Electromagnetic Density of Modes for a Finite-Size Structure

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Abstract

We give a simple and physically pleasing definition of the electromagnetic density of modes for finite-size structures filled with a generic dielectric material. The role of absorption and dispersion of the material is also investigated.

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Several attempts have been made to generalize the notion of local density of modes (LDOM)-or local density of states- and the density of modes (DOM)-or density of states- to the case of open cavities, i.e., structures of finite size where electromagnetic energy can flow in and out of the volume bounded by the surface S of the cavity . Quite surprisingly, the concept of density of states for a finite-size structure still lacks a simple, concise definition. Our results suggest that the LDOM may be generally defined as [1]:

$$\rho_\omega \equiv \rho_{\omega,\,free\ space}\,\frac{\left\langle \overline{W}_{emitted}(\vec{r})\right\rangle}{\overline{W}_{emitted\ ,\ free\ space}}, \qquad (1)$$

where $\overline{W}_{emitted}(\vec{r})$ is the mean power emitted by the point source located inside the cavity at a point $\vec{r}$, regardless of the specific nature of the source, and $\rho_{\omega free\ space}$ is the DOM of the free space, and the symbol $\langle\ \rangle$ denotes a spatial average over the volume V of the cavity. In the case of a point dipole the LDOM is directly linked to the imaginary parts of the Green's dyadic as :

$$\rho_\omega = -\frac{6\pi c}{\omega}\rho_{\omega,\,free\,space}\left\langle \mathrm{Im}[G_{\omega,\hat{x}\hat{x}}(\vec{r},\vec{r})]\right\rangle \qquad (2)$$

where $G_{\omega,\hat{x}\hat{x}}(\vec{r}_0,\vec{r}_0)$ is the $\hat{x}\hat{x}$ component of the dyadic Green's function $\overline{\overline{G}}_\omega(\vec{r},\vec{r}_0)$ calculated at $\vec{r}=\vec{r}_0$, where $\hat{x}$ is the unit vector along which the dipole is oriented. In the case of 1-D , finite structures, Eq.(2) can be recast in the following form:

$$\rho_\omega = -\frac{2k_0}{cL}\int_0^L \mathrm{Im}[G_\omega(\xi,\xi)]d\xi \qquad , \qquad (3)$$

where L is the length of the structure, k_0 the vacuum wavevector, and $G\omega(\xi,\xi)$ is in this case the scalar Green's function. In Fig.1 we compare the DOM as calculated using Eq.(3), and the DOM calculated via Wigner's time: $\tau_\omega = d\varphi_t / d\omega$ [2]. The density of modes is $\rho_{\varphi,\omega} = (1/L)(d\varphi_t / d\omega)$, where L is the length of the PC, and $\phi_t(\omega)$ is the phase of the transmission function [3]. The structure is a 1-D photonic band gap structure (PBG) whose details are described in the caption of the figure. Quite surprisingly, the two definitions differ from each other, and the DOM calculated trough the Wigner's time becomes negative near the absorption line of the dielectric material. The DOM calculated trough the Green's function remains always positive. Our results show that the true DOM for 1-D structures should always be calculated through Eq.(3), while the Wigner's time can not generally be used to evaluate the DOM. Of course, the Wigner's time retains its validity as the time that the peak of an incident, unchirped pulse needs to traverse a barrier [4].

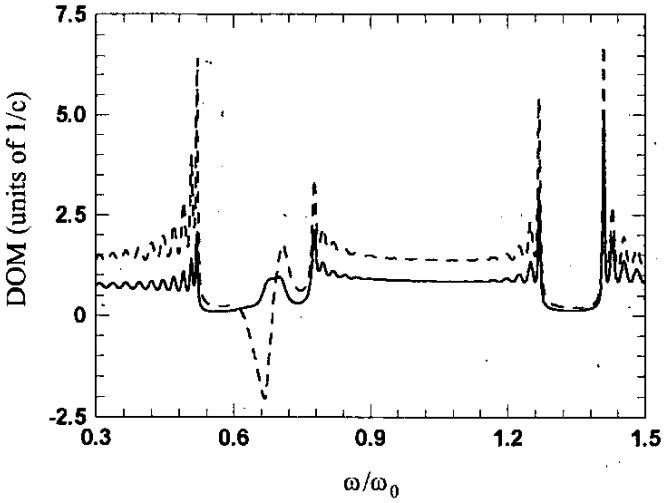

Fig.1. DOM calculated via the Green's function (solid line) and using the Wigner's time (dashed line) vs. ω/ω_0 ($\omega_0=2\pi c/\lambda_0$ where $\lambda_0=1\mu m$). The PBG structure is composed of 40alternating layers of air and a dielectric material, which has a lorentzian absorption line centered around $\omega/\omega_0=0.65$, and a refractive index approximately of 1.42 in the visible range. The layers have thicknesses a=250nm (air) and b=350nm (dielectric material).

References

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