Signal velocity and group velocity for an optical pulse propagating through a GaAs cavity

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We present measurements of the signal and group velocities for chirped optical pulses propagating through a GaAs cavity. The signal velocity is based on a specified signal-to-noise ratio at the detector. Under our experimental conditions, the chirp substantially modifies the group velocity of the pulse, but leaves the signal velocity unaltered. At unity transmittance, the velocities are equal. In general, when the transmittance is less than unity, the group velocity is faster than the signal velocity. While the group velocity can be negative, the signal velocity is always less than c/n, where c is the speed of light in vacuum and n is the refractive index of GaAs. To our knowledge, this is the first measurement of both the group velocity and the signal velocity in any system.

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How much time is required for a Gaussian-shaped optical pulse to traverse a cavity? This is a reasonable question that does not have a short answer. The problem can be simplified somewhat if we impose the condition that the shape of the pulse is not distorted upon transit except for the possibility of a reduction in amplitude (we neglect gain). Even with this condition we are left with transit times related to the front velocity V_F, the group velocity V_G, and the signal velocity V_S. Of these three quantities to measure, the logical choice would be V_G, which is the velocity of the peak of the pulse. It is easy to define the peak position and, in addition, the peak has the largest number of photons available to measure. Some of the early theoretical work on pulse propagation dealt with pulses having a leading edge shaped like a step function [1]. For pulses with a sharp leading edge, V_F has meaning. For a Gaussian pulse, V_F is not a useful concept. This leaves the all important, but little discussed, V_S. A rough definition of V_S is the velocity at which the minimum detectable signal propagates. Based on this definition, if one were asked to measure V_S, a first reasonable reaction might be to say that V_S is not well defined and it is difficult to detect single photons. Yet, the speed at which the signal travels is by all accounts an important quantity. Therefore, in our view, an attempt should be made to measure V_S, provided an acceptable definition is found. A starting point would be to define an "operational V_S" [2]. In every apparatus for transmitting a signal there will be a variety of problems in detecting the signal at the receiver. Noise sources and drift are just a few potential problems to minimize. In the end, the apparatus will have a minimum threshold level at the detector in which a signal can be distinguished from the noise at a specified error probability. For the experimental setup described below, the amplitude of the optical pulse propagating through free space provided a 71 mV detector level and we chose a threshold level of 2 mV. The rms noise level was 0.2 mV for a signal-to-noise ratio of 10. Already we have imposed a somewhat arbitrary condition on V_S by defining a 2 mV threshold level at the detector. However, in a real communication system, a threshold level is usually imposed on the system. An example would be a digital wireless communication system in which the signal attenuation and receiver noise determine the optimum threshold for discriminating the bits. Our approach is consistent with Kuzmich *et al.* [2] who discuss the arrival time of a signal in terms of a threshold level. The particular threshold level is determined by the noise and allowed error rate of the system.

In spite of the limitations of an operational V_S, several observations can be made regarding V_G and V_S for pulses traversing a cavity. In the experiment described below, we find that for the case of unity transmittance: (a) $V_G = V_S$, (b) the pulse propagating through the sample is delayed in time with respect to a pulse propagating in free space, (c) V_S reaches its maximum value at unit transmittance, and (d) the measured V_S is independent of the threshold level. For values of transmittance less than unity: (a) V_S decreases as the transmittance decreases and reaches a minimum at the minimum value of transmittance, (b) for transform limited pulses, $V_S < V_G$, (c) V_S does not exceed c/n, where c is the speed of light in vacuum and n is the refractive index of GaAs, and (d) as the threshold level is increased, the minimum V_S becomes smaller. While the above observations specifically refer to the operational V_S, we believe the observations are indicators of the behavior of the true V_S. This expectation is based on the behavior of the measured V_S close to the rms noise levels.

The experimental apparatus and sample are described in detail in a previous work [3]. Briefly, the GaAs cavity consists of a GaAs substrate, 450 μm thick with the faces uncoated. The index difference between GaAs ($n = 3.3737$ @ 1550 nm) and air resulted in a 30% reflectivity at each interface. The optical pulses were obtained from a tunable laser diode (New Focus, Model 6328) modulated by a pulse generator (Picosecond Pulse Labs, Inc., Model 3500D). The pulser was upgraded from the previous work to provide pulses with a leading edge that is nearly Gaussian in

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shape. The generated optical pulse is slightly under 1 ns full width at half maximum (FWHM) and propagates through free space to a high-speed photodetector. Note that the optical pulse is more than two orders of magnitude longer than the sample and interference effects due to reflections at the GaAs-air boundaries are important. The detector output is fed into a HP 54750A digitizing oscilloscope having a 20 GHz module, which provided a combined trigger and time base jitter of <2.5 ps. The roundtrip time for the pulse without the sample in the optical path served as the baseline. The sample was then inserted into the optical path at normal incidence and the new wave form was recorded and stored for analysis. The digitizing oscilloscope was set to average 64 wave forms and each wave form consisted of 1024 data points. The averaging was performed in order to be able to lower the threshold level and analyze V_S for points as far out on the leading edge of the pulse as possible.

V_G for a given wavelength was determined by finding the peak position of the pulse that propagated through free space and the peak position of the pulse transmitted through the GaAs cavity. The time difference between the peak positions is the group delay time. The actual transit time is then the measured delay time plus D/c, where D is the thickness of the sample. From the transit time of the peak of the pulse and the sample thickness, $V_G = D/t$, where t is the transit time. For most materials, even those having anomalous dispersion, the delay time is positive because the refractive index of the material is greater than air and most values of anomalous dispersion are not large enough to offset the refractive index.

A similar procedure is used to determine the transit time of the signal. For the transmitted pulse and the free space pulse, the time at which the leading edge of the pulse rose to the 2 mV detector level was recorded. Their difference is the delay time of the signal. The actual transit time of the signal is the measured delay time plus D/c. We emphasize that the transmitted pulse is not rescaled in amplitude when determining the signal transit times. The amplitude of the free space pulse was 71 mV, the amplitude of the transmitted pulse ranged from 22 mV up to 71 mV, and the threshold level was set to 2 mV independent of the pulse height.

For our experimental conditions, the transit times of the peak of the pulse, τ_G, can be positive, zero, or negative depending on the particular wavelength. For this reason we plot the experimental transit times instead of V_G and V_S. A negative value of τ_G indicates that the peak of the transmitted pulse has exited the sample before the incident pulse peak arrives at the sample. In this situation the transmittance is less than unity and the leading edge of the transmitted pulse lags behind the leading edge of a pulse propagating the same distance through free space. As we will see from the experimental data, this implies that the signal propagates at velocities less than c even in the case of negative τ_G.

The measured transit times τ_G and τ_S are plotted in Fig. 1 along with the transmittance of the GaAs cavity. Also plotted are the associated theoretical spectra, which will be described below. Figure 1 illustrates some of the obvious features regarding the relationship between τ_G and τ_S. For example, τ_G and τ_S are equal when the transmittance of the cavity is unity and for all other values of transmittance, $\tau_G < \tau_S$. Note the exception to this in Fig. 1 for chirped pulses at wavelengths just shorter than the cavity resonance where τ_G slightly exceeds τ_S. Also, τ_S is longest when the transmittance reaches a minimum.

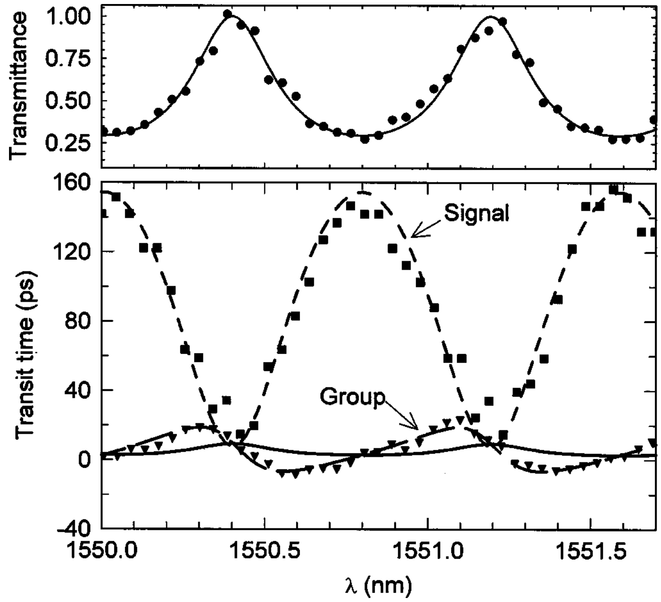

FIG. 1. Experimental and theoretical spectra for τ_G and τ_S, and transmittance of the GaAs cavity. Two theoretical plots are shown for τ_G, one is for a transform-limited pulse (solid line) and the other (which agrees with the experimental data) is for a chirped pulse (long dash). The calculated values of τ_S are the same for a chirped and transform-limited pulse.

The values of τ_G were observed to be negative, zero, or positive depending on the particular wavelength, Fig. 1. Also, the spectral dependence of τ_G is asymmetric. The asymmetry is a result of the chirp inherent in an optical pulse generated by modulating the injection current of a laser diode. The details regarding the effects of chirp on V_G are described in Refs. [3], [4]. If we consider Gaussian pulses of the type $A_0(t) = A_0 \exp[-(t^2/2\tau_0^2) - i\gamma t^2]$, where $\tau_0 = \tau_{\mathrm{FWHM}}/(2\sqrt{\ln/2})$, the chirp ($\gamma$) affects τ_G through the interaction with the transmittance spectrum by [4]

$$\tau_G = \tau_{G:\mathrm{transform\ limited}}(\omega_0) + \delta\gamma\tau_0^2, \tag{1}$$

where $\tau_{G:\mathrm{transform\ limited}}$ is the group transit time for a Fourier limited pulse and $\delta = (dT(\omega)/d\omega)|_{\omega=\omega_0}/T(\omega_0)$ is a parameter that depends on the transmittance T. As seen in Eq. (1), if the transmittance is not uniform across the spectral content of the pulse, then there is an additional component to the transit time of the peak of the pulse. To a first order approximation, this extra component does not distort the pulse shape but leads only to a wider range of group transit times compared with a transform limited pulse. As we will see below, the chirp on the optical pulse only affects V_G and not V_S.

There are several methods to calculate τ_G and τ_S. Perhaps the easiest method, and the one used to generate the theoretical data in Fig. 1, is to take a Fourier decomposition of the incident pulse, use the matrix transfer method to calculate the transmittance and phase of the various compo-

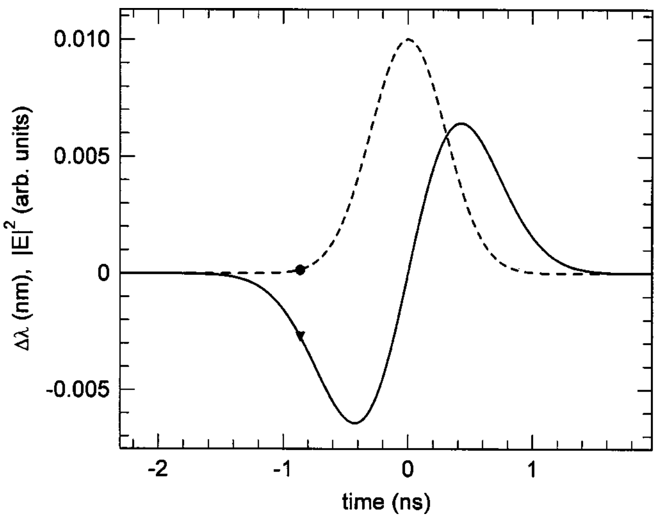

FIG. 2. Theoretical chirp profile and envelope of the optical pulse calculated from the laser rate equations. The markers indicate values at the 2 mV threshold level.

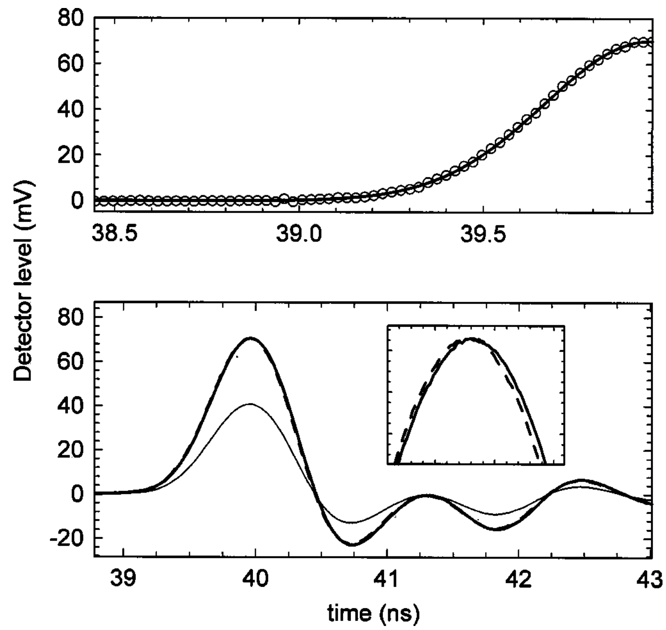

FIG. 3. Lower figure: Experimental wave forms for the free space pulse and a pulse transmitted through the cavity near 55% transmittance (where V_G is negative). The dashed line is the transmitted pulse rescaled to the amplitude of the free space pulse. The pulse was not rescaled in time, but only in amplitude. The inset shows that τ_G of the pulse transmitted through the sample is 5 ps ahead of the free space pulse. The fact that the inset shows the transmitted pulse to be ahead of the free space pulse is an artifact of the amplitude rescaling. The full pulse profiles clearly show that the leading edge of the free space pulse is ahead of the pulse transmitted through the sample. The upper figure shows the leading edge of the free space pulse fitted to a Gaussian.

nents, and then reconstitute the transmitted pulse [3]. We tested this method against beam propagation method techniques [5] and found it to be highly efficient and accurate. The chirp profile for the incident pulse, Fig. 2, was calculated from the laser rate equations [6].

The calculations reveal that τ_G is dependent on the chirp, as seen in Eq. (1). However, τ_S is not dependent on the chirp, at least up to the levels illustrated in Fig. 2. Larger values may have some effect and we cannot say that a frequency chirp will never modify V_S. To help clarify the effects of the chirp on the group velocity, Fig. 1 shows the theoretical τ_G for a transform-limited pulse and a pulse having the chirp profile shown in Fig. 2. The experimental τ_G agrees quite well with the theory for the chirped pulse. The τ_G for the chirped and transform-limited pulses are equal when the transmittance is flat, namely, at the transmittance maxima and minima. For other wavelength regions, the nonzero slope of the transmittance spectrum causes τ_G of the chirped pulse to deviate from that of the transform-limited pulse.

Comparing τ_G of the transform-limited pulse with τ_S we see the symmetries emerge. At unity transmittance, τ_S is a minimum while τ_G is a maximum and vice-versa at the transmittance minima. Also, τ_G and τ_S are equal only at 100% transmittance, for all other values of transmittance, τ_S is longer than τ_G. Looking closely at Fig. 1, there is actually a very small wavelength region near unity transmittance where $V_S > V_G$ for the chirped pulse, but not for the transform limited pulse.

Some experimental pulse profiles are shown in Fig. 3. The three wave forms plotted show a pulse that propagated through free space, a pulse that propagated through the sample near 55% transmittance where the measured V_G is negative, and the same pulse rescaled in amplitude to determine if the pulse was distorted upon transit. Several features are evident in Fig. 3. The leading edge of the pulse that propagated through free space is ahead of the leading edge of a pulse that propagated through the sample, even though the measured V_G is negative. As the peak of the transmitted pulse moves forward in time to velocities exceeding c, the amplitude decreases such that the transmitted wave form is contained within the free space pulse. Also the transmitted pulse shows minimal distortion compared with the free space pulse.

Due to the curvature of the leading edge of the pulse, τ_S will have some dependence on the value of the threshold level. This is evident from the pulses shown in Fig. 3. As the threshold level increases, say from 10 mV to 20 mV, the time interval between the leading edge of the transmitted and free space pulse increases. However, the minimum value of τ_S, which occurs at 100% transmittance, remains independent of the threshold value. The dependence of V_S on the threshold value is a cause for concern because it is imperative that different investigators be able to repeat and confirm similar measurements of V_S. In order to meet the criteria for independent evaluation, at least three features of the pulse must be reported: the shape of the leading edge of the pulse, the amplitude of the free space pulse, and the value of the threshold.

Figure 4 illustrates the dependence of τ_S on the threshold value and on the exact shape of the leading edge of the pulse. The upper figure shows a slight deviation in the experimental wave form from a true Gaussian wave form around 39.0 ns. This small deviation is responsible for rise in τ_S for threshold levels below 2 mV. Above the 2 mV level, and for a true

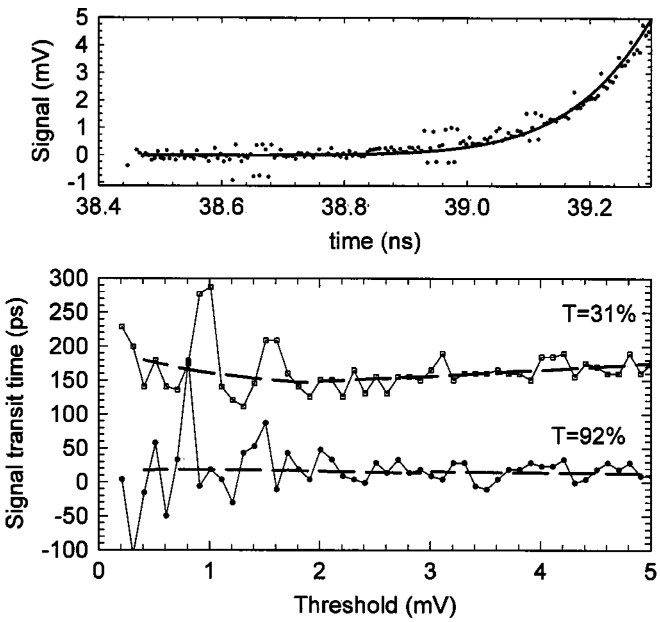

FIG. 4. Upper figure: The dots are the data from the digitizing oscilloscope showing the leading edge of the free space optical pulse. The solid line is the Gaussian fitted to the entire leading edge of the pulse up to the peak (also shown in the upper figure of Fig. 4). Note the slight discrepancy between the data and the theoretical fit in the region around 39.0 ns. Lower figure: τ_S as a function of threshold value for a transmittance of 31% and 92%. The dashed line is τ_S obtained by performing a local Gaussian fit to a small portion of the data about a particular threshold level. The data points are taken directly from the digitized wave form without any fit.

Gaussian-shaped pulse, the trend is for τ_S to increase, as the threshold level is increased. By comparing the results for the data fitted to a half Gaussian with a local Gaussian fit to a small portion of the data about a particular threshold level, it is possible to quantify any deviations from the expected pulse shape. This is part of the reason we selected a 2 mV threshold level. It was the lowest threshold we could use before the wave form deviated from a true Gaussian. There remains a question as to how to determine the level of the minimum detectable signal in an experiment. In Fig. 4, the plots of the signal transit time versus threshold level show a marked increase in the scatter of the data points just below the 2 mV level. We propose this as a reasonable indicator of the minimum detectable signal level in an experiment, the transition region from low scatter in the transit time data to a region of high scatter in the transit time data.

The plots on the lower portion of Fig. 4 for the two different values of transmittance show that the variation in the transit times as a function of threshold becomes less pronounced for higher values of transmittance. For 100% transmittance, the pulse is just translated in time and there is no dependence of τ_S on the threshold.

In a typical measurement of the threshold velocity, one would normally use a threshold detector that determines when the signal rises to a specified threshold value. If the results are averaged over many events and plotted as a function of threshold value, we expect to get data equivalent to that reported here. This will provide the shape of the wave form, which is needed to quantify the results. The averaging provides a method to improve the signal-to-noise ratio and is used in ultrawide band wireless data links.

In this paper we have reported, to the best of our knowledge, the first measurements of V_G and operational V_S for any system. In spite of the limitation of an operational signal velocity, we believe that much insight has been gained into the relationship of V_G and operational V_S for pulses propagating through a cavity. In particular, we find that V_G and operational V_S are equal only when the transmittance is unity. For all other values of transmittance, $V_S < V_G$, at least for transform-limited pulses. At some wavelengths, V_G becomes negative but the measured operational V_S are always less than c/n.

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