Birefringence in one-dimensional finite photonic bandgap structures

Mandatori, Antonio, Sibilia, Concita, Centini, Marco, D'Aguanno, Giuseppe, Bertolotti, Mario, et al.

Antonio Mandatori, Concita Sibilia, Marco Centini, Giuseppe D'Aguanno, Mario Bertolotti, Michael Scalora, Mark J. Bloemer, Charles M. Bowden, "Birefringence in one-dimensional finite photonic bandgap structures," Proc. SPIE 5000, Photonic Crystal Materials and Devices, (9 July 2003); doi: 10.1117/12.479817

Event: Integrated Optoelectronics Devices, 2003, San Jose, CA, United States

Birefringence in One Dimensional Finite Photonic Band Gap Structure

A. Mandatori, C. Sibilia, M. Centini, G. D'Aguanno, M. Bertolotti
INFM at Dipartimento di Energetica – Università "La Sapienza" di Roma – Via A. Scarpa 16, 00161 Roma – Italy, tel +39 06 4991 6541 , fax +39 06 442 40 183, concita.sibilia@uniroma1.it

M. Scalora, M. Bloemer , C.M. Bowden
Weapons Sciences Directorate, U.S. Army Aviation & Missile Command, AMSAM-RD-WS-ST, Bldg. 7804, Redstone Arsenal, Alabama 35898-5000 USA

Abstract

The spectral and dispersive behaviour of anisotropic layered structures forming a one dimensional " polarization dependent" Photonic band gap (PD-PBG) are discussed. The finite dimension of the structure has been taken into account. Interesting field localization properties are found when the optical axis of layers are not aligned each with the other one, i.e. principal axis of layers are rotated one with respect to the other. The field localization behaviour has been also discussed through a suitable definition of density of modes for the anisotropic layered structure .

A discussion about the behaviour of dispersion law for such finite periodical structure is also presented.

1. INTRODUCTION

Birefringent layered media play an important role in a number of applications. These include narrow-band birefringent filters, multistage electro-optic modulators, and polarizers. The design and the characteristics of these devices are strongly dependent upon the understanding of electromagnetic propagation in birefringent layered media.

The past two decades have witnessed, an intense investigation of electromagnetic wave propagation phenomena at optical frequencies in periodic structures. Usually referred to as photonic band gap (PBG) crystals [1-6], these structures exhibit allowed and forbidden frequency bands and gaps, in analogy with energy bands and gaps of semiconductors. The one-dimensional systems consist of alternate dielectric multilayer stacks of large contrast of the dielectric constant along the propagation direction. These structures present efficient nonlinear optical interactions , as

Photonic Crystal Materials and Devices, Ali Adibi, Axel Scherer, Shawn Yu Lin, Editors, Proceedings of SPIE Vol. 5000 (2003) © 2003 SPIE · 0277-786X/03/$15.00

second harmonic generation (SHG), thanks to the high field localization inside the structure and the possibility to achieve appropriate phase matching conditions [7, 8]. These properties have been experimentally observed in different situations [9, 10, 11, 12, 13]. Revisiting the usual interferential filters methods to derive the properties of field localization and "effective index" has proven very useful to understand the nonlinear properties of the structures.

One dimensional PBG exhibits " geometrical" birefringence, also in presence of isotropic layers [14] , when the input field direction forms an angle with the surface of the structure, and when the polarization of the input light is taken into account.

The aim of this work is to analyse the modification of the light propagation when each layer is anisotropic, assuming a normal incidence of the light. A general theory of electromagnetic propagation in periodic anisotropic layered media has been treated by a number of authors. [15, 16]. Here starting from the well known considerations of refs. 15 and 16, we retrace the formalism that has proven so useful to understand the nonlinear properties of isotropic PBG's, to set the stage for a further study of nonlinear properties of anisotropic PBG's.

2. PROPAGATION OF PLANE WAVES IN ANISOTROPIC MEDIA

We start by briefly reviewing the propagation of monochromatic plane waves in homogeneous anisotropic media [16]. In what follows the Cartesian co-ordinate system is chosen such that the z axis is normal to the interface. Since the medium is not isotropic, the propagation characteristics depend on the direction of the propagation. The orientation of the crystal axes are described by the Euler's angles θ, φ, ψ with respect to a fixed xyz co-ordinate. The dielectric tensor in the xyz co-ordinate system is given by

$$\tilde{\varepsilon} = A \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} A^{-1} \tag{1}$$

Where ε_1, ε_2, ε_3 are the principal dielectric constants and A is the co-ordinate rotation matrix given by

$$A = \begin{pmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & \sin\theta\sin\phi \\ \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & -\sin\theta\cos\phi \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{pmatrix} \tag{2}$$

Since A is orthogonal, the dielectric tensor $\tilde{\varepsilon}$ in xyz co-ordinate must be symmetric. The electric field is assumed to have $e^{i(\alpha x+\beta y+\gamma z-\omega t)}$ dependence in each crystal layer, which is assumed to be homogeneous. With the last assumption α and β remain the same throughout the layered medium, so given α and β, the z component of the propagation vector that is γ is determined directly from the wave equation in momentum space

$$k \times (k \times E) + \omega^2 \mu \tilde{\varepsilon} E = 0 \tag{3}$$

or

$$\begin{pmatrix} \omega^2\mu\varepsilon_{xx} - \beta^2 - \gamma^2 & \omega^2\mu\varepsilon_{xy} + \alpha\beta & \omega^2\mu\varepsilon_{xz} + \alpha\gamma \\ \omega^2\mu\varepsilon_{yx} + \alpha\beta & \omega^2\mu\varepsilon_{yy} - \alpha^2 - \gamma^2 & \omega^2\mu\varepsilon_{yz} + \beta\gamma \\ \omega^2\mu\varepsilon_{zx} + \alpha\gamma & \omega^2\mu\varepsilon_{zy} + \beta\gamma & \omega^2\mu\varepsilon_{zz} - \alpha^2 - \beta^2 \end{pmatrix} \times \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \tag{4}$$

In order to have non trivial plane-wave solutions, the determinant of the matrix in eq. (4) must vanish. If we call p_σ the unit vector associated to the root γ_σ we can write the electric field inside each layer as

$$E = \sum_{\sigma=1}^{4} A_\sigma \vec{p}_\sigma e^{i(\alpha x + \beta y + \gamma_\sigma z - \omega t)} \tag{5}$$

3. MATRIX METOD

In this section we introduce the matrix method for analysing the propagation of monochromatic plane waves in birefringent layered media [15, 16]. The materials are assumed to be nonmagnetic so that μ=*constant* throughout the whole layered medium. The whole structure has a dielectric tensor for each layer and we can write in the general case

$$\tilde{\varepsilon} = \begin{cases} \tilde{\varepsilon}(0) & z < z_0 \\ \tilde{\varepsilon}(1) & z_0 < z < z_1 \\ \tilde{\varepsilon}(2) & z_1 < z < z_2 \\ \vdots & \vdots \\ \tilde{\varepsilon}(N) & z_{N-1} < z < z_N \\ \tilde{\varepsilon}(s) & z_N < z \end{cases} \tag{6}$$

If we remember equation (5) we can assume that the field in each anisotropic layer can be written as a sum of four partial waves. The electromagnetic field in the nth layer of the birefringent layered medium can thus be represented by a column vector $A_\sigma(n)$, $\sigma = 1, 2, 3, 4$. As a result, the electric field distribution in the same layer can be written

$$E = \sum_{\sigma=1}^{4} A_\sigma(n)\vec{p}_\sigma(n) e^{i\left[\alpha x+\beta y+\gamma_\sigma(n)(z-z_n)-\omega t\right]} \tag{7}$$

The magnetic field is related to the electric field as

$$H = \sum_{\sigma=1}^{4} A_\sigma(n)\vec{q}_\sigma(n) e^{i\left[\alpha x+\beta y+\gamma_\sigma(n)(z-z_n)-\omega t\right]} \tag{8}$$

where

$$\vec{q}_\sigma(n) = \frac{c\vec{k}_\sigma(n)}{\omega\mu} \times \vec{p}_\sigma(n) \tag{9}$$

and

$$\vec{k}_\sigma(n) = \alpha\vec{i} + \beta\vec{j} + \gamma_\sigma(n)\vec{k} \tag{10}$$

Imposing the continuity of E_x, E_y, H_x, and H_y, at the interface $z=z_{n-1}$ leads to

$$\sum_{\sigma=1}^{4} A_\sigma(n-1)\vec{p}_\sigma(n-1)\cdot\vec{x} = \sum_{\sigma=1}^{4} A_\sigma(n)\vec{p}_\sigma(n)\cdot\vec{x}\ e^{-i\gamma_\sigma(n)t_n} \tag{11}$$

$$\sum_{\sigma=1}^{4} A_\sigma(n-1)\vec{p}_\sigma(n-1)\cdot\vec{y} = \sum_{\sigma=1}^{4} A_\sigma(n)\vec{p}_\sigma(n)\cdot\vec{y}\ e^{-i\gamma_\sigma(n)t_n} \tag{12}$$

$$\sum_{\sigma=1}^{4} A_\sigma(n-1)\vec{q}_\sigma(n-1)\cdot\vec{x} = \sum_{\sigma=1}^{4} A_\sigma(n)\vec{q}_\sigma(n)\cdot\vec{x}\ e^{-i\gamma_\sigma(n)t_n} \tag{13}$$

$$\sum_{\sigma=1}^{4} A_\sigma(n-1)\vec{q}_\sigma(n-1)\cdot\vec{y} = \sum_{\sigma=1}^{4} A_\sigma(n)\vec{q}_\sigma(n)\cdot\vec{y}\ e^{-i\gamma_\sigma(n)t_n} \tag{14}$$

where $t_n=z_n-z_{n-1}$, $n=1,2,\ldots,N$.

These four equations can be rewritten as a matrix equation

$$\begin{pmatrix} A_1(n-1) \\ A_2(n-1) \\ A_3(n-1) \\ A_4(n-1) \end{pmatrix} = D^{-1}(n-1)D(n)P(n)\begin{pmatrix} A_1(n) \\ A_2(n) \\ A_3(n) \\ A_4(n) \end{pmatrix} \tag{15}$$

where

$$D(n) = \begin{pmatrix} \vec{x}\cdot\vec{p}_1(n) & \vec{x}\cdot\vec{p}_2(n) & \vec{x}\cdot\vec{p}_3(n) & \vec{x}\cdot\vec{p}_4(n) \\ \vec{y}\cdot\vec{q}_1(n) & \vec{y}\cdot\vec{q}_2(n) & \vec{y}\cdot\vec{q}_3(n) & \vec{y}\cdot\vec{q}_4(n) \\ \vec{y}\cdot\vec{p}_1(n) & \vec{y}\cdot\vec{p}_2(n) & \vec{y}\cdot\vec{p}_3(n) & \vec{y}\cdot\vec{p}_4(n) \\ \vec{x}\cdot\vec{q}_1(n) & \vec{x}\cdot\vec{q}_2(n) & \vec{x}\cdot\vec{q}_3(n) & \vec{x}\cdot\vec{q}_4(n) \end{pmatrix} \tag{16}$$

and

$$P(n) = \begin{pmatrix} e^{-i\gamma_1(n)t_n} & 0 & 0 & 0 \\ 0 & e^{-i\gamma_2(n)t_n} & 0 & 0 \\ 0 & 0 & e^{-i\gamma_3(n)t_n} & 0 \\ 0 & 0 & 0 & e^{-i\gamma_4(n)t_n} \end{pmatrix} \tag{17}$$

The *D(n)* matrix depends only on the direction of polarization of the four partial waves. The matrices *P(n)* depend only on the phase of these four partial waves. We define the transfer matrix as

$$T_{n-1,n} = D^{-1}(n-1)D(n)P(n) \tag{18}$$

Equation (15) can thus be written

$$\begin{pmatrix} A_1(n-1) \\ A_2(n-1) \\ A_3(n-1) \\ A_4(n-1) \end{pmatrix} = T_{n-1,n}\begin{pmatrix} A_1(n) \\ A_2(n) \\ A_3(n) \\ A_4(n) \end{pmatrix} \tag{19}$$

The matrix equation which relates $A(0)$ and $A(s)$ is therefore given by

$$\begin{pmatrix} A_1(0) \\ A_2(0) \\ A_3(0) \\ A_4(0) \end{pmatrix} = T_{0,1}T_{1,2}\cdots T_{N-1,N}T_{N,s}\begin{pmatrix} A_1(s) \\ A_2(s) \\ A_3(s) \\ A_4(s) \end{pmatrix} \tag{20}$$

where $s \equiv N+1$ and $t_{N+1} \equiv 0$. We can write eq.(20) in a more compact way where T is the product matrix of all $T_{n-1,n}$.

$$\begin{pmatrix} A_{x_1} \\ R_{x_1} \\ A_{y_1} \\ R_{y_1} \end{pmatrix} = T \cdot \begin{pmatrix} T_{x_2} \\ 0_{x_2} \\ T_{y_2} \\ 0_{y_2} \end{pmatrix} \tag{21}$$

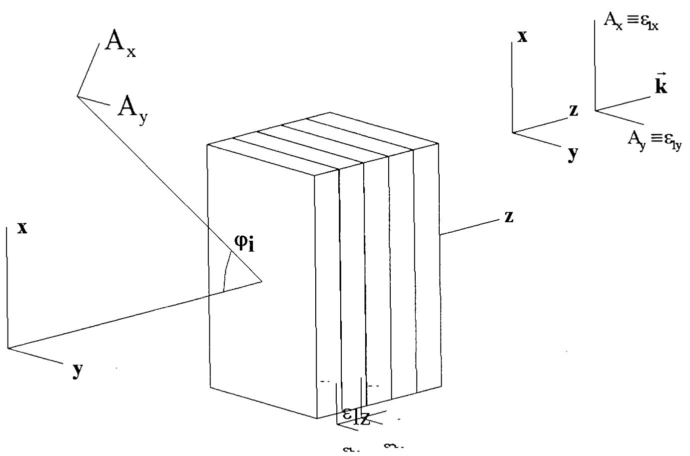

Fig.1 - Multilayer structure with aligned optical

If we refer to fig.1 we can state that the input amplitudes A_{x1}, and A_{y1} are related respectively with the directions of principal axis of the first layer with dielectric constants ε_{1x} and ε_{1y} , the same is for the reflected fields R_{x1}, R_{y1}; being the structure periodical , the last layers has a dielectric constant designed by the subscript 2, then the output amplitude T_{x2} is related with the direction of the axis ε_{2x} while T_{y2} with ε_{2y}.

4. ANISOTROPIC MULTILAYER

We now discuss some results assuming that the elementary cell of the structure is birefringent, while the external materials are isotropic. Consider first the case in which the principal axis of each layer are aligned parallel to each other (see fig.1). The input wave is separated into two polarizations one along the axis with dielectric constant ε_{1x} and the other along the axis of ε_{1y}, we named these axis respectively x_1-axis and y_1-axis, the same consideration for the output where we named x_2-axis and y_2-axis the axis respectively of dielectric constant ε_{2x} and ε_{2y}. We assign the subscript 1 to the first layer and the subscript 2 to the second layer (the output layer is designed with the subscript 2). If the principal axis of the second layer are parallel to the ones of the first layer the polarization is conserved (fig.3). An example is shown in figure 1 when the principal axis x and y of the second layer are rotated by same angle φ with respect to the z axis (fig.4).

We discuss some examples where the cladding and the substrate are air, the parameters of each layer are in the figures. The incident wave is a plane wave normal to the structure.

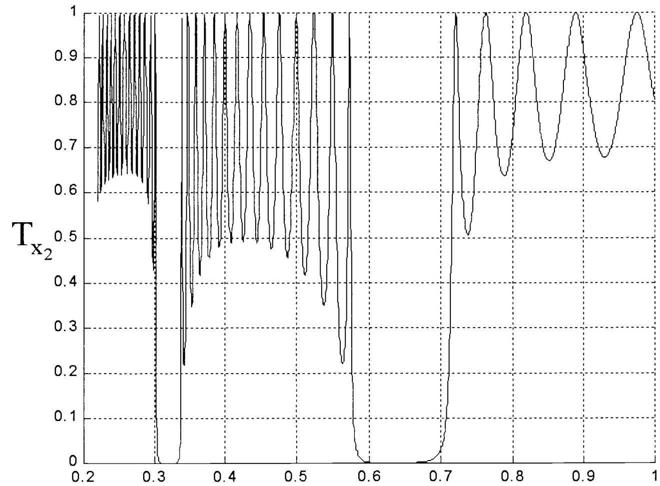

Fig.3 - Transmission spectrum for input polarized field along x1 direction. The rotation angle among ε1x and ε2x is equal to zero. Same alignment for ε1z and ε2z. ε1xx = 6, ε1yy = 2, ε1zz = 4, ε2zz = 3, ε2yy = 7, ε2zz = 4, d1 = 41.7 nm, d2 =125 nm, N = 15. Output polarized field is the same direction as the input.

Now let us introduce the rotation of the axis using the same geometrical parameters as in fig.3. The spectrum now manifests clearly the coupling between the two orthogonal polarizations.

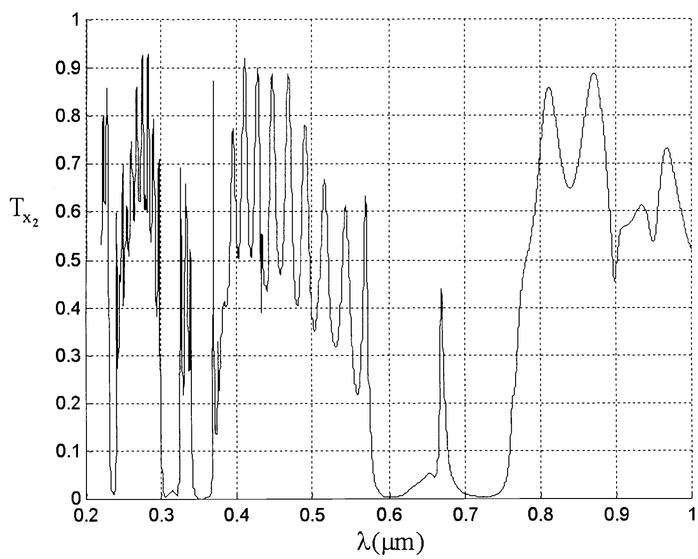

Fig.4 - Transmission spectrum for the x2-axis output with an input filed polarized with the electric field parallel to the x1-axis . Same geometrical parameters as figs.3. The spectrum is for an angle φ=20° of rotation among ε1x and ε2x.

6. CONCLUSIONS

We have discussed spectral and dispersive behaviour of layered structures when a strong anisotropy is introduced in each layer. Interesting spectral behaviour is obtained when rotation angle is introduced among principal axis of the elementary cell.

Spectral modification as a function of the angle is presented. Defects modes appear in the spectrum giving rise to a strong field spatial localization. The localization of the field can be obtained through the definition and calculation of density of modes.

REFERENCE

1. E. Yablonovitch, "Inhibited Spontaneous Emission in Solid-State Physics and Electronics" , *Phys. Rev. Let.* **58** 2059 -2061(1987)

2. S. John, "Strong localization of photons in certain disordered dielectric superlattices ", *Phys. Rev. Let.* **58**, 2486-2488 (1987)

3. E. Yablonovitch and K. M. Leung, "Hope for photonic band gaps " , *Phys. Rev. Let.* **351**, 278 –280 (1991)

4. Yablonovitch E, Optics – " Liquid versus photonic crystals ", NATURE 401, 539- 541 (1999)

5. J. Martorell and N. M. Lawandy, "Observation of inhibited spontaneous emission in a periodic dielectric structure ", *Phys. Rev. Lett.* **65**, 1877-1879 (1990)

6. J. D. Joannopoulos, R. D. Mead and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, 1994)

7. M. Scalora, M. J. Bloemer, A. S. Manka, J. P. Dowling, C. M. Bowden, R. Viswanathan, and J. W. Haus, "Pulsed second-harmonic generation in nonlinear, one-dimensional, periodic structure" *Phys. Rev. A* **56,** 3166-3174 (1997).

8. G. D'Aguanno , M. Centini, M. Scalora, C. Sibilia, M. J. Bloemer, C. M. Bowden, J. W. Haus and M. Bertolotti, "Group velocity, energy velocity, and superluminal propagation in finite photonic band-gap structures " , *Phys. Rev. E* **63**, 036610 – 036615 (2001).

9. Y. Dumeige, P. Vidakovic, S. Sauvage, I. Sagnes, J. A. Levenson, C. Sibilia, M. Centini, G. D'Aguanno, and M. Scalora, Enhancement of second-harmonic generation in a one-dimensional semiconductor photonic band gap, *Appl. Phys. Lett* **78**, 3021 –3023 (2001)

10. J. Martorell, R. Vilaseca and R. Corbalan, Second harmonic generation in a photonic crystal, *Appl. Phys. Lett.* **70**, 702 –704 (1997);

11. J. Martorell, R. Vilaseca and R. Corbalan, Scattering of second-harmonic light from small spherical particles ordered in a crystalline lattice, *Phys. Rev. A* **55**, 4520 –4525 (1997)

12. C. Simonneau, J. P. Debray, J. C. Harmand, P. Vidakovic, D. J. Lovering and J. A. Levenson, Second-harmonic generation in a doubly resonant semiconductor microcavity, *Opt. Lett.* **22**, 1775 –1777 (1997).

13. V. Balakin, D. Boucher, V. A. Bushev, N. I. Koroteev, B. I. Mantsyzov, P. Masselin, I. A. Ozheredov and A. P. Shurinov, Enhancement of second-harmonic generation with femtosecond laser pulses near the photonic band edge for different polarizations of incident light, *Opt. Lett.* 24, 793 -795(1999).

14. E.Rugolo , G.Daguanno, C.Sibilia, M.Centini, M.Scalora , M.Bertolotti – " Phase independent nonlinear amplification regime in one dimensional photonic band gap "- Pure and *Apl .Optics 3* , N.6 , S 196-S200 (Nov. 2001).

15. P. Yeh, A. Yariv and C. S. Hong, Electromagnetic propagation in periodic stratified media – I General Theory, J. Opt. Soc. Am. **67**, 423 (1977); P. Yeh, A. Yariv and C. S. Hong, Electromagnetic propagation in periodic stratified media – II Birefringence, Phase matching and X-ray lasers, *J. Opt. Soc. Am.* **67**, 438- 441 (1977).

16. P. Yeh, *Optical Waves in Layered Media* (Wiley, New York, 19880)

17. M. Centini, C. Sibilia, M. Scalora, G. D'Aguanno, M. Bloemer, C. Bowden, M. Bertolotti, Dispersive properties of finite, one-dimensional photonic band gap structures: Applications to nonlinear quadratic interactions *Phys. Rev.* **E 60** 4891-4895 (1999)

18. J.M. Bendickson, J.P. Dowling, M.Sclora " Analytic expression for the lectromagnetic mode density in finite one-dimensional photonic band gap structures " *Phys. Rev. E* **53** , 4107-4112 (1996)