Spatiotemporal Mode-Locking as Multidimensional Optimization

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Abstract: We outline a theoretical framework to understand the multitude of new mode-locked states possible in multi-transverse mode resonators. Full-3D measurements of mode-locked states comprising roughly 30 million modes agree with theoretical expectations.
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Spatiotemporal mode-locking (STML) is mode-locking within a resonator supporting more than one transverse spatial mode [1]. That is, STML is 3+1 dimensional mode-locking. In contrast, for many years mode-locking in lasers, and more recently in parametric oscillators like coherently-driven Kerr microresonators, has been considered as a 1+1 dimensional scenario. While the history of mode-locking has scattered examples of researchers venturing just beyond this 1+1 dimensional regime [2], most notably in the context of Kerr lens mode-locked Ti:sapphire lasers, the modern research focus has only just started to shift into higher dimensions. In fiber optics, this interest has motivated and grown from research on multimode nonlinear fiber optics, where effects such as Kerr beam cleanup suggest obvious advances in fiber-based laser technology [3], and in works that have begun to explore ways to control the many degrees of freedom available in multimode fibers and fiber amplifiers [4-5]. In the case of coherently-driven passive resonators, mode-locking involving a few modes has been explored in a number of forms [6]. Recently, we showed that multimode fiber oscillators could exhibit STML provided the total modal dispersion in the cavity was comparable to, or smaller than, the chromatic dispersion [1]. We observed that a spatial filter could, by introducing dissipative coupling between spatial modes, undo to some extent the effects of modal dispersion broadening, somewhat analogously to how a narrow spectral filter counteracts the broadening of a chirped pulse within a normal-dispersion mode-locked oscillator. However, subsequent investigations revealed the design conditions to be much more general, with the number of possible ways for STML to occur clearly stretching beyond our low-dimensional physical intuition.

Here, we introduce a general theoretical framework to understand mode-locking in high dimensions. We show that despite its simplicity, the theory provides a qualitative understanding of mode-locking under a wide range of cavity conditions, allowing us to predict several new types of mode-locking and related phenomena (that have no analogue in 1+1 D), and elucidate the design considerations needed for producing high-power, stable oscillators. Using a 3D electric field measurement technique, we experimentally verify the model in a fiber oscillator that supports (including in mode-locked operation) tens of millions of distinct 3D modes.

Mathematically, the basic approach we take is that of an iterated mapping on the field within the cavity. This approach is not new: it forms the basis for modern descriptions of optical mode-locking, including those in the continuous limit such as the complex cubic-quintic Ginzburg-Landau equation, the so-called master mode-locking equation, and Lugiato-Lefever equation. However, while our mathematical procedure is merely a generalization of low-dimensional models, the phenomenology is hardly so simple. On making the generalization we are faced with a bewildering multitude of new, emergent phenomena that simply cannot be described using the low-dimensional intuition or taxonomies.

Thus, our main theoretical contribution is to reframe the physics of self-organization of light in nonlinear laser cavities into mode-locked (or non-mode-locked) states in terms of intuitively-understood nonlinear attractors associated with distinct physical processes, and to study the interaction of these attractors to make up the global attractor that is the STML steady-state (Fig. 1). This shows that STML may universally be understood in terms of a 3D generalization of the "minimum-loss" gain competition heuristic that is often used to intuit why a saturable absorber causes mode-locking. The conceptualization is also closely analogous to multidimensional optimization, e.g. in neural networks, or throughout condensed matter and statistical physics [7].

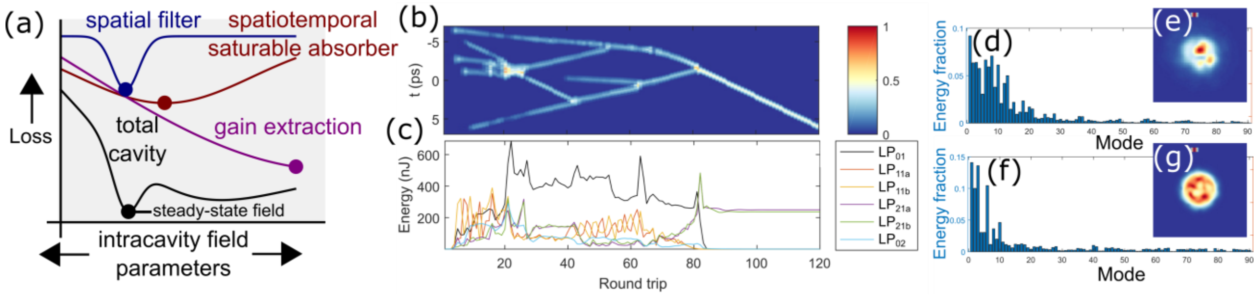

Fig. 1: Understanding STML through competing and cooperating 3D field attractors. (a) shows a cartoon of the optimization problem solved by the physics of the 3D laser cavity. Intuitively, the total attractor that describes the steady-state field can be understood as being composed of simpler attractors representing other effects. (b-c) show competition among multiple attractors in the route to steady-state in a few-mode step-index multimode fiber laser (simulation), with (b) showing the temporal pulse intensity integrated across the modes, and (c) showing the energy in each mode, both versus the round trip number and starting from noise. Competition between multiple pulses with distinct modal compositions leads to a single pulse existing exclusively in a degenerate mode family (LP_{21}) and provides another means by which STML allows power-scaling over single-mode oscillators. (d-g) show the experimental mode decomposition (d) and beam profile (e) of a highly-multimode STML state, while (f-g) show the same for a simplified theoretical model based on the intuition described in (a).

As an example, in the presence of a narrow spatial filter, minimization of loss occurs when the lasing spatial modes excite the minimum-loss transverse eigenmode of the filter (Fig. 1a). When the spatial filter has a Gaussian profile smaller than the fundamental mode of the laser cavity, then the minimization of loss occurs when the laser operates in a coherent superposition of LP_{0N} modes, such that the modes approximate a Gaussian beam, smaller than the fundamental cavity mode, similar to the filter's profile. In regimes where the spatial filter is important in determining the loss experienced by circulating light in the cavity, the steady-state mode-locked field (in experiment and simulations with the full, 3+1-D iterated nonlinear wave equations) is closely-approximated by the attractor (the steady-state laser field) of a simplified laser cavity that includes only the spatial filter.

As more, and different, spatiotemporal effects are considered within the cavity, more complex STML states, or other phenomena that happen either as mode-locking builds up or fails to occur, can be understood in terms of interactions between distinct idealized attractors (e.g., of the spatiotemporal saturable absorber or the 3D gain medium). These interactions may in some instances cooperate to form a qualitatively-new attractor, wherein one constraint may balance another one, or they may give rise to competition. An interesting example of competition occurs in a regime where little or no spatial filtering is applied, and hence on starting from noise, the laser initially forms many different multimode pulses (Fig. 1b-c). Due to the differing group velocities of these spatiotemporal pulses, they inevitably collide with one another inside the cavity. These collisions allow the intracavity field to jump from a multitude of metastable mode-locked pulses (local solutions to its optimization problem) into the global attractor. As Fig. 1b-c illustrate, this global attractor may be designed to be spatially single-mode, emerging from the violent suppression of other less-optimal pulses. This multi-pulse competition provides a recipe for designing STML oscillators that produce spatially-single-mode pulses at steady state, at power levels many times the usual multi-pulsing threshold. This pulse competition is one of several ways in which STML oscillators should allow for higher performance than single-mode oscillators (including with the ability to produce single-mode output). Finally, this conceptualization of the physics also permits us to understand the extremely multimode (~30 million modes) STML states which we can experimentally observe in some regimes. The experimental mode-decomposition of such a pulse obtained by scanning off-axis digital holography (Fig. 1 d-e) agrees reasonably with the composition predicted by a cooperative attractor (Fig. 1f-g). This mode-locking can be understood as a compromise between minimizing the pulse duration in the presence of dispersion and mode coupling (similar to principle modes [5]), while maximizing the field's overlap with the 3D-distributed gain medium to extract maximum energy each round trip.

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