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Neutrino constraints on long-lived heavy dark sector particle decays in the Earth

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Recent theoretical work has explored dark matter accumulation in the Earth and its drift towards the center of the Earth that, for the current age of the Earth, does not necessarily result in a concentration of dark matter (χ) in the Earth's core. We consider a scenario of long-lived ($\tau_\chi \sim 10^{28}$ s), super heavy ($m_\chi = 10^7 - 10^{10}$ GeV) dark matter that decays via $\chi \rightarrow \nu_\tau H$ or $\chi \rightarrow \nu_\mu H$. We show that an IceCube-like detector over 10 years can constrain a dark matter density that mirrors the Earth's density or has a uniform density with density fraction ϵ_ρ combined with the partial decay width $B_{\chi \rightarrow \nu_\tau H} \Gamma_\chi$ in the range of $(\epsilon_\rho/10^{-10}) B_{\chi \rightarrow \nu_\tau H} \Gamma_\chi \lesssim 3 \times 10^{-29} - 3 \times 10^{-28} \text{ s}^{-1}$. For $\chi \rightarrow \nu_\mu H$, $m_\chi = 10^8 - 10^{10}$ GeV and $E_\mu > 10^7$ GeV, the range of constraints is $(\epsilon_\rho/10^{-10}) B_{\chi \rightarrow \nu_\mu H} \Gamma_\chi \lesssim 6 \times 10^{-29} - 1.4 \times 10^{-27} \text{ s}^{-1}$.

I. INTRODUCTION

The identification and characterization of dark matter has been a central effort to understand the composition of 27% of the present day Universe. The weakly-interacting massive particle (WIMP) paradigm, in its simplest form, has weak-scale interacting particles in thermal equilibrium in the early Universe. As the expansion rate of the Universe exceeds their interaction rate, dark matter decouples ("freezes out") at a number density to account for the dark matter fraction of the energy density of the Universe [1–3]. Efforts for direct and indirect detection of dark matter yield upper bounds on the WIMP-nucleon cross section as a function of WIMP mass [4, 5]. Concurrently, LHC searches for WIMP candidates exclude a region of parameter space [6–8].

In the most straightforward approach, there is a single species of stable WIMPs. In the WIMP paradigm, if dark matter capture and dark matter annihilation are in equilibrium, for example in the Sun, the annihilation rate is related to the capture rate and dark matter concentrates at the Sun's core [9, 10]. However, equilibrium times for WIMP capture and annihilation in the Earth in the standard WIMP scenario are longer than the age of the solar system [11, 12]. Indirect constraints on dark matter come from, for example, IceCube searches for dark matter annihilation or in the case of an extended dark sector or small couplings to standard model particles, searches for dark matter decay [13–16]. Constraints come from the absence of signals of annihilation or decay at the

center of the Earth, in the core of the Sun and from the dark matter halo in the galaxy.

As constraints on the simplest WIMP sectors are tightened, theories of a more complex dark sector or long-lived, but unstable, dark matter have been proposed. In some of these models, the relic abundance of dark matter does not follow from the standard picture of thermal equilibrium followed by freeze-out. For example, non-thermal production of super-heavy dark matter (e.g., [17–19] and references therein) can evade mass limits that follow from unitarity constraints on cross sections in the thermal freeze-out picture [20]. Cosmological and astrophysical considerations permit large dark matter-baryon interaction cross sections (see, e.g., [21, 22]).

In constraints on dark matter accumulation in the Earth [23], it is conventionally assumed that dark matter is concentrated at the core of the Earth. With sufficiently large dark matter interactions with baryons in the Earth and absent a significant self-annihilation cross section, the dark matter density at or near the surface of the Earth can be considerably enhanced relative to the local halo density [24]. In ref. [24], for a dark matter mass equal to twice the proton mass, the dark matter density in the Earth would be nearly uniform, but it becomes more concentrated in the Earth's core for heavier masses. In more complex dark matter scenarios, the drift of heavy dark matter particles to the center of the Earth may be very slow or arrested [25–29]. Non-standard dark matter density distributions have been proposed [30] to account for two unusual events with large elevation angles reported

by the ANITA collaboration [31, 32]. This model would require almost all dark matter particles intercepted by the Earth during its lifetime to be captured [33]. Recent results from ANITA-IV do not show additional events at such large angles [34]. Nevertheless, these theoretical and observational developments emphasize the opportunity for neutrino telescopes to search for dark matter annihilation or decay that originate from a wider density distribution of dark matter throughout the Earth, rather than originate only from the center of the Earth [35].

In this paper, we consider the case of long-lived super-heavy dark matter (SHDM) χ with mass in the range of $M_\chi = 10^7 - 10^{10}$ GeV. This mass range is chosen to illustrate the capabilities of IceCube to constrain a combination of dark matter density in the Earth and its lifetime in a nearly background-free energy regime. We focus on long-lived, but unstable fermionic dark matter with decays to $\chi \rightarrow \nu_\tau H$. We also show results for $\chi \rightarrow \nu_\mu H$, and it is straightforward to extrapolate to other decay channels. We take a model independent approach to the origin of the dark matter density profile in the Earth and use two simplified dark matter density profiles: a uniform density and one that scales with the Earth's density of ordinary matter. We compare IceCube limits on the partial decay width of SHDM decays to neutrinos in the Galactic Center to our limits on the fraction of the Earth's density comprised of dark matter time the partial decay width for DM density distribution profiles that include dark matter far from the Earth's core.

The outline of the paper is as follows. In Sec. II, we briefly discuss our assumptions for the dark matter density distributions in the Earth. In Sec. III, we describe the evaluation of the number of events using an approximation of IceCube's acceptance for these events. Section IV shows our estimates of the sensitivity of IceCube to these SHDM decays. The paper wraps up with discussion and conclusions presented in Sec. V.

II. DARK MATTER IN THE EARTH

Dark matter accumulation in the Earth depends on the Earth's DM collection efficiency and on DM losses through annihilation and decay. With weak interaction scale DM-nucleon cross sections, DM accumulation is hampered by the Earth's low escape velocity $v_{\text{esc}} = 11.2$ km/s, small compared to the average velocity of the Earth relative to the DM background, $v_{\text{rel}} \approx 220$ km/s for a Maxwellian distribution of velocities. This is in contrast to the solar collection efficiency of DM given $v_{\text{esc}} = 615$ km/s for the Sun.

Proposals of efficient collection of dark matter rely on strong interactions of dark matter. If the hidden sector is multi-component and self-interacting, the distribution function is still Maxwellian but the velocity dispersion depends on the mass spectrum of the hidden sector and could be smaller than v_{esc} [36]. For example, consider a two-component dark matter model with a large mass hi-

erarchy $m_a \ll m_b$. The particles can be taken as two heavy sterile neutrinos charged under a hidden $U(1)$ gauge group. The charges Q_a and Q_b are opposite in sign and satisfy $n_a Q_a + n_b Q_b = 0$, where n_a and n_b are the number densities of the sterile neutrinos. In the Galactic halo the dark photon interactions would keep the sterile neutrinos at a common temperature, and so in the proximity of the Earth we could have $f_\oplus(v_b) \propto e^{-v_b^2/v_{0,b}^2}$, with a velocity dispersion $v_{0,b} \ll v_{\text{rel}}$. Thus, in principle, a large mass hierarchy could allow for both sufficient trapping of particle of type a if $m_p \gtrsim m_a \ll m_b$ and sufficient trapping of particle of type b via self interaction if $v_{0,b} \ll v_{\text{esc}}$. It is clear that even if this toy model can accommodate the data it would require a large amount of fine-tuning.

Another approach is to consider DM with strong interactions with nucleons. For $m_\chi \sim 1 - 10$ GeV with cross sections in the $\sim 10^{-29} - 10^{-26}$ cm² range, Neufeld, Farrar and McKee [24] have shown that over times much shorter than the age of the Earth $t_\oplus = 4.55$ Gyr [37], the "hadronically interacting" dark matter particles equilibrate thermally with the material in the Earth and result in enhancements of the average DM mass density in the Earth relative to its density in the Galactic plane. For $m_\chi \sim 1$ GeV, the enhancement is $\sim 10^{14}$ times larger than the ambient DM mass density in the Galactic plane, $\rho_{\text{DM,gal}} \approx 0.5$ GeV/cm³ [38–41] and the DM is distributed throughout the Earth. Dark matter accumulation and slow drift times to the center of the Earth for a range of masses are discussed in ref. [29]. Direct detection experimental constraints on these DM particles do not apply when the DM does not reach the underground detector or is thermalized such that it cannot provide a sufficient interaction recoil energy to be detected.

Larger DM masses are also considered in a series of papers on milli-charged relics [25–28]. Depending on the fraction of DM comprised of virialized milli-charged relics, the milli-charged dark matter density in the Earth relative to the DM mass density in the Galactic plane can be comparably enhanced and may be distributed throughout the Earth.

Rather than referring to a specific model or scenario of DM interactions in the Earth, we rely on experiment to resolve the issue of the DM distribution in the Earth. We assume that there is no dark matter annihilation. The evaporation rate of SHDM should be low [42]. We discuss here the capability of neutrino telescopes to constrain a scenario of efficient dark matter accumulation by the Earth and slow drifting towards the Earth's core through a constraint on the fraction of the Earth's density comprised of DM. We focus on long-lived DM which decays to a neutrino.

In this paper, we consider dark matter distributed uniformly in the Earth and a density that mirrors the Earth's mass density. In both cases, our starting point is that the dark matter mass within the Earth accounts for ϵ_ρ of the Earth's mass. For ease of notation, we define $\rho_\chi \equiv \rho_{\text{DM,Earth}}$. With this notation, we consider $\rho_\chi = \epsilon_\rho \rho_\oplus^{\text{avg}}$, and we consider a DM density proportional

to the Earth's matter density distribution as parameterized in the Preliminary Earth Reference Model (PREM) [43], $\rho_\chi = \epsilon_\rho \rho_\oplus^{\text{PREM}}$.

Efficient capture of DM by the Earth allows ϵ_ρ to be as large as $\epsilon_\rho \sim 2 \times 10^{-10}$ [29]. Constraints on ϵ_ρ come from considerations of thermal energy release in the DM decay. Following the discussion in [44], we consider the internal heating rate of the Earth due to DM decay. Conservatively, we assume that the χ rest mass energy goes into standard model particles upon decay of each DM particle, with the energy given up to heat the Earth. The rate of heating is $\Gamma_{\text{heat}} = m_\chi N_\chi \Gamma_\chi$ where $\tau_\chi = 1/\Gamma_\chi$ and N_χ is the number of DM particles in the Earth. Taking $m_\chi N_\chi = \epsilon_\rho M_\oplus$, the heating rate is

$$\Gamma_{\text{heat}} = \epsilon_\rho \left(\frac{10^{28} \text{ s}}{\tau_\chi} \right) \times 54 \text{ TW} < 20 \text{ TW}. \quad (1)$$

The upper bound on Γ_{heat} comes from estimating the heat flow that is not modeled by radioactivity and processes in the Earth's core, given a total internal heat flow of 44 TW [44]. As noted in ref. [44], kinetic heating from DM capture in the Earth is of order a few times 10^{-3} TW, independent of mass for SHDM. For reference regarding ϵ_ρ , the uncertainty in the mass of the Earth is $\epsilon_\rho = 10^{-4}$. The annual net loss of conventional matter from the Earth, primarily hydrogen and helium gas that escapes the atmosphere, is of order $\epsilon_\rho \sim 10^{-17}$ per year.

III. NEUTRINOS FROM DARK MATTER DECAY IN THE EARTH

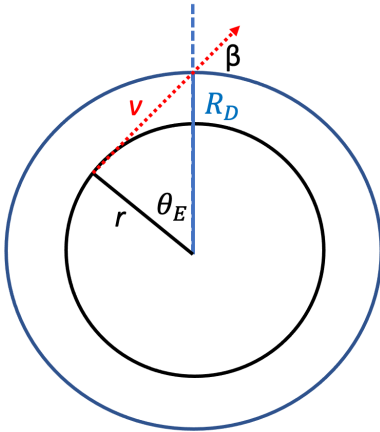


FIG. 1: Dark matter decay a distance r from the Earth's center, with detector at radial distance $R_D = R_\oplus - d$, where d is the detector distance below the Earth's surface. The distance from the decay point to the detector is denoted v .

We evaluate the number of tau neutrino-induced charged leptons produced in or arriving at a detector a distance d below the surface of the Earth, so a distance

$R_D = R_\oplus - d$ from the Earth's center, as shown in Fig. 1. We assume that $d \ll R_\oplus$. Tau neutrinos come from the decays of DM particles distributed throughout the Earth.

We begin with an evaluation of signals of upward-going taus, so DM decays can occur a distance $r \leq R_D$ at angles θ_E relative to the direction of the nadir of the detector. We label the line-of-sight distance from the detector to the decaying χ by v , where

$$v^2 = r^2 + R_D^2 - 2rR_D \cos \theta_E. \quad (2)$$

The total number of τ 's that can be detected depends on the DM number density in the Earth, $n_\chi(r) = \rho_{\chi, \text{Earth}}(r)/m_\chi$, the total decay probability over the lifetime T_0 of the detector $P_{\chi \rightarrow \nu_\tau}(T_0)$, and the probability that the τ is detected, $P^{\nu_\tau \rightarrow \tau}$. The probability $P^{\nu_\tau \rightarrow \tau}$ depends on the location of the DM decay and on the neutrino energy, determined by the mass of the DM particle. Suppressing the energy dependence, the number of events is

$$N_\tau = \int d^3r n_\chi(r) P_{\chi \rightarrow \nu_\tau}(T_0) P^{\nu_\tau \rightarrow \tau}(r, \theta_E) \frac{\Delta \Omega_{\text{obs}}}{4\pi}. \quad (3)$$

The factor $\Omega_{\text{obs}}/(4\pi)$ accounts for the fraction of the isotropic χ decays that arrive at the detector. For a detector with cross sectional area of radius r_d , the fraction χ decays that occur a distance v and have neutrinos that point to the detector is

$$\frac{\Delta \Omega_{\text{obs}}}{4\pi} \simeq \frac{r_d^2}{4v^2}. \quad (4)$$

The decay probability for DM is

$$P_{\chi \rightarrow \nu_\tau} = B(\chi \rightarrow \nu_\tau) \frac{T_0}{\tau_\chi} = B(\chi \rightarrow \nu_\tau) \Gamma_\chi T_0. \quad (5)$$

In what follows, we use an analytic or semi-analytic approximation of $P^{\nu_\tau \rightarrow \tau}$. For ν_τ interactions that occur inside the detection region (labeled with "start"), for example in IceCube, $P^{\nu_\tau \rightarrow \tau}$ depends on the neutrino charged current cross section, Avogadro's number, the length of the detector ℓ and the density of the detector ρ_{det} . It also depends on the neutrino attenuation $S(v)$ that depends on the location of the DM decay.

$$P^{\nu_\tau \rightarrow \tau} \simeq S(v) \sigma_{\text{CC}} N_A \ell \rho_{\text{det}} P_{\text{detect}}. \quad (6)$$

The neutrino attenuation factor is approximated by

$$S(v) \simeq \exp(-\sigma_{\text{CC}} N_A \rho_{\text{avg,E}} v), \quad (7)$$

where v , defined in eq. (2), is a function of r and θ_E . For near surface detectors, almost all of the signal will come from decays in the upper half of the Earth. Consequently, we assign to $\rho_{\text{avg,E}}$ the average density of the Earth along a vertical trajectory from decay point r to r_{max} . The quantity P_{detect} is the detection probability. For through-going taus or taus that decay in the detector,

P_{detect} depends on the tau lifetime, energy and the size of the detector.

When τ 's are produced outside of the detector, we use approximate formulas for the tau survival probability once it is produced. We can reasonably approximate the exit probability as discussed in ref. [45]. We assume that the distribution of tau energy E_τ^i from the neutrino charged current interaction can be approximated by a δ function with $E_\tau^i = 0.8E_\nu$. With continuous energy loss characterized by

$$\left\langle \frac{dE_\tau}{dz'} \right\rangle = -b_\tau \rho E_\tau, \quad (8)$$

the distance $z' = v - z$ that the tau propagates relates the initial and final tau energy, where the tau is produced at point z along the chord of length v . A constant energy loss parameter b_τ gives a particularly simple form of the probability of the neutrino to produce a tau that arrives at the detector and is detected, for a detector in water or ice, is

$$P^{\nu_\tau \rightarrow \tau} \simeq \int_{E_\tau^{\min}}^{E_\tau^i} dE_\tau S(z) \frac{N_A \sigma_{\text{CC}}(E_0)}{b_\tau E_\tau} \times \exp \left[-\frac{m_\tau}{c\tau b_\tau \rho_w} \left(\frac{1}{E_\tau} - \frac{1}{E_\tau^i} \right) \right] P_{\text{detect}}(E_\tau), \quad (9)$$

where $E_\tau^i = 0.8E_0$, $E_\tau^{\min} \geq E_\tau^i \exp(-b_\tau \rho_w v)$ for water density ρ_w , and the $z = v - \ln(E_\tau^i/E_\tau)/(b_\tau \rho_{\text{avg,E}})$ is the distance the neutrino travels before it interacts to produce a τ with energy E_τ . For the taus that do arrive at a detector in water or ice, they are mainly produced in the last 5–10 km of the neutrino trajectory, which except for near vertical angles, is water. Thus, we use the water density in the exponential written explicitly in eq. (9).

For our results, we use

$$b_\tau = b_\tau(E_\tau^i) = (1.2 + 0.16 \ln(E_\tau^i/10^{10} \text{ GeV})) \times 10^{-6} \text{ cm}^2/\text{g}. \quad (10)$$

for $E_\tau^i > 10^8 \text{ GeV}$ and $b = b(10^8 \text{ GeV})$ for lower energies where, in any case, the decay of the tau dominates over electromagnetic energy loss.

To illustrate how well the approximation of eq. (9) works, we show a comparison of a Monte Carlo evaluation of $P^{\nu_\tau \rightarrow \tau}$ (solid curves) to the approximate expression (dashed curves) for three shells of DM decays with $R_D = R_E$ with $P_{\text{detect}} = 1$. In fig. 2, thin DM density shells are fixed at $r = R_\oplus \cos(\beta_{\min})$, where $\beta_{\min} = 15^\circ$, 25° and 30° refers to the minimum angle β relative to the horizon of an Earth-emerging tau that comes from a shell of DM decays a distance r from the center of the Earth. These angles correspond to DM shells a distance $0.966R_\oplus$, $0.906R_\oplus$ and $0.866R_\oplus$. As the DM decays are deeper in the Earth, tau neutrino regeneration plays a role. For $\beta_{\min} = 30^\circ$, the dot-dashed curve shows the Monte Carlo result without regeneration processes in

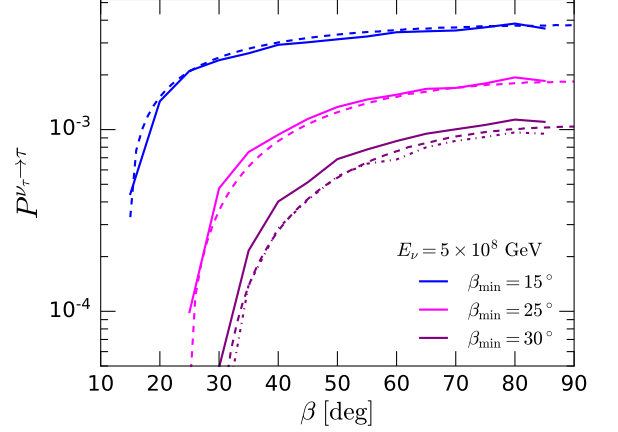


FIG. 2: $P^{\nu_\tau \rightarrow \tau}$ for neutrinos originating on a shell a distance $r = R_\oplus \cos(\beta_{\min})$ as a function of angle β of the trajectory relative to the tangent to the Earth's surface, from Monte Carlo evaluations (solid) and the approximate formula in eq. (9) (dashed). The dot-dashed line for $\beta_{\min} = 30^\circ$ shows the Monte Carlo result without regeneration.

which $\nu_\tau \rightarrow \tau \rightarrow \nu_\tau \rightarrow \tau$ in a series of neutrino interactions and tau decays. The analytic approximation does well neglecting regeneration and will yield conservative bounds on the DM matter density fraction in the Earth, ϵ_ρ .

For reference, for muon neutrinos that produce muons in neutrino interactions outside of the detector,

$$P^{\nu_\mu \rightarrow \mu} \simeq \int_{E_\mu^{\min}}^{E_\mu^i} dE_\mu S(z) \frac{N_A \sigma_{\text{CC}}(E_0)}{b_\mu E_\mu} P_{\text{detect}}(E_\mu), \quad (11)$$

with the energy loss parameter approximated as

$$b_\mu = b_\mu(E_\mu^i) = (2.2 + 0.195 \ln(E_\mu^i/\text{GeV})) \times 10^{-6} \text{ cm}^2/\text{g}. \quad (12)$$

Equation (3) shows that the unknown parameters, the DM mass fraction of the Earth ϵ_ρ and the DM lifetime and branching fraction to tau neutrinos come through the combination

$$\epsilon_\rho \frac{B_{\chi \rightarrow \nu_\tau}}{\tau_\chi} = \epsilon_\rho B_{\chi \rightarrow \nu_\tau} \Gamma_\chi.$$

We set limits on this combination of unknown parameters based on a lack of tau neutrino events in this energy range.

IV. DISCOVERY REACH

A. Events in an IceCube-like detector

The IceCube detector with $\sim 1 \text{ km}^3$ of instrumented ice can be used to constrain $\epsilon_\rho B_{\chi \rightarrow \nu_\tau} \Gamma_\chi$. We approximate

IceCube as an isotropic detector of cross sectional area of $\pi(f_D \times 0.5 \text{ km})^2$, length $\ell = f_D \times 1 \text{ km}$. Thus, the fiducial volume is $\sim 0.8 f_D^3 \text{ km}^3$. The average detection efficiency over the $T_0 = 10$ years of running time is denoted by ε_{det} .

When signals are through-going tau tracks from taus produced by tau neutrinos outside of the detector, the integrand of the probability $P^{\nu_\tau \rightarrow \tau}$ in eq. (9) has $P_{\text{detect}} = \varepsilon_{\text{det}} \exp[-\ell/(\gamma c \tau_\tau)]$ accounting for the tau lifetime τ with $\gamma = E_\tau/m_\tau$. For tracks that convert to showers (decay within IceCube, “decay” events), we take $P_{\text{detect}} = \varepsilon_{\text{det}}(1 - \exp[-\ell/(\gamma c \tau_\tau)])$ in the integrand of eq. (9). When neutrinos produce taus inside the detector, we take $P_{\text{detect}} = \varepsilon_{\text{det}}$.

For this energy range of ν_τ , between $5 \times 10^6 \text{ GeV}$ and $5 \times 10^9 \text{ GeV}$ ($E_{\nu_\tau} = m_\chi/2$), we set limits on $\varepsilon_\rho B_{\chi \rightarrow \nu_\tau} \Gamma_\chi$ assuming $f_D = 0.5$ and $\varepsilon_{\text{det}} = 1$ to illustrate IceCube’s capability to constrain the DM density in the Earth. As noted above, we take the DM mass density distributed uniformly in the Earth according to the Earth’s average density, $\rho_\chi = \varepsilon_\rho \rho_\oplus^{\text{avg}} = \varepsilon_\rho (5.5 \text{ g/cm}^3)$, and a distribution that follows the PREM density, $\rho_\chi = \varepsilon_\rho \rho_\oplus^{\text{PREM}}$.

As a first demonstration of our results, we show in fig. 3 the number of events with these approximations of the detector for $\rho_\chi = \varepsilon_\rho (5.5 \text{ g/cm}^3)$, $\varepsilon_\rho = 10^{-10}$ and $B_{\chi \rightarrow \nu_\tau} \Gamma_\chi = 10^{-28} \text{ s}^{-1}$. The number of events has two types of $m_\chi = 2E_0$ dependence. The green curve shows the number of events as a function of m_χ for events that start with a ν_τ interaction in the detector. The energy dependence associated with the neutrino comes through the neutrino interaction cross section, which grows as $\sim E^{0.3}$ with energy and the neutrino attenuation factor which suppresses the $P^{\nu_\tau \rightarrow \tau}$ as energy increases in eq. (6). An additional dependence on v in the factor $\Delta\Omega/(4\pi)$ complicates a simple discussion, however, the green line in fig. 3 demonstrates the dominant dependence on mass in this case. The number of events scales roughly as $\sim 1/m_\chi$ because we have fixed $n_\chi = \varepsilon_\rho \rho_{\text{Earth}}/m_\chi$. An increase in the mass of m_χ has a commensurate decrease in n_χ that appears in eq. (3).

With the approximations used here for $P^{\nu_\tau \rightarrow \tau}$, neglecting neutrino attenuation, it is straightforward to show that at high energy, the number of events for $\nu_\tau \rightarrow \tau$ outside the detector that have tau decays in the detector equals the number of events for the ν_τ to produce a tau directly in the detector. The approximate equality holds at high energy as long as the detection efficiencies are equal for ν_τ starting and τ decay events.

The number of events for through-going taus has a different energy behavior than for decaying taus, but the combined number of tau events, shown with the dashed line, is roughly constant at the lower end of the mass scale, then begins to decrease with increasing mass. At low energies, the probability that a tau is produced outside the detector and makes it into the detector scales with the time dilated decay length $\gamma c \tau$, so it increases with $E_0 = m_\chi/2$. This directly compensates for the decrease in n_χ that scales as $1/m_\chi$. For $E_\tau \sim$ a few times 10^8 GeV , so for m_χ at twice the scale, the tau range is mod-

ified by electromagnetic energy loss [46] so the range grows more slowly than linearly with energy, thus, the turnover in the number of events.

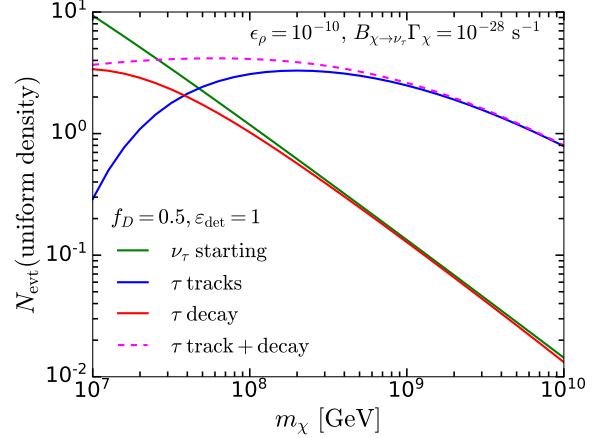


FIG. 3: The number of events for ν_τ interactions in the detector (ν_τ starting) and τ 's produced outside the detector in ice that pass through the detector (τ tracks) or decay in the detector (τ decay). We take $\varepsilon_\rho = 10^{-10}$ for constant density dark matter $\varepsilon_\rho (5.5 \text{ g/cm}^3)$, $B_{\chi \rightarrow \nu_\tau} \Gamma_\chi = 10^{-28} \text{ s}^{-1}$, detection efficiency $\varepsilon_{\text{det}} = 1$, fiducial volume fraction $(f_D)^3 = (0.5)^3$ and observing time of 10 years.

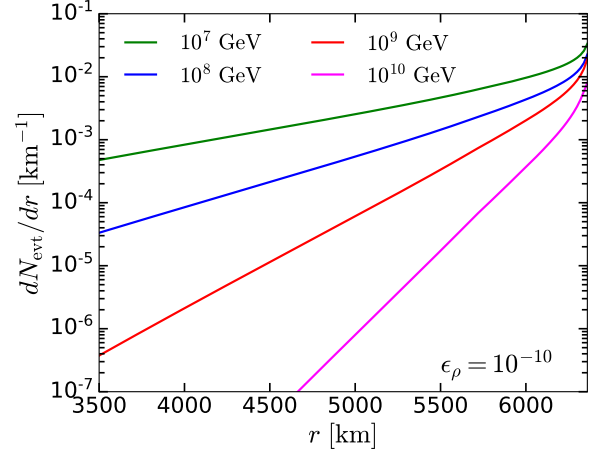


FIG. 4: The number of events for ν_τ interactions in the detector and tau tracks and decays, all combined, from $\chi \rightarrow \nu_\tau H$ as a function of radius for $\varepsilon_\rho = 10^{-10}$ and for constant density dark matter $\varepsilon_\rho (5.5 \text{ g/cm}^3)$, $B_{\chi \rightarrow \nu_\tau} \Gamma_\chi = 10^{-28} \text{ s}^{-1}$, detection efficiency $\varepsilon_{\text{det}} = 1$, fiducial volume fraction $(f_D)^3 = (0.5)^3$ and observing time of 10 years.

Figures 4 and 5 show the distribution of events coming from dark matter $\chi \rightarrow \nu_\tau H$ decays as a function of radial distance and $\sin \beta$ where β the angle of the trajectory relative to the tangent to the Earth’s surface. We have only considered upward events. Because of neutrino attenuation, the total number of events (starting, tracks and decays) decreases with energy. Higher ener-

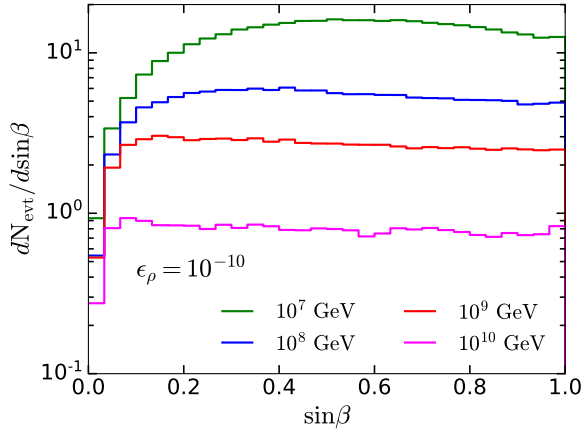


FIG. 5: The number of events for ν_τ interactions in the detector and tau tracks and decays, all combined, from $\chi \rightarrow \nu_\tau$ as a function of $\sin\beta$ for $\epsilon_\rho = 10^{-10}$ and for constant density dark matter ϵ_ρ (5.5 g/cm³), $B_{\chi \rightarrow \nu_\tau} \Gamma_\chi = 10^{-28}$ s⁻¹, detection efficiency $\epsilon_{\text{det}} = 1$, fiducial volume fraction $(f_D)^3 = (0.5)^3$ and observing time of 10 years.

gies are dominated by χ decays closer to the detector. Figure 5 shows that except for nearly horizontal incident particles, the angular distribution is nearly isotropic for the upward event rate.

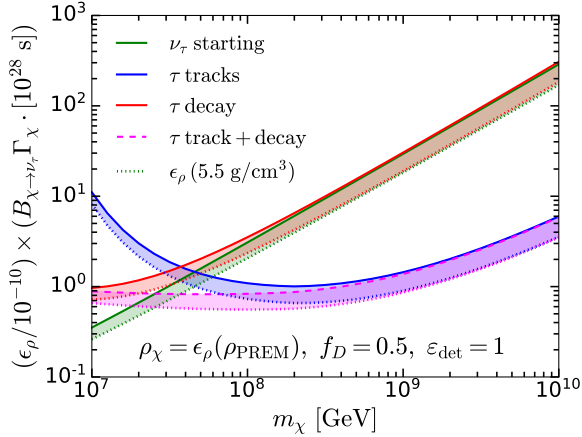


FIG. 6: Constraints on the $(\epsilon_\rho/10^{-10}) \times B_{\chi \rightarrow \nu_\tau} \Gamma_\chi [10^{28} \text{ s}]$ for constant density dark matter $\epsilon_\rho \times 5.5 \text{ g/cm}^3$ (dotted curves) and for the dark matter density that is proportional to the PREM density model of the Earth (solid and dashed curves) for an isotropic detector in ice with a fiducial volume of $f_D^3 \times 0.8 \text{ km}^3$ for $f_D = 0.5$ and detection efficiency $\epsilon_{\text{det}} = 1$. The neutrino energy that produces these events is assumed to be $E_0 = m_\chi/2$. Allowed parameters are below the curves in the figures. An observing time of 10 years is assumed.

An exclusion region for $\epsilon_\rho B_{\chi \rightarrow \nu_\tau} \Gamma_\chi$ that is achievable by an IceCube-like detector is shown in fig. 6. It is obtained by setting $N_{\text{evt}} = 2.44$, the number of events associated with a 90% CL limit assuming no background. The region below the solid curves for each of the detection

channels is allowed, for example, for $B_{\chi \rightarrow \nu_\tau} \Gamma_\chi = 10^{-28} \text{ s}^{-1}$, for $m_\chi = 10^8 \text{ GeV}$, $\epsilon_\rho \lesssim 10^{-10}$ if the distribution of dark matter in the Earth mirrors the Earth matter density, scaled by ϵ_ρ . For the low mass region, the tau events will appear as starting events or decays in the detector. At higher masses, the high energy tau track will have a similar energy loss profile in the detector as lower energy muons. We have $b_\tau/b_\mu \approx 0.08$ for charged lepton energy 10^8 GeV , so a 10^8 GeV tau will appear to be a $\sim 8 \times 10^6 \text{ GeV}$ muon of the basis of $\langle dE_\tau/dX \rangle$. Better modeling of muon stochastic energy losses [47] may ultimately help distinguish high energy taus from high energy muons.

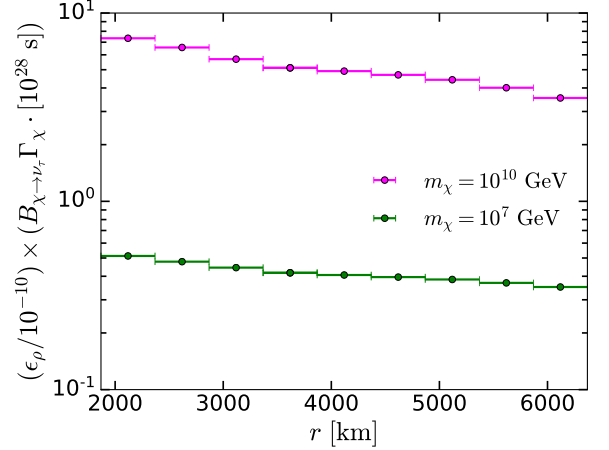


FIG. 7: For two DM masses, constraints on the $\epsilon_\rho \times B_{\chi \rightarrow \nu_\tau} \Gamma_\chi$ as in fig. 6 for constant density dark matter $\epsilon_\rho \times 5.5 \text{ g/cm}^3$ for DM in radial shells, for an isotropic detector in ice with a volume of $f_D^3 \times 0.8 \text{ km}^3$ for $f_D = 0.5$ and detection efficiency $\epsilon_{\text{det}} = 1$. The neutrino energy that produces these events is assumed to be $E_0 = m_\chi/2$. Allowed parameters are below the curves in the figures. An observing time of 10 years is assumed.

The density distribution of DM in the Earth is unknown. Figure 7 shows the potential constraints on the contributions to a tau neutrino signal from a 500 km shell of DM. For each 500 km interval in r , a predicted number of events of 2.44 sets the limits shown in the figure.

We show the corresponding results for $\chi \rightarrow \nu_\mu H$ in figs. 8 and 9. We set $E_\mu^{\text{min}} = 10^7 \text{ GeV}$ to avoid an energy region with background events. The muon track sensitivity is better than the ν_μ starting event sensitivity because the muon range is larger than $f_D \ell = 0.5 \text{ km}$ for the whole range of m_χ shown. For larger m_χ , a higher energy muon neutrino emerges, which in turn produces a higher energy muon with a longer range. The muon range scales as $\sim 1/b_\mu \cdot \ln(E_\mu^i/E_\mu^{\text{min}})$. The sensitivity curves are slightly lower for $\chi \rightarrow \nu_\mu H$ than for $\chi \rightarrow \nu_\tau H$ for $m_\chi = 10^8 \text{ GeV}$ because the muon range is larger than the tau range at these energies. For high m_χ , the tau range is longer. The relative sensitivities for layers of constant density dark matter in the Earth for $\chi \rightarrow \nu_\mu H$ compared to $\chi \rightarrow \nu_\tau H$ also shows this effect, as seen in the comparison of the results shown for $m_\chi = 10^{10} \text{ GeV}$ in figs. 7

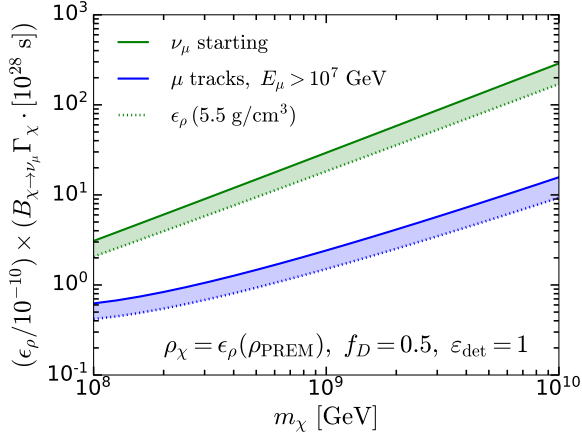


FIG. 8: Constraints on the $(\epsilon_\rho/10^{-10}) \times B_{\chi \rightarrow \nu\mu} \Gamma_\chi [10^{28} \text{ s}]$ for constant density dark matter $\epsilon_\rho \times 5.5 \text{ g/cm}^3$ (dotted curves) and for the dark matter density that is proportional to the PREM density model of the Earth (solid and dashed curves) for an isotropic detector in ice with a fiducial volume of $f_D^3 \times 0.8 \text{ km}^3$ for $f_D = 0.5$ and detection efficiency $\epsilon_{\text{det}} = 1$. The neutrino energy that produces these events is assumed to be $E_0 = m_\chi/2$. Allowed parameters are below the curves in the figures. An observing time of 10 years is assumed, and the muon energy at the detector is required to be above 10^7 GeV .

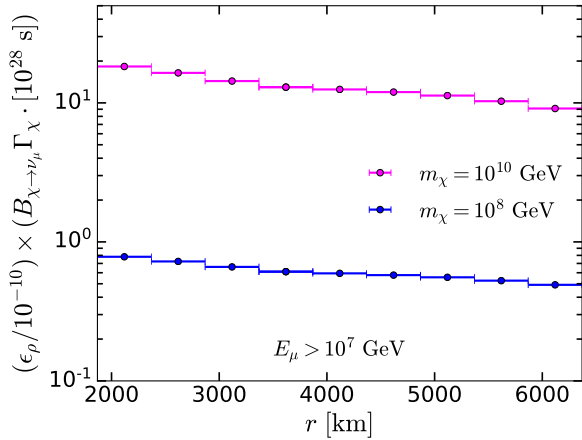


FIG. 9: For two DM masses, constraints on the $\epsilon_\rho \times B_{\chi \rightarrow \nu\mu} \Gamma_\chi$ as in fig. 8 for constant density dark matter $\epsilon_\rho \times 5.5 \text{ g/cm}^3$ for DM in radial shells, for an isotropic detector in ice with a volume of $f_D^3 \times 0.8 \text{ km}^3$ for $f_D = 0.5$ and detection efficiency $\epsilon_{\text{det}} = 1$. The neutrino energy that produces these events is assumed to be $E_0 = m_\chi/2$. Allowed parameters are below the curves in the figures. An observing time of 10 years is assumed, and $E_\mu > 10^7 \text{ GeV}$ at the detector.

and 9.

B. Constraints from the Galactic center observations

IceCube observations of the Galactic center (GC) provide an alternative probe of SHDM decay [48]. If the

lifetime of the SHDM particle is longer than the age of the Universe, $\tau_\chi > t_U$, the differential $\nu + \bar{\nu}$ flux per flavor from SHDM decay in a cone of half-angle ψ around the GC, covering a field of view $\Delta\Omega = 2\pi(1 - \cos\psi)$, is given by

$$\frac{d\Phi}{dE_\nu} = \frac{\Delta\Omega}{4\pi} \mathcal{J}_{\Delta\Omega} \frac{R_{\text{sc}} \rho_{\text{DM,gal}} \Gamma_{\chi \rightarrow \nu H}}{m_\chi} \frac{1}{3} \frac{dN}{dE_\nu} \quad (13)$$

where $dN/dE_\nu = \delta(m_\chi/2 - E_\nu)$ is the $\nu + \bar{\nu}$ spectrum produced per decay, $R_{\text{sc}} = 8.5 \text{ kpc}$ is the solar radius circle, $\Gamma_{\chi \rightarrow \nu H}$ is the partial decay width, $\rho_{\text{DM,gal}}$ is the normalizing DM density introduced in Sec. II (which is equal to the commonly quoted DM density at R_{sc}) and $\mathcal{J}_{\Delta\Omega}$ is the average in the field of view (around the GC) of the line-of-sight integration of the DM density, which is found to be

$$\mathcal{J}_{\Delta\Omega} = \frac{2\pi}{\Delta\Omega} \frac{1}{R_{\text{sc}} \rho_{\text{DM,gal}}} \int_{\cos\psi}^1 \int_0^{l_{\text{max}}} \rho(r) dl d(\cos\psi'), \quad (14)$$

where $l_{\text{max}} = \sqrt{R_{\text{halo}}^2 - R_{\text{sc}}^2 \sin^2\psi} + R_{\text{sc}} \cos\psi$ and $\rho_{\text{DM}}(r)$ is the DM density as a function of the distance from the GC, with $r = \sqrt{R_{\text{sc}}^2 - 2lR_{\text{sc}} \cos\psi' + l^2}$ [49]. For $R_{\text{halo}} \gtrsim 10 \text{ kpc}$, eq. (14) barely depends on the size of the halo. The IceCube Collaboration adopted as benchmark a dark matter distribution which follows the Burkert halo profile [50] with best-fit parameters from [51]. Other halo profiles (e.g. Navarro-Frenk-White [52]) were considered as systematic uncertainties. An in-depth analysis of the uncertainty in the DM distribution and the implications for the potential sensitivity to DM annihilation or decay in the GC appears in ref. [53].

The null result from IceCube searches can be used to set an upper bound on $d\Phi/dE_\nu$, which can be translated into an upper bound on the partial decay width as a function of m_χ . The limit from IceCube null results is approximately $B_{\chi \rightarrow \nu H} \Gamma_\chi \lesssim 10^{-28} \text{ s}^{-1}$ [48] for $m_\chi \approx 10^7 \text{ GeV}$, and decreases weakly with m_χ roughly as $(m_\chi/10^7 \text{ GeV})^{-0.33}$. From the non-observation of upward going neutrinos originated in the decay of SHDM trapped inside the Earth shown in fig. 6, for $m_\chi = 10^7 \text{ GeV}$ the limit is $(\epsilon_\rho/10^{-10}) \times (B_{\chi \rightarrow \nu\tau} \Gamma_\chi) \lesssim 3 \times 10^{-29} \text{ s}^{-1}$. For a nominal value of $\epsilon_\rho = 10^{-10}$, this gives $B_{\chi \rightarrow \nu\tau} \Gamma_\chi \lesssim 3 \times 10^{-29} \text{ s}^{-1}$, comparable to IceCube's GC limits. Our calculation of starting events when ν_τ interacts in the detector applies equally well to starting events for $\chi \rightarrow \nu_e H$ and $\chi \rightarrow \nu_\mu H$. Limits from muon tracks from $\chi \rightarrow \nu_\mu H$ are more stringent than from starting events. For $m_\chi = 10^8 \text{ GeV}$, $(\epsilon_\rho/10^{-10}) \times (B_{\chi \rightarrow \nu\mu} \Gamma_\chi) \lesssim 6 \times 10^{-29} \text{ s}^{-1}$. The limit weakens with m_χ according to $\sim (m_\chi/10^8 \text{ GeV})^{0.7}$. Should SHDM signals from the GC be observed, ϵ_ρ can be extracted.

V. DISCUSSION AND SUMMARY

Recent interest in DM models that predict that heavy DM will not immediately sink to the center of the Earth

have motivated our investigation of signals of dark matter decay from a non-standard DM density profile in the Earth. We have investigated the potential for an underground detector like IceCube to constrain the fraction of the Earth's matter density comprised of DM in a scenario where DM has a long lifetime and has a two-body decay to a neutrino, with our main focus on tau neutrino signals.

Our assumptions that the distribution of DM mass is either a fixed fraction of the Earth's density as a function of radius or a uniform density yield similar results. The limits on $(\epsilon_\rho/10^{-10})B_{\chi\rightarrow\nu_\tau}\Gamma_\chi$ are within a factor of ~ 1.6 of each other. The characteristic constraints lie in the range of $(\epsilon_\rho/10^{-10})B_{\chi\rightarrow\nu_\tau}\Gamma_\chi \lesssim 3 \times 10^{-29} - 3 \times 10^{-28} \text{ s}^{-1}$ for $m_\chi \sim 10^7 - 10^{10} \text{ GeV}$. For a DM mass range of $10^8 - 10^{10} \text{ GeV}$, we find that the constraints range between $(\epsilon_\rho/10^{-10})B_{\chi\rightarrow\nu_\mu}\Gamma_\chi \lesssim 6 \times 10^{-29} - 1.4 \times 10^{-27} \text{ s}^{-1}$.

This paper proposes a starting point for measurements of upward-going events, complementary to Galactic center observations, to constrain $\epsilon_\rho \times B_{\chi\rightarrow\nu}\Gamma_\chi$.

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