Enhancement of $\chi^{(2)}$ cascading processes in one-dimensional photonic bandgap structures

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We theoretically analyze the nonlinear phase shifts induced by cascaded $\chi^{(2)}$:$\chi^{(2)}$ processes in one-dimensional photonic bandgap structures. We find that the enhancement of the density of modes near the band edge, coupled with a suitable choice of relative phase mismatch, leads to a remarkable new effect: The relative phase shift of the fundamental field on transmission can be of the order of π over a distance of 7 μm, with input intensities of the order of only 10 MW/cm^2. © 1999 Optical Society of America

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The past two decades have witnessed an intense investigation of electromagnetic-wave propagation phenomena at optical frequencies in periodic structures. These structures are usually referred to as photonic bandgap (PBG) crystals,[1] and many practical applications of this new technology, at least in one dimension, have been suggested for nonlinear[2,3] and linear[4,5] systems. These devices[2–5] are based on the physics of the photonic band edge.

In this Letter we point out yet another effect that can occur near the band edge. Using a multiple-scale formalism,[6] we show that nonlinear $\chi^{(2)}$ cascading processes can be enhanced by several orders of magnitude with respect to a bulk medium[7]: An increase in the density of modes (DOM) and a proper choice of the phase mismatch between the fundamental and the second-harmonic (SH) fields combine to produce phase shifts that are of the order of π for input intensities of the order of 10 MW/cm^2. Very recently, preliminary experimental evidence of enhancement of SH generation near the photonic band edge was provided.[8]

Our aim in this Letter is thus twofold: First, we summarize the approach that we use. Second, we apply the formalism to discuss the specific situation related to the mismatch that is necessary for enhancement of $\chi^{(2)}$:$\chi^{(2)}$ processes near the band edge. We consider a one-dimensional (1-D), finite, N-period structure consisting of pairs of alternating layers of high and low linear refractive indices, and possibly different $\chi^{(2)}$ responses; the thicknesses of the layers are a and b, respectively, $\Lambda = a + b$ is the length of the elementary cell (see the inset of Fig. 1), and for N periods the length of the structure is $L = N\Lambda$.

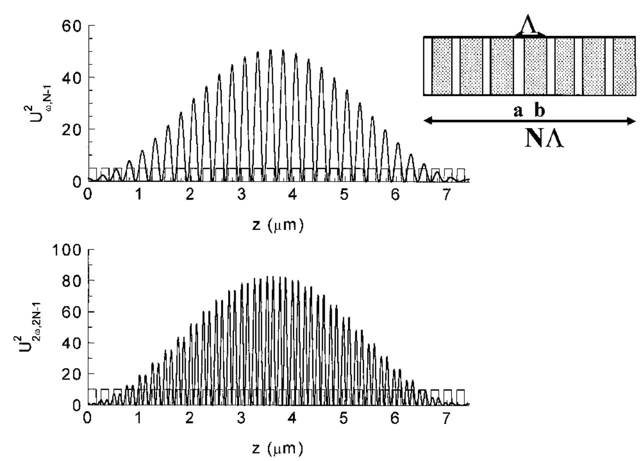

Fig. 1. Normalized intensity $U_{\omega,N-1}{}^2$ of (a) the eigenmode for the fundamental frequency (FF) field tuned at the $(N-1)$th peak of transmittance near the first band edge and (b) the eigenmode for the SH field tuned at the first resonance near the second-order band edge for the 30-period structure described in the text. The layer thicknesses are chosen to be $a = 93$ nm (Al_2O_3) and $b = 154$ nm ($Al_{30\%}Ga_{70\%}As$). Inset, schematic representation of the 1-D, finite N-period structure.

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The coupled-mode equations governing the interaction of two monochromatic plane waves at FF ω and SH 2ω in a layered 1-D structure are given by

$$(\mathrm{d}^2 A_\omega/\mathrm{d}z^2) + k_\omega{}^2 A_\omega = -\lambda\alpha A_\omega{}^* A_{2\omega}\,,$$

$$(\mathrm{d}^2 A_{2\omega}/\mathrm{d}z^2) + k_{2\omega}{}^2 A_{2\omega} = -2\lambda\alpha A_\omega{}^2\,, \tag{1}$$

where $k_\omega = k_0 n_\omega(z)$, $k_{2\omega} = 2k_0 n_{2\omega}(z)$, $n_\omega(z)$ and $n_{2\omega}(z)$ are the linear refractive indices along the propagation direction z, $k_0 = \omega/c$, and $\alpha = 2k_0{}^2 d^{(2)}(z)$ is the nonlinear coupling coefficient. λ is a dimensionless parameter that identifies the contributions of a given perturbative order.

To separate fast and slow scale variations in space we apply a suitable multiple-scale expansion.[6] We introduce a new set of $n+1$ independent variables $z_k = \lambda^k z$, with $z_0 = z$ and for $k = 0, 1, 2, \ldots, n$. The derivative operator is expanded according to the new set of coordinates, namely, $\mathrm{d}/\mathrm{d}z = \sum_{k=0}^{n} \lambda^k \mathrm{d}/\mathrm{d}z_k$, and the electric fields are also regarded as functions of the $n+1$ independent variables, namely, $A_{j\omega}(z_0, \ldots, z_n) = \sum_{k=0}^{n} \lambda^k A_{j\omega}^{(k)}(z_0, \ldots, z_n)$, with $j = 1, 2$. To first-order perturbation theory, and to zeroth order in λ, the first term in the expansion of the electric field can be expressed in the form $A_{j\omega}^{(0)}(z_0, z_1) = A_{j\omega}(z_0)\tilde{A}_{j\omega}(z_1)$, where $\tilde{A}_{j\omega}(z_1)$ is the field envelope that depends on the slowly varying variable z_1 and $A_{j\omega}(z_0)$ is the fast-oscillating linear eigenmode that solves the standard Helmholtz equation for the structure

$$\mathrm{d}^2 A_{j\omega}/\mathrm{d}z_0{}^2 + k_{j\omega}{}^2(z_0) A_{j\omega} = 0\,, \qquad j = 1, 2\,, \tag{2}$$

subject to the standard boundary condition at the input ($z_0 = 0$) and output ($z_0 = L = N\Lambda$) interfaces: $1 + r_{j\omega} = A_{j\omega}(0)$, $t_{j\omega} = A_{j\omega}(L)$, $ik_0(1 - r_{j\omega}) = \mathrm{d}A_{j\omega}(0)/\mathrm{d}z_0$, and $ik_0 t_{j\omega} = \mathrm{d}A_{j\omega}(L)/\mathrm{d}z_0$, where $r_{j\omega}$ and $t_{j\omega}$ are the linear reflectivity and transmittivity coefficients of the structure, respectively, and k_0 is the wave vector of the linear homogeneous medium that surrounds the structure. The linear eigenmodes are normalized with respect to the amplitude of the input fields.

The problem of solving Eq. (2) with the boundary conditions at the input and output interfaces can be regarded as a boundary eigenvalue problem. Under the condition that both the FF and the SH fields are tuned at a peak of transmission, the spectrum of the eigenvectors of Eq. (2) has the following form:

$$|j\omega, m_j\rangle = \exp\left(i\,\frac{m_j \pi}{N\Lambda}\, z_0\right) U_{j\omega,m_j}(z_0)\,,$$
$$m_j = 1, 2, \ldots, (N-1), (N+1), \ldots, (2N-1),$$
$$(2N+1), \ldots\,, \tag{3}$$

where $U_{j\omega,m_j}(z_0)$ are real functions of period L [i.e., $U_{j\omega,m_j}(0) = U_{j\omega,m_j}(N\Lambda)$] in orthogonality relation $\langle j\omega, l_j|\epsilon_{j\omega}|j\omega, m_j\rangle = C_{j\omega,m_j}\delta_{l_j m_j}$; here $C_{j\omega,m_j} = \int_0^{N\Lambda} \epsilon_{j\omega} U_{j\omega,m_j}{}^2 \mathrm{d}z_0$ and $\sqrt{\epsilon_{j\omega}} = n_{j\omega}(z_0)$. In Eq. (3) we skip $m_j = N, 2N, \ldots$ because the corresponding eigenmodes are tuned at the band edge, where the transmittance is zero.

In Fig. 1(a) we show the normalized eigenmode intensity $U_{\omega,N-1}{}^2$ for the FF field tuned at the $(N-1)$st peak of transmittance near the first band wedge for a 30-period stack. The structure consists of alternating layers of Al_2O_3 and $Al_{30\%}Ga_{70\%}As$. The refractive index of Al_2O_3 is taken to be approximately constant (1.62) over the range of wavelengths of interest, and we use measured data[9] for $Al_{30\%}Ga_{70\%}As$. The layer thicknesses are chosen such that when they are combined with material dispersion the SH eigenmode, shown in Fig. 1(b), is tuned at the $(2N-1)$st peak of transmittance near the second-order band edge. It is important to note that the eigenfunctions defined in Eq. (3) are similar but not identical to the Bloch functions that are generally used in the problem of nonlinear optical interactions in periodic media. The fundamental difference that we stress here is that, although Bloch functions are periodic over the elementary cell of length Λ, the family of eigenfunctions in Eq. (3) turns out to be periodic over the length of the structure $N\Lambda$ (see Fig. 1). The difference in the periodicity properties comes from consideration of a structure of finite length with boundary conditions at the input and output interfaces. This structure breaks the translational invariance of the eigenmodes over the elementary cell, introducing a long-range periodicity given by the length of the PBG, as Fig. 1 shows.

If we carry out the perturbation analysis to first order in λ, eliminating the secular terms in $A_{j\omega}^{(1)}(z_0, z_1)$, and use the orthogonality condition for the eigenstates, [Eq. (3)], for the usual rotating wave approximation the following coupled-mode equations for the field envelopes $\tilde{A}_{j\omega}(z_1)$ in the slow variable z_1 are obtained (we have formally taken $z_1 \to z$):

$$\mathrm{d}\tilde{A}_\omega/\mathrm{d}z = i\omega\rho_\omega\gamma_\omega\tilde{A}_{2\omega}\tilde{A}_\omega{}^*, \tag{4a}$$

$$\mathrm{d}\tilde{A}_{2\omega}/\mathrm{d}z = i\omega\rho_{2\omega}\gamma_{2\omega}{}^*\tilde{A}_1{}^2, \tag{4b}$$

where $1/\rho_{j\omega} = c^2\langle j\omega, m_j| - i(\mathrm{d}/\mathrm{d}z_0)|j\omega, m_j\rangle/j\omega C_{j\omega,m_j}$, $\Delta K = (m_2 - 2m_1)\pi/N\Lambda$ is the phase mismatch, and $\gamma_{j\omega} = \int_0^{N\Lambda} U_{\omega,m_1}{}^2 U_{2\omega,m_2} d^{(2)}(z_0) \times \exp(i\Delta K z_0)\mathrm{d}z_0/C_{j\omega,m_j}$. We note that a straightforward analogy connects the calculation of the bracket $\langle j\omega, m_j| - i(\mathrm{d}/\mathrm{d}z_0)|j\omega, m_j\rangle$ with the mean value of the momentum operator $\hat{p} = -i\hbar\mathrm{d}/\mathrm{d}z_0$ for electrons in a periodic potential.[10] Consequently, the eigenstate $|j\omega, m_j\rangle$ of the electromagnetic field at the peak of transmittance can be regarded as the Bloch eigenstate for the electron in an elementary cell of size $N\Lambda$. Thus we identify the term $\rho_{j\omega}$ as the DOM of the electromagnetic field inside the PBG. The coupling coefficient $\gamma_{j\omega}$ reflects the way in which the eigenmodes of the FF and the SH fields sample the distribution of the nonlinearity $d^{(2)}(z_0)$ inside the structure. In Fig. 2 we show the DOM (in units of 1/c) for the structure discussed above and shown in Fig. 1.

For the case depicted in Fig. 2, in which the eigenmodes of the FF and the SH are tuned at the peaks of transmittance near the first and the second band

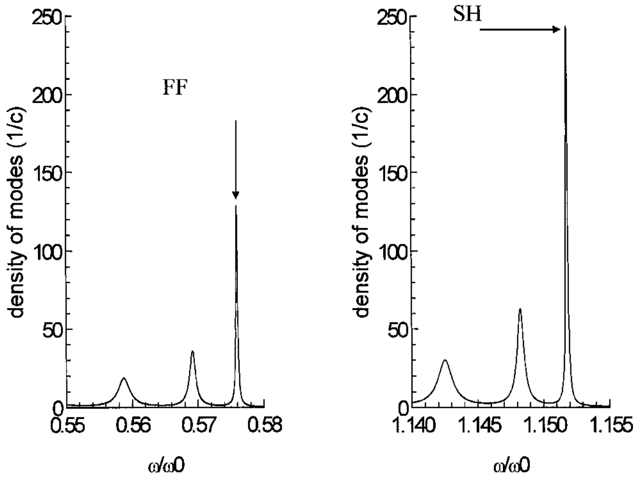

Fig. 2. DOM (in units of 1/c) for the multilayer stack in Fig. 1. ω_0 is the frequency corresponding to a wavelength of 895 nm.

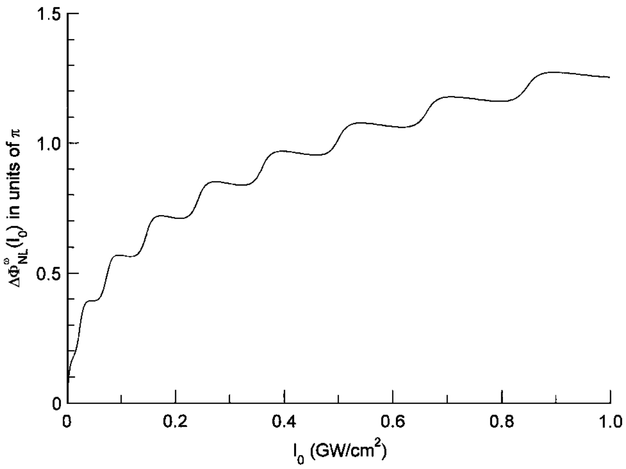

Fig. 3. Variation of the nonlinear phase of the FF field $\Delta\Phi_{\omega}{}^{\mathrm{NL}}$ versus the input FF intensity. The structure is the same as in Fig. 1.

edges, respectively, the quadratic interaction is mismatched by $\Delta K = \pi/N\Lambda$. This mismatch causes a slow oscillation over a length scale equivalent to L, the structure length. The resulting equations that govern the dynamics then take the form

$$\mathrm{d}\tilde{A}_{\omega}/\mathrm{d}z = i\sigma_{\omega}\tilde{A}_{2\omega}\tilde{A}_{1\omega}{}^{*}\exp\left(i\,\frac{\pi}{N\Lambda}\,z\right), \tag{5a}$$

$$\mathrm{d}\tilde{A}_{2\omega}/\mathrm{d}z = i\sigma_{2\omega}\tilde{A}_{\omega}{}^{2}\exp\left(-i\,\frac{\pi}{N\Lambda}\,z\right), \tag{5b}$$

where the real quantities $\sigma_{j\omega} = \omega\rho_{j\omega}\int_0^{N\Lambda} d^{(2)} \times U_{\omega,N-1}{}^{2}U_{2\omega,2N-1}\mathrm{d}z_0/\int_0^{N\Lambda}\epsilon_{j\omega}U_{j\omega,jN-1}{}^{2}\mathrm{d}z_0$ are the effective coupling coefficients. Equations (5) are formally equivalent to those governing quadratic interactions in bulk materials under moderate phase-mismatch conditions, namely, $\Delta KL = \pi$. The effect of the structure is scaled in the effective coupling coefficients $\sigma_{j\omega}$, which contain information regarding the DOM of the electromagnetic fields and the spatial average of the quadratic nonlinearity $d^{(2)}(z_0)$, which is also weighed by the amount of overlap of the linear eigenmodes. We find that the effective coupling coefficients, $\sigma_{j\omega}$, are enhanced by several orders of magnitude in a periodic structure with respect to an equivalent length of bulk material, an effect that can be traced directly to the high DOM that is available near the band edge, as Eqs. (5) suggest.

In Fig. 3 we show the nonlinear phase modulation of the FF field as a function of the input FF intensity for the same structure depicted in Fig. 2. The calculation was performed by direct numerical integration of Eqs. (5) after numerical calculation of the effective coupling coefficients $\sigma_{j\omega}$. The low-index layer (Al_2O_3) is taken to be linear, and the high-index layer ($Al_{30\%}Ga_{70\%}As$) is assumed to be nonlinear. The structure is approximately 7 μm in length. Typical values of $\chi^{(2)}$ can be of the order of 100 pm/V, which we use here. Figure 3 shows that at the exit of the medium the phase shift reaches values of the order of π, with modest FF intensities. These phase changes are remarkable because they are some 3 orders of magnitude greater than those reported for bulk materials and constitute a considerable improvement over the state of the art. Finally, although we have considered relatively short devices and $\chi^{(2)}$ interactions as an example, the enhancement of the DOM near the band edge can lead to the same effect for other processes, such as downconversion and $\chi^{(3)}$ interactions near the band edge.

In conclusion, the high DOM and the particular phase conditions that are available near the photonic band edge in our view make PBG structures the best candidates for highly efficient, all-optical devices based on cascading quadratic nonlinearities.

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References

1. C. M. Bowden, J. P. Dowling, and H. O. Everitt, eds. feature on development and applications of materials exhibiting photonic bandgaps, J. Opt. Soc. Am. B **10,** 279–359 (1993).
2. M. Scalora, J. P. Dowling, C. M. Bowden, and M. J. Bloemer, Phys. Rev. Lett. **73,** 1368 (1994).
3. M. Scalora, M. J. Bloemer, A. S. Manka, J. P. Dowling, C. M. Bowden, R. Viswanathan, and J. W. Haus, Phys. Rev. A **56,** 3166 (1997).
4. M. Scalora, R. J. Flynn, S. B. Reinhardt, R. L. Fork, M. J. Bloemer, M. D. Tocci, C. M. Bowden, H. S. Ledbetter, J. M. Bendickson, J. P. Dowling, and R. P. Leavitt, Phys. Rev. E **54,** 1078 (1996).
5. M. J. Bloemer and M. Scalora, Appl. Phys. Lett. **72,** 1676 (1998).
6. J. W. Haus, R. Viswanathan, M. Scalora, A. G. Kalocsai, J. D. Cole, and J. Theimer, Phys. Rev. A **57,** 2120 (1998).
7. R. De Salvo, D. J. Hagan, M. Sheik-Bahae, G. Stegeman, H. Vanherzeele, and E. W. Van Stryland, Opt. Lett. **17,** 28 (1992).
8. A. V. Balakin, D. Boucher, V. A. Bushev, N. I. Koroteev, B. I. Mantsyzov, P. Masselin, I. A. Ozheredov, and A. P. Shurinov, Opt. Lett. **24,** 793 (1999).
9. Courtesy of the Société de Production et Recherches Appliquées, www.sopra-sa.com/indices.htm.
10. N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart & Winston, New York, 1976), pp. 765–766.