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The Earth's Frame Dragging via Laser Ranged Satellites: a Response to "Some considerations on the present-day results for the detection of frame-dragging after the final outcome of GP-B" by L. Iorio.

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Abstract – In this note, we reply to the preceding paper by Iorio [1], hereafter referred to as I2011, where we address criticisms regarding the Lense-Thirring frame-dragging experiment results obtained from the laser ranging to the two LAGEOS satellites.

Introduction. – To put this discussion into context, a short summary of the developments to this point is needed. In the late 1980s, it was proposed to launch a satellite (LAGEOS-3) into a supplementary orbit to the existing LAGEOS satellite (LAGEOS-1) in order to separate the secular orbit node precession caused by frame-dragging (the Lense-Thirring effect predicted from General Relativity) from the much larger but similar effect due to the even zonal geopotential harmonics (J_n , where n is even) [2]. This orbit configuration would cancel out the influence of the errors in the even zonals, while the frame-dragging effect would be the same for both and thus still remain. It was understood that the orbit injection would only need to be accurate enough to cancel the uncertainty in the even zonal effects [3], not the total effect of the even zonals on the orbit. We also emphasize that even at that time, the issue of whether laser ranging could measure the frame-dragging effect was not an issue. By then, the models and data were already more than adequate to observe this signal. The problem was simply separating that signal from a similar,

but significantly larger, signal caused by errors in the even zonals. This experiment was considered important enough that it warranted a joint NASA/ASI study to evaluate the feasibility of the result [4]. The primary implementation issue was related to the cost of the launch vehicle to place the satellite in the required orbit.

Although LAGEOS-3 was not selected as a NASA mission, the determination of the Earth's gravity field continued to improve, and a second LAGEOS satellite (LAGEOS-2) was launched in late 1992, though not into the inclination that would have been best for the frame dragging experiment. In 1998, a test of frame-dragging with an accuracy on the order of 20% was announced [5]. In order to remove the effect of the two largest even zonal errors (due to J_2 and J_4), this result relied on a linear combination of the residual node rates of LAGEOS-1 and LAGEOS-2 as well as the perigee of LAGEOS-2. Frame-dragging also affects the orbit perigee in a secular way, as do the even zonals, and a particular linear combination was used to cancel the two largest

errors. However, there were significant concerns with this experiment, most notably due to the use of the uncertain perigee signal and the true accuracy of the gravity field available at the time [6].

Around this time, NASA was preparing to launch a joint gravity mission with the German space agency. Based on preliminary results of the improved gravity fields from GRACE [7], an accurate test of frame-dragging would be possible using only the node signals from the two LAGEOS satellites [8,9]. Soon after, such a result was announced [10] and confirmed independently a few years later [11]. A complete description of the combined analyses is available in [12–14]. Our joint conclusion is that the GRACE-based gravity models are indeed accurate enough that the General Relativity prediction of frame-dragging is confirmed with an error of approximately 10%.

Soundness of the Analysis Method. – We address the critique in I2011 of the analysis method first. The author indicates that the analysis of geodetic signals in time series of orbit element residuals is inadequate, and states that the frame dragging effect must be “explicitly modeled and solved-for in the data reduction process.” This ignores the value of decades of analyses that have successfully used orbit element residuals to determine geodetic parameters, dating back to the Vanguard satellite whose orbit eccentricity variations provided the first estimate of the Earth’s ‘pear shape’ [15]. Today, this technique has been developed into the more sophisticated orbit element excitation approach [16, 17]. Explicit parameter estimation is of course widely used, particularly when large numbers of parameters have to be estimated or an error covariance matrix is essential, but analysis of orbit element residuals has significant advantages as well [16]. The ability to plot the time series and see how a predicted mechanism fits provides invaluable insight into the problem that parameter estimation cannot provide. Our approach provides a valuable visual confirmation, illustrated so well in Figure 1, that the unmodeled variations in J_2 are indeed canceled in the J_2 -error-free combination time series. The time series can be band-limited filtered and outliers can be detected and edited, and all of this can be accomplished with little computational expense. Finally, simply estimating a parameter and looking at the covariance does not account for the effect of the systematic errors that typically dominate the error. As in the case for the original LAGEOS-3 analysis [4, 18], where the frame-dragging parameter was explicitly estimated, a post-fit error analysis was still required to obtain a realistic uncertainty assessment. The formal errors are often so overly optimistic that they provide no useful uncertainty information, particularly when an experiment is dominated by a few systematic errors, as is the case here.

In the case of the LAGEOS frame-dragging experi-

ment, a particularly simple but elegant approach is used. The basic time series used for our analysis is the sequence of the orbit end-point overlaps, which are the mismatch between the end of one arc and the beginning of the next arc. These are converted into orbit node residuals that are then accumulated over time to generate the long-term node drift signal for each satellite, as illustrated in Figure 1. These two node signals are combined by multiplying the LAGEOS-2 residual node drift by a constant c and adding it to the node signal from LAGEOS-1. By choosing a specific value for c , based on each satellite’s orbit inclination and altitude, the largest error due to J_2 can be eliminated. This J_2 -error-free combination, also shown in Figure 1, is compared to the same linear combination of the node precessions predicted by General Relativity. In addition to our own internal checks (described later), the use of orbit overlaps to determine a residual node time series was determined through independent analysis to work correctly for secular signals [19]. So we find it curious that the author, while criticizing our use of this method, has applied the same method for his own frame-dragging test using the Mars Global Surveyor (MGS) orbit overlaps [20, 21]. In [21], the author notes that “effects like the Lense-Thirring one, accumulating in time, are, instead, singled out” and refers to the same [19] reference; these MGS results of the author (implying a 6% confirmation of General Relativity) are particularly surprising in light of there being no dedicated gravity mission at Mars to provide the same kind of high-accuracy gravity field model that makes the Earth-based experiment possible at the 10% level. In any case, we conclude that the author’s concern over the use of orbit element overlaps to observe the drift in the node due to the frame-dragging effect is unfounded.

Residual Analysis. – Returning to the analysis method, the best available modeling is used in a least-squares fit to the laser ranging data for a continuous dynamical orbit based on the numerical integration of a complex dynamical model starting with a given set of initial conditions. There will always be some modeling error that remains. In our case, this is dominated by the even zonal errors and not modeling (by choice) the frame dragging effect. The least-squares orbit fit minimizes the variance of the laser range residuals. As long as no empirical parameters are estimated that can accommodate the errors in the out-of-plane orbit components (node and inclination), these zonal errors and unmodeled frame dragging effect must be reflected in the residuals. The author of I2011 affirms this in [20] by noting that the frame-dragging effect, by not being modeled in the MGS orbit fits, “is not included, so that it is fully accounted for by the showed residuals.” In the least-squares fit to the range data, only the mean orbit error can be accommodated (via the adjustment of the orbit initial conditions). This adjustment of the mean orbit minimizes

the overall variance, but results in approximately equal residuals near the ends of the arc and smaller residuals near the arc mid-point, as this point is where the mean (but slightly incorrect) orbit tends to be closest to the true orbit. The result is the well-known ‘bow-tie’ in the orbit error [22], which can be clearly seen in Figure 2, where we show the actual residuals for a typical 14-day fit. We note that in I2011, the author complains that no such residual plots were ever shown, but a plot of tens of thousands of residuals would have provided no useful information. Of course we gladly provide the residuals for every laser range observation if asked. Also we note that the residuals in Figure 2 are not inconsistent with Fig. 5 in I2011, after taking into account the effect of the initial conditions adjustment and that the residuals must also include the effect of the even zonal errors. The author’s concern about the LAGEOS residuals not properly reflecting the modeling errors (including not modeling the frame-dragging effect) is unfounded.

For the best orbit fits, special empirical acceleration parameters are typically estimated which are highly effective in removing what is essentially a slow drift in the orbit elements (both in-plane and out-of-plane) over the fit interval. It is these parameters that allow the determination of sub-cm-level orbits for the LAGEOS satellites in spite of imperfect force models. However, for this experiment, the cross-track (out-of-plane) accelerations are not estimated to avoid absorbing the unmodeled frame-dragging effect on the node. This also leaves in the residuals the effect due to the errors in the even zonals, with the result shown in Figure 2. These fits are not sub-cm, but rather closer to 3-4 cm (for data that is precise at the few-mm level for the best stations). The in-plane accelerations are estimated to accommodate the significant along-track non-gravitational modeling errors, but this has no effect on the cross-track (out-of-plane) component. This is derived from the well-known Lagrange Planetary Equations (in Gauss’s form), which clearly indicate that the forcing in the along-track direction has no direct effect on the orbit node. This is, in fact, recognized by the author himself, where in [20], he states that because only along-track (in-plane) empirical accelerations were estimated for MGS, “it is unlikely that the out-of-plane Lense-Thirring signal was removed from the data in the least-squares procedure.” Furthermore, in the independent tests performed by Ries [11], certain internal checks were conducted. In one case, the frame-dragging effect was modeled, and the change in the residual time series matched exactly the relativity prediction. This is a very strong internal verification; if the use of empirical in-plane parameters was in some way absorbing or creating the frame-dragging signal, a perfect shift in the resulting time series is not possible. A second test where the geodesic precession was also not modeled resulted in precisely a 57% change in the frame-dragging signal, which is exactly what would be expected since the

component of the geodetic precession in the equatorial plane is 57% of the frame-dragging effect. Consequently, the author’s concern that the frame-dragging effect (on the node) has been accommodated or distorted in some way due to the estimation of in-plane empirical parameters is unfounded.

The J_2 -Free Linear Combination. – The author of I2011 devotes considerable discussion to the need to compute the combination coefficient c with exquisite precision, but the argument presented is incorrect. Recall that the two node signals are combined by multiplying the LAGEOS-2 residual node drift by a constant c and adding it to the residual node drift from LAGEOS-1, where $c=0.545$ was used by Ciufolini et al. [12–14]. This produces a J_2 -error-free node drift signal that is then compared to the same linear combination of the frame-dragging signal predicted by General Relativity. This is the critical point. The two linear combinations are compared to each other, and the level of agreement reflects the closeness in agreement to General Relativity. Referring back to the LAGEOS-3 experiment, the orbit inclination only had to be precise enough to cancel the errors in the gravity model, not the full signal. The same is true here, as can be seen by considering the following. If we suppose that the errors in the gravity field are so small that they are negligible, then the two residual node time series will contain only the frame-dragging signal. In that case, any linear combination of the two nodes will match the same linear combination of the General Relativity predictions for the two satellites. The value of c becomes arbitrary. It should be clear that the combination constant c only needs to be known well enough compared to the residual gravity modeling errors. In the case of our 10% experiment, c needs to be known with an accuracy of only a few percent. Even considering the range between $c=0.545$ used in Ciufolini et al. [12–14] and the value of 0.54097 proposed by I2011, the impact on the result is only about 2%. Recomputing c on an arc-by-arc basis to more closely reflect the evolving orbit parameters affects the result by only 1%. This also renders irrelevant the concern about the knowledge of the orientation of the Earth’s spin axis; it is known with an accuracy that exceeds the requirement for calculating c to a few digits. We conclude that the author’s concern about requiring ultra-precise knowledge of the combination factor c is unfounded.

Reference Frame Effects. – The author of I2011 raises a vague concern that the geocentric reference frame used for the satellite data analysis is somehow determined in a favorable way for this experiment, since the LAGEOS data are used as part of the reference frame definition. However, what is primarily relevant for the laser ranging experiment is the rotation of the node with respect to inertial space, since this represents the absolute relativistic signal of interest. This is why it is necessary to model the geodesic precession since this is a similarly appearing component in the node precession, and we take as given

that geodesic precession has been validated by independent tests. While it is difficult to imagine how the use of laser ranging data for the reference frame could favorably ‘imprint’ the frame-dragging signal in these results, it is essentially impossible in any case. Earth rotation (often called UT1), over long time scales, is entirely determined by Very Long Baseline Interferometry to distant quasars. Polar motion is currently determined almost entirely by VLBI and Global Positioning System data. Furthermore, polar motion is the crust-fixed motion of the Earth’s rotation axis. As a consequence, it is modulated by the Earth’s rotation and cannot look like a drift with respect to inertial space, which is the only signal that would affect the LAGEOS experiment. It is noted that the independent analysis by Ries *et al.* [11] did not estimate polar motion corrections, but relied entirely on Earth orientation from other techniques and obtained a similar experimental result. The concerns in I2011 about the impact of laser ranging on the reference frame, resulting somehow in a favorable experiment outcome, are thus unfounded.

Closing Remarks. — Finally, we address some of the material in the introduction of I2011. The author notes the cost of the GP-B mission, that it lasted for 52 years (data was actually collected for 12 months only), that the mission was designed especially for a test of General Relativity, and that there are some comments in the literature questioning the investment in that mission. We ask how any of this is relevant to the reliability of the results from laser ranging, and we are a bit surprised about these clearly prejudicial remarks. Further, the remark by the author that the LAGEOS satellites “were originally launched for different purposes” seems to be a way to imply, without actually saying it, that the satellites are not well suited for this experiment. On the contrary, the two satellites are nearly ideal test objects. The LAGEOS satellites, being solid metal, have very low area-to-mass ratios, which strongly attenuates the surface force effects on the orbits. The satellites are high enough to attenuate the effect of the errors in all but the lowest degree zonal harmonics, but not so high that the frame-dragging signal is not clearly observable. The only way to make the experiment better would be to have launched LAGEOS-2 into the original LAGEOS-3 supplementary orbit.

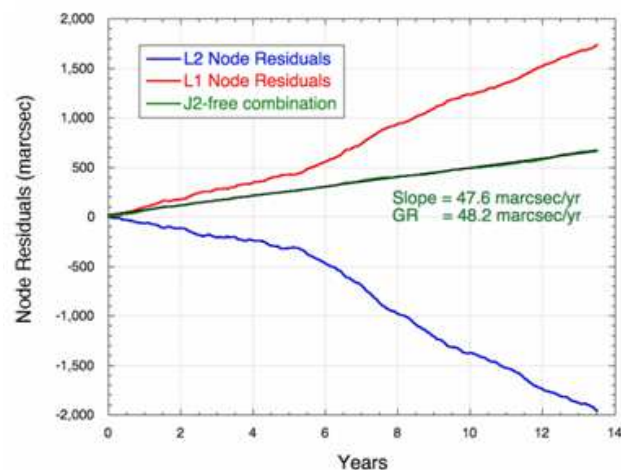


Fig. 1: Residuals of the node of LAGEOS-1 (in red), the node of LAGEOS-2 (in blue), and the J2-free linear combination of the two nodes (in green). The effect of the J2 error (which dominates the two node residual trends) is larger for LAGEOS-2 because of its lower inclination. The linear combination, based on each satellite’s altitude and inclination, removes the specific effect of all errors in J2, including its unmodeled decadal variations. Figure from Ries *et al.* [11].

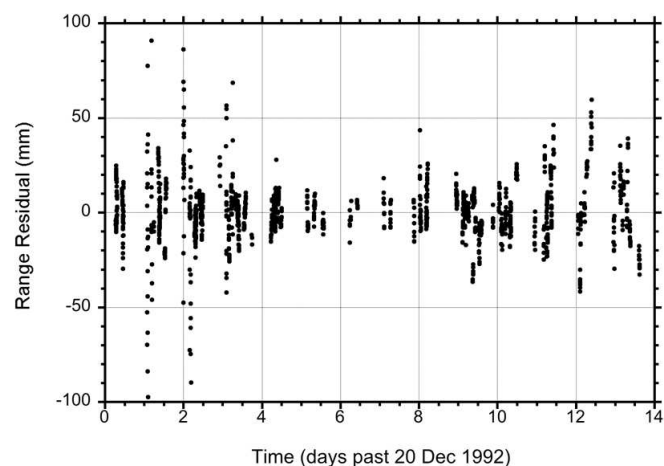


Fig. 2: Laser range residuals for a typical 14-day fit for LAGEOS-1. Because no cross-track empirical accelerations are estimated, the drift in the node (caused by errors in the even zonals and not modeling frame-dragging) over the fit interval cannot be accommodated. The fit orbit differs from the true orbit by the node drift, and this mismatch is reflected in the residuals as a bow-tie shaped modulation of the cross-track orbit errors. It is also reflected in the orbit end-point overlaps, which are used to construct the residual node time series.

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