

The Effects of Visual Representations When Problem Solving

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Abstract

The purpose of this study was to determine if the use of visual representations while problem solving would increase student's achievement. The participants in this study consisted of eight, fourth grade students, who were selected based on their low achievement on a pre-assessment. The participants participated in a 30-minute, supplemental instruction group two days a week for ten weeks. During this time, participants were introduced to two types of visual representations, bar models and diagrams. Participants were given the opportunity to apply these representations to different types of word problems. The results revealed a significant increase in mean math scored for the pre to the post-test. The results of the post-assessment suggest the use of visual representations have a positive effect on student's problem-solving achievement.

CHAPTER I

INTRODUCTION

Overview

Problem solving is a mathematical skill students are exposed to throughout their entire schooling. This skill is taught and assessed in every grade level, K through 12. Problem solving mastery is crucial to a child's mathematical success, as it is part of one of the standards for mathematical practices outlined in the Common Core State Standards, *Making sense of problems and persevering in solving them* (Flores, Hinton, & Burton, 2016).

Problem solving is taught in various forms, the most common being through word problems (Flores et al., 2016). Zhu (2015) defines word problems as “linguistically presented problems requiring arithmetic solutions” (p.608). A student's mathematic ability is often assessed through their performance on word problems, whether it is on their unit assessments, the annual PARCC assessment, and even the SATs. A student will struggle to be successful in math if they are unable to successfully and accurately solve word problems.

The use of visual representations has been found to help individuals solve different types of math related problems as they establish meaning within a problem and aid in determining problem-solving methods (van Garderen, 2006). Two forms of visual representations that have been shown to aid students when problem solving and which are used in this study are diagrams and bar models. Diagrams are a type of visual representation that can be used to help students interpret and solve word problems using spatial relationships (Chu, Rittle-Johnson, & Fyfe, 2017). A study conducted by Chu et al. found the use of diagrams increased student's accuracy when problem-solving. Morin, Watson, Hester, and Raver (2017) conducted a study to determine the effectiveness of bar models, which they believe can be used to solve a variety of problems.

Bar models were shown to increase problem-solving accuracy. Bar models can be taught at a young age and then built upon as students get older and experience more difficult, involved problems.

Solving word problems is a difficult task for many students, especially students with mathematic disabilities (Zhu, 2015). It is a task that students are not only going to encounter in their entire school career but is a task that is going to get more complicated as they go. It is imperative that students are taught strategies to solve word problems when they are younger, so they are successful when solving as they get older.

As an educator, it is apparent problem solving is not only a huge focus across grade levels, but also a huge deficit amongst students across grade levels. The level of importance and difficulty tied to problem solving is what drew interest to this topic. It is important to teach students strategies, such as using visual representations, so they can utilize them when solving word problems. Hopefully, this could lead to problem solving being less of a deficit in the future.

Statement of Problem

Does the creation of visual representations increase student mathematical achievement?

Hypothesis

Visual representations will have no effect on student mathematical achievement.

Operational Definitions

The independent variable in this study is the implementation of a supplemental instruction. Supplemental instruction was implemented via small groups that used visual representation including pictures, models, bar drawings, use of manipulatives, etc. The supplemental group consisted of students that met twice a week to practice using visual representations when solving word problems. The dependent variable in this study is math

achievement on teacher created word problems. Student achievement on word problems will be determined using the problem rubric created by the researcher's administration.

CHAPTER II

A REVIEW OF THE LITERATURE

This review of the literature explores the various ways visual representations are involved with problem-solving. Section one discusses the importance of problem-solving skills, defines word problems and briefly discusses strategies teachers can use to support students who have difficulty with word problems. Section two explains how the use of visual representations can be beneficial when problem-solving, two different types of visual representations and possible reasons visual representations might not be beneficial. Section three briefly touches on various ways the use of visual representations can be promoted in the classroom. In section four, three intervention programs designed to help students with problem-solving involving visual representations are explained. Finally, in section five, two specific types of visual representations are described. The ideas of problem-solving and solving word problems are used interchangeably throughout this review.

Introduction to Problem Solving

Problem solving is a skill that is critical to a student's mathematical success as it is taught and assessed in every grade, K through 12. "Making sense of problems and persevering in solving them" is one of the standards for mathematical practices outlined in the Common Core State Standards as a practice student's need in order to succeed in math (Flores, et al. 2016). There are many forms of problem solving which can be seen in schools, the most common being word problems. Zhu (2015) defines word problems as "linguistically presented problems requiring arithmetic solutions" (p.608).

Solving word problems is a difficult task for many students, especially students with mathematic disabilities (Zhu, 2015). When supporting students who have difficulties solving

word problems, there are several strategies that are commonly used by teachers (Flores et al., 2016). Flores et al. state these strategies include identifying key information, identifying key words, and creating a conceptual model. Of the three strategies discussed, identifying key words was the only one that proved to hinder students' performance, as key words can be misleading and remove the focus of meaning from the problem.

Using Visual Representations When Problem Solving

The use of visual representations has been found to help individuals solve different types of math related problems (van Garderen, 2006). van Garderen states two benefits of visual representations being they establish meaning within a problem and aid in determining problem-solving methods. When solving word problems, it is imperative to a student's success that they are able to determine the meaning of each part of the problem and determine an appropriate method to solve the problem.

van Garderen (2006) and Carden and Cline (2015) both discuss two different types of visual representations; pictorial imagery, images that have the visual appearance of the objects in a problem, and schematic imagery, images that have the spatial relations outlined in a problem. van Garderen and Carden and Cline both found schematic images were correlated with problem solving success, while pictorial images were correlated with problem solving failure. van Garderen (2006) also found gifted students are more likely to use visual images when problem solving than students with learning disabilities (LD) are. When gifted students used visual images they often used schematic imagery while students with LD more often used pictorial images. Therefore, not only do students with LD have difficulty solving word problems in general, but they are rarely using a strategy that will help them and when they do, they are not using the most effective strategy.

In a study conducted by Swanson, Orosco and Lussier (2014), the idea that students with LD may have a low working memory was explored. The low working memory may be affecting the effectiveness of problem solving strategies taught to LD student, including the use of visual representations (using a diagram) and verbal skills (underlining important information). This idea was proven, students with a higher working memory performed better with the intervention strategies than students with a lower working memory. Also, students with a lower working memory who did not receive the verbal or visual intervention performed higher than those who did.

Promoting Visual Representations in the Classroom

According to Carden and Cline (2015), all students, even those who do not find word problems challenging, would benefit from visual imagery/ visualization being promoted as a problem-solving strategy. Visualization strategies will increase the likelihood of successful students continuing to be successful as they come across more complex problems. These strategies should be taught to children at an early age and should be incorporated as a general practice.

Carden and Cline (2015) discuss several ways teachers can encourage and promote the use of visual imagery while problem solving. One way is to encourage communication of visual imagery by allowing students to share and explain their imagery to their peers. Another way is to start students off with simple problems and then transfer them to more difficult problems that would use the same type of imagery. Finally, Carden and Cline state the importance of students being able to build confidence and experience in creating their own types of imagery.

Problem Solving Programs Involving Visual Representations

There are many intervention programs that have been designed to help students who struggle with problem solving. Schema-Based Instruction (SBI), Concrete-Representational-Abstract Sequence (CRA), and Cognitive Strategy Instruction (CSI) are examples of three programs that involve the use of visual representations as part of their program.

Schema-Based Instruction (SBI) SBI is a problem-solving strategy that combines problem-solving and reading comprehension strategies (Spooner, Saunders, Root, & Brosh, 2017). SBI requires students to create representations that visually map out word problems in order to enhance their comprehension and understanding of the problem structure. The representations created using SBI helps reduce the chances of a student misunderstanding the problem situation, which is the most common error made when solving word problems.

Jitendra, Harwell, Im, Karl, and Slater (2018) conducted a study to determine the effectiveness of SBI for a sample of seventh grade students with varying levels of math difficulties. In this study, classrooms were randomly assigned to SBI or control conditions. Classrooms with SBI replaced the district curriculum with the SBI program and classrooms with the control conditions taught the district curriculum as is. All students completed two different pretests, posttests, and delayed posttests, the PPS (proportional problem solving) and the GMADE (Group Mathematics Assessment and Diagnostic Evaluation). On both the PPS and GMADE posttest, the SBI students outperformed the controls students. .

Concrete-Representational-Abstract (CRA) CRA is a sequence of skills/ stages students should follow when learning how to solve problems. This sequence includes concrete- using manipulatives, representational- drawing pictures, and abstract- using just numbers (Flores et al., 2016). Students should not advance to the next stage until they are developmentally ready.

If students are struggling to visually represent the problem representationally, then they should revert to the concrete stage and visually represent the problem with manipulatives. Similarly, if a student is struggling to solve a problem abstractly, they should revert to the representational stage and represent the problem with a drawing.

Flores et al. (2016) conducted a study involving a CRA intervention program with three third graders. These three students were proficient in adding and subtracting, as well as reading on at least a second-grade level. The intervention took place afterschool four days a week, for 20 minute sessions. The intervention included four phases; teaching different problems structures, concrete instruction, representational instruction and abstract instruction. At the concrete level students acted out problems, at the representational level students were prompted to choose appropriate diagrams and at the abstract level there were no diagrams. Students were given probes, a sheet with four problems, each day to assess the skills learned the previous day. By the end of the program, all three students achieved mastery, scoring 100% on three probes.

Cognitive Strategy Instruction (CSI) CSI teaches students different cognitive (visualization) and metacognitive (self-questioning) strategies to improve their problem-solving performance (Zhu, 2015). When using CSI to problem solve, students are taught the following steps; read the problem, paraphrase the problem, make a drawing/ diagram, make a plan, estimate, solve the problem, check their work. According to Zhu, CSI has been shown to improve word problem success.

Krawec, Huang, Montague, Kressler and de Alba (2013) conducted a study with 40 middle schools where schools were randomly assigned to the control group or to conduct the intervention program *Solve It!*, a CSI program. Random average achieving (AA) students and students with learning disabilities (LD) in both groups were given the Math Problem-Solving

Assessment (MPSA) as a pretest and posttest. Students in the intervention group received *Solve It!* once a week for 30-minutes and the control group received no extra instruction. It was found that students who received *Solve It!* used more strategies when completing the posttest than the students in the control group used. Additionally, *Solve It!* was found to be equally effective for AA students and students with LD and these students outperformed the students in the control group.

Specific Types of Visual Representations

As discussed earlier, a common form of visual representation when solving word problems is visual imagery, both pictorial and schematic. Two additional forms of visual representations that can be used when problem-solving are diagrams and bar models. Morin, et al. (2017) believe organizers used for SBI are limited to the types of problems they can be used to solve and therefore conducted a study to determine the effectiveness of an organizer called bar models, which they believe can be used to solve a variety of problems. In the study, bar models were proven to increase problem-solving accuracy. Bar models can be taught at a young age and then built upon as students get older and experience more difficult, involved problems.

Diagrams are a type of visual representation that can be used to help students interpret and solve word problems using spatial relationships (Chu, et al., 2017). Chu et al. discuss several benefits of using diagrams including that diagrams may describe relationships between known and unknown quantities, they may decrease the amount of thinking that needs to be done mentally, and they may connect concrete and abstract representations. A study conducted by Chu et al. found the use of diagrams increased student's accuracy when problem-solving.

Summary

Problem solving is a mathematical skill necessary for success in grades K-12, as well as throughout life. It is essential for students to be taught strategies that will increase their level of success. The use of visual representations is a common strategy used throughout different problem-solving intervention programs. Visual representations can be used and structured in many ways, but each way is designed to help students understand the problem and determine the appropriate method needed to solve. All students can benefit from being exposed to visual representations, even those who tend to do well when problem-solving.

CHAPTER III

METHODS

Design

The purpose of this study was to determine if using visual representations affects student achievement when completing word problems. Students were selected to participate in the supplemental group based on their word problem performance, making this a quasi-experimental pretest/ posttest design. The independent variable is the supplemental group. The dependent variable is math achievement on teacher created word problems. After completing a word problem, students were scored using a predetermined grading scale of 0, 1, 2, or 3. Students receiving a 0 or 1 were placed in a supplemental instruction group. After 10 weeks of intervention, the students in the supplemental instruction group completed the same word problem and were scored again using the same grading scale.

Participants

Students participating in this study were from a below level, 4th grade math class in a Title 1, Baltimore County elementary school. The supplemental instruction group was made up of eight students, five males and three females. Five of the students were African American and three of the students were Hispanic. Five of the students had Individualized Education Plans (IEPs) and three of the students are labeled as English Language Learners (ESOL). Convenience sampling was used to select these eight students, being they were already in the class taught by the researcher.

Instruments

The instrument used in this study was a math test with -0, 1, 2, 3 grading scale determined by the researcher's administrative team. In order to receive a three, students must

have completed the necessary steps to find the correct answer, as well as labeling the parts of their equation and writing their answer in a complete sentence. Students receiving a two completed the necessary steps and found the correct answer but did not label their equation or include a sentence. Students receiving a one attempted to complete the correct steps but received an incorrect answer and students receiving a zero were unable to determine the correct steps. Since this instrument was created by the researcher's administration, it was not tested for reliability or validity.

Procedure

This supplemental group met every Tuesday and Thursday for ten weeks during the student's lunch, which lasted for 30 minutes from 12:15- 12:45. During week one the intervention group was introduced to two types of visual representations- bar models and diagrams. Tuesday was spent modeling and practicing solving equations with bar models and Thursday was spent modeling and practicing solving equations with diagrams. The remaining nine weeks were spent applying these two types of visual representations to word problems. Weeks two through six were spent solving single step word problems created by the researcher. Each week had a different operational focus; week two- addition, week three- subtraction, week four- multiplication, week five- division and week six- a mix. During weeks two through six, students were required to use bar models on Tuesday's and diagrams on Thursday's. Weeks seven through ten were spent solving multi-step word problems created by the researcher, incorporating combinations of all four operations. During weeks seven and eight, students were forced to use bar models on Tuesday's and diagrams on Thursday's. During weeks nine and ten, students were able to choose the method they found most useful. Each session began with the students and researcher working through a problem together. At this time the researcher modeled

how either the bar model or diagram would be used when solving that type of problem. Then the students were instructed to work independently on another example while the teacher addressed individual needs. The students shared their work with a partner and discussed/ explained what they did. The students continued to independently solve problems and discuss them with partners for the remainder of the session. On the last day of intervention, students completed their posttest, solving the same problem they originally solved during the pretest.

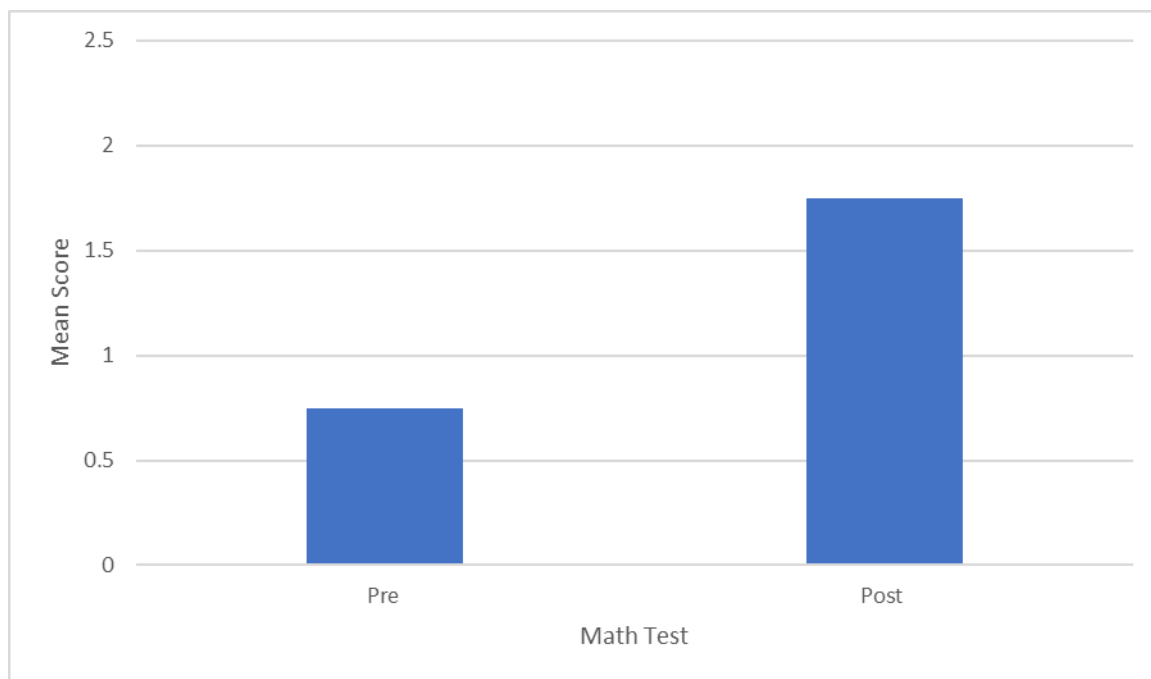
CHAPTER IV

RESULTS

This study examined the effect of supplemental instruction on math achievement.

The pre-test administered to the eight students had a mean math score of .75, with maximum score of 2.5, which significantly increased to a mean math score of 1.75, $t(7) = -2.37$, $p < .05$ (Figure 1). The standard deviation on the pre-test was .46 and .96 on the post-test. Math scores increased from the pre to post.

Figure 1: Mean Math Scores on the Pre and Post Test



CHAPTER V

DISCUSSION

The purpose of this study was to determine if the use of visual representations would improve student achievement on word problems. The hypothesis for this study, visual representations have no effect on student achievement when problem solving, was not supported.

Implications of Findings

In this study, there was significant difference between the mean math scores of the pre-assessment and the post assessment, increasing from .75 to 1.75. This significant increase indicates the use of visual representations do have an effect on student achievement when problem solving.

Theoretical Consequences

Zhu (2015) states that solving word problems is an especially difficult task for students with math disabilities. This was also supported in this study as five of the eight participants have been identified as learning disabled (LD) and have math goals on their IEPs. Research conducted by van Garderen (2006) discusses two different types of visual representations, pictorial imagery, images with visual appearance of the objects in a problem and schematic imagery, images with spatial relations outlined in a problem. van Garderen's research found students with LD more often use pictorial images, which will not support their problem solving skills. In this study, the eight participants used pictorial images on their pre-assessment and did not perform well. On the post-assessment the participants that used schematic imagery, improved their performance from the pre-assessment.

In research conducted by Flores, et al., (2016), three strategies for supporting students who have difficulties solving word problems were discussed, one of these strategies being

creating a conceptual model. van Garderen (2006) also found the use of visual representations to be helpful to students when solving math related problems, as they establish meaning and aid in determining methods to solve. In this study, the visual representations discussed in the supplemental instruction group, were designed to aid the participants in establishing the meaning and determining which operation(s) were needed to solve.

Threats to Validity

There are several factors at play in this study that threaten the validity of the results. One factor that may threaten validity is maturation. This study took place over a 10 week span, so students' progress would have improved whether or not they participated in the supplemental group. Problem solving was a huge focus in the student's everyday math instruction so in addition to the problem solving instruction and practice received within the supplemental group, they were receiving it outside of the supplemental group several times a week as well.

Another threat to validity is differential selection. The participants in this study were not randomly selected, they were selected because they did not perform well on the pre-assessment. Since the participants did not perform well initially, this left a lot of room for improvement on the post-assessment. If students who had performed well on the pre-assessment were selected for the study, they may not have shown as much improvement.

In addition, the small group setting may have threatened the validity of the results. Participants were working in a group of eight students, making it easy for the researcher to focus their attention on students in need. Also, working in a small group setting makes it easier for the students to focus as there are less distraction.

Connections to Previous Studies

There have been many studies conducted in which specific problem solving strategies were implemented and their ability to improve students problem solving abilities were analyzed. Schema-based instruction (SBI), cognitive strategy instruction (CSI) and concrete-representation-abstract (CRA) strategies were used in three different studies. All three of these strategies involved using visual representations at some point in their problem solving process. They also involved other problem solving strategies such as reading comprehension, using manipulatives, paraphrasing the problem, etc. that were not used in this study. The results of these three studies proved the strategies as being effective.

A study conducted by Chu, et al., (2017) focused specifically on diagrams, like this study. Chu et al. found the use of diagrams to be beneficial when their participants were finding the relationship between known and unknown quantities. As discussed earlier, diagrams were found to benefit participants in this study in a similar manner. Chu et al. found the use of diagrams increased students problem solving accuracy, as is the case in this study.

Implications for Future Research

A couple recommendations can be made for similar studies conducted in the future. One recommendation for future research is to only focus on diagrams, rather than diagrams and bar models. Bar models were more abstract than diagrams, causing students to struggle using them. Supplemental group sessions scheduled to use bar models when problem solving were more focused on creating the bar model, rather than solving the word problems. Also, diagrams helped create a better understanding of the actual context of the problems, aiding students as they were determining the appropriate operations needed.

Another recommendation for future research is to conduct the supplemental group instruction at a time other than lunch. Students were not able to focus all of their attention on the instruction because they were distracted by eating.

Summary

Problem solving is a mathematical skill that is difficult for many students and is a focus across grade levels. It is crucial that students are provided strategies they can use to help them be successful when problem solving. This study has shown that teaching students to use visual representations is an effective strategy that can be used when solving different types of word problems.

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