100
STUDENT 2	10	9	1	1
STUDENT 3	6	18	-12	144
STUDENT 4	6	6	0	0
STUDENT 5	12	16	-4	16
STUDENT 6	18	23	-5	25
STUDENT 7	12	12	0	0
STUDENT 8	13	19	-6	36
STUDENT 9	16	23	-7	49
STUDENT 10	6	6	0	0
STUDENT 11	4	5	-1	1
STUDENT 12	8	9	-1	1
TOTALS	120	165	-45	373

THE TOTAL NUMBER OF SUBJECTS (N)

$N = 12$

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

$Sd = 4.309$

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X} = -3.75$

THE OBTAINED VALUE FOR t

$t = 3.015$

THE CRITICAL VALUE FOR t (N-1,@.05)

$t = 1.796$

EXHIBIT 3

CHAPTER TEST SCORES ON EQUAL SECTIONS
EXPERIMENTAL GROUP

	PRE CHAPT. 5 SCORE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	76	78	-2	4
STUDENT 2	81	82	-1	1
STUDENT 3	93	90	3	9
STUDENT 4	57	80	-23	529
STUDENT 5	93	88	5	25
STUDENT 6	65	66	-1	1
STUDENT 7	69	70	-1	1
STUDENT 8	78	86	-8	64
STUDENT 9	98	88	10	100
STUDENT 10	60	74	-14	196
STUDENT 11	54	44	10	100
STUDENT 12	80	82	-2	4
TOTALS	904	928	-24	1034

THE TOTAL NUMBER OF SUBJECTS (N)

N = 12

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 9.468

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -2.00

THE OBTAINED VALUE FOR t

t = 0.732

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.796

EXHIBIT 4

CHAPTER TEST SCORES ON EQUAL SECTIONS
CONTROL GROUP

	PRE CHAPT. 5 SCORE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	75	88	-13	169
STUDENT 2	83	86	-3	9
STUDENT 3	92	90	2	4
STUDENT 4	60	78	-18	324
STUDENT 5	98	96	2	4
STUDENT 6	67	82	-15	225
STUDENT 7	69	60	9	81
STUDENT 8	79	96	-17	289
STUDENT 9	96	92	4	16
STUDENT 10	66	82	-16	256
STUDENT 11	61	62	-1	1
STUDENT 12	88	82	6	36
TOTALS	934	994	-60	1414

THE TOTAL NUMBER OF SUBJECTS (N)

N = 12

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 10.063

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -5.00

THE OBTAINED VALUE FOR t

t = 1.721

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.796

EXHIBIT 5

ACHIEVEMENT
EXPERIMENTAL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	7	7	0	0
STUDENT 2	6	15	-9	81
STUDENT 3	6	22	-16	256
STUDENT 4	2	8	-6	36
STUDENT 5	5	24	-19	361
STUDENT 6	8	12	-4	16
STUDENT 7	3	11	-8	64
STUDENT 8	4	8	-4	16
STUDENT 9	3	7	-4	16
TOTALS	44	114	-70	846

THE TOTAL NUMBER OF SUBJECTS (N)

N = 9

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 6.140

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -7.778

THE OBTAINED VALUE FOR t

t = 3.800

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.860

EXHIBIT 6

ACHIEVEMENT
CONTROL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	9	19	-10	100
STUDENT 2	10	9	1	1
STUDENT 3	6	6	0	0
STUDENT 4	4	12	-8	64
STUDENT 5	12	12	0	0
STUDENT 6	13	19	-6	36
STUDENT 7	6	6	0	0
STUDENT 8	4	5	-1	1
STUDENT 9	8	9	-1	1
TOTALS	72	97	-25	203

THE TOTAL NUMBER OF SUBJECTS (N)

N = 9

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 4.086

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -2.778

THE OBTAINED VALUE FOR t

t = 2.040

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.860

EXHIBIT 7

ACHIEVEMENT
EXPERIMENTAL GROUP

	PRE CHAPT. 5 SCORE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	76	78	-2	4
STUDENT 2	81	82	-1	1
STUDENT 3	57	80	-23	529
STUDENT 4	65	66	-1	1
STUDENT 5	69	70	-1	1
STUDENT 6	78	86	-8	64
STUDENT 7	60	74	-14	196
STUDENT 8	54	44	10	100
STUDENT 9	80	82	-2	4
TOTALS	620	662	-42	900

THE TOTAL NUMBER OF SUBJECTS (N)

N = 9

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 9.381

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -4.667

THE OBTAINED VALUE FOR t

t = 1.492

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.860

EXHIBIT 8

ACHIEVEMENT
CONTROL GROUP

	PRE CHAPT. 5 SCORE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	75	88	-13	169
STUDENT 2	83	86	-3	9
STUDENT 3	60	78	-18	324
STUDENT 4	67	82	-15	225
STUDENT 5	69	60	9	81
STUDENT 6	79	96	-17	289
STUDENT 7	66	82	-16	256
STUDENT 8	61	62	-1	1
STUDENT 9	88	82	6	36
TOTALS	648	716	-68	1390

THE TOTAL NUMBER OF SUBJECTS (N)

N = 9

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 10.466

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -7.556

THE OBTAINED VALUE FOR t

t = 2.166

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.860

EXHIBIT 9

ATTITUDES
EXPERIMENTAL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	1.71	1.71	0.00	0.00
STUDENT 2	0.57	0.57	0.00	0.00
STUDENT 3	-0.29	0.43	-0.71	0.51
STUDENT 4	-0.43	0.57	-1.00	1.00
STUDENT 5	1.29	1.00	0.29	0.08
STUDENT 6	-0.29	1.00	-1.29	1.65
STUDENT 7	-0.29	0.43	-0.71	0.51
STUDENT 8	0.86	1.14	-0.29	0.08
STUDENT 9	0.00	0.00	0.00	0.00
STUDENT 10	0.14	-0.29	0.43	0.18
STUDENT 11	-0.71	-0.14	-0.57	0.33
STUDENT 12	0.29	0.29	0.00	0.00
TOTALS	2.86	6.71	-3.86	4.35

THE TOTAL NUMBER OF SUBJECTS (N)

N = 12

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 0.531

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -0.321

THE OBTAINED VALUE FOR t

t = 2.095

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 1.796

EXHIBIT 10

ATTITUDES
CONTROL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	-1.29	0.43	-1.71	2.94
STUDENT 2	-0.86	-1.00	0.14	0.02
STUDENT 3	0.00	0.14	-0.14	0.02
STUDENT 4	-0.29	1.00	-1.29	1.65
STUDENT 5	1.00	0.57	0.43	0.18
STUDENT 6	-1.00	-1.71	0.71	0.51
STUDENT 7	-0.43	0.57	-1.00	1.00
STUDENT 8	2.43	2.29	0.14	0.02
STUDENT 9	1.00	1.14	-0.14	0.02
STUDENT 10	1.43	-1.43	2.86	8.16
STUDENT 11	-1.43	-0.86	-0.57	0.33
STUDENT 12	1.57	1.00	0.57	0.33
TOTALS	2.14	2.14	0.00	15.18

THE TOTAL NUMBER OF SUBJECTS (N)

N = 12

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 1.175

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = 0.000

THE OBTAINED VALUE FOR t

t = 0.000

THE CRITICAL VALUE FOR t (N-1,∂.05)

t = 1.796

EXHIBIT 11

POSTTEST AVERAGES

	CONTROL	EXPERIMENTAL
LEARNING STYLE:		
VISUAL	19	15
	18	10
	12	22
	6	24
	9	11
	64	82
MEAN =	12.8	16.4
AUDITORY	6	8
KINESTHETIC	9	7
	16	13
	12	8
	19	12
	23	14
	5	7
	84	61
MEAN =	14	10.2

EXHIBIT 12

VISUAL
EXPERIMENTAL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	6	15	-9	81
STUDENT 2	6	10	-4	16
STUDENT 3	6	22	-16	256
STUDENT 4	5	24	-19	361
STUDENT 5	3	11	-8	64
TOTALS	26	82	-56	778

THE TOTAL NUMBER OF SUBJECTS (N)

N = 5

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 6.140

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -11.200

THE OBTAINED VALUE FOR t

t = 4.079

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.132

EXHIBIT 13

VISUAL
CONTROL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	9	19	-10	100
STUDENT 2	6	18	-12	144
STUDENT 3	12	12	0	0
STUDENT 4	6	6	0	0
STUDENT 5	8	9	-1	1
TOTALS	41	64	-23	245

THE TOTAL NUMBER OF SUBJECTS (N)

N = 5

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 5.899

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -4.600

THE OBTAINED VALUE FOR t

t = 1.744

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.132

EXHIBIT 14

KINESTHETIC
EXPERIMENTAL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	7	7	0	0
STUDENT 2	11	13	-2	4
STUDENT 3	7	8	-1	1
STUDENT 4	8	12	-4	16
STUDENT 5	9	14	-5	25
STUDENT 6	3	7	-4	16
TOTALS	45	61	-16	62

THE TOTAL NUMBER OF SUBJECTS (N)

N = 6

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 1.966

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -2.667

THE OBTAINED VALUE FOR t

t = 3.322

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.015

EXHIBIT 15

KINESTHETIC
CONTROL GROUP

	PRE SCORE	POST SCORE	DIFFERENCE (PRE-POST)	DIFFERENCE SQUARED
STUDENT 1	10	9	1	1
STUDENT 2	12	16	-4	16
STUDENT 3	18	12	6	36
STUDENT 4	13	19	-6	36
STUDENT 5	16	23	-7	49
STUDENT 6	4	5	-1	1
TOTALS	73	84	-11	139

THE TOTAL NUMBER OF SUBJECTS (N)

N = 6

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 4.875

THE MEAN OF THE DIFFERENCES (PRE-POST)

$\bar{X}$ = -1.833

THE OBTAINED VALUE FOR t

t = 0.921

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.015

EXHIBIT 16

CHAPTER TEST AVERAGES

	CONTROL	EXPERIMENTAL
LEARNING STYLE:		
VISUAL	88	82
	90	90
	60	80
	82	70
	82	74
	402	396
MEAN =	80.4	79.2
AUDITORY	78	44
KINESTHETIC	86	78
	96	88
	82	66
	96	86
	92	88
	62	82
	514	488
MEAN =	85.7	81.3

EXHIBIT 17

VISUAL
EXPERIMENTAL GROUP

	PRE CHAPT. 5 SCORE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	81	82	-1	1
STUDENT 2	93	90	3	9
STUDENT 3	57	80	-23	529
STUDENT 4	69	70	-1	1
STUDENT 5	60	74	-14	196
TOTALS	360	396	-36	736

THE TOTAL NUMBER OF SUBJECTS (N)

N = 5

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 10.918

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -7.200

THE OBTAINED VALUE FOR t

t = 1.475

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.132

EXHIBIT 18

VISUAL
CONTROL GROUP

	PRE CHAPT. 5 SCORE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	75	88	-13	169
STUDENT 2	92	90	2	4
STUDENT 3	69	60	9	81
STUDENT 4	66	82	-16	256
STUDENT 5	88	82	6	36
TOTALS	390	402	-12	546

THE TOTAL NUMBER OF SUBJECTS (N)

N = 5

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 11.371

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -2.400

THE OBTAINED VALUE FOR t

t = 0.472

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.132

EXHIBIT 19

KINESTHETIC
EXPERIMENTAL GROUP

	PRE CHAPT. 5 AVERAGE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	76	78	-2	4
STUDENT 2	93	88	5	25
STUDENT 3	65	66	-1	1
STUDENT 4	78	86	-8	64
STUDENT 5	98	88	10	100
STUDENT 6	80	82	-2	4
TOTALS	490	488	2	198

THE TOTAL NUMBER OF SUBJECTS (N)

N = 6

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 6.282

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = 0.333

THE OBTAINED VALUE FOR t

t = 0.130

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.015

EXHIBIT 20

KINESTHETIC
CONTROL GROUP

	PRE CHAPT. 5 AVERAGE	CHAPTER 5 SCORE	DIFFERENCE	DIFFERENCE SQUARED
STUDENT 1	83	86	-3	9
STUDENT 2	98	96	2	4
STUDENT 3	67	82	-15	225
STUDENT 4	79	96	-17	289
STUDENT 5	96	92	4	16
STUDENT 6	61	62	-1	1
TOTALS	484	514	-30	544

THE TOTAL NUMBER OF SUBJECTS (N)

N = 6

THE STANDARD DEVIATION OF THE DIFFERENCE SCORES

Sd = 8.877

THE MEAN OF THE DIFFERENCES

$\bar{X}$ = -5.000

THE OBTAINED VALUE FOR t

t = 1.380

THE CRITICAL VALUE FOR t (N-1,@.05)

t = 2.015

EXHIBIT 21

POSTTEST QUESTION 4

	CONTROL	EXPERIMENTAL
LEARNING STYLE:		
VISUAL	4	5
	3	4
	2	2
	1	13
	4	1
	14	25
MEAN =	2.8	5
AUDITORY	1	4
KINESTHETIC	6	3
	7	6
	0	0
	8	5
	6	6
	2	0
	29	20
MEAN =	4.8	3.3

EXHIBIT 22

POSTTEST QUESTION 6

LEARNING STYLE:	CONTROL	EXPERIMENTAL
VISUAL	2	2
	1	2
	2	3
	1	1
	1	0
	7	8
MEAN =	1.4	1.6
AUDITORY	2	0
KINESTHETIC	0	1
	5	3
	0	1
	2	2
	1	1
	1	1
	9	9
MEAN =	1.5	1.5

QUESTION 6

State all that you know (other than $\overline{WZ} = \overline{XY}$) from the following figure:

trapezoid WXYZ

W ______________ X

Z ______________________ Y

LIMITATIONS

For a variety of reasons, several limitations exist to the validity and completeness of this research. As one constraint, changes in attitude or achievement may have occurred solely as a result of having a new (student) teacher with a different teaching style. Another constraint on the research was the difference in class size between the experimental group and the control group. For consistency, the experimenter paired the students between the two groups based on ability, as determined by their pretest scores and average scores on the previous chapter tests. Obviously, when determining the pairing of the twelve sets of students, judgment errors could have easily been made. In order to identify the students' learning styles, an instrument was designed by the experimenter and the thesis advisor. The questions on the instrument were straightforward, designed to indicate students' learning styles without ambiguity. However, this instrument had not been tested for validity before its use in this research. Therefore, this factor is another possible source of error. Finally, since the manipulatives were only implemented in one group of twelve students, the results cannot be generalized; they must remain applicable only to the experimenter's third period, experimental geometry class.

IMPLICATIONS FOR FUTURE RESEARCH

In many studies of the benefits of using manipulatives, researchers have focused on standardized tests as measures of achievement. In this research analyzing the results of the quadrilateral chapter test provided inconclusive results. However, when the improvement from the pretest to the posttest was studied, that of the experimental group was much more significant than that of the control group. For this reason, future research should be geared at analyzing benefits from many different angles in various areas. For example, observing the students, the experimenter noticed the benefits of students helping teach other students how to use the manipulatives. Through their explanations to other students, these students gained confidence and more expertise with the topics at hand. Benefits of students teaching other students have been documented by Gartner (16), and future research should address this as an added benefit of manipulative activities. As classroom teachers consider the use of manipulatives in their lessons, Gartner (24) reminds them to assess the learning styles of their students and adapt their instruction accordingly. Further research should be done to determine the type of learner most benefited by manipulatives. The results of this study revealed that the visual learners benefited the most. However, the nature of manipulatives lends itself to helping kinesthetic learners; perhaps future research would confirm this most strongly.

The experimenter's observations supported the use of manipulatives with low achieving students especially. However, the limited number of pairs in the experimental group prevented an accurate comparison of the improvements of low achievers with the improvements of high achievers. This would be a direction well worth further investigation. Finally, the use of two posttest questions to determine conclusive evidence of increased retention due to manipulatives should be supported by further measures. Some research to this effect has been done by Suydam (10). Since mathematics is a building discipline, retention is vital; more reseach in this area is justified. Thus, due to the limitations, as well as the results, of this research many directions for future research on manipulatives have been suggested.

CONCLUSION

After planning lessons to include manipulatives, teaching the unit, administering tests, pairing students, and analyzing the results of t tests, the experimenter has drawn several conclusions from the research in answer to the questions posed by the thesis statement. From the analysis of the data, the experimental group experienced significant improvements over the control group in achievement based on their scores on the pretest and the posttest. As measured by the semantic differential, the attitudes of the experimental group toward geometry were much more positive than the control group's due to their exposure to manipulatives. Finally, visual learners seemed to benefit most from the use of manipulatives. Kinesthetic learners also showed significant improvements due to the manipulatives. Thus, as supported by this research, the use of manipulatives in the geometry classroom does indeed make a difference in a variety of different areas.

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