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Nonlinear Pulse Propagation Phenomena in 1D Photonic Crystals.

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Abstract.

We numerically investigate nonlinear pulse propagation in finite, 1-d photonic crystals, and highlight novel properties that may help pave the way to a new class of high-efficiency nano-devices. We show that phase matching conditions for multiple wavelength generation and interactions can be achieved by judiciously combining material's index dispersions and geometrical features. We also show that enhanced nonlinear interactions can occur with efficiencies three orders of magnitude larger with respect to bulk materials having the same lengths and nonlinearity. Finally, we suggest potential applications as miniaturized second and third harmonic generators, nonlinear mirrors, parametric amplifiers, and optical switchers.

Keywords: frequency conversion, photonic crystals, nonlinear optical devices.

1. INTRODUCTION

In the last few years significant effort has been devoted to the investigation of nonlinear properties of photonic band gap crystals. Even though the study of electromagnetic propagation in 2-d and 3-d structures exhibits a complicated topology and demands extensive computer time, some of the physical characteristics and peculiarity of these structures can be understood by considering 1-d photonic crystals. These structures consist of alternating layers of materials with different refractive indices. For a periodic modulation, the typical transmission spectrum exhibits pass bands and forbidden gaps: the finite size of the structure is the cause of the presence of resonance peaks and of field's localization as shown in fig.1. The growing interest in second harmonic generation [SHG] is reflected in the large number of recent publications on this topic (see Ref. [2] and references therein). The importance of parametric

interactions such as SH generation, and frequency up- and down-conversion in general, is due to the unavailability of laser radiation at frequencies that are not accessible through more direct processes. Starting from a simple linear model, we show how phase matching conditions can be fulfilled for a SHG process in a finite photonic crystal. Then, we extend the same criteria to a generic three wave mixing process, and study nonlinear pulse propagation, and show some examples of possible applications of 1-D photonic crystals as highly efficient nonlinear miniaturized devices.

2. THEORETICAL MODEL

Previously we showed that in periodic structures phase matching conditions for second harmonic generation (SHG) [1,2] and frequency down conversion could be fulfilled if the fields were tuned to the appropriate resonance peaks of the transmission spectrum: the effective dispersion introduced by the geometry acts to compensate material dispersion. We emphasize that the dispersive properties of finite structures are not the same as for structures of infinite length. The expression for the real part of the effective index for the N period structure can be written as [1]:

$$\tilde{n}_{eff} = \frac{c}{\omega Na} \left\{ \tan^{-1} [z \tan(N\beta) \cot(\beta)] + \text{Int} \left[\frac{N\beta}{\pi} + \frac{1}{2} \right] \pi \right\}, \quad (1)$$

where c is the speed of light in vacuum, a is the thickness of the unit cell, β is the Bloch phase for an infinite structure having the same unit cell as the finite structure in question, and z is defined as:

$$z = \frac{\text{Im}(t)}{\text{Re}(t)}, \quad (2)$$

where t the transmittance of the unit cell. Eq (1) contains all the information necessary for the phase matching conditions of a generic multiwave mixing process. As an example, we consider a 20-period, mixed half-wave/quarter-wave PBG structure composed of air ($n_1=1$), and a dispersive ideal material to allow easy tuning of the fields near their respective band edges, and simultaneous access to a relatively high density of modes (DOM). The reference wavelength is 1 μm , and the refractive index at that wavelength is $n_2=1.42857$. The transmission spectrum is shown in fig. 1. We observe that the first order band edge resonance is tuned at 1.69 μm , and we choose it as our fundamental field frequency. From Eq.1 can be easily show that the phase matching condition for SHG is satisfied if $\beta_{2\omega} = 2\beta_{\omega}$. In order to take full advantage of the field localization we tune the fundamental field at the band edge resonance such that:

$$\beta_{\omega} = \frac{\pi}{N}(N-1); \quad (3)$$

the phase matching condition is thus fulfilled if:

$$\beta_{2\omega} = 2\beta_{\omega} = \frac{\pi}{N}(2N-2), \quad (4)$$

In other words, the second harmonic field is tuned to the second resonance near the second order band gap. This can be easily done by introducing a typical amount of dispersion in the high index material, so that $n_2=1.5188$ at $\lambda=0.845\mu\text{m}$. The same effect could have been obtained with a real material by introducing a slight imbalance in layer thickness to offset the original $\lambda/2$ - $\lambda/4$ setup. In fig. 2 we show how phase matching conditions can be achieved in terms of the effective index we defined, and note that the geometrical dispersion acts to compensate the material one. We checked our model numerically integrating the nonlinear wave equations. These are derived from Maxwell's equation for the field in a regime that we refer to as the slowly varying envelope approximation in time SVEAT [2,3]. We consider pulse propagation in the presence of large index discontinuities, and so we retain all second-order spatial derivatives. However, we assume that pulse envelopes initially have duration much greater than the optical cycle during the entire interaction. The method of integration is based on a modification of the split-step algorithm so that it can handle reflections of all orders, and is discussed in detail in Ref. [3]. The fields can arbitrarily and conveniently be written as:

$$E(z, t) = \varepsilon_{\omega}(z, t) \exp[i(kz - \omega t)] + \varepsilon_{2\omega}(z, t) \exp[2i(kz - \omega t)] + c.c. \quad (5)$$

$$P^{\text{NL}}(z, t) = P_{\omega}^{\text{NL}}(z, t) \exp[i(kz - \omega t)] + P_{2\omega}^{\text{NL}}(z, t) \exp[2i(kz - \omega t)] + c.c. \quad (6)$$

where ε and P are complex envelope functions, and $k=\omega/c$. The field at 2ω is initially zero, and is allowed to be generated in the backward and forward directions. Our choice of wave vectors corresponds to conditions consistent with a pump field initially propagating in free space, located away from the structure. Any phase modulation effects that arise from multiple reflections and nonlinear interactions are fully accounted for in the dynamics of the complex field envelopes. Therefore, assuming that pulses never become so short as to violate SVEAT, we integrate the equations of motion following the numerical model set forth in Refs. [2,3]. We tuned the pump field at the band edge resonance and then we calculated the overall converted second harmonic energy as a function of the refractive index n_2 at frequency 2ω . In fig. 3 we show that the SHG process occurs with maximum efficiency when $n_2(2\omega) = 1.5188$ thus, when the SH field is tuned at the second resonance near the second order band gap, as predicted by our effective index model. We also show that the SH field is generated in both backward and forward directions with maximum efficiency for the same value of n_2 . This is an indication that the effective index model leads to a dispersion relation that is uniquely determined. In fig. 4 we show several snapshots to follow the dynamics of the fundamental and the generated second harmonic fields for an incident short pump pulse (~ 100 fs). A non-zero value of the $\chi^{(2)}$ coefficient is assumed only for the high index layers. In this case the field is not strongly localized inside the structure because the spectrum of the pulse is somewhat wider than the transmittance resonance of the structure. The fields are normalized to show both FH and SH on the same scale; the total conversion efficiency is low, but we are using a low intensity pump and low nonlinearity coefficient (~ 1 pm/V). Nevertheless, we have shown [2] that using longer pulses (1 ps) SH conversion efficiency can be 2-3 orders of magnitude larger with respect to a phase matched bulk having the same length and same $\chi^{(2)}$ coefficient. Even though the overall conversion efficiencies that we will discuss may be obtained with current state-of-the-art devices, the gain per unit length that results from a drastic reduction in sample length due to field localization effects is several orders of

magnitude larger compared to the gain per unit length of any state-of-the-art, quasi-phase matched device. For this reason we are confident that the devices and methods we propose will become an attractive alternative as the miniaturization trend continues. In what follows we propose several applications, namely, a multiple wavelength generator and a nonlinear mirror.

3. APPLICATIONS

3.1 Nonlinear mirror:

Nonlinear interactions with second-order nonlinearities have recently been proposed as an alternative to intensity dependent changes induced in cubic nonlinear materials [4] for at least two reasons: speed, and primarily because of low-loss properties, in contrast with third-order processes. The possibility of loss-less operation is attractive because it can help revive the idea of fast control of light-by-light for all-optical signal processing. In this regard, devices that aim to control the reflectivity of a pump beam have been proposed [5,7]. These devices are based on the nonlinear interaction that arises when an intense second harmonic (SH) field is injected together with a weak fundamental field (FF). Ref. [5] refers to an optical loop mirror filled with a $\chi^{(2)}$ medium. Ref. [6] discusses the effect for a single interface, while Ref. [7] addresses reflectivity control in a micro cavity geometry. Ideally, a nonlinear mirror that is independent of the relative phase between the input beams is desirable, as Cojocaru et al. [7] clearly sought to achieve. However, phase-dependence is one of the hallmarks of second-order processes, and difficult if not impossible to override. Thus, we seek to exploit phase properties to our advantage, in order to achieve control of the reflective and transmissive properties of a 1-d photonic band gap structure doped with a $\chi^{(2)}$ material. We show that in a regime of undepleted SH intensity, a proper choice of the effective phase matching conditions between FF and SH fields (as discussed in Ref. [8]) can lead to a significant increase of nonlinear gain and dramatic reflectivity and transmissive changes of the FF over a distance of only a few microns. In Fig. 5 we show several snapshots of the dynamics at different times, focusing on the differences between the linear (a) and the nonlinear (b) regime. We consider the same structure described in fig. 2 so that the phase matching conditions are fulfilled, and pulse durations of approx 1 ps. We note that now the second harmonic beam is the stronger signal, tuned to the second resonance near the second order band gap, while the fundamental field is much less intense and tuned at the first order band edge resonance. A reflected field at the ω frequency is generated as a consequence of the nonlinear gain; amplification of the transmitted field also occurs when the SH field is turned on. The process is phase sensitive, and the maximum efficiency occurs when the relative input phase difference is $-\pi/2$. The second harmonic field intensity is of the order of 1 GW/cm²; the fundamental intensity is 6 orders of magnitude lower. The $\chi^{(2)}$ coefficient is 100 pm/V. In conclusion, group velocity reduction near the band edge can give rise to a very efficient nonlinear amplification scheme induced by the coupling between an FF and an SH fields. The high DOM and the particular

phase matching conditions that can be accessed near the photonic band edge make it possible for PBG structures to be considered highly likely candidates as efficient, phase sensitive nonlinear mirrors.

3.2 Simultaneous Second and Third Harmonic Generation:

In 1964, S. A. Akhmanov and R. V. Khokhlovnece was the first to theoretically investigate simultaneous SH and sum frequency generation process ($\omega+2\omega \rightarrow 3\omega$) in a bulk material [9]. They found that it was impossible to achieve simultaneous phase-matching conditions for both processes in ordinary materials. During the last 10 years, interest in nonlinear frequency conversion has increased thanks to a technique referred to as quasi phase matching (QPM) [10–12]. Even though QPM was devised in the 1960s by Armstrong et al. [10], multicolor generation (red, green, blue) was first reported by Yamamoto, Yamamoto, and Taniuchi [13] in a proton-exchanged, MgO doped LiNbO3 waveguide. The process exploits the Cerenkov configuration [14], which provides QPM conditions over a wide range of wavelengths. These experiments demonstrated that QPM could be used to yield good conversion efficiencies for simultaneous multiwavelength generation, and reopened the challenge for the realization of compact, multicolor laser sources for laser scanners, printers, and color display devices [15–17]. It is also possible to achieve phase matching for three-wave mixing using a periodic modulation of the linear refractive index, as first proposed by Bloembergen and Sievers [10] in reflection geometry. This technique allows interesting applications of multicolor frequency generation, as recently shown by Konotop and Kuzmiak [18]. In their work, the authors show that it is possible to achieve a double phase matching condition for second and third harmonic generation using simultaneous second- and third-order nonlinearities in infinite structures, thus neglecting effects that arise from field localization. In this section we show that a generic, finite, one-dimensional photonic band gap structure can be used to achieve efficient, simultaneous, collinear, global (both forward and backward) second and third harmonic generation using only a second-order nonlinearity, and by judiciously taking advantage of the simultaneous availability of phase-matching conditions and field localization effects inside the finite structure. Therefore, we propose a simple way to achieve optimal conditions by tuning the fields to appropriate resonance peaks, without resorting to uniaxial crystals or periodic domain inversion, as required by QPM. We then test the model on a realistic AlN/SiO2 stack, and find that large enhancements of conversion efficiencies, in excess of one order of magnitude, persist even under conditions of phase mismatch. We chose a $\lambda/8$ - $\lambda/2$ geometry because it allows phase matching and good field's localization for FF, SH and TH near three successive band gap order. From Eqs.(3-4), the phase matching condition for simultaneous third harmonic generation is achieved if:

$$\beta_{3\omega} = 3\beta_{\omega} = \frac{\pi}{N}(3N - 3), \quad (7)$$

It is possible to achieve this for a 40-period structure provided we introduce a typical amount of dispersion in the high index material, so that $n_2(\omega)=1.428$, $n_2(2\omega)=1.453$, and $n_2(3\omega)=1.481$. The low index material is $n_1=1$ as in previous examples, and $\chi^{(2)} = 144$ pm/V. In Fig.6 we show the conversion efficiency as a function of the input field intensity for the SH and TH fields. We compare these results with those of a phase-matched bulk material of the

same length and a nonlinearity uniformly distributed over its entire length ($\sim 10 \mu\text{m}$). These conditions represent the best-case scenario, described in Ref. [9], for a material that in reality does not exist. The dynamics is typical of a phase-matched interaction, though some notable differences arise. The crossing points denoted by *A* and *B* in Fig. 6 represent input peak powers that yield the same conversion efficiency for second and third harmonic generation. We note that for the ideal bulk material *B* occurs at 4 GW/cm^2 , while for the PBG structure *A* occurs at 0.2 GW/cm^2 , a factor of 20 smaller for the PBG. In addition, in the neighborhood of 0.2 GW/cm^2 , the PBG conversion efficiency is at least two orders of magnitude greater than bulk's conversion efficiency. In the case of the PBG structure, using 2 ps pulses also makes a difference. The pulse is not only finite in extent, its bandwidth is also comparable to the bandwidth of the band edge resonance, and so part of the incident energy is back reflected. To some extent, TH conversion efficiency increases as pulse duration increases, as previously observed for SHG [2]. As an example, we follow the suggestions of Ref. [20], and consider a 30-period AlN/SiO₂ multilayer stack. As pointed out in Ref. [20], this kind of stack is particularly attractive because it might provide an alternative to more expensive molecular beam epitaxy fabrication processes. The nonlinear coefficient of ALN is of the order of 10 pm/V . In Fig. 7(a) we show the tuning conditions on the fields; in Fig. 7(b) we show the results of the numerical integration of wave equations for this sample, and compare with SHG and third harmonic generation from one coherence length of AlN (approximately $7 \mu\text{m}$ for THG). Since the nonlinearity is relatively small, we consider the undepleted pump regime. The parameters and thickness for both materials are given in the caption. We found that while SHG can be exactly phase matched, i.e., $\beta(2\omega)=2\beta(\omega)$, geometrical dispersion cannot fully compensate the material dispersion of both materials. That is, $\beta(3\omega)\neq\beta(2\omega)+\beta(\omega)$, and the TWM process is slightly mismatched. Nevertheless, the TH signal is tuned to the second resonance near its band edge, as shown in Fig. 7(a), leading to a slight phase mismatch. The results depicted in Fig. 7(b) suggest that THG occurs at a rate approximately 300 times greater than bulk THG, while SHG is increased by at least one order of magnitude with respect to its bulk counterpart.

4. CONCLUSIONS

We studied pulse propagation in photonic crystals that display linear and nonlinear effects that make these structures good candidates for the realization of a new class of efficient nonlinear nano-devices. In particular, we showed two examples: a nonlinear mirror, and a multi-frequency generator. These examples highlight the fact that the efficiency per unit of length can be orders of magnitude greater compared to ideal bulk materials of similar length and nonlinear properties.

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Figure 1: Transmission spectrum for a 20 period, mixed half-wave/quarter wave stack. The refractive indices are $n_2=1.42857$ and $n_1=1$ for the $\lambda/2$ and $\lambda/4$ layers, respectively. The reference wavelength is $\lambda_0=1\ \mu\text{m}$. The band edge resonance is tuned at $\lambda=1.69\ \mu\text{m}$. Without the addition of material dispersion the second harmonic field is tuned to the fourth resonance near the second order band gap.

Figure 2: Effective index calculated for the structure described in Figure 1 (dashed) in the absence of dispersion; and for the same structure when an amount of dispersion is introduced in the high index material, so that $n_2(\omega)=1.42857$ and $n_2(2\omega)=1.5188$. We observe that the phase matching condition is fulfilled: $neff(\omega)=neff(2\omega)$.

Figure 3: Second harmonic conversion efficiency as a function of the value of refractive index at 2ω for the 20-period structure described in Figure 1. Circles: total second harmonic energy generated; Squares: SH energy generated in the forward direction; Triangles: SH energy generated in the backward direction.

Figure 4: Snapshots of the pulse dynamics for the fundamental and the generated second harmonic fields. We note field localization inside the structure, and that second harmonic generation occurs in both directions.

Figure 5: (a) linear propagation of 1 ps pulse through the structure described in fig. 2. More than 95 % of the pulse energy is transmitted, and the rest is reflected. (b) When a strong second harmonic field is switched on, nonlinear gain acts in both directions, leading to the amplification of both the transmitted and reflected fields.

Figure 6: SH and TH conversion efficiencies as functions of the pump peak intensity for the PBG structure (solid lines) and the ideal case of a plane wave propagating in a phase-matched bulk material (dashed lines) having the same length and nonlinear coefficient as the PBG structure. Points A and B identify the peak input intensities for which the conversion efficiencies of SH and TH generation are equal.

Figure 7a: Linear transmission vs. wavelength for a 30-period AlN (292.6 nm)/SiO₂ (91.4 nm) stack. Note that the third harmonic frequency (517nm) is not tuned to the third resonance from the band edge, where exact phase matching would ensue.

Figure 7b: SH and TH conversion efficiencies as functions of peak input intensity for our ALN/SiO₂ stack.