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# Two-photon X-ray Ghost Microscope

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Unlike classic imaging devices in the visible spectrum, there are no effective imaging lenses to produce the point-to-point image-forming function for high-energy (short-wavelength) X rays. The X-ray imaging that we are familiar with more closely resembles a projection or “shadow” of the object rather than a point-to-point image. Here, we present an imaging mechanism that produces true point-to-point imaging of X rays through the measurement of two-photon interference intensity fluctuation correlation, which allows for a table-top X-ray microscope by means of a magnified secondary ghost image. In principle, once some experimental barriers are overcome, this X-ray “ghost microscope” may achieve nanometer spatial resolution and open up new capabilities that would be of interest to the fields of physics, material science, and medical imaging.

## INTRODUCTION

Imaging lenses play a critical role in classic imaging with visible light. In general, an imaging lens is able to produce a diffraction-limited point-to-point correlation between the object plane and the image plane, forming a magnified or demagnified image of the object [1, 2]. If desired, an additional lens system is then able to map the primary image onto a secondary image plane for further magnification or demagnification; notably making an optical microscope possible. Unfortunately, there are no effective imaging lenses for high-energy ( $> 20$  keV) X rays due to the refractive index of X rays being approximately 1 for all known materials [3]. In fact, current X-ray imaging techniques commonly used in the medical field do not utilize true point-to-point image-forming function as done with visible light and lenses. This type of X-ray “image” is actually a projection of the X-rays that pass through the object with different materials and thicknesses of materials causing more absorption in some areas compared to others. This is more comparable to shadows than true image formation [3, 4]. In this article, we discuss a new technique that is able to (1) produce a one-to-one lensless point-to-point image-forming correlation between the object plane and the image plane through the measurement of two-photon interference induced intensity fluctuation correlation of X rays; (2) through the help of scintillators and lenses working in visible spectrum, the X-ray image can be mapped onto a secondary image plane with significant magnification; namely, an X-ray ghost microscope. Similar to lensless ghost imaging in the visible spectrum [5–7], the X-ray lensless image-forming correlation is the result of two-photon interference: two randomly created and randomly paired photons interfering with the pair itself [6, 8].

Visible-light ghost imaging was first demonstrated in 1995 using entangled photon pairs through the measurement of the second-order coherence function [9]. In this demonstration, a point-to-point image-forming correlation resulted from a pair of entangled photons interfer-

ing with the entangled pair itself. 10 years later, it was demonstrated that two-photon interference of randomly created and randomly paired photons in a thermal state can produce a similar point-to-point image-forming correlation without the need of an imaging lens. This type of ghost imaging is commonly called lensless ghost imaging [5–7]. Interestingly, the first lensless ghost imaging of thermal light was observed in the secondary ghost imaging plane with the help of a lens. An unfolded schematic diagram of the 2005 ghost imaging experiment of thermal light is illustrated in Fig. 1. The original purpose of this special setup was to confirm the observed ghost image is the result of a point-to-point image-forming correlation and not a projection or shadow [5].

X-ray ghost imaging has now been demonstrated multiple times [10–17]; however, it is still a developing field. One commonly used technique for current X-ray ghost imaging is to introduce a spatially varied material to produce a “speckle” pattern in the X-ray beam. This results in a classical speckle-to-speckle ghost imaging with resolution dependent on the speckle size. However, this ghost

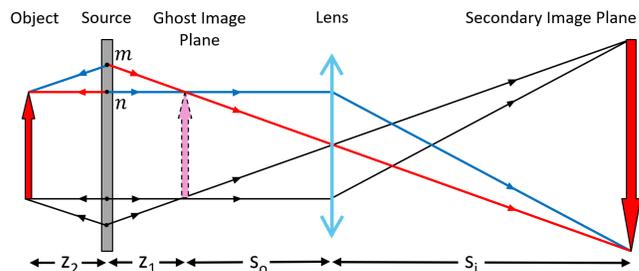


FIG. 1. Experimentally achieved with a beam splitter, this “unfolded” schematic of the 2005 thermal light ghost imaging demonstration of Valencia *et al.* helps depict the symmetry of the two-photon amplitudes. The one-to-one lensless ghost image and the magnified secondary ghost image are results of two-photon interference. Of the many two-photon amplitudes, such as the red and blue two-photon amplitudes of the random  $m$ th and  $n$ th pair of photons shown here, each superpose constructively at a corresponding  $\vec{\rho}_i$  to form an image.

imaging technique differs from the mechanism of the two-photon ghost imaging experiments. Classified according to their experimental setup and working mechanism, we find two classes of ghost imaging: (I) The observed ghost image is produced from a natural point-to-point image forming correlation that is the result of two-photon interference. This class of ghost imaging follows the mechanism of the entangled state ghost imaging of Pittman *et al.* [9] and the thermal light ghost imaging of Valencia *et al.* [5]. With thermal light ghost imaging, the point-to-point image-forming correlation is able to achieve spatial resolution of  $\lambda/\Delta\theta_s$ , where  $\Delta\theta_s$  is the angular diameter of the light source and has the potential to be turbulence-free [8, 18–20]. Pelliccia *et al.* use type (I) ghost imaging in their first demonstration of X-ray ghost imaging [11]. Alternatively, (II) the observed ghost image is produced from a classical speckle-to-speckle correlation shown with visible light [21–23] and with a variety of X-ray sources [12, 14–17]. Here, a ghost image of the object is obtained from the coincidences between two sets of identical “speckles” formed by either spatially correlated laser beams or an aperture mask following the light source to produce shadows and bright spots distributed on the object plane and on the ghost image (detector) plane. Included as a classical speckle-to-speckle correlation is computational ghost imaging which removes the need of a beam splitter by measuring the object plane and ghost image plane at separate times with an identical speckle pattern [10, 13, 24]. These classical projected speckles are different from the two-photon interference that produces a point-to-point image-forming correlation and, correspondingly, this type of ghost imaging has a resolution dependent on the size of the classically formed speckles. This type of ghost imaging observes a projection or a shadow of the object, comparable to how classic X-ray imaging technology is a projection of the object. Both classes of ghost imaging use the measurement of intensity fluctuation correlation, utilizing either changes in the speckle distribution over time or fluctuations resulting from two-photon interference, to obtain an image.

The X-ray ghost microscope studied in this article belongs to type (I) ghost imaging via two-photon interference. As with lensless ghost imaging, the spatial resolution of X-ray ghost imaging before magnification is determined by  $\lambda/\Delta\theta_s$ , where  $\lambda$  is the wavelength and  $\Delta\theta_s$  is the angular diameter of the X-ray source. This unique characteristic of lensless ghost imaging makes the resolving potential of an X-ray microscope a point of interest. As an example, using a high-energy ( $>20$  keV) X-ray source with a relatively large angular diameter may produce a one-to-one (no magnification or demagnification) lensless ghost image with nanometer resolution. It should be noted that this is the opposite of what is desired for classic projectional X-ray imaging which sees an increase in resolution when using a smaller, point-like source, such as modern X-ray tubes and synchrotron X-ray sources

[3, 25]. Type (I) ghost imaging is still possible with these small sources, but the resolution will be reduced. When expecting one-to-one ghost imaging at nanometer resolution, the image becomes unresolvable by any state-of-the-art CCD or CMOS detector arrays. However, this primary ghost image can be magnified significantly onto the secondary ghost image plane through a scintillator-lens system, potentially making nanometer sized features resolvable by a CCD or CMOS with micrometer pixels. Importantly, the secondary ghost image, either magnified or demagnified, is produced directly by the point-to-point image-forming correlation between the object plane,  $\vec{\rho}_o$ , and the secondary ghost image plane,  $\vec{\rho}_i$ , where  $\vec{\rho}_o$  and  $\vec{\rho}_i$  represent the transverse coordinates of the object plane and image plane, respectively. This peculiar feature preserves a two-photon diffraction limited spatial resolution. An additional benefit of two-photon interference is that this X-ray ghost microscope can be set up such that it is insensitive to any rapid phase variations along the optical path due to random changes in composition, density, length, index of refraction, or medium vibration, namely “turbulence-free” [18–20]. The turbulence-free nature is especially important for the extremely high resolution imaging obtainable with the X-ray ghost microscope.

This article is organized in four sections: (1) we introduce the concept of two-photon interference induced intensity fluctuation correlation, (2) we prove a point-to-point image-forming correlation for producing lensless ghost image and its secondary image via two-photon interference, and (3) we discuss how to obtain observable intensity fluctuation correlation of X-ray with broad spectrum.

## TWO-PHOTON INTERFERENCE AND MEASURING INTENSITY FLUCTUATION CORRELATION

Recall, in 1905, Einstein introduced a granularity to radiation, abandoning the continuum interpretation of Maxwell [26]. This led to a microscopic picture of radiation and a statistical view of light, including X rays. In Einstein’s picture a light source consists of many point-like sub-sources, each of which emit their own subfields. Originally labeled in German by “strahlenbündel,” which translates to “bundle of ray” in English, subfields are now labeled as “photons” in modern language. For a thermal light source, these subfields are emitted in a random manner such that the  $m$ th subfield (photon) emitted from the  $m$ th point-like sub-source may propagate in all possible directions with a random initial phase. It has been proven that an effective wavefunction can be defined from the quantum theory of optical coherence [27, 28] to specify the space-time behavior of a photon [8, 29]. The effective wavefunction of a photon in a thermal state is mathematically the same function as Einstein’s subfield. In

Einstein's picture, the radiation measured at coordinate  $(\mathbf{r}, t)$  is the result of a superposition of a large number of subfields,

$$E(\mathbf{r}, t) = \sum_m E_m(\mathbf{r}, t) = \sum_m E_m(\mathbf{r}_m, t_m) g_m(\mathbf{r}_m, t_m; \mathbf{r}, t),$$

where  $E_m(\mathbf{r}_m, t_m)$  labels the subfield emitted from the  $m$ th sub-source at coordinate  $(\mathbf{r}_m, t_m)$ , and  $g_m(\mathbf{r}_m, t_m; \mathbf{r}, t)$  represents the field propagator or Green's function that propagates the  $m$ th subfield from coordinate  $(\mathbf{r}_m, t_m)$  to coordinate  $(\mathbf{r}, t)$ . Shortening the notation, we will replace  $E_m(\mathbf{r}_m, t_m)$  with  $E_m$  and  $g_m(\mathbf{r}_m, t_m; \mathbf{r}, t)$  with  $g_m(\mathbf{r}, t)$ . In Einstein's picture of light, the expectation value of the intensity corresponds to a statistical ensemble average which takes into account *all possible realizations of the field* or, more specifically, takes into account *all possible relative phases of the subfields*:

$$\begin{aligned} \langle I(\mathbf{r}, t) \rangle &= \langle E^*(\mathbf{r}, t) E(\mathbf{r}, t) \rangle \\ &= \left\langle \sum_m E_m^*(\mathbf{r}, t) \sum_n E_n(\mathbf{r}, t) \right\rangle \\ &= \left\langle \sum_m |E_m(\mathbf{r}, t)|^2 \right\rangle + \left\langle \sum_{m \neq n} E_m^*(\mathbf{r}, t) E_n(\mathbf{r}, t) \right\rangle \\ &= \sum_m |E_m(\mathbf{r}, t)|^2. \end{aligned} \quad (1)$$

The expectation value of  $\sum_{m \neq n} E_m^*(\mathbf{r}, t) E_n(\mathbf{r}, t)$  goes to zero when taking into account all possible relative phases of the subfields. We may conclude the theoretical expectation value of intensity  $\langle I(\mathbf{r}, t) \rangle = \sum_m |E_m(\mathbf{r}, t)|^2$  is the result of the  $m$ th subfield interfering with the  $m$ th subfield itself, while the  $m \neq n$  term is a result of the  $m$ th subfield interfering with a different  $n$ th subfield. In a realistic measurement in which only a limited number of subfields contribute to the measurement, all possible phases may not be present, meaning the  $m \neq n$  term may not vanish and contribute noise to the measurement. We may name this term a two-photon interference induced intensity fluctuation,

$$\Delta I(\mathbf{r}, t) = \sum_{m \neq n} E_m^*(\mathbf{r}, t) E_n(\mathbf{r}, t). \quad (2)$$

The two-photon induced intensity fluctuation is different from other traditional "intensity fluctuations." For instance, the number of subfields contributing to a measurement may vary from measurement to measurement and thus the value of  $\sum_m |E_m(\mathbf{r}, t)|^2$  may change from measurement to measurement. These variations may be from a classical mask in the path of the light or variations in the intensity of the source. Note that in the following discussion, no classical intensity fluctuations are involved, neither from a spatial intensity distribution nor temporal intensity distribution, but only taking the two-photon induced intensity fluctuations of thermal light.

The X-ray ghost microscope studied in this article utilizes the two-photon induced intensity fluctuation correlation  $\langle \Delta I(\mathbf{r}_1, t_1) \Delta I(\mathbf{r}_2, t_2) \rangle$  for type (I) ghost imaging. Although the expectation or ensemble average of the intensity fluctuation measured by a single detector  $D_j$ ,  $j = 1, 2$ , is zero,  $\langle \Delta I(\mathbf{r}_j, t_j) \rangle = \langle \sum_{m \neq n} E_m^*(\mathbf{r}_j, t_j) E_n(\mathbf{r}_j, t_j) \rangle = 0$ , the expectation or ensemble average of the correlation of the intensity fluctuations measured by  $D_1$  and  $D_2$ , jointly, may not equal zero,

$$\begin{aligned} &\langle \Delta I(\mathbf{r}_1, t_1) \Delta I(\mathbf{r}_2, t_2) \rangle \\ &= \left\langle \sum_{m \neq n} E_m^*(\mathbf{r}_1, t_1) E_n(\mathbf{r}_1, t_1) \sum_{p \neq q} E_p^*(\mathbf{r}_2, t_2) E_q(\mathbf{r}_2, t_2) \right\rangle \\ &= \sum_{m \neq n} E_m^*(\mathbf{r}_1, t_1) E_n(\mathbf{r}_1, t_1) E_n^*(\mathbf{r}_2, t_2) E_m(\mathbf{r}_2, t_2). \end{aligned} \quad (3)$$

Due to the random relative phases between the subfields canceling in a specific case, when  $m = q$  and  $n = p$ , there is a surviving term in the above summation. Mathematically, the result of Eq. (3) can be represented as the cross term of the following superposition,

$$\begin{aligned} &G^{(2)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) \\ &= \sum_{m \neq n} |E_m(\mathbf{r}_1, t_1) E_n(\mathbf{r}_2, t_2) + E_n(\mathbf{r}_1, t_1) E_m(\mathbf{r}_2, t_2)|^2 \\ &= \langle I(\mathbf{r}_1, t_1) \rangle \langle I(\mathbf{r}_2, t_2) \rangle + \langle \Delta I(\mathbf{r}_1, t_1) \Delta I(\mathbf{r}_2, t_2) \rangle \end{aligned} \quad (4)$$

corresponding to the superposition of two different yet indistinguishable alternatives of joint photodetection: (1) the  $m$ th subfield (photon) is measured at  $(\mathbf{r}_1, t_1)$  while the  $n$ th subfield (photon) is measured at  $(\mathbf{r}_2, t_2)$ ; (2) the  $n$ th subfield (photon) is measured at  $(\mathbf{r}_1, t_1)$  while the  $m$ th subfield (photon) is measured at  $(\mathbf{r}_2, t_2)$ . The cross term of this superposition models the concept of randomly paired photons interfering with the pair itself, namely two-photon interference.

## IMAGE-FORMING CORRELATION FROM TWO-PHOTON INTERFERENCE

To better visualize the image-forming correlation in the X-ray ghost microscope, we consider a simple correlation measurement of two detectors directed at an X-ray source, as done in the original Hanbury Brown-Twiss experiment that started the practice of optical correlation measurements and also demonstrated at X-ray synchrotron sources [30–35]. At a certain distance from our X-ray source (perhaps an X-ray tube), two point-like X-ray photodetectors,  $D_1$  and  $D_2$ , are transversely scannable on the same  $z = d$  plane in the Fresnel near field (unlike the original Hanbury Brown-Twiss experiments in the Fraunhofer far field). The X-ray photodetectors may include scintillators to convert X-ray photons to visible photons. The intensity fluctuation correlation

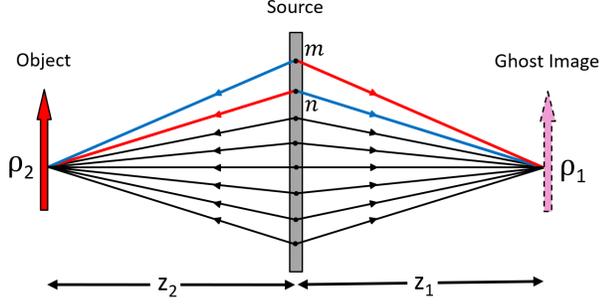


FIG. 2. “Unfolded” schematic setup for ghost imaging. The pair of two-photon amplitudes highlighted in red and blue are different, yet indistinguishable paths for joint photodetection from the  $m$ th and  $n$ th sub-sources that superpose constructively when  $z_1 = z_2 = d$  and  $\vec{\rho}_1 \approx \vec{\rho}_2$ . Adding the contributions of all random pairs at  $\vec{\rho}_1 \approx \vec{\rho}_2$ , the intensity fluctuation correlation measurement, or coincidence measurement ( $t_1 = t_2$ ), yields a point-to-point correlation limited by diffraction.

measured by  $D_1$  and  $D_2$ , jointly, is thus

$$\begin{aligned} & \langle \Delta I(\mathbf{r}_1, t_1) \Delta I(\mathbf{r}_2, t_2) \rangle \\ &= \sum_{m \neq n} E_m^*(\mathbf{r}_1, t_1) E_n(\mathbf{r}_1, t_1) E_n^*(\mathbf{r}_2, t_2) E_m(\mathbf{r}_2, t_2) \\ &\simeq I_0^2 \left| \sum_m g_m^*(\mathbf{r}_1, t_1) g_m(\mathbf{r}_2, t_2) \right|^2 \end{aligned} \quad (5)$$

where the approximation is made by assuming a large number of randomly distributed subfields,  $m \sim \infty$ , participate to the joint measurement. Applying the Fresnel near field approximation to propagate the field from each sub-source to the photodetectors by means of the following Green’s function [8]

$$g_m(\omega; \vec{\rho}_j, z_j) = \frac{c_0}{z_j} e^{-i\omega\tau_j} e^{i\frac{\omega}{2cz_j} |\vec{\rho}_j - \vec{\rho}_m|^2}, \quad (6)$$

where  $c_0$  is a normalization constant and  $\tau_j \equiv t_j - z_j/c$ . Assuming a disk-like source and randomly distributed and randomly radiated point-like sub-sources, we can approximate the sum of  $m$  into an integral of  $\vec{\rho}_s$  on the source plane. Assuming perfect temporal correlation,  $t_1 = t_2$ , the intensity fluctuation correlation measurement between  $D_1$  and  $D_2$  yields a diffraction limited correlation between the planes of  $z_1 = d$  and  $z_2 = d$ ,

$$\begin{aligned} & \langle \Delta I(\vec{\rho}_1, z_1) \Delta I(\vec{\rho}_2, z_2) \rangle \Big|_{z_1=z_2} \\ &\propto \left| \int d\vec{\rho}_s g_{\vec{\rho}_s}^*(\omega; \vec{\rho}_1, z_1) g_{\vec{\rho}_s}(\omega; \vec{\rho}_2, z_2) \right|^2 \\ &\simeq \text{somb}^2 \frac{\pi \Delta\theta_s}{\lambda} |\vec{\rho}_1 - \vec{\rho}_2|, \end{aligned} \quad (7)$$

where the somb-function is defined as  $2J(x)/x$  and  $\Delta\theta_s \approx 2R/d$  is the angular diameter of the radiation source viewed at the photodetectors. For a large value of  $\Delta\theta_s$ ,

the point-to-“spot” sombrero function can be approximated as a delta-function,  $\delta(\vec{\rho}_1 - \vec{\rho}_2)$ . For instance, if we use an X-ray source of  $\Delta\theta_s \sim 0.1$  with wavelength around  $10^{-10}$  meter, the correlation is restricted within  $|\vec{\rho}_1 - \vec{\rho}_2| \sim 10^{-9}$  meter. We thus effectively have a point-to-point intensity fluctuation correlation between the measurement planes  $z_1 = d$  and  $z_2 = d$ .

As usual for spatial correlation measurements, the above spatial correlation is calculated for one frequency  $\omega$  (or wavelength  $\lambda$ ) under perfect second-order temporal correlation. It is easy to show that, in a vacuum, under the same experimental conditions indicated in Fig. 2, all wavelengths produce the same two-photon diffraction limited point-to-point correlation of Eq. (7). Although the width of the sombrero-like correlations are slightly different due to different values of  $\lambda$ , all of them can be approximated as a delta-function

$$\langle \Delta I(\vec{\rho}_1, z_1) \Delta I(\vec{\rho}_2, z_2) \rangle \Big|_{z_1=z_2} \propto \delta(|\vec{\rho}_1 - \vec{\rho}_2|). \quad (8)$$

By placing an object in the plane of  $z_2 = d$ , represented by an aperture function  $|A(\vec{\rho}_o)|^2$ , and moving  $D_2$  a short distance from the plane of  $z_2 = d$  to collect all possible X-rays transmitted from the aperture in the form of a bucket detector, the intensity fluctuation correlation between the point-like photodetector  $D_1$  and the bucket photodetector  $D_2$  produces a lensless ghost image when scanning  $d_1$  in the  $z_1 = d$  plane [8]

$$\begin{aligned} & \int d\vec{\rho}_o |A(\vec{\rho}_o)|^2 \langle \Delta I(\vec{\rho}_1, z_1) \Delta I(\vec{\rho}_o, z_o) \rangle \Big|_{z_1=z_o} \\ &\simeq \int d\vec{\rho}_o |A(\vec{\rho}_o)|^2 \delta|\vec{\rho}_1 - \vec{\rho}_o| = |A(\vec{\rho}_1)|^2 \end{aligned} \quad (9)$$

where  $\text{somb}^2(\pi\Delta\theta_s/\lambda)|\vec{\rho}_1 - \vec{\rho}_o| \simeq \delta|\vec{\rho}_1 - \vec{\rho}_o|$  is the image-forming point-spread function or the point-to-point image-forming function. In order to allow the two detectors to be directly in the beam line and still be equidistant from the source, a beam splitter is often used [36].

In Eq. (9), we have assumed  $z_o = z_1 = d$  for the object to be in focus. One interesting result of this is that if we move  $D_1$  from the plane of  $z_1 = d$  to the plane of  $z_1 = d'$ , the ghost image observed in the intensity fluctuation correlation becomes the new aperture function of the slice of the object that coincides with the plane of  $z_o = d'$  (i.e. the image is in focus where the cross section of the object corresponds to the same location as  $D_1$ ). By scanning  $D_1$  from one plane to another along  $z_1$ , a set of slices of the object can be grouped together to form a 3-D image of the object. This differs from traditional X-ray computerized tomography (CT) imaging [38] and ghost tomography (GT) demonstrated by Kingston *et al.* [16] which rely on rotating the object or revolving the detectors around the object 360° [39].

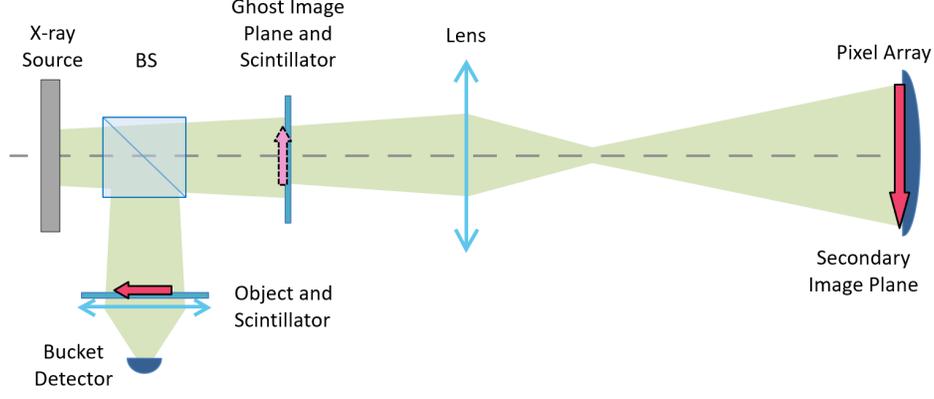


FIG. 3. X-ray ghost microscope. A beam splitter (most likely a crystal aligned to utilize Laue diffraction which would not provide  $90^\circ$  separation as depicted), creates two paths for the beam, one directed at the object-scintillator pairing and the second directed at a scintillator. This independent scintillator is placed on the ghost image plane to convert the X-ray ghost image into the visible spectrum. A lens (or lens system) produces a magnified secondary ghost image which allows for a high resolution image of the object to be resolvable by a standard CCD or CMOS after correlation.

### X-RAY GHOST MICROSCOPE

It is easy to find from Eq. (9) that the spatial resolution of this X-ray ghost image is too high to be resolvable by any state-of-the-art 2-D photodetector array, such as CCD or CMOS sensors. Fortunately, the ghost image can be converted into the visible spectrum by a scintillator and magnified by a lens system onto a secondary image plane. In fact, the working principle of producing secondary ghost image in visible spectrum by a lens system, schematically illustrated in Fig. 1, has been demonstrated by Valencia *et al.* in 2005 [5]. The successful observation of the secondary ghost image in the visible spectrum inspired the design of the X-ray ghost microscope. The schematic design of the X-ray microscope is shown in Fig. 3. With the help of a scintillator and a visible spectrum lens system, a magnified secondary ghost image is observable from the measurement of the intensity fluctuation correlation. To model this we adjust Eq. (7) to include the lens system,

$$\begin{aligned}
 \langle \Delta I(\vec{\rho}_o, z_o) \Delta I(\vec{\rho}_i, z_i) \rangle &\propto \left| \int d\vec{\rho}_s g_{\vec{\rho}_s}^*(\omega; \vec{\rho}_o, z_o) g_{\vec{\rho}_s}(\omega; \vec{\rho}_i, z_i) \right|^2 \\
 &= \left| \int d\vec{\rho}_s g_{\vec{\rho}_s}^*(\omega; \vec{\rho}_o, z_o) \{ g_{\vec{\rho}_s}(\omega; \vec{\rho}_1, z_1) \right. \\
 &\quad \times \left. \left[ \int d\vec{\rho}_1 g_{\vec{\rho}_1}(\omega; \vec{\rho}_L, z_L) \right] [g_{Lens}] \left[ \int d\vec{\rho}_L g_{\vec{\rho}_L}(\omega; \vec{\rho}_i, z_i) \right] \right\} \right|^2 \\
 &\simeq \left| \int d\vec{\rho}_1 \delta(|\vec{\rho}_o - \vec{\rho}_1|) \text{somb}^2 \frac{\pi D}{s_o \lambda} |\vec{\rho}_1 - \vec{\rho}_i / \mu|^2 \right. \\
 &\quad \left. \simeq \text{somb}^2 \frac{\pi D}{s_o \lambda} |\vec{\rho}_o - \vec{\rho}_i / \mu|, \right. \\
 &\tag{10}
 \end{aligned}$$

where  $g_{\vec{\rho}_s}(\omega; \vec{\rho}_1, z_1)$ ,  $g_{\vec{\rho}_1}(\omega; \vec{\rho}_L, z_L)$ ,  $g_{Lens}$ , and  $g_{\vec{\rho}_L}(\omega; \vec{\rho}_i, z_i)$  are the Green's functions propagating the field from the source plane to the one-to-one primary

ghost image plane, from the primary ghost image plane to the lens plane, from the input plane of the lens to the output plane of the lens, and from the lens plane to the secondary ghost image plane, respectively [8]. In Eq. (10),  $D$  is the diameter of the lens,  $s_o$  is the distance from the primary ghost image to the lens,  $s_i$  is the distance from the lens to the secondary ghost image, satisfying the Gaussian thin lens equation  $1/s_o + 1/s_i = 1/f$ , where  $f$  is the focal length of the lens and  $\mu = s_i/s_o$  is the magnification factor of the secondary ghost image. We then place an object at  $z_o$  with an aperture function  $|A(\vec{\rho}_o)|^2$  in the  $z_o$  plane, and move the bucket detector a distance from the  $z_o$  plane to collect all possible X rays transmitted from  $|A(\vec{\rho}_o)|^2$ . A magnified secondary ghost image of the aperture function is then observed from the joint detection between the CCD (CMOS) and the bucket detector

$$\begin{aligned}
 &\int d\vec{\rho}_o |A(\vec{\rho}_o)|^2 \langle \Delta I(\vec{\rho}_o, z_o) \Delta I(\vec{\rho}_i, z_i) \rangle \\
 &\propto \int d\vec{\rho}_o |A(\vec{\rho}_o)|^2 \text{somb}^2 \frac{\pi D}{s_o \lambda} |\vec{\rho}_o - \vec{\rho}_i / \mu| \\
 &\simeq |A(\vec{\rho}_i / \mu)|^2. \\
 &\tag{11}
 \end{aligned}$$

Considering a CCD (CMOS) with  $10^{-6}$  meter pixels, it would be desirable to use a microscope working in the visible spectrum and achieve  $\mu > 1000$ . For visible light,  $\lambda \sim 0.5 \mu\text{m}$  and the angular resolution of a centimeter diameter lens is roughly  $0.5 \times 10^{-4}$  rad. Coupling a microscope with the primary high-resolution ghost image at  $s_o = 50 \mu\text{m}$  could theoretically provide  $> 1000$  magnification and nanometer resolution but, due to physical constraints, tens of nanometers resolution is more easily achievable. A factor that may aid in the high resolving capabilities of the X-ray ghost microscope is the stability of two-photon interference. It has been shown that

ghost imaging and other two-photon interference phenomena can be set up to achieve turbulence-free measurements [18–20, 31]. This is achieved when the superposed two-photon amplitudes experience the same turbulence and medium vibrations along their optical paths, meaning any composition, density, length, refractive index, or medium vibration induced random phase variations along the optical paths do not have any effect on each individual two-photon interference.

### ACHIEVING CORRELATION WITH BROADBAND X-RAY SOURCES

The intensity fluctuation correlation utilized for the X-ray ghost microscope is directly related to the second-order correlation function,  $G^{(2)}(\vec{\rho}_o, t_o; \vec{\rho}_i, t_i)$ , of X-rays in a thermal state. Up to this point, we have assumed perfect temporal correlation to allow us to focus on the spatial portion of this function. Now we will do the opposite and focus on the temporal aspect of the measurement. Similar to ghost imaging with sunlight, we have to face the problems caused by the extremely broad spectrum of X-ray sources and relatively slow detectors. Due to the limited ability of the photodetectors in determining the registration time of a photoelectron, the response time of the photodetectors and the associated electronics will affect the measurement of  $G^{(2)}(t_1 - t_2)$ , where  $t_j$  is the registration time of photodetectors  $D_1$  and  $D_2$ . For instance, due to the slow response time of the photodetectors, relatively speaking, the measured second-order correlation may have a much wider temporal width and much smaller amplitude. The annihilation of a photon at time  $t_j$  produces a pulse of electric current to an electronic circuit which is able to analyze the pulse and to determine the electronic registration time  $\tilde{t}_j$  within a particular uncertainty. The jitter of the leading edge of the pulse as well as the fluctuations of the pulse height both contribute to the uncertainty of the electronic registration times, or the measurement times, of  $\tilde{t}_1$  and  $\tilde{t}_2$ . We may characterize this uncertainty as a response function of the photodetector,  $D(\tilde{t}_j - t_j)$ , where  $t_j$  is the photon annihilation time and  $\tilde{t}_j$  is the electronic registration time. Thus, the joint photodetection measurement of  $D_1$  and  $D_2$  can be treated as a convolution between the response functions and the second-order correlation function  $G^{(2)}(t_1 - t_2)$ ,

$$\begin{aligned} & G^{(2)}(\tilde{t}_1 - \tilde{t}_2) \\ &= \frac{1}{t_c^2} \int dt_1 \int dt_2 G^{(2)}(t_1 - t_2) D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2) \end{aligned} \quad (12)$$

where we have normalized the function by applying

$$\int dt_j D(\tilde{t}_j - t_j) = t_c, \quad D(0) = 1, \quad (13)$$

and  $t_c$  is defined as the response time, or characteristic time, of the photodetector. When using fast detectors, the width of the response functions are much narrower than the temporal width of  $G^{(2)}(t_1 - t_2)$  so the response functions can be treated as delta functions,  $D(\tilde{t} - t) \sim t_c \delta(\tilde{t} - t)$ . In this extreme case, the measured second-order correlation function will reveal the theoretical expectation of  $G^{(2)}(t_1 - t_2)$ ,

$$\begin{aligned} & G^{(2)}(\tilde{t}_1 - \tilde{t}_2) \\ &= \int dt_1 \int dt_2 G^{(2)}(t_1 - t_2) \delta(\tilde{t}_1 - t_1) \delta(\tilde{t}_2 - t_2) \\ &= G^{(2)}(t_1 - t_2). \end{aligned} \quad (14)$$

However, when the response times are larger, the situation is different; especially when the temporal widths of the response functions are much wider than that of  $G^{(2)}(t_1 - t_2)$ . In this extreme case, the second-order correlation function itself can then be treated as a delta function and Eq. (12) turns into the following convolution between the response functions of the two photon counting detectors,

$$\begin{aligned} & G^{(2)}(\tilde{t}_1 - \tilde{t}_2) \\ &= G^{(2)}(0) \left( \frac{\tau_0}{t_c^2} \right) \int d\tau D(\tilde{t}_1 - \tilde{t}_2 - \tau) D(\tau), \end{aligned} \quad (15)$$

where  $\tau = t_1 - t_2$  and we have normalized the delta function by applying

$$\int dt G^{(2)}(\tau) \simeq \int dt G^{(2)}(0) \tau_0 \delta(\tau) = G^{(2)}(0) \tau_0, \quad (16)$$

and  $\tau_0$  is a constant named as the second-order correlation time. Eq. (15) indicates two things: (1) the width of the observed  $G^{(2)}(\tilde{t}_1 - \tilde{t}_2)$  is now determined by the response function of the photodetectors, which could be significantly broadened compared to the original correlation and (2) the relative slow response time of the photodetectors may reduce the magnitude of the measured second-order correlation. For Gaussian response functions the convolution yields a Gaussian function of  $G^{(2)}(\tilde{t}_1 - \tilde{t}_2)$  with a reduced central value of  $G^{(2)}(0)$ . The reduction factor is roughly  $\tau_0/t_c$ .

The above result for slow detectors may not be a problem in the measurement of entangled states; however, it may affect the measurement of a  $G^{(2)}$  function for thermal or pseudo-thermal light significantly. The  $G^{(2)}$  function of thermal or pseudo-thermal field has two terms, the product of two measured mean photon numbers (trivial part) and the photon number fluctuation correlation

(nontrivial part) [8],

$$\begin{aligned}
& G^{(2)}(\tilde{t}_1 - \tilde{t}_2) \\
& \propto \frac{1}{t_c^2} \int dt_1 \int dt_2 \langle n_1 n_2 \rangle D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2) \\
& = \frac{1}{t_c^2} \int dt_1 \int dt_2 [\langle n_1 \rangle \langle n_2 \rangle \\
& \quad + \langle \Delta n_1 \Delta n_2 \rangle] D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2). \tag{17}
\end{aligned}$$

The time average over  $t_c$  has no effect on the first term (trivial part) for a CW thermal or pseudo-thermal field with broad spectrum, because  $\langle n_1 \rangle \langle n_2 \rangle = \bar{n}_1 \bar{n}_2$  is a constant

$$\begin{aligned}
& \frac{1}{t_c^2} \int dt_1 \int dt_2 [\langle n_1 \rangle \langle n_2 \rangle D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2)] \\
& = \frac{1}{t_c^2} [\bar{n}_1 \bar{n}_2 \int dt_1 D(\tilde{t}_1 - t_1) \int dt_2 D(\tilde{t}_2 - t_2)] \\
& = \bar{n}_1 \bar{n}_2. \tag{18}
\end{aligned}$$

However, it reduces the magnitude of the second term (nontrivial part) significantly when  $\tau_0 \ll t_c$

$$\begin{aligned}
& \frac{1}{t_c^2} \int dt_1 \int dt_2 \langle \Delta n_1 \Delta n_2 \rangle D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2) \\
& \simeq \frac{\bar{n}_1 \bar{n}_2}{t_c^2} \int dt_1 \int dt_2 \tau_0 \delta(t_1 - t_2) D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2) \\
& \simeq \frac{\bar{n}_1 \bar{n}_2}{t_c^2} \tau_0 \int d\tau D(\tilde{t}_1 - \tilde{t}_2 - \tau) D(\tau). \tag{19}
\end{aligned}$$

For Gaussian response functions, the magnitude of the second term (nontrivial part) is thus roughly  $\tau_0/t_c$  times smaller than that of the first term (trivial part), which may reach  $10^{-6}$  (one part of a million) for an X-ray source with  $10^{15}$  Hz bandwidth and a nanosecond photodetector.

How does one distinguish the relatively small, nontrivial second-order correlation of thermal field with short coherence time  $\tau_0$ ? The obvious approach is to use fast photodetectors with  $t_c \sim \tau_0$ . However, the state-of-the-art technology has not been able to produce photodetectors fast enough to achieve this for broadband light sources yet. One realistic approach is using a pulsed thermal or pseudo-thermal radiation source with pulse width of  $\tau_p$  and recording a single pulse per frame. In ideal cases, the pulse width is approximately the same as the coherence time,  $\tau_p \approx \tau_0$ , such that all of the photons in the pulse are considered second-order coherent. In this case, the time integral in Eq. 17 shall have similar effects on the first term (trivial part) of  $G^{(2)}$  as it had in Eq. (19),

$$\begin{aligned}
& \frac{1}{t_c^2} \int dt_1 \int dt_2 \langle n(t_1) \rangle \langle n(t_2) \rangle D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2) \\
& \simeq \frac{\bar{n}_1 \bar{n}_2}{t_c^2} \int dt_1 \int dt_2 \tau_0 \delta(t_1 - t_2) D(\tilde{t}_1 - t_1) D(\tilde{t}_2 - t_2) \\
& \simeq \frac{\bar{n}_1 \bar{n}_2}{t_c^2} \tau_0 \int d\tau D(\tilde{t}_1 - \tilde{t}_2 - \tau) D(\tau). \tag{20}
\end{aligned}$$

where we have approximated the short pulsed  $\langle n(t_1) \rangle \langle n(t_2) \rangle$  as a delta-function such that  $\langle n(t_1) \rangle \langle n(t_2) \rangle \sim \bar{n}_1 \bar{n}_2 \tau_0 \delta(t_1 - t_2)$ . This result allows the intensity fluctuation correlation to become more distinguishable and is suitable for measuring the second-order correlation of X rays. Unfortunately, even ‘‘monochromatic’’ pulsed X-ray sources still have a relatively broad bandwidth such that the pulse width is still significantly greater than the coherence,  $\tau_p \gg \tau_0$ . In past two-photon X-ray correlation measurements, high-resolution monochromators have been introduced to make the coherence time,  $\tau_0$ , larger and more comparable to  $\tau_p$  [33–35]. This introduces experimental challenges because of how much intensity is lost in the monochromators. Single detectors may easily be able to detect these low light levels, but this will prove more difficult with current sensor array (CCD or CMOS) technology. As sensor array technology advances, highly monochromatic short pulsed X-ray sources where the pulse width is comparable to the coherence time,  $\tau_0$ , are ideal candidates to fully realize the X-ray ghost microscope.

## SUMMARY

In summary, we have analyzed the working mechanism of a table-top X-ray ghost microscope. By applying Einstein’s granularity picture of radiation, which includes X rays, we found that two-photon interference may produce an image-forming correlation that forms a lensless one-to-one X-ray ghost image with high spatial resolution. Utilizing a high-energy ( $> 20$  keV) source with a relatively large angular diameter, we may produce an observable lensless one-to-one ghost image with nanometer resolution. Furthermore, with the help of a scintillator and an optical lens system, the primary ghost image can be mapped onto a secondary image plane with significant magnification. We have also found that a short pulsed X-ray source and measuring a single pulse per frame is preferred for observing the intensity fluctuation correlation of X rays due to the wide bandwidth of the spectrum. The main experimental restriction that currently applies to this microscope is the intensity of the pulsed X-ray source paired with the sensitivity of the cameras used. Even with measuring a single pulse per frame, in order to achieve even stronger correlation it is preferred to have a more monochromatic beam such that the coherence time,  $\tau_0$ , is larger and more comparable to the pulse width,  $\tau_p$ . In addition to this, the desire for a large angular diameter makes synchrotron X-ray sources less favorable as the resolution of the primary ghost image would be greatly reduced. This is unfortunate because the high intensity of synchrotron radiation is much better suited for using a monochromatic beam with a camera. As cameras and X-ray sources see technological advance-

ments, this table-top X-ray ghost microscope will open up new capabilities that would be of interest to the fields of physics, material science, and medical imaging.

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