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<b>14. ABSTRACT</b> We describe the results of a project to carry out theoretical and computational modeling of frequency comb sources in support of experimental efforts in the DARPA PULSE program. This work had two thrusts. The first was to model the SESAM fiber lasers that were used by the Newbury team at NIST to carry out free-space frequency transfer experiments. The second was to model frequency comb generation in microresonators, focusing on solitons and cnoidal waves. We developed a unique set of computational algorithms based on dynamical systems theory that allowed us to rapidly and unambiguously determine the stability and noise performance of the lasers and microresonators. We applied these tools to explain the performance limitations and optimize the performance of the SESAM lasers. We also used these tools to explain in part the difficulty in accessing solitons and to find a deterministic approach for generating large-bandwidth soliton trains, as a special limit of cnoidal waves.
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# **Modeling Optical Time and Frequency Generation and Transfer Systems**

*Final Report*

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## Descriptive Summary

The goals of this project changed over the lifetime of the grant. The originally proposed work was closely tied to the research effort of the time that was led by Nathan Newbury at NIST, and the principal goal was to support the development of advanced laser sources. In order to develop robust, carrier-envelope-phase-locked sources that can be transported without losing lock, the Newbury team used semiconductor saturable absorbing mirrors (SESAMs) to provide saturable absorption in their laser systems. This technology replaced nonlinear polarization rotation using the Kerr effect in optical fibers to provide the saturable absorption. As we and others had demonstrated theoretically and the Newbury group had seen experimentally, nonlinear polarization is not a stable source of saturable absorption because the polarization state of standard optical fibers varies randomly due to environmental perturbations. The use of SESAMs, combined with polarization-preserving fibers, solved this problem. However, it led to some unexplained issues. First, it was found that it was not possible to operate too close to the zero dispersion wavelength. Second, it was found that in some cases, parasitic frequency sidebands appeared. We were tasked with explaining both of these phenomena and determining how to avoid them. We accomplished this goal within the first three years of the project. We also proposed examining atmospheric effects that would limit the performance of the free-space optical frequency transfer system that the Newbury team developed. This proposed goal turned out to be superfluous because atmospheric effects were not a significant limit. In the last two years of the grant, we were asked to optimize the laser parameters to obtain higher pulse energies and to increase the wall plug efficiency. We achieved both these goals.

Unfortunately, this work did not have a strong impact on the experimental work of the Newbury team. At a fairly early stage in the DARPA PULSE project, the team effectively froze their laser development and focused on developing good control over their multiple laser combs using field-programmable gate arrays (FPGAs) that could be computer controlled and controlled remotely and on the application of their laser systems to free-space frequency transfer.

Nonetheless, the techniques that we developed to characterize the stable operating regimes of the lasers and their noise performance appear likely to be of significant future value. We have developed algorithms that perform many orders of magnitude faster than standard techniques that are based on brute-force solution of the evolution equations.

An additional goal of the project that we initiated after two years with the support of the program manager (Dr. Prem Kumar) was to model solitons in microresonators. The goal in this case was to examine alternatives to the conventional approach to obtaining solitons, which is to start the system in a chaotic state and move to a parameter regime in which solitons will randomly appear. While engineering solutions have been found that can produce solitons with reasonable reliability, they are still randomly generated, and the search for a deterministic path to obtain solitons continues at the time of this writing. At the suggestion of Dr. Andrew Weiner of Purdue University, we pursued the study of transverse mode interactions and soliton molecules. We also pursued work on cnoidal waves (also known as Turing rolls) that we had earlier started in collaboration with Dr. Yanne Chembo of FEMTO-ST in France. The approaches that we

had earlier developed to determine stable operating regimes in laser systems was of great assistance in our work on microresonators.

The work on transverse mode coupling led to two publications and a meeting presentation. While successful, this approach required the use of two pumps, which was not pursued experimentally during the project lifetime. However, this approach has been examined by two research groups within the past year.

We identified the dark solitons that had been observed by the Weiner group as soliton molecules. However, we did not solve the accessibility problem, and we laid this work aside due to lack of interest from the Weiner group in pursuing it further.

The work on cnoidal waves has been highly successful. We have identified a path that can produce a broadband train of solitons deterministically. We are currently working with experimental groups to develop an experimental demonstration and a comprehensive publication is in presentation. Summary presentations have already appeared in conferences. This work led a grant from the National Science Foundation to pursue further advances.

All the work that we carried out under this grant has appeared in publications or meeting presentations, with the exception of two papers that are currently in preparation. A detailed listing of all publications and presentations that were carried out with support from this grant are in the remainder of this report.

Additionally, this grant supported two PhD students. One has graduated and is now working in the US insurance industry. The computational techniques that he learned while a PhD student and helped to develop are directly relevant to his work. Another PhD student is currently supported by the NSF grant that was a spinoff of this project.

### **Significant Accomplishments**

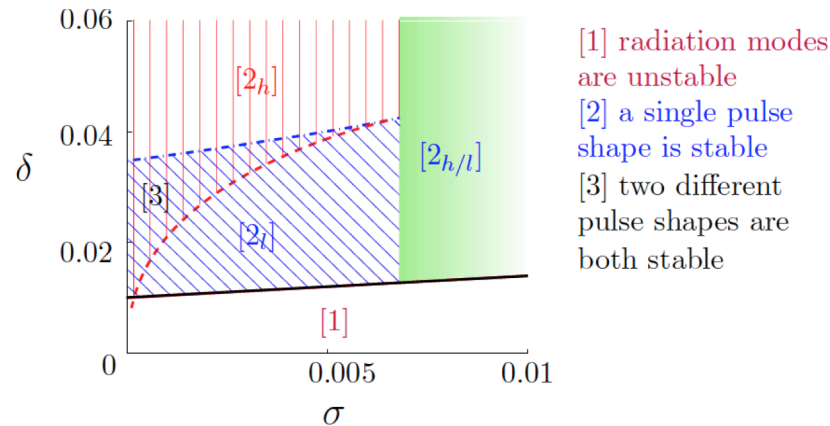
- The development of computational methods, based on dynamical systems theory, that allow us to determine the stable operating regimes of lasers and other resonators many orders of magnitude faster than conventional computational techniques.
- Identification of wake modes as the source of experimentally observed sidebands in the SESAM laser and the wake mode instability as the effect that prevents operation of the SESAM laser near the zero dispersion point.
- Determination of the combination of fiber amplifier length and output coupling that can increase the pulse energy and increase the wall plug efficiency in the SESAM laser without increasing the sideband energy and without a substantial increase in the background noise.
- Development of coupled equations for two transverse modes in microresonators, including the impact of avoided crossings.
- Theoretical explanation of the dark soliton structures observed by the Weiner group as soliton molecules.
- Development of analytical expressions for the cnoidal wave solutions in the lossless limit and their computational extension to the case with loss.
- Determination of the regions of stable operation for cnoidal waves with different periodicities and identification of the conditions for obtaining broadband solutions.

## Task-by-Task Account

NOTE: The tasks changed significantly over the lifetime of the grant from the tasks that were originally proposed in response to requests from members of the Newbury team and discussions with Dr. Newbury (the team leader) and Dr. Prem Kumar (the program manager during most of the project lifetime).

### Task 1: Development of efficient computational methods for determining the stability and the noise performance of lasers

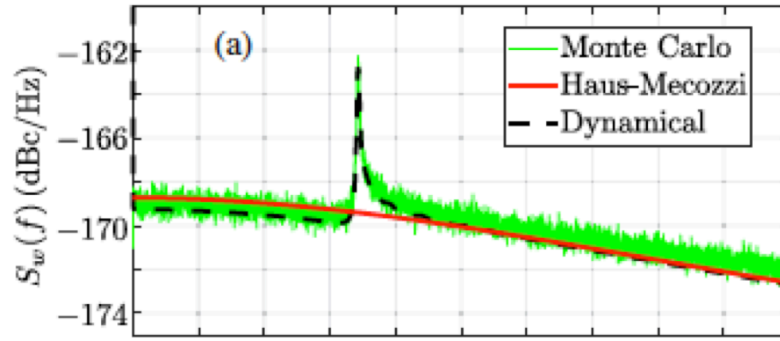
- The basic development of the boundary tracking algorithms was completed prior to the beginning of the DARPA program, and we applied it to lasers that are locked using nonlinear polarization rotation. That allowed us to unambiguously determine where the laser operates stably and identify a parameter regime in which two different modelocked solutions can exist. That was later verified experimentally in the group of Dr. Steven Cundiff.



Task 1, Figure 1: Stable regions in a laser that is locked using nonlinear polarization rotation. In the region labeled [3], two modelocked solutions can exist [Wang et al, J. Opt. Soc. Am. B **31**, 2914 (2014)].

- The development of the algorithms for determining the noise performance was completed over three years. We compared our algorithms to Monte Carlo simulations and to the analytical Haus-Mecozzi theory. Our algorithms run 3–4 orders of magnitude faster than the Monte Carlo simulations and are highly accurate unlike the Haus-Mecozzi theory that completely misses the contribution of the wake mode sidebands to the power spectral density.

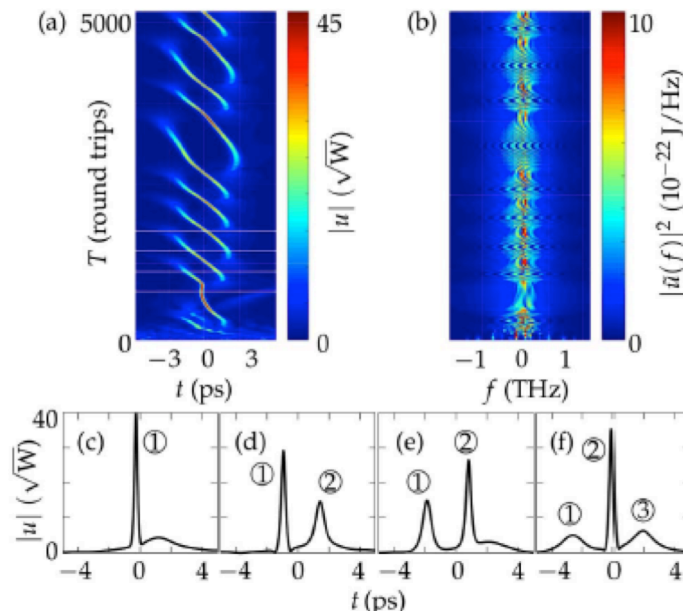
**Conclusions:** We have developed powerful algorithms that in our view should be far more widely used in the optical laser and resonator community. We have begun making a serious effort to publicize them. They form the basis for several recent articles and invited talks, as well as a successful National Science Foundation grant proposal. However, much more still needs to be done to develop these algorithms.



Task 1, Figure 2. Comparison of our dynamical approach to Monte Carlo simulations and the analytical Haus-Mecozzi model. [Wang et al., J. Opt. Soc. Am. B **35**, 2521 (2018).]

## Task 2: The wake mode sidebands and their impact

- Because the SESAM has a response time of several seconds, a gain window opens up behind a modelocked pulse. Within that gain window, radiation grows. This radiation has a frequency offset with respect to the modelocked pulse, leading to observable sidebands under some operating conditions. The dispersion of this radiation must be large enough to sweep it out of the wake before it can grow; otherwise it is unstable. That sets a lower limit on the dispersion and/or an upper limit on the pulse energy. We precisely identified the stable operating conditions using our boundary tracking algorithms.

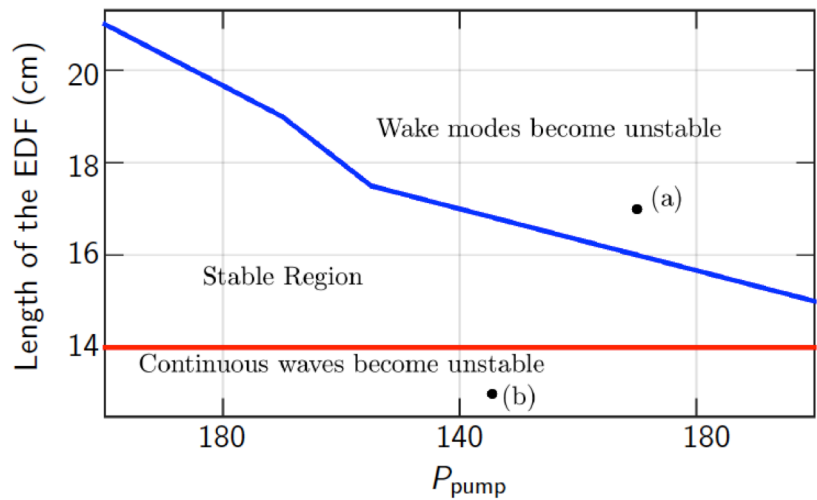


Task 2, Figure 1. Progression of the wake mode instability. A leapfrog evolution is observed in which a new pulse grows up in the wake of the existing pulse. [Wang et al., Opt. Lett. **42**, 2364 (2017).]

**Conclusions:** We identified the mechanisms that lead to sidebands and prohibit operation near the zero dispersion wavelength of SESAM lasers. We note that this mechanism does not prohibit operation with normal dispersion, which may be the subject of future work.

### Task 3: Optimization of the SESAM laser

- In the last two years of the project, the Newbury team asked us to look at parameter optimization with the goal of increasing the wall plug efficiency and/or increasing the output pulse energy and average power. Using our dynamical algorithms, we were able to do global searches of the parameter space and accomplish both tasks. The key to increasing the wall plug efficiency is to increase the length of the amplifier to make more efficient use of the pump while at the same time increasing the output coupling to avoid instabilities. The increase in the pulse energy more than makes up for the increase in amplifier noise, measured in dBc/Hz. The key to increasing the pulse energy all by itself without increasing the length of the gain fiber is to increase the output coupling, although the optimal output is somewhat smaller than when increasing the gain fiber length.



Task 3, Figure 1. The tradeoff between pump power and the length of the fiber amplifier [Wang et al., CLEO 2019, paper SF1C.]

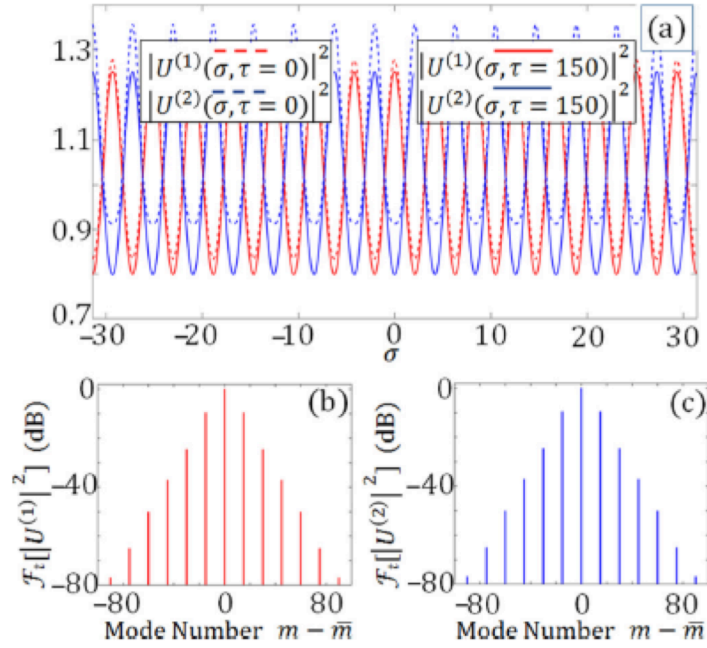
**Conclusions:** The operation of the NIST SESAM laser can be significantly improved by making relatively minor adjustments in the operating parameters. Optimizing the laser operation is not currently a high priority for the Newbury team, but we anticipate that at some point these results will become useful.

### Task 4: Development of coupled mode equations

- At the request of the Weiner group, we developed coupled equations that describe the nonlinear interaction of two transverse modes, including the effect of avoided crossings. We found that it is possible to take advantage of the avoided crossings



and the mode couplings to generate soliton trains in both the normal and anomalous dispersion regimes.



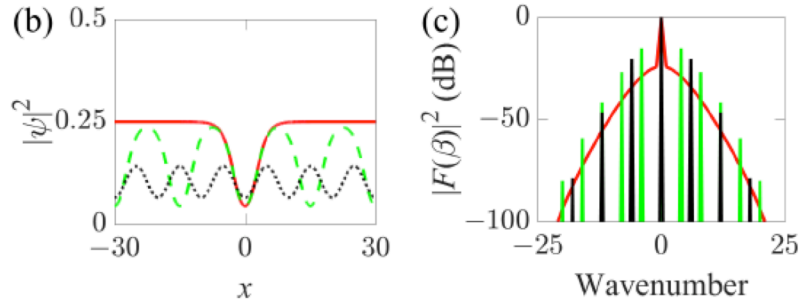
Task 4, Figure 1. Development of a combined soliton train in which two transverse modes are both present. [D'Aguanno and Menyuk, Phys. Rev. A **93**, 043820 (2016).]

**Conclusions:** In order to develop this mode, it is necessary to use two frequency pumps, which implies that two laser sources are being used. That adds to the complexity of the system and limits interest in this approach. Nonetheless, a group at NIST has examined this approach within the past year as a way of deterministically accessing solitons. The conditions for stable operation have yet to be determined. That will be a topic for future work if experimental interest in these systems increases.

### Task 5: Dark soliton molecules

- Using snaking diagrams, we were able to identify the dark soliton structures that the Weiner group had observed in the normal dispersion regime as soliton molecules. There is significant current interest in soliton molecules in the nonlinear optics community, but they have yet to be carefully examined in the context of microresonators. They have the advantage that they can be obtained in the normal dispersion regime, where many more material systems exist and bright solitons cannot be accessed. On the other hand, the spectra that they produce are not clean. While we identified the solutions, we did not determine the conditions under which they are accessible. Since there was no additional experimental work, we decided not to pursue this study.

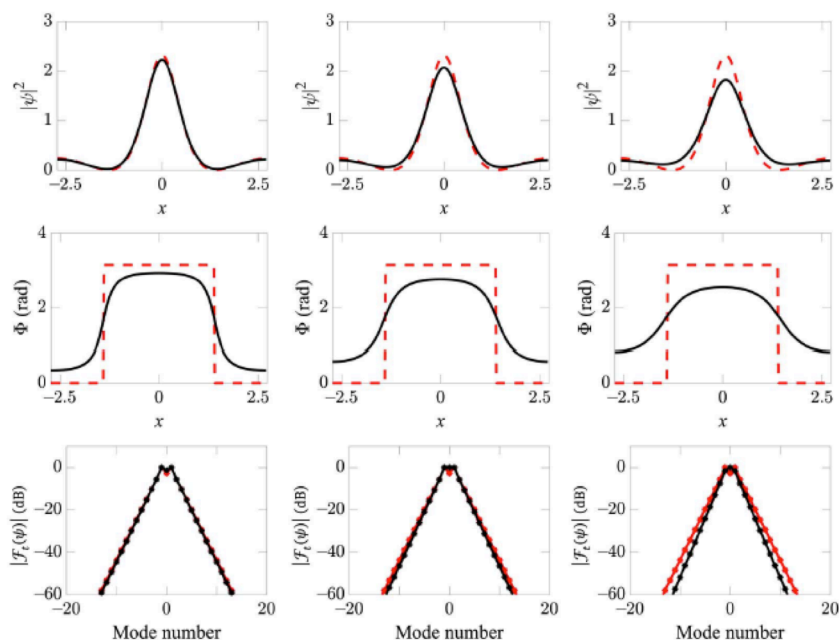
**Conclusions:** While we did not pursue this study beyond our preliminary findings, we are in a good position to pursue this topic if experimental findings make it worthwhile.



Task 5, Figure 1. Dark soliton molecule profiles corresponding to different numbers of elements in the molecules and the corresponding spectra. [Qi et al., FIO 2017, paper JTU3A.14.

## Task 6: Cnoidal Wave Solutions

- Single bright solitons are a special class of a much larger class of pulse trains, generally referred to in the nonlinear dynamics and fluid literature as cnoidal waves, although the community that works on microresonators has for the most part called them Turing rolls. The entire family shares many of the properties of single bright solitons and, in particular, they have clean spectra that fall off exponentially from the center peak. In certain parameter regimes, they can be as broadband as single solitons. Frequency combs based on single bright solitons have made remarkable progress in the past decades, but the single bright solitons remain difficult to access, and they use the pump inefficiently. Our research has indicated that using cnoidal waves can solve both these problems, while at the same time remaining broadband. In the regime in which cnoidal waves are broadband, they are essentially a train of solitons. They are accessed in a parameter regime in which they have a high pedestal, which locks them in place. As the detuning is increased and the pedestal diminishes, they remain locked in place. This process is completely deterministic.
- In the lossless limit, it is possible to find analytical solutions for cnoidal waves of any periodicity that can be expressed in terms of Jacobi elliptic functions. Just like single bright solitons in microresonators, these solutions have a pedestal. We note that single bright solitons do not have analytical solutions except in the lossless case, but the solutions are a good approximation to the actual solutions. In the case of the cnoidal waves, we found that the analytical solutions are only a good solution to stable cnoidal wave solutions when the periodicity is small. Above a periodicity that depends on the size of the microresonator, the cnoidal wave solutions of a given periodicity are unstable until the loss exceeds a minimum value.

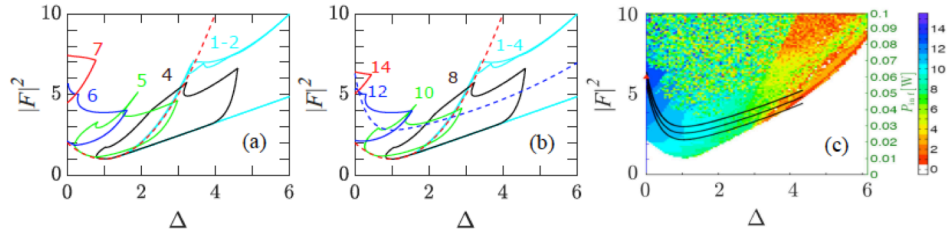


Task 6, Figure 1. Comparison of cnoidal wave solutions with (black-solid) and without (red-dashed) loss for a five-period solution. The solutions with and without loss resemble each other in this case. [Qi et al. J. Opt. Soc. Am. B **34**, 785 (2017).]

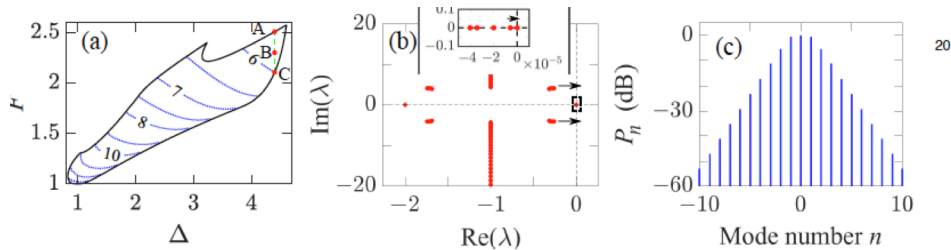
- In more recent work that served as the basis of a successful National Science Foundation, we used our boundary-tracking algorithms to identify the stable operating regimes for a wide range of periodicities and with parameters that correspond to bright soliton experiments that were conducted in the Weiner group. We found that the region in which cnoidal waves are stable occupy a U-shaped belt in the (pump power)  $\times$  (frequency detuning) parameter space. At pump powers below the belt, continuous waves are stable. Above the belt, there are no stable stationary solutions. Only breathers or chaotic solutions exist. The region where single solitons are stable coincides almost entirely with the regions where continuous waves are stable and where higher-order cnoidal waves are stable. That explains in large measure why single solitons are hard to access. Near the bottom of the belt and at lower detunings, the continuous waves are unstable so that the cnoidal waves can be simply accessed by raising the pump power. By then increasing the pump power and the detuning in an appropriate ratio, the pedestal of the cnoidal wave solution diminishes and the solution gradually transforms into a soliton train. It is important to note that there is no sharp transition from non-soliton cnoidal waves to soliton cnoidal waves. With the same periodicity, they are all part of the same solution family.

**Conclusions:** Cnoidal waves are a very promising avenue for obtaining large-bandwidth frequency combs deterministically. The only drawback is that the comb line spacing is no longer given by the free spectral range, but instead equals the free spectral range multiplied by the periodicity. However, the power in each line is multiplied by the periodicity-squared, which will be useful for some applications. We expect to publish

shortly a comprehensive study of cnoidal waves, and we have also begun to examine the impact of thermal effects on these solutions. So far, the results appear to hold once thermal effects are taken into account.



Task 6, Figure 2. Regions in the parameter space at which cnoidal waves are stable as a function of the detuning ( $\Delta$ ) and the pump power ( $|F|^2$ ). Sub-figures (a) and (b) show the stable regions for two different microresonator lengths. Sub-figure (c) shows earlier simulations that were carried out in the Weiner group. [Qi et al., CLEO 2018, paper SF2A.6.]



Task 6, Figure 3. This shows the falloff of the spectrum in dB/line for an 8-period cnoidal wave with parameters that correspond to those of the Weiner group experiments. The spectrum in sub-figure (c) corresponds to the red dot in sub-figure (a). With these parameters, the period-8 cnoidal waves is effectively a train of solitons. [Qi et al., CLEO 2018, paper SF2A.6.]

## Summary

With support from DARPA via AMRDEC, we carried out research in two principal areas. The first was laser modeling, which was carried out in collaboration with the Newbury group at NIST. As part of that work, we developed computational algorithms for determining the stability and noise performance of lasers and other resonators, based on dynamical systems theory. We used these algorithms to find the regions in which the SESAM laser developed at NIST is expected to be stable. We were able to use these algorithms to explain the presence of sidebands that were observed in some parameter regimes and to find parameter regimes in which they are no longer present. We also used these algorithms to optimize the performance of the laser—finding regions in which the wall-plug efficiency and/or the output energy can be improved with little or no noise cost.

The second research area that we explored was microresonators. Using our boundary-tracking algorithms, we carried out an extensive study of where single solitons and other cnoidal wave solutions are expected to be stable. These studies provided important insight into why single solitons are hard to access and pointed us toward higher-periodicity cnoidal wave solutions that can be accessed deterministically and that have a bandwidth that is as large as the bandwidth of soliton solutions.

We are currently developing additional applications of our boundary-tracking and noise performance algorithms. We are working with experimental partners to explore the effectiveness of the cnoidal wave solutions in accessing broad bandwidth frequency combs.

## Publications

1. S. Wang, A. Docherty, B. S. Marks, and C. R. Menyuk, “Boundary Tracking Algorithms for Determining the Stability of Mode-Locked Pulses,” *J. Opt. Soc. Am. B* **31**, 2914–2930 (2014).  
[doi:10.1364/JOSAB.31.002914]
2. G. D’Aguanno and C. R. Menyuk, “Nonlinear Mode Coupling in Whispering-Gallery-Mode Resonators,” *Phys. Rev. A* **93**, 043820 (2016).  
[doi:10.1103/PhysRevA.93.043820]
3. C. R. Menyuk and S. Wang, “Spectral Methods for Determining the Stability and Noise Performance of Passively Modelocked Lasers,” *Nanophot.* **5**, 332–350 (2016).  
[Invited paper]  
[doi:10.1515/nanophot-2016-033]
4. Y. Shen, J. Zweck, S. Wang, and C. R. Menyuk, “Spectra of Short-Pulse Solutions of the Cubic-Quintic Complex Ginzburg-Landau Equation Near Zero Dispersion,” *Stud. Appl. Math.* **137**, 238–255 (2016).  
[doi:10.1111/sapm.12136]
5. S. Wang, B. S. Marks, and C. R. Menyuk, “Comparison of Models of Fast Saturable Absorption in Passively Modelocked Lasers,” *Opt. Express* **24**, 20228–20244 (2016).  
[doi:10.1364/OE.24.020228]
6. S. Wang, B. S. Marks, and C. R. Menyuk, “Nonlinear Stabilization of High-Energy and Ultrashort Pulses in Passively Modelocked lasers,” *J. Opt. Soc. Am. B* **33**, 2596–2601 (2016).  
[doi:10.1364/JOSAB.33.002596]
7. Z. Qi, G. D’Aguanno, and C. R. Menyuk, “Nonlinear Frequency Combs Generated by Cnoidal Waves in Microring Resonators,” *J. Opt. Soc. Am. B* **34**, 785–794 (2017).  
[doi:10.1364/JOSAB.34.000785]
8. G. D’Aguanno and C. R. Menyuk, “Coupled Lugiato-Lefever Equation for Nonlinear Frequency Comb Generation at an Avoided Crossing of a Microresonator,” *Eur. Phys. J. D* **71:74**, 70705 (2017).  
[doi:10.1140/epjd/e2017-70705-x]
9. S. Wang, S. Droste, L. C. Sinclair, I. Coddington, N. R. Newbury, T. F. Carruthers, and C. R. Menyuk, “Wake Mode Sidebands and Instability in Mode-Locked Lasers with Slow Saturable Absorbers,” *Opt. Lett.* **42**, 2362–2365 (2017).  
[doi:10.1364/OL.42.002362]
10. J. Zweck and C. R. Menyuk, “Computation of the Timing Jitter, Phase Jitter, and Linewidth of a Similariton Laser,” *J. Opt. Soc. Am. B* **35**, 1200–1210 (2018).  
[doi.org/10.1364/JOSAB.35.001200]

11. S. Wang, T. F. Carruthers, and C. R. Menyuk, “Efficiently Modeling the Noise Performance of Short-Pulse Lasers With a Computational Implementation of Dynamical Methods,” *J. Opt. Soc. Am. B* **35**, 2521–2531 (2018).  
[doi.org/10.1364/JOSAB.35.002521]
12. X. Zhang, S. Wang, N. Guo, F. Li, C. R. Menyuk, and P. K. A. Wai, “Design of a Dual-Channel Modelocked Fiber Laser that Avoids Multi-Pulsing,” *Opt. Exp.* (accepted for publication).

### Conference Presentations (Contributed)

1. S. Wang and C. R. Menyuk, “Soliton Wake Instability in a SESAM Modelocked Fiber Laser,” Student Poster Competition in Optics and Photonics, College Park, MD (Apr. 4, 2014).
2. S. Wang, C. R. Menyuk, L. Sinclair, I. Coddington, and N. R. Newbury, “Soliton Wake Instability in a SESAM Modelocked Fiber Laser,” Conference on Lasers and Electro-Optics, San Jose, CA (June 8–13, 2014), paper SW3E.
3. S. Wang and C. R. Menyuk, “Stability of Modelocked Lasers With Slow Saturable Absorbers,” Ninth IMACS International Conference on Nonlinear Evolution Equations and Wave Theory, Athens, GA (Apr. 1–4, 2015), session 17.
4. S. Wang and C. R. Menyuk, “Optimization-Oriented Modeling of a Modelocked SESAM Fiber Laser,” IEEE Photonics Conference, Reston, VA (Oct. 4–8, 2015), paper WP2.
5. Z. Qi, G. D’Aguanno, and C. R. Menyuk, “Cnoidal Waves in Microresonators,” Conference on Lasers and Electro-Optics, San Jose, CA (June 5–10, 2016), paper FM2A.7.
6. S. Wang, C. R. Menyuk, S. Droste, L. C. Sinclair, I. Coddington, and N. R. Newbury, “Wake Mode Sidebands and Instability in Comb Lasers with Slow Saturable Absorbers,” Conference on Lasers and Electro-Optics, San Jose, CA (June 5–10, 2016), paper SM3H.5.
7. G. D’Aguanno and C. R. Menyuk, “Coupled Bright Solitons in the Normal Dispersion Regime in Whispering-Gallery-Mode Resonators,” Conference on Lasers and Electro-Optics, San Jose, CA (June 5–10, 2016), paper JW2A.48.
8. S. Wang and C. R. Menyuk, “Calculation of the Power Spectral Density in Passively Modelocked Lasers With Slow Saturable Absorbers,” OSA Frontiers in Optics, Rochester, NY (Oct. 17–21, 2016), paper JW4A.189.
9. S. Wang, C. R. Menyuk, S. Droste, L. Sinclair, I. Coddington, and N. Newbury, “Optimizing the Power Efficiency of a SESAM Comb Laser,” Conference on Lasers and Electro-Optics, San Jose, CA (May 14–19, 2017), paper SF1C.2.
10. Z. Qi, G. D’Aguanno, and C. R. Menyuk, “Dark Solitons and Cnoidal Waves in Microresonators with Normal Dispersion,” OSA Frontiers in Optics, Washington, DC (Sept. 17–20, 2017), paper JTu3A.14.

11. C. R. Menyuk, S. Wang, and T. F. Carruthers, “A Dynamical Perspective on Noise in Passively Modelocked Lasers,” OSA Frontiers in Optics, Washington, DC (Sept. 17–20, 2017), paper JTU3A.57.
12. X. Zhang, S. Wang, F. Li, C. R. Menyuk, and P. K. A. Wai, “Design of a Dual-Channel Modelocked Fiber Laser that Avoids Multipulsing,” Conference on Lasers and Electro-Optics, San Jose, CA (May 13–18, 2018), paper JTh2A.125.
13. Z. Qi, S. wang, J. A. Jaramillo-Villegas, M. Qi, A. M. Weiner, G. D’Aguanno, and C. R. Menyuk, “Stability of Cnoidal Wave Frequency Combs in Microresonators,” Conference on Lasers and Electro-Optics, San Jose, CA (May 13–18, 2018), paper SF2A.6.

### Conference Presentations (Invited)

1. C. R. Menyuk and S. Wang, “Boundary Tracking Algorithms for Determining the Stability of Modelocked Lasers,” American Mathematical Society Western Spring Sectional Meeting, Albuquerque, NM (Apr. 4–6, 2014), paper 1099-78-140.
2. S. Wang and C. R. Menyuk, “Stability of Modelocked Lasers With Slow Saturable Absorbers,” SIAM Nonlinear Waves Meeting, Cambridge, United Kingdom (Aug. 11–14, 2014), session MS-35.
3. C. R. Menyuk, “Solitons, Self-Induced-Transparency, and Quantum Cascade Lasers,” SIAM Nonlinear Waves Meeting, Cambridge, United Kingdom (Aug. 11–14, 2014), session MS-72.
4. C. R. Menyuk, “Maximizing the Bandwidth While Minimizing the Spectral Fluctuations Using Supercontinuum Generation in Photonic Crystal Chalcogenide Fibers,” OMICS Lasers, Optics, and Photonics Conference, Philadelphia, PA (Sep. 8–10, 2014).
5. C. R. Menyuk, “Quantitative Modeling of Comb Sources: Short- Pulse Lasers and Microresonators,” OSA Incubator on Quantitative Modeling of Frequency-Comb Sources, Washington, DC (Nov. 19–21, 2014).
6. C. R. Menyuk and N. R. Newbury, “Quantitative Modeling of Comb Sources: Short-Pulse Lasers and Microresonators,” OSA Optical Metrology Technical Group, July 1, 2015 [Webinar].
7. C. R. Menyuk and S. Wang, “Stability and Noise Performance of Modelocked Lasers: A Status Report,” Nonlinear Waves, Theory and Applications International Conference, Beijing, China (June 25–28, 2016), session MS6.
8. C. R. Menyuk and S. Wang, “Spectral Methods for Determining the Stability and Noise Performance of Modelocked Laser Pulses,” SIAM Nonlinear Waves and Coherent Systems Conference, Philadelphia, PA (Aug. 7–11, 2016), session MS29.
9. J. Zweck, Y. Sehn, C. R. Menyuk, and S. Wang, “Spectra of Short Pulse Solutions of the Cubic-Quintic Complex Ginzburg Landau Equation Near Zero Dispersion,”



- SIAM Nonlinear Waves and Coherent Systems Conference, Philadelphia, PA (Aug. 7–11, 2016), session MS38.
10. V. R. Besse, S. Wang, and C. R. Menyuk, “Stability of Short-Pulse Solutions of the Complex Cubic-Quintic Ginzburg-Landau Equation,” SIAM Nonlinear Waves and Coherent Systems Conference, Philadelphia, PA (Aug. 7–11, 2016), session MS38.
  11. S. Wang and C. R. Menyuk, “Modeling Modelocked Fiber Lasers With Slow Saturable Absorbers,” SIAM Nonlinear Waves and Coherent Systems Conference, Philadelphia, PA (Aug. 7–11, 2016), session MS38.
  12. Z. Qi, G. D’Aguanno, and C. R. Menyuk, “The Lugiato-Lefever Equation and Cnoidal Waves in Microresonators,” SIAM Nonlinear Waves and Coherent Systems Conference, Philadelphia, PA (Aug. 7–11, 2016), session MS38.
  13. Z. Qi and C. R. Menyuk, “Cnoidal Wave Solutions to the Lugiato-Lefever Equations with Applications to Microresonators,” Tenth IMACS International Conference on Nonlinear Evolution Equations and Wave Theory, Athens, GA (Mar. 29–Apr. 1, 2017), session 18.
  14. C. R. Menyuk, Z. Qi, and S. Wang, “Instability and Noise in Nonlinear Optical Waveguides,” Progress in Electromagnetics Research Symposium, Singapore (Nov. 19–22, 2017), session 2P5a.
  15. C. R. Menyuk, Z. Qi, and S. Wang, “Instability and Noise in Nonlinear Optical Resonators,” SIAM Conference on Nonlinear Waves and Coherent Structures, Orange, CA (June 11–14, 2018), session MS16.
  16. V. K. Shinglot, J. Zweck, C. R. Menyuk, and Y. Shen, “Dynamics and Noise Performance of Periodically-Stationary Pulses in Fiber Lasers,” SIAM Conference on Nonlinear Waves and Coherent Structures, Orange, CA (June 11–14, 2018), session MS31.
  17. C. R. Menyuk, Z. Qi, and S. Wang, “Stability and Noise in Frequency Combs: Harnessing the Music of the Spheres,” Workshop on Spatiotemporal Multimode Complex Optical Systems, Dubai, UAE (Oct. 16–19, 2018).

### Dissertation

1. S Wang, *Analysis of Stability and Noise in Passively Modelocked Comb Lasers*, (PhD Dissertation, University of Maryland Baltimore County, 2018).