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Global Social Interactions with Sequential Binary Decisions: The Case of Marriage, Divorce, and Stigma

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Abstract

This paper studies global social interactions in a stylized model of marriage and divorce with complementarities across agents. The key point of departure from traditional models of social interactions is that actions are interrelated and sequential. We establish existence and uniqueness results akin to those in traditional models. In contrast to these models, however, we show that the presence of strategic complementarities is no longer sufficient to generate a social multiplier that exceeds one in this environment. Self-fulfilling conformity, whereby a greater desire to conform at the individual level leads to greater homogeneity of choices in the aggregate, is not retained either. Some empirical implications are also discussed.

Keywords: social interactions, social multiplier, self-fulfilling conformity, uniqueness under moderate social influence

1 Introduction and Summary

Canonical models of social interactions examine agents who select an action from a unidimensional finite choice set or an interval of the real line (e.g., Brock and Durlauf, 2001 or Glaeser and Scheinkman, 2003). More recently, Horst and Scheinkman (2006) generalized the analysis by allowing for local and global social interactions in a model with multidimensional, continuous actions. In this paper we study global social interactions with interdependent and binary sequential actions in more than one dimension. As in previous papers, we prove the existence of an equilibrium under general conditions and uniqueness under tighter conditions that limit the influence of peer groups on individual preferences. However, this is the first paper to show that strategic complementarities are not sufficient to generate a social multiplier greater than one in an environment where agents face a multidimensional choice set. We also identify and investigate the robustness of self-fulfilling conformity, a property which states that choices become more homogeneous when individuals' taste for conformity increases. Quite interestingly, while the wider literature on social interactions implicitly takes this property as given, we show that self-fulfilling conformity need not hold in our environment.

We develop these results in a concrete model of marriage and divorce to demonstrate their relevance and to clarify their intuition rather than to illuminate any facts about marriage and divorce *per se*, although the latter may be possible. Marriage and divorce decisions are clearly sequential and interdependent; the decision to marry or not depends on the likelihood of divorce, and a couple are less likely to divorce if they had been more selective about whom to marry.

Moreover, the actions of other agents can influence individual decisions through the search market, or by changing the social stigma (or reward) attached to any action. For example, the opportunity cost of marriage increases as the size of the single pool increases because one is more likely to meet attractive mates in a "thicker" market. Similarly, one is more inclined to divorce as the size of the divorcé market grows since the chance of successful remarriage increases.[1] Finally, for some people marriage and divorce decisions are affected by prevailing norms. If most people marry relatively young, individuals may feel compelled to also marry young. If divorce is

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[1] These search externalities have long been incorporated into job search models (e.g., Rogerson, Shimer, and Wright, 2005 or Pissarides, 2000) as well as models of mate search (e.g., Mortensen, 1988).

rare, a troubled couple may decide to stay together to avoid the stigma of divorce.

These mechanisms give rise to strategic complementarities on each dimension of decision making. That is, the incentive to marry young increases in the proportion of the population that marries young, and the incentive to divorce increases in the divorce rate. As such, we adopt a reduced form modeling approach where we directly assume the existence of strategic complementarities.

The Social Multiplier. *Strategic complementarities* arise when the marginal util- ity to one person of taking an action increases in the average level of the action taken by members of the agent's peer group. (An agent's peer group is the set of peo- ple whose actions influence the agent's preferences.) Thus, each agent's behavior is affected by exogenously given fundamentals and by the endogenously determined behavior of his peers.[2] If equilibrium is unique or there is some selection device, in canonical models complementarities guarantee that a change in fundamentals has both a direct effect and an indirect effect on behavior that work in the same direc- tion. This results in a *social multiplier,* as in Becker and Murphy (2001). "This social multiplier can also be thought of as a ratio $\Delta P/\Delta I,$ where I is the average response of an individual action to an exogenous parameter (that affects only that person) and P is the (per capita) response of the peer group to a change in the same parameter that affects the entire peer group." (Scheinkman, 2008) Framed this way, strategic complementarities produce a social multiplier that exceeds one (e.g., Glaeser and Scheinkman, 2003).

Surprisingly, strategic complementarities are not sufficient to generate a social multiplier larger than one when multiple decisions are interdependent. To see this, consider what happens to the long run likelihood of divorce when the private benefit to marriage increases, say because of a tax break for married couples.

The typical social multiplier intuition would tell us that if the tax break applied to only one couple and that couple's probability of divorce decreases as a result of the tax break, then a tax break applied to everybody would further decrease the couple's probability of getting divorced as their peer group of divorced couples in the economy is now smaller. But this intuition ignores the fact that strategic complementarities are also acting on the decision to marry (i.e., the interdependent decision). As more

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[2] See Manski (1993) for an excellent exposition on the challenges of empirically identifying en- dogenous social effects.

people marry, the social incentive to marry is raised and thus selectivity into marriage is lowered. The tax break therefore increases the number of low quality marriages, which could actually increase the divorce rate. In this case the probability of divorce would decrease by less than if the tax break was only applied to one couple so that the social multiplier on the probability of divorce is less than one.

Similarly, the social multiplier on the probability of remaining single can be less than one when individuals are forward-looking. If the probability of divorce falls with a marriage tax break after accounting for all the direct and indirect effects, this will increase the social cost of divorce and force unhappy couples to stay together. The net effect may be that the expected social value of marriage in the future life periods actually falls. Anticipating this unpleasant outcome, singles may lower their selectivity into marriage by less than they would when the tax break applies to only them. We illustrate this possibility with a numerical example in Section 5.

Self-fulfilling conformity. In many models of social interactions with strategic complementarities, the degree of homogeneity is an increasing function of the individual desire to conform. As a useful expedient, let us call this seemingly tautological relationship *self-fulfilling conformity*.[3] To date the literature has focused on how even a small desire to conform can lead to a relatively large degree of homogeneity (e.g., Bernheim, 1994; Schelling, 1971). As Bernheim (1994: 844) puts it: "When status is sufficiently important relative to intrinsic utility..., many individuals conform to a single, homogeneous standard of behavior, despite heterogeneous underlying preferences." We take the analysis in a different direction and show that self-fulfilling conformity is not a tautology.

In the model below, self-fulfilling conformity breaks down because of the interdependence between marriage and divorce decisions. If young marriage is common and divorce is rare, the self-fulfilling conformity intuition indicates that an increase in the desire to conform would result in more young marriages and a lower divorce rate. However, more young marriages occur only when selectivity into marriage falls. If this drop in selectivity is significant, the divorce rate may actually increase because of the larger share of low quality marriages. In other words, an increase in the taste for conformity may increase heterogeneity in divorce decisions. A numerical example in Section 5 illustrates that a greater taste for conformity can increase heterogeneity

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[3] A precise definition is given in Section 4.3 in the context of the model.

in marriage decisions when agents are forward looking as well.

Uniqueness under Moderate Social Influence. The social multiplier and self-fulfilling conformity are well-defined only if equilibrium is unique or an appropriate selection device is used. This is an important qualification in models with strategic complementarities as they tend to generate multiple equilibria. A general finding in the literature is that equilibrium is unique under conditions that limit the influence of peers on individual preferences (Brock and Durlauf, 2001; Glaeser and Scheinkman, 2003; Horst and Scheinkman, 2006)[4], a property Glaeser and Scheinkman (2003) call *uniqueness under moderate social influence (MSI)*. Horst and Scheinkman (2006) show that uniqueness under MSI obtains in a model where actions are continuous and the choice set is multidimensional.[5] This paper proves this result for discrete and sequential actions in more than one dimension.

Equilibrium is characterized by a pair of equations which implicitly define the selectivity into marriage and the divorce threshold. To prove uniqueness, we first recast this pair of equations as an implicitly defined discrete-time dynamic system. Then we find a sufficient condition under which a fixed point of such a system is globally asymptotically stable (and hence unique). This result, Lemma 2, may be of independent interest and generalizes Proposition 2 in Fujimoto (1986) to the case of implicitly defined systems. When applied to the model, this sufficiency condition limits the influence of peer groups on individual preferences, that is, uniqueness under MSI obtains. A benefit of this approach is that if the condition is met at equilibrium then the equilibrium is locally asymptotically stable. Like Brock and Durlauf (2001), we use local (asymptotic) stability as a selection device. This ensures that the social multiplier and self-fulfilling conformity are well-defined.

The paper proceeds as follows. The next section describes the model and defines the peer groups. Section 3 characterizes equilibrium and proves existence. Section 4 is the main section of the paper. It contains the proof of uniqueness under MSI and the conditions for local asymptotic stability. The section continues by formally showing that when agents are myopic, the social multiplier on the probability of

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[4] This statement presumes that a benchmark model without social interactions has a unique equilibrium.

[5] Horst and Scheinkman (2006) do not investigate the robustness of self-fulfilling conformity nor the social multiplier to a multidimensional choice set, however.

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divorce may be less than one and that self-fulfilling conformity may not hold among the old and married. Section 5 presents numerical examples with forward-looking agents where the social multiplier on the probability of marriage is less than one and where self-fulfilling conformity fails to obtain among the young. Section 6 discusses some empirical implications of the analysis and Section 7 concludes.

2   The Model

2.1   Meetings, Choices, and Match Quality

A countably infinite number of identical agents indexed by $i \in \{1, ..., \infty\}$ begin life single and advance together through two stages of life, young (y) and old (o).

When young, agent i is paired with agent $i + 1$ for i odd. A pair of matched singles is called couple i for brevity even though i refers to just one person. A couple decides to remain single or marry based on their *match quality*, which is an observable random variable Θ_y assigned by nature and common to both agents. Let θ^i_y be couple

$i's$ realization of Θ_y. Assume Θ_y is identically and independently distributed across couples according to the distribution F_y. Assume F_y is continuously differentiable, has finite mean, and has a support large enough so that a positive measure of singles chooses to marry and a positive measure chooses to remain single.

When agents are old, those who did not marry while young must remain single when old. Those who married when young must decide to remain married or divorce based on a new match quality Θ_o assigned by nature. Couple $i's$ realization of Θ_o, denoted θ^i_o, is observed by and common to both agents. The distribution of Θ_o given $\Theta_y = \theta_y$ is $F_o(.|\theta_y)$ and is *i.i.d.* across couples. Assume F_o is continuously differentiable in both arguments, has finite mean, and is increasing in θ_y in the sense of first order stochastic dominance: for any z, $\frac{\partial F_o(z|\theta_y)}{\partial \theta_y} < 0$. This assumption captures the intuition that expected match quality when old should be increasing in match quality when young. The support of $F_o(.|\Theta_y)$ is assumed large enough so that a positive measure of married couples divorces and a positive measure remains married.

An individual's *marital status* is denoted *s, m,* and *d* for single (never married), married, and divorced, respectively. The possible life paths are illustrated in Figure 1. The assumptions that singles may marry only when young and that remarriage is impossible are made for the sake of parsimony. A richer model would not change
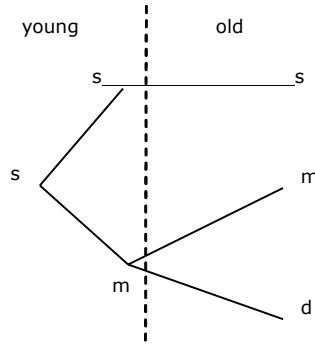
Figure 1: Possible life paths.

the qualitative nature of the results because the number of decision nodes equals the number of equations in the system that describes the equilibrium and the results are independent of the system's dimensionality, so long as it is at least two and finite.

2.2 Payoffs

Let l index the life stage, $l \in \{y, o\}$. Payoffs depend on marital status η, realized match quality θ_l^i (if married), and the proportion of an agent's peer group that shares his marital status $p = p(\eta, l)$. Specifically,

$$(1) \qquad U(\eta, l, \theta_l^i) = u(\eta, \theta_l^i) + \lambda v(p(\eta, l)).$$

Payoffs are received at the end of a life stage and are a weighted sum of the private payoff u (determined exogenously) and the social payoff v (determined endogenously). The parameter $\lambda \geq 0$ can be interpreted as the *taste for conformity*; social payoffs affect preferences more as the taste for conformity increases. The additive separation between private and social payoffs, and the quasi-linearity imposed below, are assumptions designed to increase the transparency of the results. The results hold for the more general payoff function $U(\eta, l, \theta_l^i) = U(\eta, \theta_l^i, \lambda v(p(\eta, l)))$.

2.2.1 Private Payoffs (u)

The private payoff to singlehood is normalized to zero: $u(s, .) = 0$. The private payoff to marriage is $u(m, \theta_l^i) = \gamma + \theta_l^i$ [6] The constant $\gamma \in \mathbb{R}$ represents the gains or losses for the typical couple from marriage versus singlehood, which may arise from things like economies of scale in household production, the tax treatment of married people versus singles, etc. Match quality represents the idiosyncratic payoffs to marriage, including love. A divorcé receives the payoff to singlehood but must also incur the cost of divorce, $c \in \mathbb{R}_+$. Thus, $u(d, .) = -c$.

2.2.2 Social Payoffs (v)

The *social payoff* captures the impact that peer groups have on preferences. Since $p = p(\eta, l)$ is the proportion of an agent's peer group that shares his marital status, assume $v(p)$ is strictly increasing in p to get strategic complementarities.[7] We also make the technical assumption that v is smooth on its compact domain $[0, 1]$. This guarantees that v and its derivatives are bounded. Let $\bar{v} \equiv v(1)$ denote the maximum value of the function v.

An agent's peer group depends on his marital status and life stage. Peer groups are specific to the life stage; social payoffs when young are based only on what other young people are doing whereas social payoffs when old are defined relative to the behavior of other old people. Since everyone starts out single and must choose to remain single or marry when young, the peer group when young is simply the entire population. Letting α be the proportion of young people who remain single and $1 - \alpha$ be the proportion who marry young (the marriage rate), we have $p(s, y) = \alpha$ and $p(m, y) = 1 - \alpha$. We will sometimes refer to α as the size of the single pool.

While the whole population is the natural peer group for young people, there is not a clear choice for the old because individuals can begin this life stage either single or married. At least two reasonable peer groups may be specified. One option is to define it as the subpopulation of individuals who share the same marital status at the beginning of the second stage of life. As justification, one might argue that the

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[6] Of course, this assumes that the privately consumed components of marital surplus are split equally in each period. For example, one can think of match quality as a public good and define 2γ to be the private component of total marital surplus.

[7] Since social payoffs depend only on aggregate statistics, the social interactions in this paper are global.

Table 1: The payoff function $U(\eta, l, \theta^i)_t$

Life Period	Marital Status	Payoff
Young (y)	s	$\lambda v(\alpha)$
	m	$\theta_y^i + \gamma + \lambda v(1-\alpha)$
Old (o)	s	λv
	m	$\theta_o^i + \gamma + \lambda v(1-\delta)$
	d	$-c + \lambda v(\delta)$

stigma of divorce should depend only on the divorce rate δ (i.e., the proportion of *marriages* that end in divorce) and not on the proportion of the *population* that is divorced, $(1-\alpha)\delta$. Alternatively, the entire population might be chosen as a peer group so that what matters is only the partition of the population at the end of the period. In the absence of any theory or evidence that supports one approach over the other, we assume the former primarily because it facilitates the analysis. This gives $p(d, o) = \delta$ and $p(m, o) = 1 - \delta$. Also, $p(s, o) = 1$ since individuals who chose to remain single when young must remain single when old. The total payoff function is summarized in Table 1.

The key results in the paper are driven by the fact that marriage and divorce decisions are interdependent. This interdependence arises regardless of the peer group specified, so the qualitative nature of the results is not sensitive to the choice of peer group.

2.3 Expectations and Timing

To complete the model, assume that individuals are expected payoff maximizers. Expectations about match quality are calculated from the distributions F_y and $F_o(\cdot|\theta_y)$. Expectations about peer group behavior are rational and determined in equilibrium. Figure 2 summarizes the within period timing. It should be emphasized that, within life stages, agents make decisions simultaneously.

3 Equilibrium

A strategy for an agent maps the set of possible realized match qualities into a feasible action at each of the two decision nodes. A strategy profile constitutes an *equilibrium* if, for every agent and at every decision node, the expected payoff to an agent's
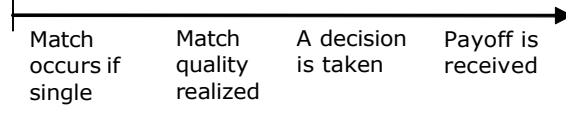
Figure 2: Within period timing.

strategy is at least as good as any alternative strategy, holding the strategies of all other agents fixed.

The behavior of any single agent (or pair of agents) has no impact on the *state variables* α and δ since the population is infinitely large. Consequently, one can derive a particular agent's optimal strategy taking the state variables as given. Recall also that agents are identical and two agents in a couple receive the same payoffs to marital status choices. Thus there will never be disagreement about which action to take and we may analyze the couple's choice as an individual decision problem. We use the logic of backward induction to show that the unique optimal strategy is a cutoff strategy at each decision node.

If an individual begins the second life stage married, he may either remain married or obtain a divorce. The agent, taking the strategies of other agents as given (and hence the values of α and δ), chooses to remain married if and only if[8]

$$\theta_o + \gamma + \lambda v(1 - \delta) \geq -c + \lambda v(\delta),$$

where we have suppressed the i superscript on θ_o. The left hand side of the inequality is the payoff to remaining married while the right hand side is the payoff to getting a divorce. Thus, the optimal strategy is a cutoff strategy such that an individual remains married if and only if the realized match quality, θ_o, exceeds some *divorce threshold*, z, where z solves

(2) $$z + \gamma + \lambda v(1 - \delta) = -c + \lambda v(\delta).$$

A young individual must decide to remain single or marry. Letting $0 \leq \beta < \infty$

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[8]This condition implies that the agent will choose marriage if he is indifferent between marriage and divorce. This tie-breaking rule is arbitrary and has no consequence for equilibrium because the continuity of the distributions that generate match quality implies only a null set of agents will receive a match quality that makes them indifferent between these two options.

be the time preference factor, a young agent marries her match iff

$$\theta_y + \gamma + \lambda v(1 - \alpha) + \beta E[V(m)|\theta_y] \geq \lambda v(\alpha) + \beta\lambda\bar{v},$$

where $V(m) = \max\{\theta_o + \gamma + \lambda v(1 - \delta), -c + \lambda v(\delta)\}$. The left hand side of this inequality is the expected value of marriage while the right hand side is the value of singlehood. The term $\beta E[V(m)|\theta_y]$ is the discounted expected value of being married in the second period given $\Theta_y = \theta_y$. Since this term is increasing in θ_y by first order stochastic dominance, $\theta_y + \beta E[V(m)|\theta_y]$ traverses the real line as θ_y traverses the real line. The unique optimal strategy is therefore a cutoff strategy such that an individual marries iff $\theta_y \geq x$, where x is defined implicitly by

(3) $$x + \gamma + \lambda v(1 - \alpha) + \beta E[V(m)|x] = \lambda v(\alpha) + \beta\lambda\bar{v}.$$

Call x the *selectivity when young*.

Since all agents are optimally using the same cutoff strategy in any equilibrium, o and δ can be straightforwardly calculated for any set $\{x, z\}$ of cutoff values.

Lemma 1 *For any set of cutoff values $\{x, y\}$,*

$$\alpha = \alpha(x) = \Pr(\Theta_y < x) \text{ almost surely and}$$
$$\delta = \delta(x, z) = \Pr(\Theta_o < z|\Theta_y \geq x) \text{ almost surely.}$$

Proof. See Appendix. ∎

On a technical note, this is the key observation that allows the population equilibrium to be characterized (*a.s.*) by a system of deterministic equations. In particular, Lemma 1 allows equations (2) and (3) to be rewritten as follows:

(4a) $$z = -\gamma - c + \lambda[v(\delta(z, x)) - v(1 - \delta(z, x))],$$
(4b) $$x = -\gamma + \lambda[v(\alpha(x)) - v(1 - \alpha(x))] - \beta E(x, z) + \lambda\beta\bar{v},$$

where $E(x, z) \equiv \Pr(\Theta_o \geq z|x)[E(\Theta_o|\Theta_o \geq z, x) + \gamma + \lambda v(1 - \delta(z, x))] + \Pr(\Theta_o < z|x)[-c + \lambda v(\delta(z, x))]$. It follows that the equilibrium cutoff strategies are a fixed point of system (4). We prove in the Appendix that an equilibrium always exists.

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The proof first shows that we can restrict our search for an equilibrium to a compact and convex set, and then applies Brouwer's fixed point theorem.

Theorem 1 *An equilibrium exists.*

Proof. See Appendix. ∎

4 Analysis

From the discussion in the introduction, three questions will guide the analysis: (i) Under what conditions is there a unique equilibrium? (ii) How does the population react to a change in the fundamentals (e.g., a change in γ)? (iii) How does an increase in the taste for conformity, λ, affect homogeneity? These questions relate to uniqueness under MSI, the social multiplier, and self-fulfilling conformity, respectively.

4.1 Uniqueness under MSI

Recall that uniqueness under moderate social influences (MSI) says that the population will arrive at a unique equilibrium if individual preferences are not too sensitive to changes in the social environment. To show that this property holds in the present model, we follow Brock and Durlauf (2001) by first recasting the equilibrium system (4) as a dynamic one. Then we look for conditions that guarantee asymptotic stability. This is a fruitful approach because when the condition is applied globally we guarantee uniqueness; applied locally, we ensure the equilibrium is locally asymptotically stable. The comparative statics exercises to follow are meaningful at a locally asymptotically stable equilibrium even if there are multiple equilibria.

The cutoff values that determine the state variables are lagged in the dynamic analogue to system (4):

$$z_t + \gamma + c - \lambda \left[v(\delta(x_{t-1}, z_{t-1})) - v(1 - \delta(x_{t-1}, z_{t-1})) \right] = 0$$

(5)

$$x_t + \gamma - \lambda \left[v(\alpha(x_{t-1})) - v(1 - \alpha(x_{t-1})) \right] + \beta E(x_t, x_{t-1}, z_t, z_{t-1}) - \lambda \beta \bar{v} = 0,$$

where $E(x_t, x_{t-1}, z_t, z_{t-1}) \equiv \Pr(\Theta_o \geq z_t | x_t) [E(\Theta_o | \Theta_o \geq z_t, x_t) + \gamma + \lambda v(1 - \delta(x_{t-1}, z_{t-1}))] +$
$\underline{\Pr(\Theta_o < z_t | x_t)} [-c + \lambda v(\delta(x_{t-1}, z_{t-1}))]$.[9] A crucial observation is that system (5) has

[9] While we use this formulation solely as a technical device, one may think of this dynamic

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the same set of fixed points as system (4). Thus, equilibrium is unique in the dynamic version iff it is unique in the static version.

One difficulty with this approach is that the dynamic system (5) is implicitly defined, so standard stability results for dynamic systems of the form $a_{t+1} = H(a_t)$ do not directly apply. We resolve this issue in the next lemma by extending a standard global asymptotic stability result (e.g., Proposition 2 in Fujimoto, 1986) to implicitly defined discrete dynamic systems.

For convenience, we first review some concepts from stability theory. Given an initial condition a_0 and letting $H : \mathsf{R}^n \to \mathsf{R}^n$, the dynamic system $a_{t+1} = H(a_t)$ generates a sequence of points $\{a_0, a_1, a_2, ...\}$ called the forward orbit. A point $\bar{a} \in \mathsf{R}^n$ is an equilibrium or fixed point of this system if $\bar{a} = H(\bar{a})$. A fixed point $\bar{a}$ is said to be *stable* if for any $\varepsilon > 0$ there exists $\sigma > 0$ such that whenever $\|a_0 - \bar{a}\| < \sigma$, the points a_t in the forward orbit satisfy $\|a_t - \bar{a}\| < \varepsilon$ for $t > 0$. A fixed point $\bar{a}$ is *(locally) asymptotically stable* if it is stable and, in addition, there exists $r > 0$ such that for all a_0 satisfying $\|a_0 - \bar{a}\| < r$, the iterates a_t satisfy $\lim_{t \to \infty} a_t = \bar{a}$. A fixed point $\bar{a}$ is a *global attractor* on an interval I if $a_0 \in I$ implies $\lim_{t \to \infty} a_t = \bar{a}$. A fixed point $\bar{a}$ is *globally asymptotically stable* if it is stable and is a global attractor.

Lemma 2 *Let $H : A \times A \to A$ for $A \subset \mathsf{R}^n$ define the dynamic system*

(6)
$$H(a_{t+1}; a_t) = 0$$

with component functions $H^i(a_{i,t+1}; a_t)$, where $a_{i,t+1} \in A^i \subset \mathsf{R}$, $i = 1, ..., n$, and $A^1 \cup \cdots \cup A^n = A$. Assume for all i

   a) *H^i is strictly monotone in $a_{i,t+1}$ and continuously differentiable,*

   b) *$\forall\, a_t \in A,\ \exists\, a_{t+1} \in A$ such that $H(a_{t+1}, a_t) = 0$, and*

   c) *A is non-empty and convex.*

   If H has a fixed point, it is globally asymptotically stable (on A) so long as, for any (a_{t+1}, a_t) in $A \times A$ and some matrix p-norm $\|.\|_p$

(M)
$$\left\| 1 - [D_{a_{t+1}} H]^{-1} D_{a_t} H \right\|_p \leq 1.$$

Proof. See Appendix. ∎

$D_y H$ is the matrix of partial derivatives induced by differentiating H with respect to the vector y. A matrix p-norm $|.|_p$ is a matrix norm induced from a ℓ_p vector norm.[10] If condition (M) holds, we can find a Liapunov function which then implies that a globally asymptotically stable equilibrium exists.

A little more notation is needed to apply the result. Let $\bar{v}'$ be the maximum value that the derivative of the social payoff function v can take. (Recall that $v' \geq 0$ by assumption). Similarly, let $\bar{s}$ be the supremum of the set of absolute values that the derivatives of $\alpha(x)$ and $\delta(z, x)$ can take with respect to any variable. Since $\alpha_x, \delta_z \geq 0$ and $\delta_x \leq 0$, this means $\alpha_x, \delta_z, |\delta_x| \leq \bar{s}$.[11] Finally, let $\lambda \leq \bar{\lambda}$.

Theorem 2 (Uniqueness under MSI) *Some combination of the bounds $\bar{v}'$, $\bar{s}$ and $\bar{\lambda}$ can be tightened to ensure that condition (M) is satisfied in system (5). In this case equilibrium is unique.*

It is important to note that the bounds restrict the magnitude of the *changes* in the social payoff function and not necessarily the *size* of the social payoffs. It is possible to have a unique equilibrium if social payoffs strongly affect preferences (i.e., λ is large) if the bounds $\bar{v}'$ or $\bar{s}$ are small. However, a small taste for conformity (λ small) is also sufficient to guarantee uniqueness.

Qualitatively, this is exactly what drives uniqueness under (average) moderate social influence in Glaeser and Scheinkman (2002) and Horst and Scheinkman (2006). The first of these papers derives this result when the choice set is unidimensional while the second accommodates an n-dimensional choice set. However, both papers focus on continuous actions whereas actions are discrete in this paper. Consequently, the uniqueness result in Horst and Scheinkman (2006) does not directly apply here.

The proof of Theorem 2, which is an application of Lemma 2, is instructive and elements of it will be used later in the paper. Observe that we have already proven or assumed that system (5) satisfies the three regularity conditions of Lemma 2, and the existence of a fixed point was established in Theorem 1. It remains to show that condition (M) is satisfied under the hypotheses of Theorem 2.

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[10] Recall that for each ℓ_p vector norm $||_p$ on R^k one may define an associated matrix norm of a $k \times k$ matrix A by $|A| = \max_{|x|_p \neq 0} \frac{|Ax|_p}{|x|_p}$.

[11] Here and throughout the rest of the paper, y_x is defined as the partial derivative of y with

respect to x, $y_x \equiv \frac{\partial y}{\partial x}$

The matrix in condition (M), suppressing time subscripts, equals[12]

$$
\begin{vmatrix} 1 & 0 \\ -f(z|x)(z-R) & 1+\beta\dfrac{\partial E}{\partial x_t} \end{vmatrix}^{-1} \times \begin{vmatrix} -\lambda\delta_z T & -\lambda\delta_x T \\ \beta\lambda\delta_z Q & -\lambda\alpha_x S + \beta\lambda\delta_x Q \end{vmatrix}
$$

$$
= \begin{vmatrix} \lambda\delta_z T & -\lambda|\delta_x|T \\ \dfrac{\lambda\delta_z[f(z|x)(-z+R)T+\beta Q]}{1+\beta\frac{\partial E}{\partial x_t}} & \dfrac{-\lambda\alpha_x S-\lambda|\delta_x|[f(z|x)(-z+R)T+\beta Q]}{1+\beta\frac{\partial E}{\partial x_t}} \end{vmatrix}.
$$

where

$$
\begin{aligned}
Q &\equiv \Pr(\Theta_o < z|x)\,v'(\delta) - \Pr(\Theta_o \geq z|x)\,v'(1-\delta), \\
R &\equiv -c - \gamma + \lambda(v(\delta) - v(1-\delta)), \\
S &\equiv v'(\alpha) + v'(1-\alpha), \\
T &\equiv v'(\delta) + v'(1-\delta),\ \text{and} \\
\frac{\partial E}{\partial x_t} &= \int_z^{\varphi(x)}(\theta_0 - R)\frac{\partial f(\theta_o|x)}{\partial x}d\theta_o + (\varphi(x) - R)f(\varphi(x)|x)\frac{\partial\varphi(x)}{\partial x}.
\end{aligned}
$$

Condition (M) is satisfied if each of the entries in this matrix is small enough. For example, the matrix 1−norm for a $k \times k$ matrix A is $\max_{1\leq j\leq k}\sum_{i=1}^{k}|a_{ij}|$, in which

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[12] This calculation allows for the upper limit of the support of $F_o(.\,\theta_y)$ to depend on θ_y. The upper limit of integration $\varphi(x)$ in the integrals below is therefore a function of x. The calculations for the matrix elements use Leibniz's Rule and these two facts:

1. $\Pr(\Theta_o \geq z|x)\,E(\Theta_o|\Theta_o \geq z, x) = \Pr(\Theta_o \geq z|x)\dfrac{\int_z^{\varphi(x)}\theta f(\theta|x)\,d\theta}{\Pr(\Theta_o \geq z|x)} = \int_z^{\varphi(x)}\theta f(\theta|x)\,d\theta,$

2. $\int_{\varphi(x)}^z \Pr(\Theta_o \geq z|x)[E(\Theta_o|\Theta_o \geq z, x) + \gamma + \lambda v(1-\delta(x,z))] + \Pr(\Theta_o < z|x)[-c + \lambda v(\delta(x,z))]$

$= \int_z^{\varphi(x)}\theta_o f(\theta_o|x)\,d\theta_o + \gamma + \lambda v(1-\delta) - \Pr(\Theta_o < z|x)[c + \gamma - \lambda(v(\delta) - v(1-\delta))]$

$= \int_z^{\varphi(x)}(\theta_o + c + \gamma - \lambda(v(\delta) - v(1-\delta)))f(\theta_o|x)\,d\theta_o - c + \lambda v(\delta)$

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case condition (M) is satisfied if for all z and x

$$(7) \qquad \lambda \delta_z T + \frac{|\lambda \delta_z \left(f(z|x)(-z+R)T+\beta Q\right)|}{1 + \beta \frac{\partial E}{\partial x_t}} \;<\; 1 \text{ and}$$

$$(8) \qquad \lambda |\delta_x| T + \frac{|-\lambda \alpha_x S - \lambda |\delta_x| \left(f(z|x)(-z+R)T+\beta Q\right)|}{1 + \beta \frac{\partial E}{\partial x_t}} \;<\; 1.$$

First-order stochastic dominance implies $\frac{\partial E}{\partial x_t} > 0$, which in turn implies that $1 +$

$\beta \frac{\partial E}{\partial x_t}$ is bounded below by one. Thus, inequalities (7) and (8) are satisfied if some

combination of the bounds in Theorem 2 is tight enough. This completes the proof

of Theorem 2.[13]

Fortunately, when we wish to apply condition (M) only locally around an equi-
librium we can take advantage of the fact that $R = z$. This simplifies the matrix to

$$(9) \qquad \begin{bmatrix} \lambda \delta_z T & -\lambda |\delta_x| T \\ \dfrac{\lambda \delta_z \beta Q}{1+\beta \frac{\partial E}{\partial x_t}\big|_{R=z}} & \dfrac{-\lambda \alpha_x S - \lambda |\delta_x| \beta Q}{1+\beta \frac{\partial E}{\partial x_t}\big|_{R=z}} \end{bmatrix}.$$

The corresponding inequalities become

$$(10) \qquad \lambda \delta_z T + \frac{|\lambda \delta_z \beta Q|}{1 + \beta \frac{\partial E}{\partial x_t}\big|_{R=z}} \;<\; 1 \text{ and}$$

$$(11) \qquad \lambda |\delta_x| T + \frac{|-\lambda \alpha_x S - \lambda |\delta_x| \beta Q|}{1 + \beta \frac{\partial E}{\partial x_t}\big|_{R=z}} \;<\; 1.$$

4.2   The Social Multiplier

This subsection investigates the direct and indirect effects on behavior of a marginal
increase in the private benefit to marriage γ. The goal is to show that the social
multiplier can be less than one despite the presence of strategic complementarities. It
is understood that the analysis takes place at an equilibrium where condition (M) is
satisfied.

The two choice sets agents potentially face in life are {remain single, marry} when young and {divorce, remain married} when old and married. Since choice sets are

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discrete, a "behavior" is interpreted as the probability of taking an action. As there are two choice sets, we are interested in the reaction of two behaviors to an increase in γ: the probability of remaining single, $\Pr(\Theta_y < x)$, and the expected probability of divorce given marriage, $\Pr(\Theta_o < z | \Theta_y \geq x)$.[14]

The social multiplier on any given behavior is the equilibrium response to a change in γ divided by the individual response that would occur if the change in γ applied to only one couple. For example, the equilibrium change in the probability of remaining single is $\frac{\partial \Pr(\Theta_y < x)}{\partial x}\frac{dx}{d\gamma}$. If γ increases for just one pair of matched singles, the resulting change in their behavior has no effect on α or δ (the size of the single pool and the divorce rate, respectively) since the couple has measure zero. Consequently the pair of matched singles for whom γ increases lowers their selectivity into marriage by $\frac{dx^I}{d\gamma}$, where $\frac{dx^I}{d\gamma} = \frac{dx}{d\gamma}$ when α_x, δ_x, and δ_z are set equal to zero. Thus, the social multiplier on the probability of remaining single is $\left(\frac{\partial \Pr(\Theta_y < x)}{\partial x}\frac{dx}{d\gamma} \middle/ \frac{\partial \Pr(\Theta_y < x)}{\partial x}\frac{dx^I}{d\gamma} \right) = \frac{dx}{d\gamma}\middle/\frac{dx^I}{d\gamma}$.

Call this the *singlehood multiplier*.

Similarly, the equilibrium response to an increase in γ on the probability of divorce if married is $\frac{\partial \Pr(\Theta_o < z | \Theta_y \geq x)}{\partial z}\frac{dz}{d\gamma} + \frac{\partial \Pr(\Theta_o < z | \Theta_y \geq x)}{\partial z}\frac{dx}{d\gamma} = \delta_z\frac{dz}{d\gamma} + \delta_x\frac{dx}{d\gamma}$ (*a.s.*) where we have

used the fact that $\delta = \Pr(\Theta_o < z | \Theta_y \geq x)$ *a.s.* to ease notation. The individual response is $\delta_z\frac{dz^I}{d\gamma} + \delta_x\frac{dx^I}{d\gamma}$. The social multiplier on the probability of divorce given marriage, or the *divorce multiplier*, is then $\left(\delta_z\frac{dz}{d\gamma} + \delta_x\frac{dx}{d\gamma} \middle/ \delta_z\frac{dz^I}{d\gamma} + \delta_x\frac{dx^I}{d\gamma} \right)$.

Totally differentiating system (4) gives

$$\frac{dz}{d\gamma} = -\frac{1 - \lambda\alpha_x S + \lambda\delta_x T(1 + \beta E_y) + \beta E_x}{(1 - \lambda\delta_z T)(1 - \lambda\alpha_x S + \beta E_x) + \lambda\delta_x T\beta E_z} \quad \text{and}$$

$$\frac{dx}{d\gamma} = -\frac{(1 - \lambda\delta_z T)(1 + \beta E_y) - \beta E_z}{(1 - \lambda\delta_z T)(1 - \lambda\alpha_x S + \beta E_x) + \lambda\delta_x T\beta E_z}.$$

Setting $\alpha_x = \delta_x = \delta_z = 0$, letting $E^I_x = E_x$ when $\alpha_x = \delta_x = \delta_z = 0$ and noting that $E^I_z = 0$[15], we have

$$\frac{dz^I}{d\gamma} = -1 \quad \text{and} \quad \frac{dx^I}{d\gamma} = -\frac{1 + \beta E_y}{1 + \beta E^I_x}$$

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[14]One may also think of the probability of divorce if married as the probability of divorce given

an initial match quality, or $\Pr(\Theta_o < z \; \Theta_y = \theta_y)$. We have done the formal analysis for this case and have found that the social multiplier on this divorce behavior may be less than one as well. These results are available upon request.

[15] By Leibniz's Rule, $E'_z = f(z|x)(-z - c - \gamma + \lambda(v(\delta) - v(1 - \delta)))$. Using $z = -c - \gamma + \lambda z(v(\delta) - v(1 - \delta))$ in equilibrium, we have $E' = 0$.

Hence, the singlehood multiplier and the divorce multiplier are

$$(12) \qquad \frac{dx}{d\gamma} \Big/ \frac{dx^I}{d\gamma} = \frac{\frac{(1-\lambda\delta_z T)(1+\beta E_y)-\beta E_z}{(1-\lambda\delta_z T)(1-\lambda\alpha_x S+\beta E_x)+\lambda\delta_x T\beta E_z}}{\frac{1+\beta E_y}{1+\beta E_x^I}}, \quad \text{and}$$

$$(13) \qquad \frac{\delta_z \frac{dz}{d\gamma} + \delta_x \frac{dx}{d\gamma}}{\delta_z \frac{dz^I}{d\gamma} + \delta_x \frac{dx^I}{d\gamma}} = \frac{\frac{-\delta_z(1-\lambda\alpha_x S+\lambda\delta_x T(1+\beta E_y)+\beta E_x)-\delta_x((1-\lambda\delta_z T)(1+\beta E_y)-\beta E_z)}{(1-\lambda\delta_z T)(1-\lambda\alpha_x S+\beta E_x)+\lambda\delta_x T\beta E_z}}{-\delta_z - \delta_x \frac{1+\beta E_y}{1+\beta E_x^I}}.$$

The only restrictions on the values of these multipliers are imposed by the requirement that matrix (9) satisfies condition (M). In general, these restrictions are not enough to guarantee that each of these multipliers is greater than one. In fact, we may conclude that the presence of strategic complementarities is not sufficient to generate a social multiplier that is greater than one if there exists a locally asymptotically stable equilibrium in which *either* of the multipliers (12)-(13) is less than one. This possibility is most easily illustrated with myopic agents, $\beta = 0,$ in which case the divorce multiplier may be less than one. We show with a numerical example in Section 5 that the singlehood multiplier may be less than one when $\beta > 0$.

4.2.1 The Case of Myopic Agents ($\beta = 0$)

When $\beta = 0$ condition (M) is satisfied at equilibrium using the matrix $1-$norm if $\lambda\delta_z T < 1$ and $\lambda |\delta_x| T + \lambda\alpha_x S < 1$. In this case $\lambda\delta_z T, \lambda |\delta_x| T,$ and $\lambda\alpha_x S$ are each guaranteed to be between zero and one. The social multipliers if $\beta = 0$ are

$$(14) \qquad \left(\frac{dx}{d\gamma} \Big/ \frac{dx^I}{d\gamma} \right) \Big|_{\beta=0} = \frac{1}{1 - \lambda\alpha_x S}, \quad \text{and}$$

$$(15) \qquad \frac{\delta_z \frac{dz}{d\gamma} + \delta_x \frac{dx}{d\gamma}}{\delta_z \frac{dz^I}{z\,d\gamma} + \delta_x \frac{dx^I}{x\,d\gamma}} \Big|_{\beta=0} = \frac{-\delta_z \frac{1-\lambda\alpha_x S+\lambda\delta_x T}{(1-\lambda\delta_z T)(1-\lambda\alpha_x S)} \overline{z} \delta_x \frac{1}{1-\lambda\alpha_x S}}{-\delta_z - \delta_x}.$$

Proposition 1 *Suppose $\beta = 0, \lambda > 0,$ and that condition (M) is satisfied at equilibrium using the ℓ_1 norm.*
   a) The singlehood multiplier, $\frac{dx}{d\gamma}/\frac{dx^I}{d\gamma}$, always exceeds one.

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b) *The divorce multiplier,* $-\dfrac{\delta_z \frac{dy}{dz} + \delta_x \frac{dy}{dx}}{\delta_z \frac{dz}{dy} + \delta_x \frac{dx}{dy}}$, *is less than one if* $|\delta_x| \notin (\rho\delta_z, \delta_z)$, *where*

$\rho = \dfrac{\lambda\delta_z T(1-\lambda\alpha_x S)}{\lambda\alpha_x S + \lambda\delta_z T\,(1-\lambda\alpha_x S)} < 1.$ *It is undefined if* $|\delta_x| = \delta_z$ *and is otherwise greater than or equal to one.*

Proof. Part (*a*) follows immediately from $\lambda \alpha_x S \in (0, 1)$. The analysis for the divorce multiplier depends on $\frac{d\delta^I}{d\gamma}$, the individual response to an increase in γ. If $\frac{d\delta^I}{d\gamma} = -\delta_x - \delta_z > 0$ a little algebra shows that the divorce multiplier exceeds one if

$|\delta|_x > \rho\delta$ where $\rho = \frac{\lambda\delta_z T(1-\lambda\alpha_x S)}{\lambda\alpha_x S+\lambda\delta_z T (1-\lambda\alpha_x S)}$, but this is always true since $\rho < 1$. When $\frac{d\delta^I}{d\gamma} = -\delta_x - \delta_z < 0$ it is easy to see that the divorce multiplier is *less* than one if and

only if $|\delta_x| \in (\rho\delta_z, \delta_z)$. ∎

First concentrate on the effect of an increase on γ on the decision to remain single. An increase in γ directly lowers the probability of remaining single since agents are myopic and ignore outcomes when old. But there is also an indirect effect because the single pool shrinks, and this further decreases the incentive to remain single. Since the direct and indirect effects of an increase in γ go in the same direction, the singlehood multiplier exceeds one.

Turning to the divorce multiplier, first consider the case where the individual response to an increase in γ is an increase in the probability of divorce, $\frac{d\delta^I}{d\gamma} = -\delta_z - \delta_x > 0$. The equilibrium response takes into account both the indirect effect on selectivity into marriage discussed in the previous paragraph and the indirect effect on the divorce decision. These indirect effects make divorce more likely since selectivity into marriage falls even further and, respectively, the divorce threshold increases since the divorce rate is higher. The divorce multiplier exceeds one since these indirect effects reinforce the direct effect.

On the other hand, the divorce multiplier may be less than one if $\frac{d\delta^I}{d\gamma} < 0$. In fact, the multiplier will be negative when $(1 - \lambda\alpha_x S) \delta_z < |\delta_x| < \delta_z$, for then the numerator is positive while the denominator is negative. The key mechanism behind this result is the presence of strategic complementarities acting on the decision to marry. Setting $\lambda\alpha_x S = 0$ is equivalent to eliminating strategic complementarities on the decision to marry, and one can verify that when we do this the divorce multiplier exceeds one.

To better grasp the intuition, allow $\lambda\alpha_x S$ to vary from zero and suppose $(1 - \lambda\alpha_x S) \delta_z < |\delta_x| < \delta_z$. In this case, an increase in γ that affects only one couple (the "special" couple) will *lower* their probability of divorce given marriage while an increase in γ that affects all couples will *increase* the probability of divorce given marriage for the typical couple (even though the direct effect is negative). When the increase in γ

applies to only the special couple they lower their selectivity into marriage. De- spite this lower selectivity, though, the likelihood of divorce when old actually falls

because the divorce threshold also decreases, and the fall in the divorce threshold is large enough to counteract the decrease in selectivity. When the increase in γ applies to all couples, the drop in selectivity is larger and outweighs the lower divorce threshold, causing the likelihood of divorce when old to increase. The reason for the larger drop in selectivity is that the size of the single pool shrinks and, because of complementarities in the marriage decision, this increases the social incentive to marry.

4.3 Self-Fulfilling Conformity

Recall that self-fulfilling conformity refers to the monotonic relationship between individual taste for conformity and homogeneity of behavior. Typically this means that if in equilibrium the marginal social utility of taking an action is positive, an increase in the taste for conformity increases the average level of an action taken within a reference group. When the choice set contains two actions, as is the case in this application, the marginal social utility to action 1 is positive if the difference in social payoffs between actions 1 and 2 is positive. The average level of the action refers to the proportion of the reference group that takes the action.

Let MSU^d be the marginal social utility of divorce (i.e., the social payoff difference between divorcing and remaining married) and let MSU^s be the marginal social utility of remaining single (i.e., the social payoff difference between remaining single and marrying). The proportion of the old and married that divorces is δ and the proportion of the young that remains single is α. The taste for conformity is parameterized by λ. Self-fulfilling conformity thus obtains among the young if and only if $sign\,(MSU^s) = sign\left(\frac{d\alpha}{d\lambda}\right)$ and obtains among the old and married if and only if $sign(MSU^d) = sign\left(\frac{d\delta}{d\lambda}\right)$ For example, when the social payoff difference between divorce and remaining married is positive ($MSU^d > 0$), self-fulfilling conformity obtains among the old and married if the divorce rate increases as the taste for conformity increases ($\frac{d\delta}{d\lambda} > 0$).

Definition 1 Self-fulfilling conformity obtains among the young *if and only if sign* $(MSU^s) = sign\left(\frac{d\alpha}{d\lambda}\right)$ *at any equilibrium.* Self-fulfilling conformity obtains among the old and married *if and only if sign*$(MSU^d) = sign\left(\frac{d\delta}{d\lambda}\right)$ *at any equilibrium.* Self-fulfilling conformity obtains *if and only if sign* $(MSU^s) = sign\left(\frac{d\alpha}{d\lambda}\right)$ *and sign*$(MSU^d) = sign\left(\frac{d\delta}{d\lambda}\right)$ *at any equilibrium.*

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Notice that

(16) $\quad MSU^d = \lambda B$ and

$$MSU^s = \lambda v(\alpha) + \lambda \beta \bar{v} \left(\begin{array}{l} \lambda v(1-\alpha) + \lambda \beta \Pr(\Theta_o > z|x) v(1-\delta) \\ + \lambda \beta \Pr(\Theta_o < z|x) v(\delta) \end{array} \right)$$

(17) $\qquad = \lambda A + \beta \lambda (\bar{v} - v(1-\delta) - \Pr(\Theta_o < z|x) B),$

where $A \equiv v(\alpha) - v(1-\alpha)$ and $B \equiv v(\delta) - v(1-\delta)$.[16] The effect on δ and α of an increase in the desire to conform is $\frac{d\delta}{d\lambda} = \delta_x \frac{dx}{d\lambda} + \delta_z \frac{dz}{d\lambda}$ and $\frac{d\alpha}{d\lambda} = \alpha_x \frac{dx}{d\lambda}$, respectively. Straightforward calculations on the equilibrium system (4) show that

$$\frac{dz}{d\lambda} = \frac{B(1 - \lambda \alpha_x S + \beta E_x) + \lambda \delta_x T(A + \beta \bar{v})}{(1 - \lambda \delta_z T)(1 - \lambda \alpha_x S + \beta E_x) + \lambda \delta_x T \beta E_z} \text{ and}$$

$$\frac{dx}{d\lambda} = \frac{(A - \beta E_\lambda + \beta \bar{v})(1 - \lambda \delta_z T) - B \beta E_z}{(1 - \lambda \delta_z T)(1 - \lambda \alpha_x S + \beta E_x) + \lambda \delta_x T \beta E_z}.$$

so that

(18) $$\frac{d\delta}{d\lambda} = \frac{\left(\begin{array}{l} \delta_x((A - \beta E_\lambda + \beta \bar{v})(1 - \lambda \delta_z T) - B \beta E_z) \\ + \delta_z(B(1 - \lambda \alpha_x S + \beta E_x) + \lambda \delta_x T(A - \beta E_\lambda + \beta \bar{v})) \end{array} \right)}{(1 - \lambda \delta_z T)(1 - \lambda \alpha_x S + \beta E_x) + \lambda \delta_x T \beta E_z} \text{ and}$$

(19) $$\frac{d\alpha}{d\lambda} = \alpha_x \frac{(A - \beta E_\lambda + \beta \bar{v})(1 - \lambda \delta_z T) - B \beta E_z}{(1 - \lambda \delta_z T)(1 - \lambda \alpha_x S + \beta E_x) + \lambda \delta_x T \beta E_z}.$$

As with the social multipliers, the only restrictions on the values of $M\,SU^s$, $M\,SU^d$, $\frac{d\delta}{d\lambda}$, and $\frac{d\alpha}{d\lambda}$ come from the requirement that matrix (9) satisfies condition (M). In general, these restrictions are not enough to guarantee that self-fulfilling conformity obtains. When $\beta > 0$ a numerical example in Section 5 illustrate that self-fulfilling conformity may fail among the young. But the analysis and intuition are simpler when $\beta = 0$. In this case, self-fulfilling conformity may fail to obtain among the old and married.

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[16]Because the marginal social utility of singlehood takes into account the expected social payoff of being married when old, and this quantity depends on the probability of divorce, MSU^s in general depends on match quality when young θ_y. Self-fulfilling conformity, however, hinges on the response

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of the marginal couple to an increase in the taste for conformity, where the marginal couple is the one just indifferent between remaining single and marrying (i.e., for whom $\theta_y = x$); the key question is whether this couple strictly prefers marriage or singlehood after a marginal increase in λ. It is therefore sufficient to consider only MSU^s for this marginal couple.

4.3.1 The Case of Myopic Agents ($\beta = 0$)

The analysis is restricted to equilibria where condition (M) is satisfied locally using the matrix 1-norm. As before, this ensures that $\lambda\alpha_x S$, $\lambda |\delta_x| T$, and $\lambda\delta_z S$ are each between zero and one. We have

$$
\begin{aligned}
MSU^s|_{\beta=0} &= \lambda A, \\
MSU^d|_{\beta=0} &= \lambda B, \\
\frac{d\alpha}{d\lambda}\Big|_{\beta=0} &= \alpha_x \frac{A}{1 - \lambda\alpha_s S} \text{ and} \\
\frac{d\delta}{d\lambda}\Big|_{\beta=0} &= \frac{\delta_x A \left(1 - \lambda\delta_z T\right) + \delta_z \left(B \left(1 - \lambda\alpha_x S\right) + A\lambda\delta_x T\right)}{\left(1 - \lambda\delta_z T\right)\left(1 - \lambda\alpha_x S\right)}.
\end{aligned}
$$

Proposition 2 *Suppose $\beta = 0$, $\lambda > 0$, and that condition (M) is satisfied at equilibrium using the ℓ_1 norm.*

(a) Self-fulfilling conformity obtains among the young.

(b) If $\text{sign}(A) \neq \text{sign}(B)$, self-fulfilling conformity obtains among the old and married. But if $\text{sign}(A) = \text{sign}(B)$ self-fulfilling conformity fails to obtain among the old and married iff $|\delta_x| > \delta_z(1 - \lambda\alpha_x S)\frac{A}{B}$.

(c) Hence, self-fulfilling conformity fails to obtain iff $\text{sign}(A) = \text{sign}(B)$ and $|\delta_x| > \delta_z(1 - \lambda\alpha_x S)\frac{A}{B}$.

Proof. Part (a) follows immediately from $\alpha_x > 0$ and $\lambda\alpha_x S \in (0, 1)$. Turning to the old and married, algebra shows that $\frac{d\delta}{d\lambda} \gtrless 0$ as $|\delta_x| A \lessgtr \delta_z(1 - \lambda\alpha_x S)B$. From this it follows that self-fulfilling conformity obtains if $sign(A) \neq sign(B)$ but may fail to obtain if $sign(A) = sign(B)$.

To see this, suppose $B < 0$ and $A > 0$. Self-fulfilling conformity obtains iff $\frac{d\delta}{d\lambda} < 0$ since $B < 0$ implies $MSU^d < 0$. In this case $|\delta_x|A > \delta_z(1 - \lambda\alpha_x S)B$, which implies $\frac{d\delta}{d\lambda} < 0$ always. A similar analysis applies if $B > 0$ and $A < 0$.

But now suppose $B < 0$ and $A < 0$. Self-fulfilling conformity obtains iff $\frac{d\delta}{d\lambda} < 0$ since $B < 0$. But $\frac{d\delta}{d\lambda} > 0$ whenever $|\delta_x| A < \delta_z(1 - \lambda\alpha_x S)B$, or $|\delta_x| > \delta_z(1 - \lambda\alpha_x S)\frac{B}{A}$ since $A < 0$. It is easy to see that self-fulfilling conformity fails under the same condition when $B > 0$ and $A > 0$. ∎

Self-fulfilling conformity may fail to obtain among the old and married when $sign(A) = sign(B)$ because an increase in the taste for conformity has an impact on marriage selectivity that pushes the divorce rate in the socially penalized direction.

For example, $A, B < 0$ corresponds to an equilibrium where young marriage is so- cially rewarded and divorce is socially penalized, perhaps because of an "old maid" stigma and a divorce stigma. An increase in the taste for conformity lowers a young person's selectivity into marriage which exerts upward pressure on the divorce rate. If selectivity falls enough or the divorce rate is relatively sensitive to changes in selec- tivity, in particular if $\frac{|\delta_x|}{1-\lambda \alpha_x S} > \delta_z \frac{B}{A}$, the divorce rate will increase. In other words, an increase in the taste for conformity *lowers* homogeneity among the old and married.

In contrast, the effect that an increase in the taste for conformity has on selectivity when $sign(A) \not= sign(B)$ pushes the divorce rate in the socially rewarded direction. For example, $A > 0$ and $B < 0$ means young marriage and divorce are socially penalized in equilibrium. An increase in the taste for conformity increases marriage selectivity which makes divorce less likely.

5   Numerical Examples with Forward-Looking Agents

The preceding section has already accomplished the goals of this paper: to prove uniqueness under MSI, and to show that when agents take interrelated and sequential actions in more than one dimension, the presence of strategic complementarities is not sufficient to generate a social multiplier greater than one nor to ensure that self-fulfilling conformity obtains. We demonstrated the last two points with the divorce multiplier and self-fulfilling conformity among the old and married when $\beta = 0$. This section strengthens the argument by showing that with forward-looking agents the singlehood multiplier can be less than one and that self-fulfilling conformity can fail among the young.

We parameterize the model in the following way. Let match quality when young Θ_y be distributed uniformly on $[0, 1]$ and match quality when old $\Theta_o | \Theta_y$ be distributed uniformly on $[-1, 1 + \Theta_y]$. The distribution of $\Theta_o | \Theta_y$ is clearly increasing in Θ_y in the sense of first-order stochastic dominance. Moreover, each distribution is continuously differentiable on the interior of its support and has finite mean. Letting $v(p) = p^4$,

Table 2: Parameterizations for the Examples.

Parameters	Value	
	Example 1	Example 2
β	9.73	0.04
c	0.0006	0.0077
γ	-0.8498	-0.4702
λ	0.0622	0.010

a detailed derivation in the Appendix shows that equilibrium is given by

$$
\begin{aligned}
z &= -\gamma - c + \lambda \left(\delta^4 - (1-\delta)^4 \right) \\
x &= \boxminus \quad \frac{1}{2+x}\left(\frac{1}{(1+x)^2 - z^2} \left(-\gamma + \lambda \left(\alpha^4 - (1-\alpha)^4 \right) \right) - \frac{1}{2}\left(c + \gamma - \lambda \left(\delta^4 - (1-\delta)^4 \right) \right)(1+x-z) \right) \\
&\quad \square \; -\beta \qquad\qquad -c - \lambda \left(1 - \delta^4 \right)
\end{aligned}
$$

Since $\alpha = \Pr\left(\Theta_y < x \right)$ and $\delta = \Pr(\Theta_o < z | \Theta_y \geq x)$ a.s., we have (a.s.)

$$
\delta\,(x,z) = \frac{z+1}{1-x}\ln\left(\frac{3}{2+x} \right) \quad \text{and}
$$

$$
o\,(x) = x.
$$

The expression for α is straightforward; the calculation for δ is in the Appendix.

We are searching for a locally asymptotically stable equilibrium in which the singlehood multiplier is less than one, and a second stable equilibrium where self-fulfilling conformity fails to obtain among the young. The equilibrium is stable if condition (M) is satisfied locally using matrix (9). The singlehood multiplier is equation (12). For self-fulfilling conformity, MSU^s and $\frac{d\alpha}{d\lambda}$ are expressed in equations (17) and (19).[17]

The parameters for the examples are shown in Table 2, and the outcomes are displayed in Table 3. We emphasize that the parameter values reflect preferences and are chosen for illustrative purposes only. We compute the matrix ∞-norm to verify an asymptotically stable equilibrium.

Figure 3 helps to explain the intuition for why the singlehood multiplier is less

[17] The details of the derivations and of the numerical algorithm can be found at http://pages.towson.edu/fchriste/research. The results have been confirmed using three different computational software packages.

Table 3: Benchmark Results.

Outcome	Value	
	Example 1	Example 2
x	0.51	0.46
z	0.86	0.46
α	0.51	0.46
δ	0.68	0.54
$\lvert M \rvert_\infty$	0.09	0.01
dx / dx'	0.98	1.01
$-\dfrac{\delta_z \frac{d\gamma}{dz} + \delta_x \frac{d\gamma}{dx}}{\delta_z \frac{d\gamma}{dz'} + \delta_x \frac{d\gamma}{dx'}}$	1.04	1.00
$MSU^d \times \frac{\frac{d\gamma}{d\delta}}{d\delta} \, \frac{d\delta}{d\lambda}$	-0.004	$2.2E-6$
$MSU^s \times \frac{\frac{d\gamma}{d\alpha}}{d\alpha} \, \frac{d\alpha}{d\lambda}$	1.36	$-5.8E-17$

than one in Example 1. While panel A shows that an increase in the private benefit to marriage γ shrinks the single pool (α), in panel B we see that the marginal social utility of remaining single (MSU^s) increases for the marginal pair of young matched singles (the pair for whom $\theta_y = x$). This counterintuitive result occurs because the expected social value of marriage ($ESVM$) in the second period falls (panel B).[18] As a result, forward-looking singles lower their selectivity into marriage by less than they would if the increase in γ applied to only them.

The intuition driving the failure of self-fulfilling conformity among the young in Example 2 is similar. Early marriage is socially rewarded in equilibrium ($MSU^s > 0$). As the taste for conformity λ increases, conventional intuition would suggest that fewer people marry young. But panel A of Figure 4 illustrates that the opposite is true; the size of the single pool (α) decreases. An increase in the taste for conformity raises the expected value of marriage when old for the marginal pair of young matched singles (panel B). Forward-looking singles take this into account and become more likely to marry when young.

6   Some Empirical Implications

The social multiplier, and peer effects more generally, is useful for explaining how small differences in fundamentals can lead to large changes in behavior. This multi-

[18] The expected social value of marriage is $\Pr\left(\Theta_o \geq z|x\right) \lambda v\left(1 - \delta\right) + \Pr\left(\Theta_o < z|x\right) \lambda v\left(\delta\right).$

plier has been used to help explain variation in crime across cities (Glaeser, Sacerdote, and Scheinkman, 1996), student outcomes (Sacerdote, 2001; Cippolone and Rosolia, 2007; among others), smoking and substance abuse (DeCicca, Kenkel, and Mathios, 2008; Gaviria and Raphael, 2001; Harris, González López-Valcárcel, 2008; among others), obesity (Trogdon, Nonnemaker, and Pais, 2008), benefit plan enrollment (Duflo and Saez, 2003), and productivity (Falk and Ichino, 2006; Mas and Moretti, 2009), just to name a few.

This paper introduces a new concern in identifying peer effects. The social multiplier may be equal to or near one even when the taste for conformity is strong, so linear tests may fail to detect the presence of social interactions when in fact they ex- ist. That is, peer groups may have a strong influence on preferences even if empirical tests suggest they do not. In fact, carrying the analysis to its limit, empirical tests may find evidence of negative spillovers even in the presence of strategic complemen- tarities since the social multiplier can be less than one or even negative. This may be an especially important consideration in policy interventions which affect many aspects of an individual's life, such as the Moving to Opportunity program (see Kling, Leibman, and Katz, 2007), since there are likely to be many interdependent decisions which are influenced by multiple peer groups.

In addition, self-fulfilling conformity is implicitly assumed when interpreting empirical tests for the strength of peer, neighborhood, or spillover effects. All else equal, greater homogeneity is often interpreted as evidence of a stronger social interactions effect. Underlying this interpretation is that the social multiplier is larger when the taste for conformity is greater and self-fulfilling conformity holds. Glaeser, Sacerdote, and Scheinkman (2003) use this insight as a theoretical basis for estimating the size of the multiplier in several different contexts. This paper shows that this may be a dubious interpretation in certain applications.

7   Concluding Remarks

Despite a vast literature on peer effects, this is the first paper to show that the social multiplier may be less than one in the presence of strategic complementarities when decisions are multidimensional and interdependent. This paper also formalizes a notion of self-fulfilling conformity and demonstrates that this property does not hold uniformly across the parameter space. However, we show that uniqueness under

moderate social influence is retained. We obtain these results in a concrete model of marriage and divorce in order to show that these results are more than a theoretical curiosity.

An important extension for this research is to find general conditions under which the social multiplier is greater than one and conditions under which self-fulfilling conformity holds in a general multidimensional choice model, and to further explore the relationship between the two concepts. The insights developed in this paper and some of the formal results are likely to be helpful in this endeavor.

8 Appendix

8.1 Proofs Omitted from Text

Proof of Lemma 1. The model described in the text is a hierarchical probability model where a joint distribution of the random vector (Θ_o, Θ_y) is implied. In particular, if F is the (implicit) distribution of (Θ_o, Θ_y) then the conditional distribution of Θ_o given that $\Theta_y = \theta_y$ is F_o $(. | \Theta_y = \theta_y)$ and the marginal distribution of Θ_y is F_y. While only couples that decide to marry receive a draw from F_o, it is a useful and innocuous abstraction to proceed in the proof as if nature draws a realization of the random vector (Θ_o, Θ_y) when each pair of singles is initially matched, but only reveals the realization of Θ_o in the second period if $\theta_y \geq x$.

Let F^m be the empirical distribution associated with m draws from F. As there are infinitely many couples, the empirical distribution of (Θ_o, Θ_y) in the population (for a given set of draws) is given by $\lim_{m \to \infty} F^m = F^\infty$. But the Glivenko-Cantelli lemma implies $\sup_q |F(q) - F^m(q)| = 0$ almost surely whenever $m \to \infty$, or $F = F$ (a.s.)[19]

The proportion of couples that remain single in the first life stage is equal to the proportion for whom $\theta_y < x$, but since $F^\infty = F$ a.s. this is almost surely equal to F_y (x). In other words, $\alpha = \Pr(\Theta_y < x)$ almost surely. (Note that the proportion of couples that remain single equals the proportion of individuals that remain single).

By definition the divorce rate δ equals the fraction of the population that marries and divorces, divided by the fraction that marries. In other words, δ equals the fraction of couples for whom $\theta_y \geq x$ and $\theta_o < z$, divided by the fraction for whom θx. But since $F^\infty = F$ a.s., δ is almost surely equal to $\frac{\Pr(\Theta_o < z, \Theta_y \geq x)}{\Pr(\Theta_y \geq x)}$. Bayes' rule implies $\Pr(\Theta_o < z | \Theta_y \geq x) = \frac{\Pr(\Theta_o < z, \Theta_y \geq x)}{\Pr(\Theta_y \geq x)}$, so that $\delta = \Pr(\Theta_o < z | \Theta_y \geq x)$ a.s., as desired. ∎

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[19] Let $\theta^1, \theta^2, ...$ be an infinite sequence of (realized) draws from the distribution F. Let F^m be the empirical distribution associated with m such draws. The Glivenko-Cantelli Lemma states that with probability 1, F^m converges uniformly to F as m increases without bound:

$$\sup_x |F^m(x) - F(x)| \to 0 \; a.s.$$

See Durrett (1996).

Proof of Theorem 1.   For ease of exposition, rewrite system (4) as

(20) $z_2 = -\gamma - c + \lambda[v(\delta(z_1, x_1) - v(1 - \delta(z_1, x_1))]$

(21) $x_2 + \beta \Pr(\Theta_o > z_1 | x_2) E(\Theta_o | \Theta_o > z_1, x_2) =$

$$-\gamma(1 + \beta \Pr(\Theta_o > z_1 | x_1)) + \beta \Pr(\Theta_o \le z_1 | x_1) c$$

$$= \quad v(\alpha(x_1)) - v(1 - \alpha(x_1)) + \beta\bar{v}$$

$$+\lambda \begin{bmatrix} -\beta \Pr(\Theta_o > z_1 | x_1) v(1 - \delta(x_1, z_1)) \\ -\beta \Pr(\Theta_o \le z_1 | x_1) v(\delta(x_1, z_1)) \end{bmatrix}.$$

Proving that there exists $(x_1, z_1) = (x_2, z_2)$ that satisfies (20)-(21) is equivalent to proving the existence of a fixed point in system (4).

Since v is bounded, it is simple to verify from equation (20) that for any $(x_1, z_1) \in \mathsf{R}^2$, we must have $z_2 \in [-\gamma - c - \lambda(\bar{v} - \underline{v}), -\gamma - c + \lambda(\bar{v} - \underline{v})] \equiv \mathsf{Z}$. We can therefore restrict our search for an equilibrium to $(x_1, z_1) \in \mathsf{R} \times \mathsf{Z}$.

Notice that for any $(x_1, z_1) \in \mathsf{R}^2$ the right hand side of equation (21) must lie in a compact interval since v and $\Pr(\Theta_o \le z_1 | x_1)$ are bounded functions. Let $\underline{a}$ and $\bar{a}$ be the associated greatest lower and least upper bounds. In contrast, the left hand side is strictly increasing in x_2 and traverses the real line as x_2 traverses the real line. To see this, notice that $\Pr(\Theta_o > z_1 | x_2) E(\Theta_o | \Theta_o > z_1, x_2)$ is the expected value of the function

$$h(\Theta_o; z_1) = \begin{cases} \Theta_o & \text{if } \Theta_o > z_1, \\ 0 & \text{else.} \end{cases}$$

As $h(\Theta_o)$ is increasing in Θ_o it follows from first-order stochastic dominance that $E(h(\Theta_o; z_1) | x_2) = \Pr(\Theta_o > z_1 | x_2) E(\Theta_o | \Theta_o > z_1, x_2)$ is increasing in x_2. Hence for any z_1, $x_2 + \beta E(h(\Theta_o; z_1) | x_2)$ passes through $[\underline{a}, \bar{a}]$ as x_2 traverses the real line. Moreover, continuity ensures that for any z_1 there exists a unique x'_2 and x''_2 such that $x'_2 + \beta E(h(\Theta_o; z_1) | x'_2) = \underline{a}$ and $x''_2 + \beta E(h(\Theta_o; z_1) | x''_2) = \bar{a}$. Given $z_1 \in \mathsf{Z}$, it follows that $x_2 \in [\underline{x}, \bar{x}] \equiv \mathsf{X}$, where $\underline{x} = \inf\{x'_2 | z_1 \in \mathsf{Z}\} \in (-\infty, \infty)$ and $\bar{x} = \sup\{x'_2 | z_2 \in \mathsf{Z}\} \in (-\infty, \infty)$ for any $x_1 \in \mathsf{R}$. Thus we can restrict our search for an equilibrium to $(x_1, z_1) \in \mathsf{X} \times \mathsf{Z}$.

Hence, for any $(x_1, z_1) \in \mathsf{X} \times \mathsf{Z}$ we have $(x_2, z_2) \in \mathsf{X} \times \mathsf{Z}$, that is, system (20)-(21) defines a continuous mapping $\mathsf{X} \times \mathsf{Z} \to \mathsf{X} \times \mathsf{Z}$. Finally, since $\mathsf{X} \times \mathsf{Z}$ is a compact and convex set, Brouwer's Fixed Point Theorem gives the desired result. ∎

Proof of Lemma 2.   As is standard in stability theory, we can assume that the

fixed point $\bar{x}$ of (6) is the zero solution. To see this, set $y_t = x_t - \bar{x}$. Then (6) takes the form $H(y_{t+1} + \bar{x}; y_t + \bar{x}) = 0$ which has the zero solution as the equilibrium that corresponds to the equilibrium solution $\bar{x}$ of (6).

We wish to apply the logic of a Liapunov function to prove this so we must show that condition (M) implies that A is compact. To this end, notice that the assumptions in the theorem and the implicit function theorem imply that, for any given vector x_t, we can write $x_{t+1} = G_t(x_t)$, where G_t is the implicit function induced

by H. Let L be the line segment joining any two points x_t, $x'_t \in A$. The multivariable mean value theorem states that[20]

(22) $$ \mathbf{I} G_t(x_t) - G_t(x'_t) \mathbf{I} \leq \mathbf{I} x_t - x'_t \mathbf{I} \max_{q \in L} \mathbf{I} D_{x_t} G_t(q) \mathbf{I}, $$

where $D_{x_t} G_t(x_t)$ is the Jacobian of G_t. $D_{x_t} G_t(x_t)$ is a matrix whereas the other terms are vectors. The matrix norm in this inequality is that which is induced by the vector norm. The implicit function theorem gives

$$ D_{x_t} G_t(x_t) = -[D_{x_{t+1}} H(x_{t+1}, x_t)]^{-1} D_{x_t} H(x_{t+1}, x_t). $$

By convexity of A, condition (M) implies that $\max_{q \in L} \mathbf{I} D_{x_t} G_t(q) \mathbf{I} < 1$. So selecting $x_t \neq x'_t$, $x'_t = 0$ and noting that $G_t(0) = 0$, inequality (22) implies

$$ \mathbf{I} G_t(x_t) \mathbf{I} < \mathbf{I} x_t \mathbf{I}, $$

which holds for all t. Since this holds for all t, we have $\mathbf{I} x_0 \mathbf{I} > \mathbf{I} x_1 \mathbf{I} > \mathbf{I} x_2 \mathbf{I} > \cdots$. Consequently, we can restrict attention to $B_{x_0}(0) \subset A$, the closed ball contained in A that is centered at 0 and has radius $\mathbf{I} x_0 \mathbf{I}$. Note that $B_{x_0}(0)$ is compact.

Since the mapping $x_t \longrightarrow -\mathbf{I} x_t \mathbf{I}$ is continuous, $-\mathbf{I} x_t \mathbf{I} \leq 0$ with equality iff $x = 0$, and $-\mathbf{I} G_t(x_t) \mathbf{I} \geq -\mathbf{I} x_t \mathbf{I}$ with equality iff $x = 0$, we can use this vector norm as a Liapunov function to prove that the equilibrium of (6) is globally asymptotically stable. See Lemma 6.2 in Stokey and Lucas with Prescott (1989) for the details of this argument. ∎

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[20] See Edwards (1973).

8.2 Calculations for the Numerical Example

The generic equilibrium equations are

$$z = -\gamma - c + \lambda[v(\delta(z, x)) - v(1 - \delta(z, x))]$$
$$x = -\gamma + \lambda[v(\alpha(x)) - v(1 - \alpha(x))] - \beta E(x, z) + \lambda\beta\bar{v},$$

where $E(x, z) \equiv \Pr(\Theta_o \geq z|x)[E(\Theta_o|\Theta_o \geq z, x) + \gamma + \lambda v(1 - \delta(z, x))] + \Pr(\Theta_o < z|x)[-c + \lambda v(\delta(z, x))]$. Following the calculations in footnote 12, we write

$$E(x, z) = \int_z^{\varphi(x)} (\theta_o - (\gamma + c - \lambda[v(\delta) - v(1 - \delta)]))f(\theta_o|x)d\theta_o - c + \lambda v(\delta).$$

Using our distributional assumptions and $v(p) = p^4$ we get

$$E(x, z) = \int_z^{1+x} \frac{\theta_o - z}{2 + x}d\theta_o - c + \lambda v(\delta)$$

$$= \frac{1}{2+x}\left(\frac{(1+x)^2 - z^2}{2} - \left(c + \gamma - \lambda\left(\delta^4 - (1 - \delta)^4\right)\right)(1 + x - z)\right) - c + \lambda\delta^4$$

Thus,

$$-\beta(E(x, z) - \lambda\bar{v}) = -\beta\left[\frac{1}{2+x}\left(\frac{(1+x)^2 - z^2}{2} - \left(c + \gamma - \lambda\left(\delta^4 - (1 - \delta)^4\right)\right)(1 + x - z)\right) - c - \lambda\left(1 - \delta^4\right)\right]$$

The remaining terms use the fact that $v(p) = p^4$ to get

$$z = -\gamma - c + \lambda\left(\delta^4 - (1 - \delta)^4\right)$$

$$x = \boxminus \left[\frac{1}{2+x}\left(\frac{(1+x)^2 - z^2}{2} - \left(c + \gamma - \lambda\left(\delta^4 - (1 - \delta)^4\right)\right)(1 + x - z)\right)\frac{-\gamma + \lambda\left(\alpha^4 - (1 - \alpha)^4\right)}{}\right] \quad \square$$

$$\square\, -\beta \qquad\qquad -c - \lambda\left(1 - \delta^4\right) \qquad\qquad \square$$

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To calculate the divorce rate, let $f(\theta_o, \theta_y)$ be the *pdf* of the random vector (Θ_o, Θ_y), $f(\theta_o|\theta_y)$ the conditional *pdf* given that $\Theta_y = \theta_y$, and $f_{\Theta_y}(\theta_y)$ the marginal *pdf* of Θ_y. Note that $f(\theta_o|\theta_y)$ is the *pdf* associated with the Uniform$(-1, 1 + x)$ distribution

while $f_{\Theta_y}(\theta_y)$ is the *pdf* associated with the uniform$(0, 1)$ distribution. Then

$$
\begin{aligned}
\delta &= \Pr(\Theta_o < z \,|\, \Theta_y \geq x) \\
&= \frac{\Pr(\Theta_o < z,\ \Theta_y \geq x)}{\Pr(\Theta_y \geq x)} \\
&= \frac{1}{1-x} \int_x^1 \int_{-1}^z f(\theta_o, \theta_y)\, d\theta_o d\theta_y \\
&= \frac{1}{1-x} \int_x^1 \int_{-1}^z f(\theta_o|\theta_y)\, d\theta_o\ f_{\Theta_y}(\theta_y)\, d\theta_y \\
&= \frac{1}{1-x} \int_x^1 \frac{z+1}{2+\theta_y}\, d\theta_y \\
&= \frac{z+1}{1-x} [\ln(3) - \ln(2+x)] \\
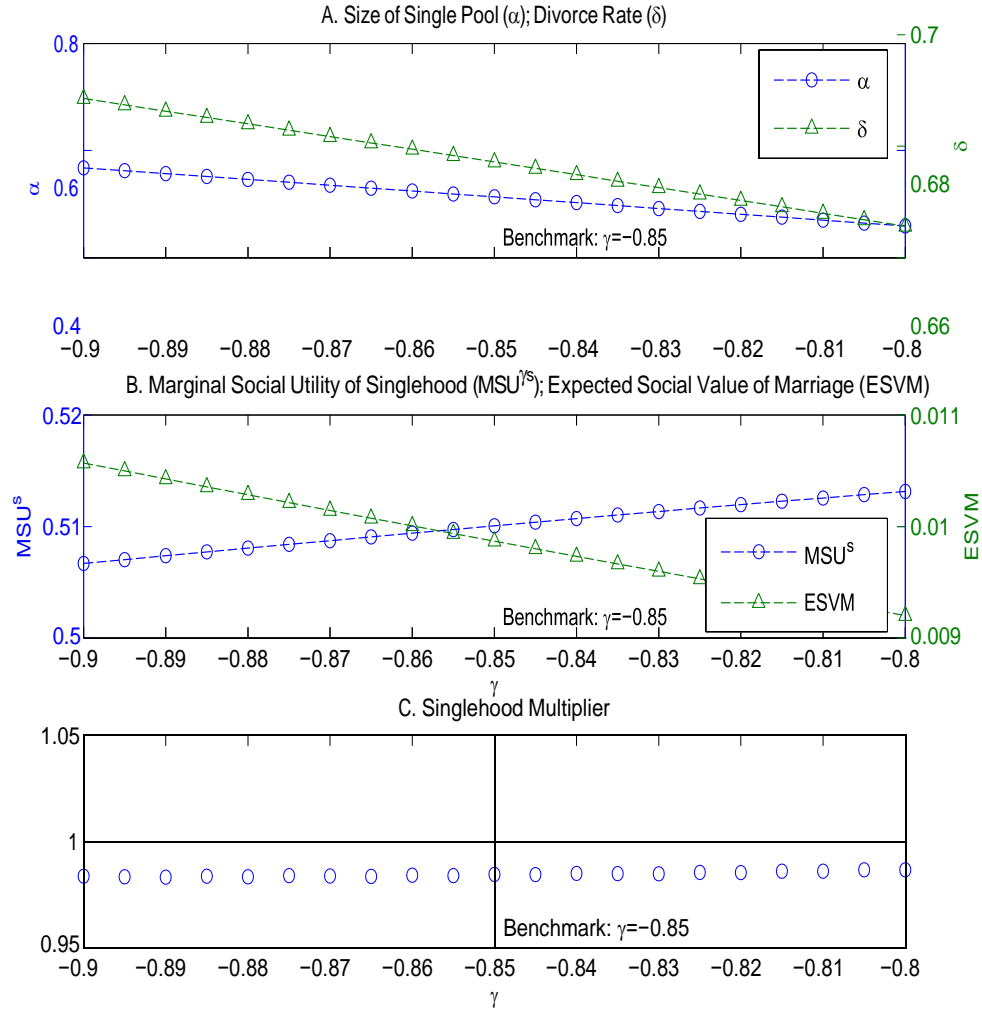&= \frac{z+1}{1-x} \ln\!\left(\frac{3}{2+x}\right).
\end{aligned}
$$

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8.3   Figures

Figure 3: Example 1: The singlehood multiplier $\frac{dx}{d\gamma}/\frac{dx^I}{d\gamma}$ can be smaller than one when agents are forward looking.

A. Size of Single Pool (α); Divorce Rate (δ)

B. Expected Value of Marriage: E(x,z)

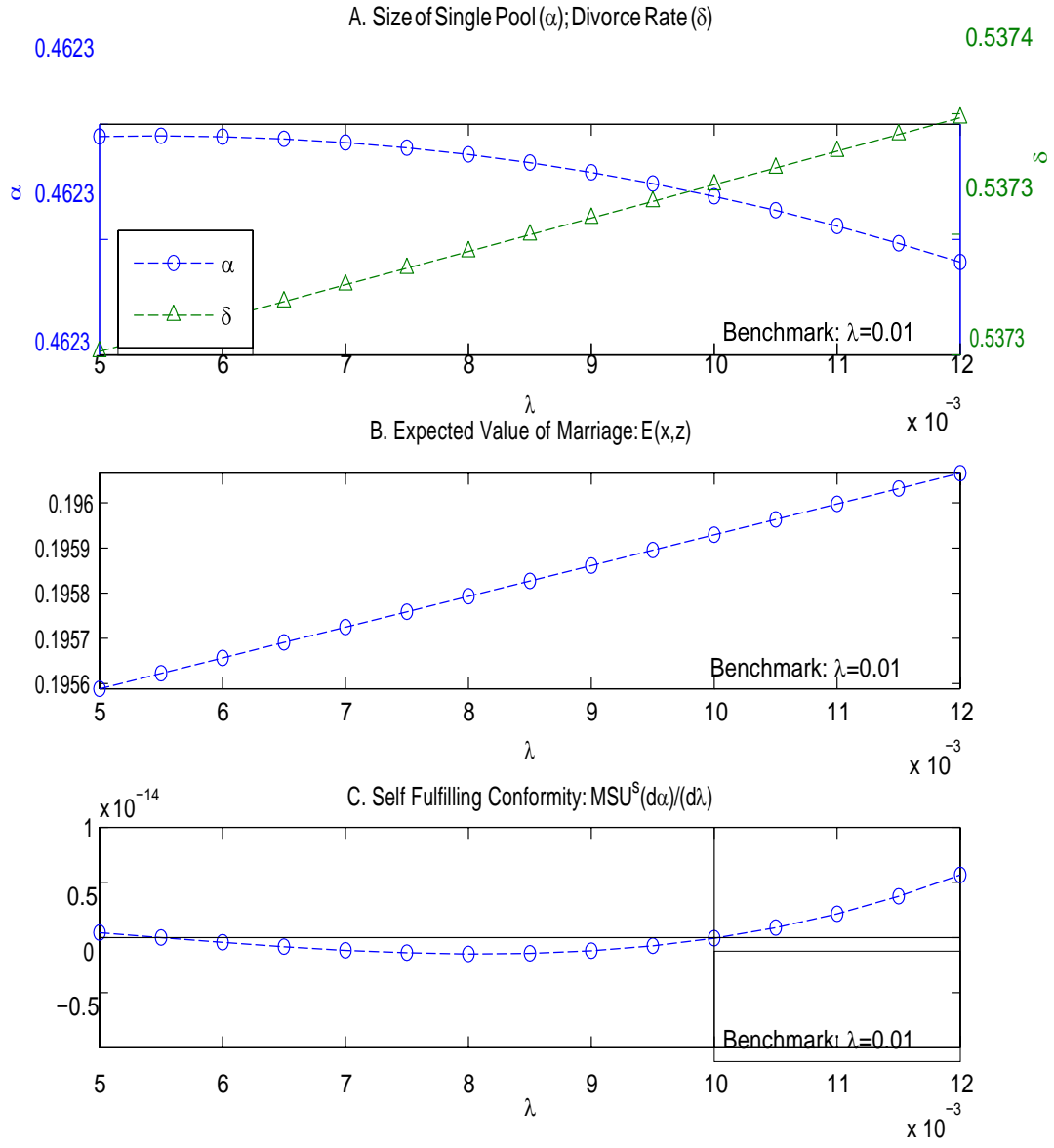C. Self Fulfilling Conformity: $MSU^s(d\alpha)/(d\lambda)$

Figure 4: Example 2: Self-fulfilling conformity among the young can fail to hold $\left(MSU^{\ s} \times \frac{d\alpha}{d\lambda} < 0 \right)$ when agents are forward looking.

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