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# LARES Satellite Thermal Forces and a Test of General Relativity

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**Abstract**—We summarize a laser-ranged satellite test of frame dragging, a prediction of General Relativity, and then concentrate on the estimate of thermal thrust, an important perturbation affecting the accuracy of the test. The frame dragging study analysed 3.5 years of data from the LARES satellite and a longer period of time for the two LAGEOS satellites. Using the gravity field GGM05S obtained via the Grace mission, which measures the Earth's gravitational field, the prediction of General Relativity is confirmed with a 1- $\sigma$  formal error of 0.002, and a systematic error of 0.05. The result for the value of the frame dragging around the Earth is  $\mu = 0.994$ , compared to  $\mu = 1$  predicted by General Relativity. The thermal force model assumes heat flow from the sun (visual) and from Earth (IR) to the satellite core and to the fused silica reflectors on the satellite, and reradiation into space. For a roughly current epoch (days 1460 – 1580 after launch) we calculate an average along track drag of  $-0.50\text{pm/sec}^2$ .

## I. DRAGGING OF INERTIAL FRAMES

In 1918, Lense and Thirring [1] described the frame-dragging of an orbiting body, in the limiting case valid around Earth of slow rotation of the central body and weak gravitational field:  $\dot{\Omega} = 2\mathbf{J}/(a^3(1 - e^2)^{3/2})$ . Here  $\Omega$ ,  $a$  and  $e$  are the longitude of the *ascending node*, the semimajor axis and the eccentricity of the orbiting body, while  $\mathbf{J}$  is the angular momentum of the central body. We recall that the orbital plane intersects the equator in two nodes, one of which, the ascending node, is crossed by the orbiting body from southern to northern hemisphere [2]. The Earth satisfies the conditions of “weak gravitational field” and “slowly rotating”, so this formula describes the behavior of Earth satellites. This *Eastward* precession has been observed and measured using the two LAGEOS satellites [3]–[7] and a model of the gravitational field of the Earth provided by the GRACE mission [8], [9], with improving errors to about 10%. (Satellite Laser Ranging allows range measurement with an accuracy that can reach a few millimeters [10].) The LAGEOS tests

of frame-dragging were used to set limits on some string theories equivalent to Chern-Simons gravity [11]. Another satellite experiment, *Gravity Probe B* (GPB), was put into orbit in 2004 and verified frame dragging (via a related gyroscope precession effect) with approximately 20% accuracy [12]. Very recently, preliminary analysis of the LARES laser ranged satellite along with the two LAGEOS satellites and latest GRACE results, produced a frame dragging consistent with that predicted by GR, with estimated 5% errors [13]. We summarize the results of that analysis below and then go on to model thermally induced forces on LARES. LARES is a satellite with a spherical tungsten alloy core which carries 92 *cube corner reflectors* (CCRs) for laser ranging.

## II. LARES FRAME-DRAGGING MISSION

The node shift of an orbiting body is dominated by the classical Newtonian effect due to the axially symmetric components of deviations of the gravity field of the central body from sphericity. These deviations are specifically due to the *even zonal harmonics*. Their effect decreases with increasing order, so the main perturbing effect is due to the even zonal of degree two,  $J_2$ , (Earth's quadrupole moment) [2]. The Newtonian effects precess the orbital plane of LARES westward at a rate of  $\approx 1.7$  degrees/day [14]!

The LARES experiment combines data from the three satellites (Table I) LARES (Italian Space Agency - ASI), LAGEOS (NASA) and LAGEOS 2 (NASA and ASI) to obtain three observable quantities provided by the nodal rates of the three satellites [15]. The three observables can then be used to determine three unknowns: frame-dragging and the two uncertainties in the two lowest degree even zonal harmonics,  $J_2$ , and  $J_4$ . We analyzed data provided by the International Laser Ranging Service (ILRS) for the

TABLE I  
MAIN CHARACTERISTICS AND ORBITAL PARAMETERS OF THE SATELLITES OF THE LARES EXPERIMENT.

	LARES	LAGEOS	LAGEOS 2	GRACE
Semimajor axis [km]	7821	12270	12163	6856
Eccentricity	0.0008	0.0045	0.0135	0.005
Inclination	69.5°	109.84°	52.64°	89°
Launch date	13 Feb, 2012	4 May, 1976	22 Oct, 1992	17 Mar, 2002
Mass [kg]	386.8	406.965	405.38	432
Number of CCRs	92	426	426	4
Diametre [cm]	36.4	60	60	

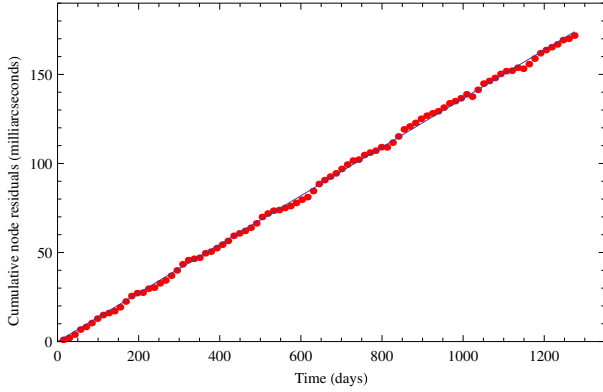


Fig. 1. Fit of the combined orbital residuals of LARES, LAGEOS and LAGEOS 2 with six periodical terms corresponding to six main tidal perturbations observed in the orbital residuals removed. The slope is  $\mu = 0.994$  where  $\mu = 1$  is the GR result [13].

three satellites from 26 February 2012 to 6 September 2015. The laser-ranging normal points were analysed using NASA's orbital determination program GEODYN II [16], including the Earth gravity model GGM05S, Earth tides, solar radiation pressure, Earth albedo, thermal thrust, lunar, solar and planetary perturbations and Earth rotation from Global Navigation Satellite Systems and Very Long Baseline Interferometry. GGM05S is a state-of-the-art Earth gravity model released in 2013, based on approximately 10 years of GRACE data. It describes the Earth's spherical harmonics up to degree 180 [17].

The orbital residuals of a satellite are obtained by subtracting the computed orbital elements of the satellite from the observed ones. The residuals of the satellite's node are due to the errors in the Earth's even zonal harmonics and to the Lense-Thirring effect which is not included in GEODYN II's modelling. General Relativity predicts frame-dragging of about 30.7, 31.5, and 118.4 milliarcsec/year on LAGEOS, LAGEOS 2 and LARES, respectively. At the LARES orbit this corresponds to a nodal displacement of about 4.5 meters/year.

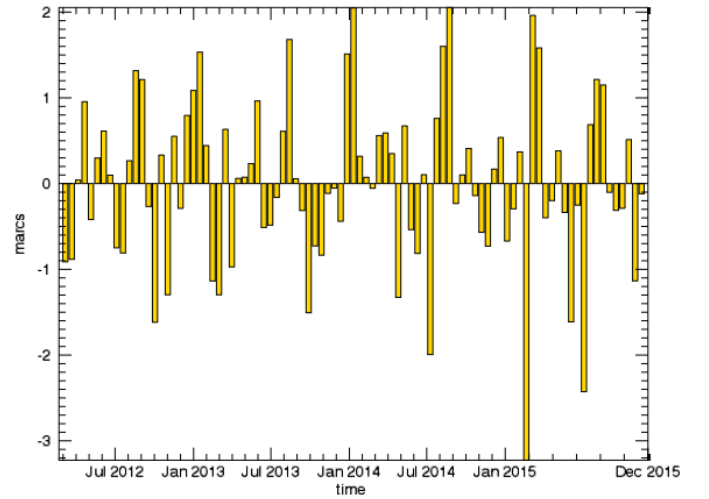


Fig. 2. Residuals after the removal of the six tidal signals and the constant trend [13].

To have a significantly accurate fit of the observed residuals it is necessary to include in the modeling the main periodic tidal signals that were singled out not only analytically but also by a Fourier analysis. The resulting fit, including the secular trend and six tides, produced (Fig. 1):

$$\mu = (0.994 \pm 0.002) \pm 0.05 \quad (1)$$

Here  $\mu = 1$  is the value of frame-dragging normalized to its GR value, 0.002 is the formal 1-sigma error (the postfit residuals in Figure 2 show a normal –Gaussian– distribution to good approximation) and 0.05 is the estimated systematic error caused by uncertainties in GGM05S in the even zonal harmonics with degree greater than four, and other error sources. For a conservative estimate of the systematic errors we took three times the published values of the calibrated errors of the coefficients of the harmonic expansion of the Earth gravitational field. This is a common approach in space geodesy for Earth gravity field error estimation (sometimes

taking a factor of two instead of three). Those tripled errors were then propagated to the nodes of the three satellites of the LARES experiment providing a systematic error of about 0.04 in frame-dragging caused by the uncertainties in the knowledge of the Earth's gravity field even zonal harmonics. Previous error analyses [5], [18]–[27] have shown that the systematic errors associated with all the other perturbations, i.e. tides, non-gravitational perturbations and other unidentified effects, amount to about 0.03. The combination of all those errors, using the total Root Sum Squared (RSS), provides a value of 5%<sup>1</sup> Note that the analysis described above and reported in [13] solves for thermally generated forces from the data, rather than predicting them from a model. The rest of this paper describes a model for those forces [28].

### III. THERMAL THRUST AND TEMPERATURE DISTRIBUTION ON LARES' SURFACE

Thermal thrust is the most subtle of the non-gravitational perturbations acting on LARES. It is small and difficult to model because is influenced by many parameters that affect the temperature distribution on the body. First the two sources: solar radiation and Earth infrared (IR) radiation, that are strongly dependent on the orbit and sun position, which affect the occurrence and duration of eclipses. As a consequence the heat exchange situation will change constantly thus making it necessary to evaluate the temperature distribution and the relevant thermal thrust day by day. Then we have the influence of the material body of the satellite and the way the parts are mounted. Because the metal sphere is a very good heat conductor, its temperature is quite uniform, so it radiates heat isotropically and produces little thermal thrust.

A different situation occurs for the CCRs, which are mounted loosely between plastic mounting rings. This mounting, designed to avoid thermal stresses on the CCRs, makes each CCR practically thermally isolated from the satellite body and from the other CCRs. In addition each CCR is made of Suprasil 311, a special grade glass that has a substantial heat capacity (though slightly lower than conventional optical glass) and significant absorption of infrared radiation. Thus during exposure to heat source it will heat with a delay and when in the shadow it will cool with a delay. Each of the CCRs can have a different temperature, so unequal radiation from CCRs can lead to an effective nonisotropic thermal radiation and an acceleration of the satellite.

Analysis of data of the first 4 months in orbit showed an anomalous along-track acceleration of LARES that on average was about  $-0.4pm/s^2$  ( $pm := picometer$ )<sup>2</sup> An earlier algorithm was able to closely match this observation by using parameter values for the glass IR absorbance

<sup>1</sup>A reduction of the systematic errors can be obtained by an increase of the number of years of observations (preferably more than 7 years from launch date). Furthermore robustness of the results can be increased by using different and updated Earth gravity field models and independent orbital determination programs for the orbital analysis.

<sup>2</sup>This remarkably small residual acceleration is smaller than that of any other satellite, including the LAGEOS satellite, whose mean along-track acceleration is about  $-2pm/s^2$ .

$\alpha_{gl,IR} \equiv$  emissivity  $\epsilon_{gl,IR}$  of the CCRs equal to 0.60 and visual absorbance  $\alpha_{gl,vis}$  equal to 0.15 [29]. These are consistent with surface-contaminated glass on orbit, and with experiments carried out on orbit to determine  $\alpha_{gl,vis}$  [30], [31].

### IV. SATELLITE STRUCTURE

The CCRs are distributed uniformly on LARES surface along each parallel. They are disposed in each parallel as shown in the following table where  $I$ ,  $n_I$  and  $\theta_I$  are the parallel number, the number of CCRs in the parallel and the colatitude of the parallel.

Parallel	$n_I$	$\theta_I(deg)$	Parallel	$n_I$	$\theta_I(deg)$
I	1	0	-V	16	100
II	5	20	-IV	14	120
III	10	40	-III	10	140
IV	14	60	-II	5	160
V	16	80	-I	1	180

A CCR is basically a corner of a cube. Its shape is completed with a cylindrical part that provides a circular front face of diameter of 1.5 inches. The CCR is loosely maintained in place by two plastic mounting rings (made of KEL-F) that engage three protruding tabs of the custom made CCRs. The mounting system is completed with a retainer ring and three screws that lock the plastic rings in place. Screws and retainer rings are made from the same alloy as the entire satellite. This design allows a stress free mounting for the CCRs but introduces uncertainties in the temperature estimation of the CCRs. We regard the tungsten alloy retainer rings as being at the same temperature as the satellite body [29].

### V. ORBIT

The European Space Agency VEGA launcher, developed by ELV and Avio, lifted off with the LARES satellite on-board on February 13, 2012 at 10:00 UTC (07:00 local time) from Kourou spaceport in French Guyana [32]. In the following celestial coordinates are used.<sup>3</sup> The longitude of the ascending node on day  $k$  is  $\Omega(k) \approx (220 - 1.7k)\pi/180$ , and this nodal drift is caused mainly by the Newtonian gravitational effect of  $J_2$ . Time  $t$  spans between 0 and the orbital period  $T$  where at time  $t = 0$  the satellite is at the ascending node.

The experimental values of orientation and frequency of the spin of LARES are found the Ref. [33]. The orientation of the spin is at  $RA = 12^h22^m48^s$  ( $RMS = 49^m$ ) and  $Dec = -70.4$  degrees ( $RMS = 5.2$  degrees). The spin axis is taken coincident with the south-north axis of the satellite. The eddy currents induced in the satellite body by the Earth's magnetic field cause a spin down torque

<sup>3</sup>The right-handed system aligned with the Earth's pole ( $z$ ) and the vernal equinox ( $x$ ).

that reduces the spin frequency of the satellite, as [33]:  $\omega(k) = (0.546 \text{ rad/s}) \exp -0.00322509k$ . This expression predicts the spin rate to be now ( $\approx$  day 1500 in orbit) of order  $\approx 5/\text{orbit}$ , so CCR temperature equilibration rates are comparable to the satellite spin rate, and large individual-CCR effects may arise. We use direct numerical integration (built in functions in *Mathematica* [34]) of the coupled nonlinear heat-transfer equations for the 93 separate pieces of the satellite (the tungsten alloy core and 92 CCRs). Each piece is considered separately isothermal. Our analysis is close to Slabinski's [35] LAGEOS study. He modeled the thermal behaviour of LAGEOS and integrated the nonlinear heat balance equations with the aim of modeling the along track component of the thermal thrust of the satellite. In the present work all the components of the thermal forces are estimated as a function of time along the orbit. We also use a comparison Fourier code [29], which linearizes the equations and assumes the spin frequency is an integer multiple (here, exactly 5) of the orbital frequency. The Fourier code provides initial data for the nonlinear code, which uses the exact satellite spin rate. The nonlinear set is then run for several orbits (here 8) with fixed orbital parameters to relax out any effect from the Fourier provided initial conditions. The thermal force on the  $i$ th CCR is perpendicular to the face (inward toward the center of the satellite) and with magnitude  $F_i = 2\pi R^2 \epsilon_{gl,IR} \sigma T_i^4 / (3c)$ . The vector sum of the forces of all the CCRs produces the net force on LARES satellite. The along track force is a unit vector along the orbit dotted into this vector sum. For details concerning the satellite design, the determination of the view factors between the CCRs and the tungsten alloy cavities housing the CCRs, the orbit geometry and relevant eclipses (Fig. 3) as well as the spin orientation, refer to Ref. [29].

In the current slow spin regime, Fourier methods produce valuable average values for CCR temperatures (which in that method are assumed to be equal in parallels), and for the daily along-track average drag. However there are significant deviations from the Fourier results during the orbit. The calculated average along track drag swings between  $-0.70 \text{ pm/s}^2$  and  $-0.25 \text{ pm/s}^2$  and averages  $-0.50 \text{ pm/s}^2$  over days 1460 - 1580; Fig. 4. But the calculated instantaneous accelerations have excursions that are about an order of magnitude larger than the resulting orbit-averaged drag (Figs. 5 - 8). Figures 9 and 10 show the time dependence of the core temperature and of the individual CCRs in parallel  $V$ .

#### ACKNOWLEDGMENT

We thank Victor Slabinski for very useful comments. This material is supported by the Texas Cosmology Center (TCC). TCC is supported by the College of Natural Sciences, the Department of Astronomy at the University of Texas at Austin, and the McDonald Observatory.

We gratefully acknowledge Italian Space Agency grants I/034/12/0, I/034/12/1 and 2015-021-R.0 and the International Laser Ranging Service for providing high-quality laser ranging

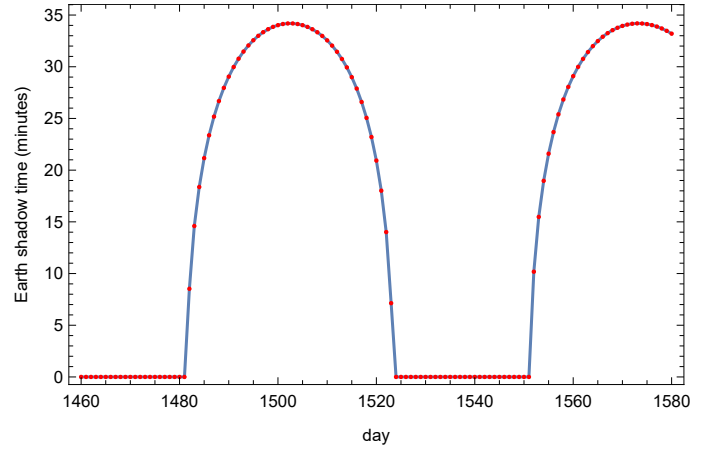


Fig. 3. The eclipse duration as a function of number of days since launch for days 1460 - 1580.

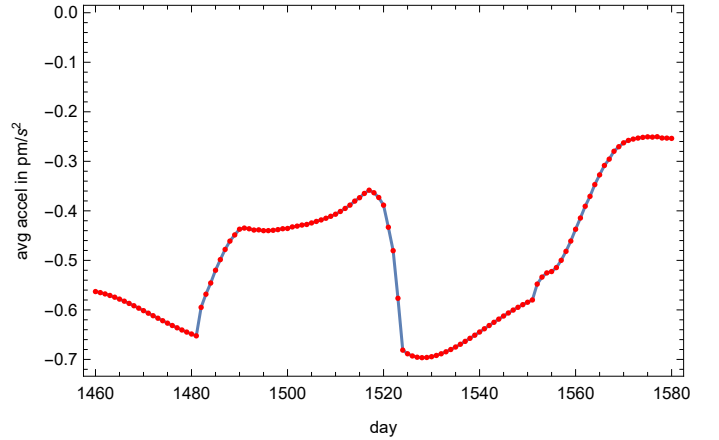


Fig. 4. Time series of the calculated daily average thermal drag; negative values indicate drag. The average calculated thermal drag (average of daily averages) over these 120 days is  $-0.50 \text{ pm/s}^2$ .

tracking of the LARES satellites. E.C. Pavlis acknowledges the support of NASA Grants NNX09AU86G and NNX14AN50G. R. Matzner acknowledges NASA Grant NNX09AU86G and J.C. Ries NASA Contract NNG12VI01C.

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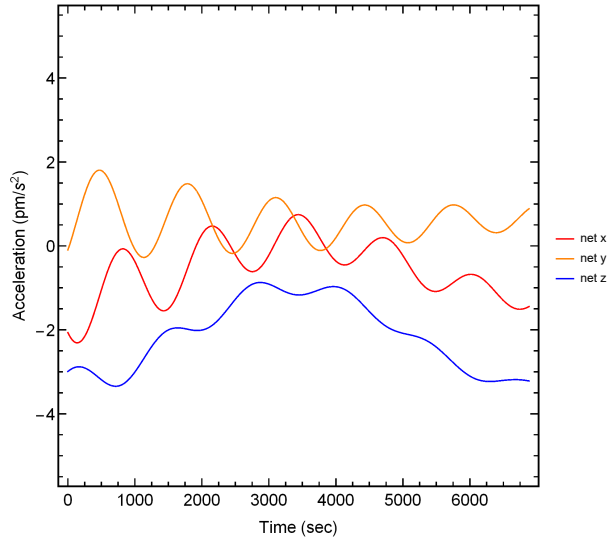


Fig. 5. Day 1470; there is no eclipse on this day. Calculated net acceleration ( $pm/s^2$ ) in components (in celestial coordinate frame). The spin rate is approximately  $5/orbit$ , and this introduces obvious variations into the accelerations.

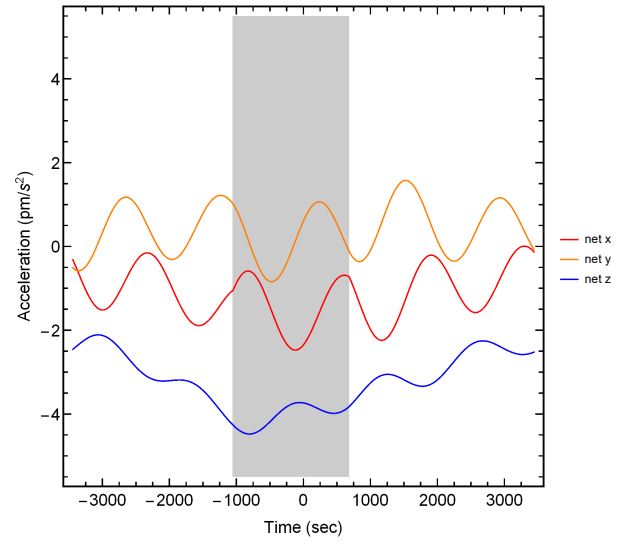


Fig. 7. Day 1490: Calculated net acceleration in components. An eclipse occurred between about  $-1050$  and  $700$  seconds in the orbit on this day, and the satellite crossed the ascending node during the eclipse. We plot one full orbital period, with the node crossing in the center of the plot to more clearly show eclipse-induced effects. The onset and the end of the eclipse introduce noticeable features into time derivative of some of the force components and into the drag (Fig 8), and there are clearly visible  $\approx 5/orbit$  spin effects.

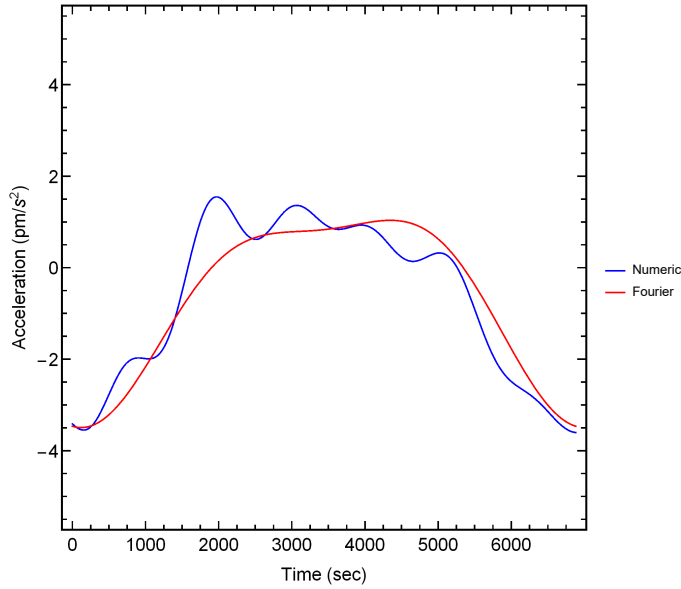


Fig. 6. Day 1470: Along-track acceleration in numerical simulation (blue curve). Negative values indicate *drag*. Comparison fast-spin (assuming all CCRs in a row have the same temperature; see [29]) Fourier computation (sum of the constant, first and second harmonics of the orbital frequency): smoother, red curve, over one orbital period. Note that amplitudes of the force components (Fig 5) and of the along-track acceleration are much larger (almost an order of magnitude larger) than the average drag for that day ( $-0.60pm/s^2$ ; see Figure 4).

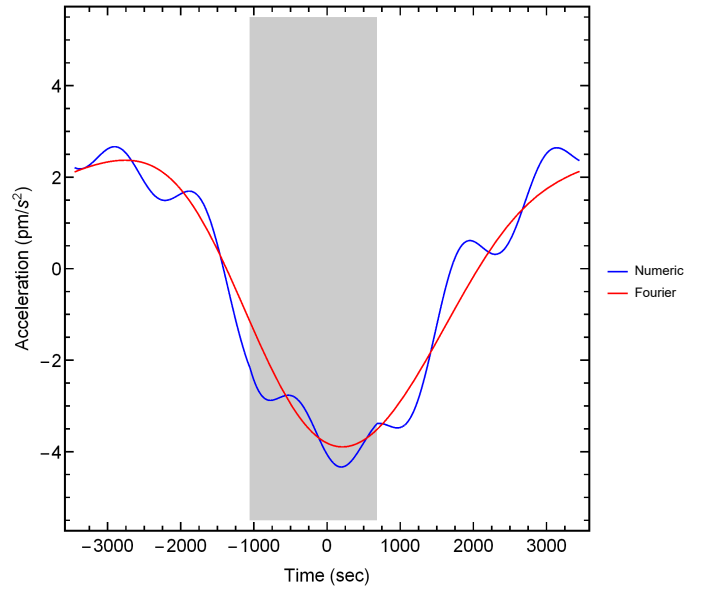


Fig. 8. Day 1490: Calculation of along-track acceleration in numerical simulation (blue curve); Comparison fast-spin Fourier computation (smoother, red curve). Average drag for that day :  $-0.40pm/s^2$ ; see Figure 4.

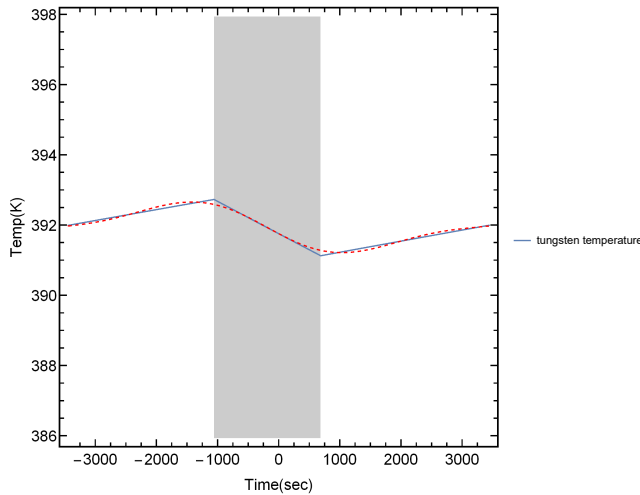


Fig. 9. Day 1490: Computed tungsten alloy core temperature. The dotted line is a Fourier result (constant plus first and second orbital harmonic). The eclipse introduces noticeable sudden changes in the time derivatives of the temperature of the metal and of the CCRs (Fig. (10)). The range of both the Figs. 9 and 10 is 12K.

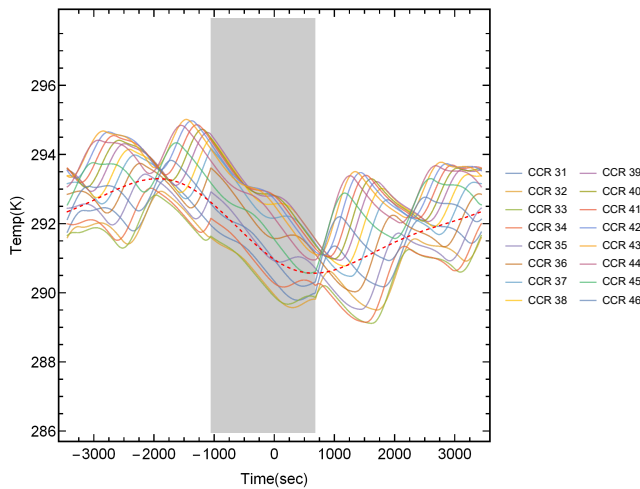


Fig. 10. Day 1490: Calculated individual CCR temperatures for parallel  $V$  (see Section IV).

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