

Access to this work was provided by the University of Maryland, Baltimore County (UMBC) ScholarWorks@UMBC digital repository on the Maryland Shared Open Access (MD-SOAR) platform.

Please provide feedback Please support the ScholarWorks@UMBC repository by emailing [scholarworks-group@umbc.edu](mailto:scholarworks-group@umbc.edu) and telling us what having access to this work means to you and why it's important to you. Thank you.

# Generation of entangled cats from photon number states

S.U. Shringarpure and J.D. Franson

Physics Department, University of Maryland Baltimore County, MD 21250 USA  
saurabh.s@umbc.edu, jfranson@umbc.edu,

**Abstract:** A phase-entangled cat state is created when a number state passes through a beamsplitter. This entanglement is used to violate Bell's inequality using large Kerr media. This approach may have applications in quantum communications. © 2020 The Author(s)  
**OCIS codes:** 270.0270, 270.4180, 270.5565

## 1. Introduction

A photon number state can be used to generate optical cat states [1]. This is because the phase of a number state is completely uncertain and contains an equally weighted superposition of coherent states on a circle [2]. Passing a number state through a beamsplitter creates an entangled state in which the phases of the two output beams are highly correlated as shown in Fig. 1. Measuring the phase of one collapses the state of the other into an approximate coherent state with a specific phase [3]. However, as the initial phase is completely uncertain, it corresponds to a superposition with all possible phases. Therefore, this can be viewed as the form of entangled-cat state.

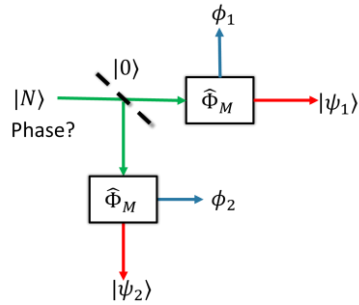


Fig. 1. A number state incident on a balanced beam splitter will produce an entangled state in which the phases of the two output beams are highly correlated [3].

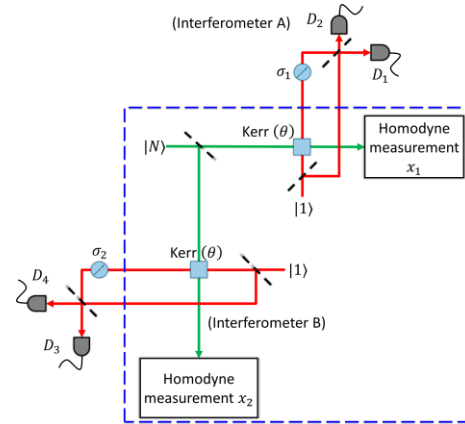


Fig. 2. Two single-photon interferometers use a Kerr medium along with a constant offset to apply a phase shift of  $\pm\theta$  depending on the path taken by the photons. We post select on a final phase of  $\pi/2$  in both beams. Quantum interference between the two probability amplitudes for such a result can violate Bell's inequality [4].

This suggests that Bell's inequality could be violated using two distant interferometers as illustrated in Fig. 2 [4]. Here each beam passes through a single-photon interferometer with a Kerr medium in one path. The Kerr medium, combined with a constant phase shift in both paths, produces a phase shift of  $\pm\theta$  depending on the path taken by the single photons. If we were to measure the phases of the two beams and post-select on the case in which both phases have a specific value such as  $\pi/2$ , then this result would occur if the phases of the two beams initially had a value of  $\pi/2 \pm \theta$  followed by a phase shift of  $\mp\theta$  in the two interferometers. Quantum interference between these two probability amplitudes would produce a violation of Bell's inequality. In our approach, we replace the complete phase measurements with single quadrature measurement to obtain a Bell's inequality violation as described below.

## 2. Violation of Bell's inequality

It can be shown that the state of the system after the initial beam splitter but before the two single-photon interferometers can be written in the form

$$|\psi\rangle = \int_0^{2\pi} d\phi f_\phi \left| \frac{R}{\sqrt{2}} e^{i\phi}, \frac{R}{\sqrt{2}} e^{i\phi} \right\rangle, \quad (1)$$

where  $R$  and  $\phi$  correspond to the amplitude and phase of a coherent state [4]. Eq. (1) clearly shows the phase-entangled nature of the quantum state. Here, the phase shift due to reflection has been compensated by additional phase shifts so that both the beams have identical phases. The homodyne measurements can be described most easily in the position representation where the wave function can be divided into four components such as  $\psi_{+-}(x_1, x_2)$ , which corresponds to a positive Kerr phase shift in beam 1 and a negative phase shift in beam 2. Here  $x_1$  and  $x_2$  are the results of the homodyne measurements. It can be shown that quantum interference between  $\psi_{++}(x_1, x_2)$  and  $\psi_{--}(x_1, x_2)$  will give a post-selected output that is proportional to  $\cos^2[(\sigma_1 + \sigma_2)/2]$ , where  $\sigma_1$  and  $\sigma_2$  are phase shifts introduced into the two single-photon interferometers in Fig. 2. This will violate Bell's inequality provided that the amplitudes  $\psi_{+-}(x_1, x_2)$  and  $\psi_{-+}(x_1, x_2)$  are negligible [2].

Fig. 3a shows a plot of  $|\psi_{++}(x_1, x_2)|^2 = |\psi_{--}(x_1, x_2)|^2$  as a function of  $x_1$  and  $x_2$  while Fig. 3b shows  $|\psi_{+-}(x_1, x_2)|^2 = |\psi_{-+}(x_1, x_2)|^2$ . It can be seen that there are many values of  $x_1$  and  $x_2$  where  $\psi_{+-}$  and  $\psi_{-+}$  are negligible, which allows a violation of Bell's inequality with high visibility. These results correspond to  $N = 24$  and  $\theta = \pi/4$ , but similar results can be obtained for  $N = 2$  as well.

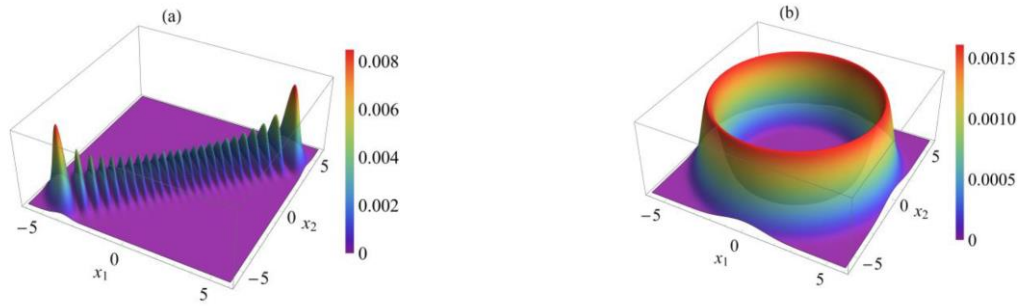


Fig. 3. Plots of the wave function in the position representation. (a)  $|\psi_{++}(x_1, x_2)|^2 = |\psi_{--}(x_1, x_2)|^2$  (b)  $|\psi_{+-}(x_1, x_2)|^2 = |\psi_{-+}(x_1, x_2)|^2$ . Quantum interference between  $\psi_{++}(x_1, x_2)$  and  $\psi_{--}(x_1, x_2)$  can violate Bell's inequality [2].

Entangled state that violate Bell's inequality have applications in quantum communications and quantum key distribution. In addition, a number state is one of the simplest examples of a nonclassical state, and the fact that Bell's inequality can be violated in this way is of fundamental importance.

## 3. References

- [1] Alexei Ourjoumtsev, Hyunseok Jeong, Rosa Tualle-Brouiri and Philippe Grangier, "Generation of optical 'Schrödinger cats' from photon number states." *Nature* **448**, 784-786 (2007).
- [2] J. Janszky, P. Domokos and P. Adam, "Coherent states on a circle and quantum interference." *Phys. Rev. A* **48**, 2213 (1993)
- [3] J. D. Franson, "Nonlocal reduction of the wave function by quantum phase measurements." *Phys. Rev. A* **49**, 3221 (1994).
- [4] S.U. Shringarpure and J.D. Franson, "Violating Bell's inequality using a number state and a beam splitter." arXiv: 1911.02468.