

Extending the D-Wave with support for Higher Precision Coefficients

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Abstract

D-Wave only guarantees to support coefficients with 4 to 5 bits of resolution or precision. This paper describes a method to extend the functionality of the D-Wave to solve problems that require the support of higher precision coefficients.

1 Introduction

The D-Wave[3] is an adiabatic quantum computer[2, 4] which supports the following objective function:

$$F = \sum_i a_i q_i + \sum_i \sum_j b_{ij} q_i q_j \quad (1)$$

where $q_i \in \{-1, 1\}$ are the qubit values returned by the D-Wave, and $a_i \in [-2, 2]$ and $b_{ij} \in [-1, 1]$ are the coefficients given to the D-Wave associated with the qubits and the qubit couplers respectively. This however does not tell the whole story.

An algorithm has been developed, multi-qubit correction, MQC, that reduces a set of D-Wave result samples to a single sample that has a objective function value that is less-than or equal-to the objective function value of any sample of D-Wave result sample set. This algorithm presumes that the D-Wave result samples contain groups of qubits that can be used to construct a more optimal solution to the objective function. This has borne out to be true, as has been presented in the paper [1].

This presumption that D-Wave result samples contain information that can be utilized to construct a better solution to the objective function is utilized here to extract information from multiple variations of the objection function to find an optimal solution to an objective function that is based on coefficients of greater precision.

The precision of the coefficient values that are passed to the D-Wave hardware influences the values of the qubits that are returned by the D-Wave hardware. Currently, D-Wave only guarantees to take advantage of 4 to 5 bits precision of the floating point coefficient values passed to it by the user. This paper presents an algorithm, HPE (High Precision Enhancement), that will allow the D-Wave to in effect support higher precision coefficient values. This means that more bits of the values passed to it will be effectively utilized.

Precision here is measured in terms of bits of resolution. For example, 4 bit resolution represents the ability to represent the numbers 0 to 15 in integer increments, or the values from zero to one in

increments of $\frac{1}{16}$. In general, n bit resolution can represent the values between 0 and 1 in increments of $\frac{1}{2^n}$. And since D-Wave values can be positive or negative, an additional bit is needed as a sign bit. Thus, $n + 1$ bit resolution represents the values between 1 and -1 in increments of $\frac{1}{2^n}$.

HPE constructs multiple versions of the objective function, all of which have the same minima and maxima. This is done by multiplying the objective functions by different constants (Eq. 2).

$$F^k = \sum_i c_k a_i q_i + \sum_i \sum_j c_k b_{ij} q_i q_j \quad (2)$$

where k is the objective function version and c_k is the version constant. Note that if a_i and b_{ij} are multiplied by the same constants c_k the minima of F and F^k are the same. Thus D-Wave should return a valid set of q_i for either F and F^k . We are assuming here that a_i and b_{ij} are floating point numbers and that the D-Wave support coefficients with precision up to that of double precision floating point numbers. This is not however the case as has already been pointed out. And arguably, F and F^k are not the same once the coefficients a_i and b_{ij} are truncated to a range of values $[-2, 2]$ and $[-1, 1]$ respectively and are reduced to the precision supported by the hardware.

So in reality each F^k represents a different function to the D-Wave hardware. The previously stated presumption is used here, that D-Wave result samples, though they are not the global solution, may contain information that lead to the global minimum. For the purpose of this algorithm the presumption is that the different versions of F^k will return sample sets that contain information that lead to a better solution, if not the global solution, of the given higher precision objective function F .

2 The Higher Precision Enhancement Algorithm (HPE)

2.1 The Algorithm

For each function version F^k a sample set S^k is obtained from the D-Wave. Each sample set may contain anywhere from 1000 to 10,000 samples, s_i^k . Each sample s_i^k is a set of qubit values, q_j , that represent a solution to F^k .

HPE extensively utilizes MQC to reduce sample sets to single samples. The algorithm starts by taking a sample, s_i^k , from each sample set, S^k , to make a sample set, T^h . Then MQC is used to reduce T^h to a single sample t^h . These samples, t^h , are formed into a sample set H . And finally, MQC is used to reduce H to a single sample, h , which is HPE's solution for F .

Although the coefficients of F^k are used to provide coefficients to the D-Wave to obtain result samples, MQC always uses the coefficients of F to reduce the sample sets.

2.2 The Scaling

HPE is simple, but the choice of the scaling parameters are critical to the success of the algorithm, but may not be difficult to determine.

The common scaling factor, d , is the maximum absolute value of magnitude of the minimums and maximums of the a_i and b_{ij} . The initial scaling factor, c_0 , is a multiple of the inverse of d , so as to allow the resolution of the higher resolution of larger value coefficients. The choice of c_k is very much dependent on the greater precision desired as well as the dynamic range of the coefficients.

In the following test cases $c_0 = \frac{1}{8d}$ and $c_{k+1} = c_k \sqrt{2} = c_0 (\sqrt{2})^k$ (i.e. F^k scales by a half a bit of precision as k is incremented by 1). In the following tests, HPE used 20 versions of F^k where k varies from 0 to 19.

3 The Test Results

The tests are based on pseudo-random number generated coefficients, values of a_i and b_{ij} . These coefficients are quantized to the level of precision required for the test. A ground truth coefficient set for F is generated at a specified precision, then a series of coefficient sets for F^k are generated at a lower precision. The ground truth set for F represents the higher precision problem to be solved. While the lower precision sets F^k represent a set of problems which are solved by lower precision quantum hardware. This is done to allow the comparison of how well a high precision problem can be solved with lower precision hardware.

The objective function, F , is defined as the base or anchor problem, the problem that defines mathematical ground truth of the problem for which the minimum is to be found, the solution. The base problem consists of both the coefficients and the precision to which the coefficients are to be quantized. The solution sample returned by the D-Wave with the minimum value for F is defined as dw . The solution sample obtained from MQC applied to the samples return by the D-Wave for F is defined as m . The solution sample obtained from HPE when applied to the samples returned by the D-Wave from solving all of F^k objective functions is h .

The feasibility tests(section 3.1) is to test the feasibility of solving high precision problems with low precision hardware. The feasibility tests simulates very low precision virtual-hardware by creating very low resolution quantization of F^k . The base problem F is quantized at a resolution that represents the level that the D-Wave should be able to support.

The high precision test(section 3.2) is intended to show how the D-Wave results alone or the D-Wave results post-processed with MQC compares with HPE utilizing the D-Wave. The high precision test is to test problems with a much higher precision requirement than the D-Wave supports, while HPE utilizes the D-Wave to solve the problem with F^k solved on the D-Wave's lower precision hardware.

And finally, the unconstrained test(section 3.3) neither quantizes the problem coefficients or the HPE solver coefficients, (i.e. all coefficients are unmodified double precision numbers).

Note that, even though D-Wave only claims that the D-Wave hardware only supports 4 to 5 bit precision, the D-Wave is assumed in this paper to optimistically support 9 bit precision.

In Tables 1 thru 3, 'BP' is the base/anchor problem precision, 'HP' is the hardware precision (real or pseudo hardware), 'Cases' is the number of difference sets of coefficients used to define the problem to be solved, 'Samples' is the number of sets of qubit values requested from the D-Wave for each problem set of coefficients to be solved, ' $dw < h$ ' represents the number of cases where the raw D-Wave results have a lower value than the HPE result, ' $m < h$ ' represents the number of cases where the MQC result value had a lower value than the HPE result, ' $m = h$ ' represents the number of cases where the MQC result value had the same value as the HPE result, and ' $h < m$ ' represents the number of cases where the HPE result value had a lower value than the MQC result.

In all tests case presented in this paper the MQC results had a lower value than the raw D-Wave results. Due to the nature of the MQC algorithm MQC will always have as low or lower value than the raw D-Wave results.

3.1 Feasibility Test

These feasibility tests are the initial test to show that the algorithm successfully uses lower precision hardware to solve a higher precision problem. For this test the ground truth precision is 9 bits, one sign bit and 8 bit resolution, and the pseudo-quantum hardware machine precision is 3 bits, one sign bit and 2 bit resolution.

Table 1 shows the results of the feasibility tests. If the base or ground truth problem is of order

BP	HP	Cases	Samples	$dw < h$	$m < h$	$m = h$	$h < m$
9	3	1000	1000	9	775	106	119
9	3	100	10000	1	56	28	16
17	3	100	10000	0	59	30	11
25	3	100	10000	0	56	34	10
33	3	100	10000	0	52	38	10

Table 1: Various Precision Problems Running on 3 Bit Precision Pseudo-hardware.

the precision of the actual D-Wave, the D-Wave does well. In some cases it did do better than HPE though never better than MQC. In most cases however HPE does better than the D-Wave alone even though it was working with far less precision. Arguably, the D-Wave with MQC post processing did do better in most cases than HPE with much lower precision. HPE with the lower precision did, in some cases, better than the D-Wave with MQC post processing. This shows that it is feasible to obtain a solution of a high precision problem with much lower precision hardware.

3.2 High Precision Test

The purpose of the high precision test is to show how well HPE performs using the D-Wave over the D-Wave with out HPE processing. Table 2 shows the results of the high precision tests.

BP	HP	Cases	Samples	$dw < h$	$m < h$	$m = h$	$h < m$
9	9	100	1000	0	0	37	63
17	9	100	1000	0	0	48	52
25	9	100	1000	0	0	45	55
33	9	100	1000	0	0	45	55
41	9	100	1000	0	0	44	56
49	9	100	1000	0	0	48	52

Table 2: Various Precision Problems Running on 9 Bit Precision Hardware.

In no case was the D-Wave alone or with MQC better than HPE and in more than half the cases HPE was better than the D-Wave with MQC, otherwise it was the same as MQC.

3.3 Unconstrained Test

Table 3 shows the results of the unconstrained tests. In the unconstrained test, neither the coefficients of F or F^k are preprocessed by quantization. All procession is by sending non-quantized double precision floating point coefficients to be solved by only D-Wave processing. The results vary little from the high precision tests. 'dbl' refers to full double precision floating point.

BP	HP	Cases	Samples	$dw < h$	$m < h$	$m = h$	$h < m$
dbl	dbl	1000	1000	0	0	39	61

Table 3: 49 bit Precision Problem Running on Unconstrained D-Wave Hardware.

4 Conclusion

HPE (high precision enhancement) is a simple algorithm for processing objective functions which require higher precision coefficient than the current or possibly any future quantum hardware will be able to support. The feasibility test results indicate that if all you need is the precision that the hardware supports, then quantum plus MQC is possibly sufficient. However if the support of precision greater than that supported by the hardware is needed, then HPE may give a significant edge over quantum hardware with MQC post-processing.

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