Microscopic bath effects on noise spectra in semiconductor quantum dot qubits

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When a system is thermally coupled to only a small part of a larger bath, statistical fluctuations of the temperature (more precisely, the internal energy) of this “sub-bath” around the mean temperature defined by the larger bath can become significant. We show that these temperature fluctuations generally give rise to 1/f power spectral density from even a single two-level system. We extend these results to a distribution of fluctuators, finding the corresponding modification to the Dutta-Horn relation. Then we consider the specific situation of charge noise in silicon quantum dot qubits and show that recent experimental data can be modeled as arising from as few as two two-level fluctuators, and accounting for sub-bath size improves the quality of the fit.

Charge noise, particularly the so-called 1/f noise ubiquitous in electronic devices, is currently the most significant roadblock to the successful development of semiconductor-based scalable solid state qubits. It is well known that an ensemble of thermally activated two-level fluctuators (TLFs) with a broad range of switching rates gives rise to a 1/f power spectral density (PSD) with a linear temperature dependence. This is the standard hand-waving explanation given to explain wide-spread observations of pink noise in solid state qubit devices, with some sort of charged defects playing the role of the TLFs. Early data from laterally-defined quantum dots in silicon showed that the noise power indeed appears to increase with temperature, but with large error bars that preclude a more detailed conclusion. Recent experiments, however, have shown striking deviation from the expected linear temperature dependence. Ref. 1 measures a temperature dependence which is not only nonlinear, but in some cases non-monotonic, qualitatively consistent with the Dutta-Horn model of a large ensemble of TLFs with a non-uniform distribution of switching rates, although any expected quantitative consistency varies widely within the data set. Meanwhile, Ref. 2 finds a quadratic temperature dependence. On the other hand, Ref. 3 observes a $T_2^*$ decoherence time that is approximately constant over a range of temperatures from 0.45K-1.2K, suggesting that their charge noise comes from a few TLFs with activation energies much smaller than $k_B T$, rather than a broad distribution. The situation, particularly in the measured noise temperature dependence, is thus quite confusing.

In this work we show that, in principle, 1/f noise with nonlinear temperature dependence can be produced by even a single TLF coupled to a small subsection of the thermal bath. Although we cannot assert that our proposed mechanism is operational in semiconductor qubits (in fact, the physical mechanism underlying 1/f noise is still obscure), we show that the experimental data of Ref. 1 can be reasonably fit as arising from a small number of TLFs, and that the fit is improved by incorporating this microscopic thermal bath effect via an additional fit parameter.

The essential narrative here is that, even within the TLF model, observation of 1/f noise over some broad frequency range need not imply an ensemble of TLFs. One (or few) TLFs can suffice, in which case a nonlinear temperature dependence is natural. This conclusion of the adequacy of just a few TLFs obviously has important implications.

We assume a stochastic TLF with activation energy $E$ and a thermally activated transition time $\tau \exp(E/k_B T_{sb})$. Here we take $T_{sb}$ to be the effective temperature of the microscopic portion of the thermal bath to which the fluctuator is coupled (i.e., the “sub-bath”). $T_{sb}$ is then a stochastically fluctuating quantity whose average is the same as the macroscopic bath temperature, $T$, but with variance

$$\sigma_{sb}^2 = \frac{k_B T^2}{C_V}$$

where $C_V$ is the heat capacity of the microscopic sub-bath. Since $C_V$ is an extrinsic quantity, $\sigma_{sb}$ is proportional to one over the square root of the volume of the sub-bath. However, it is quite physical to assume that a microscopic two-level fluctuator may be coupled to only a microscopic sub-bath, so the variance could be non-negligible. For instance, in semiconductor quantum dots one typically imagines the charge noise as arising from charged TLFs coupled via Coulomb interaction to a thermal electrostatic environment, but since the Coulomb interaction is screened by the nearby two-dimensional electron gas (2DEG) and the dense array of metallic top gates, each TLF will only interact strongly with its immediate surroundings. Generally, $C_V$ is also a function of $T$; at low temperatures, it is dominated by the electronic heat capacity, which is linear in $T$. For example, considering a 2DEG sub-bath of area $A$, the variance is

$$\sigma_{sb}^2 = \frac{3\hbar^2 T}{\pi m A k_B}$$

where $m$ is the effective mass of the electrons. This can certainly be non-negligible, since for Si, with an effective
mass of $m = 0.19 m_e$ (where $m_e$ is the electron mass), and
an area of one square micron, at a typical temperature of
50 mK one would have sub-bath fluctuations of 14 mK.

Averaging over the sub-bath statistical temperature
distribution $f(T_{sb})$, the PSD is

\[
S(\omega) = \frac{\Delta^2}{\omega} \int_{-\infty}^{\infty} dT_{sb} f(T_{sb}) \frac{4\tau \exp(E/k_BT_{sb})}{1 + \omega^2 \tau^2} \exp(2E/k_BT_{sb})
\]

where $\Delta$ is the total variance of the signal produced by
the switching events. For convenience, we will integrate
terms $T_{sb} e^{-(\tau_{sb} - \tau)^2/2\sigma_{sb}^2}$

\[
\int_{-\infty}^{\infty} dT sb e^{-(\tau_{sb} - \tau)^2/2\sigma_{sb}^2} \text{sech} \left( \frac{E}{k_B T_{sb}} + \ln(\omega/\tau) \right),
\]

where we have also defined

\[
T_{\omega} = \frac{E}{k_B \ln(1/\omega) - \tau}.
\]

Note that we implicitly assume that $\sigma_{sb} < T$, with
unphysical negative sub-bath temperatures nonetheless per-
mitted in the tail of the distribution. That can trivially
be remedied by truncating the distribution on the lower
side, but that produces more complicated expressions
without affecting the conclusions below (see Supplementary
Information).

The integrand in Eq. (4) contains two peaks, one at
the distribution’s mean, $T$, and one from the Lorentzian
at $T_{\omega}$. While the integral is easily carried out numeri-
cally for any set of parameters, the qualitative behavior
is illuminated by the following approximation. Assuming
the peaks are well separated, approximate the sech term
as a constant in the vicinity of $T$ and as a gaussian in
the vicinity of $T_{\omega}$, with width

\[
\sigma_{\omega} = \frac{E}{k_B \ln(1/\omega) - \tau},
\]

so as to obtain

\[
S(\omega) \approx \frac{2\Delta^2}{\omega} \sech \left( \frac{E}{k_B} \left( \frac{1}{T_{sb}} - \frac{1}{T_{\omega}} \right) \right) + \frac{2\Delta^2}{\omega} \sigma_{\omega} \ln(1/\omega) - \frac{1}{\sigma_{\omega}^2} \frac{1}{\omega^2} e^{-2(\tau_{sb} - \tau_{\omega})^2 / \sigma_{sb}^2}.
\]

This has nontrivial frequency and temperature de-
pendence, but in the low-frequency limit $T_{\omega} \ll T$ and
$\sigma_{\omega} \ll \sigma_{sb}$,

\[
S(\omega) \approx \frac{4\tau \Delta^2}{1 + \omega^2 \tau^2} + \frac{2E \Delta^2}{k_B \sigma_{sb} \omega^2} \ln(1/\omega) - \frac{1}{\omega^2} e^{-2(\tau_{sb} - \tau_{\omega})^2 / \sigma_{sb}^2}.
\]

The first term is the typical Lorentzian spectrum of a
single TLF. The second term rises above this Lorentzian

data in Ref. [6] is believed to
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\]

The first term is the typical Lorentzian spectrum of a
single TLF. The second term rises above this Lorentzian

\[
\omega_{1/f} \sim 0.01 \tau^{-1} \frac{E}{2k_B \sigma_{sb}} \exp(\frac{T^2}{2\sigma_{sb}^2}),
\]

transitioning to white noise at intermediate frequencies,
and finally falling as $1/\omega^2$ at frequencies above $\tau^{-1}$. Al-
though no quantum dot experiment to our knowledge has
measured the noise spectrum at high enough frequency to
conclusively observe the roll-off to $1/\omega^2$ (although Ref. [7]
finds suggestions of it in the data of Ref. [8]), some have
observed a whitening of $1/\omega^2$ noise with increasing fre-
quency [1, 9].

Turning our attention now to the temperature depen-
dence, we can compute Eq. (4) numerically for a given
set of parameters, as shown in Fig. [2]. The PSD is peaked
around $T_{\omega}$ and is qualitatively similar to the PSD in the
absence of temperature fluctuations. It is instructive to
look at the limiting cases analytically. At temperatures
well above $T_{\omega}$, Eq. (4) holds, so one has an exponen-
tial decay to a constant value as temperature increases.

FIG. 1: Power spectral density vs frequency from
Eq. (4) for $E/k_BT = 1$, $\sigma_{sb} = 0.3$.

floor at low frequencies as $1/\omega \ln^2 \omega \tau$, which over a broad
frequency range centered around $\omega_0$ is practically indistin-
guishable from $1/\omega^\alpha$ with $\alpha = 1 + 2/\ln(\omega_0 \tau)$. An ex-
ample of this behavior is shown in Fig. [1].

Thus, if temperature fluctuations are significant, one
can expect to find $1/\omega^\alpha$ noise at frequencies below
roughly

\[
S(\omega) \approx \frac{2\Delta^2}{\omega} \sech \left( \frac{E}{k_B} \left( \frac{1}{T_{sb}} - \frac{1}{T_{\omega}} \right) \right) + \frac{2\Delta^2}{\omega} \sigma_{\omega} \ln(1/\omega) - \frac{1}{\sigma_{\omega}^2} \frac{1}{\omega^2} e^{-2(\tau_{sb} - \tau_{\omega})^2 / \sigma_{sb}^2}.
\]
So one obtains
\[ S(\omega) \approx \sqrt{2 \Delta} e^{-\frac{\omega T}{\omega_T}} \sqrt{2} \frac{1}{\sqrt{2\pi\sigma_{sb}}} \int_{-\infty}^{\infty} d\tau e^{-\frac{(\tau - \tau_0)^2}{2\sigma_{sb}^2}}, \] (10)
where the expressions for \( T_* \) and \( \sigma_* \) in terms of the physical parameters are algebraically cumbersome, but simplify in the low-temperature limit if we assume \( \lim_{T \rightarrow 0} \frac{\tau}{(\sigma_{sb}^2 E/k_B)^{1/3}} = 0 \) (as is the case for electrons [2]) to
\[ T_* = \frac{1}{3} T + \left( \frac{E}{k_B\sigma_{sb}^2} \right)^{1/3}, \quad \sigma_* = \frac{1}{\sqrt{3}} \sigma_{sb}. \] (11)
So one obtains
\[ S(\omega) \approx \frac{4\sqrt{2} \Delta^2}{\sqrt{3}\omega_T \omega_{sb}^2} \exp \left( -\frac{1}{2} \left( \frac{-2T}{3\sigma_{sb}} + \left( \frac{E}{k_B\sigma_{sb}} \right)^{1/3} \right)^2 \right) \times \exp \left( -\frac{3E/k_B}{T + 3(\sigma_{sb}^2 E/k_B)^{1/3}} \right). \] (12)

So, instead of the typical linear temperature dependence [2], it is exponentially flat at low temperatures, going like
\[ \exp \left( -\frac{3\sqrt{\frac{2mE}{k_B\hbar}}}{2\sqrt{2k_B\hbar}} \right)^{2/3} \frac{T^{-1/3}}{T}. \]

This flatness at low temperatures is qualitatively consistent with some of the data sets of Ref. [1], and we show below that superposing PSDs as in Fig. 2 of a few TLFs with different activation energies can provide an alternate way to understand and fit the nonlinear behavior in Ref. [1]. However, at this point it suffices to note that by dropping the assumption that 1/f noise must imply a continuous distribution of TLFs, we have preserved the nonlinear temperature dependence of a single TLF, and, depending on the parameter values, one could obtain seemingly very different behaviors if one observes over only a narrow range of temperatures.

We now discuss briefly how the PSD of a continuous distribution of TLFs would be affected by sub-bath temperature fluctuations. The classic Dutta-Horn model [4] for a temperature-independent distribution of activation energies, \( F(E) \), gives
\[ S = \frac{2\pi k_B T}{\omega} F(E_\omega), \]
where \( E_\omega \equiv k_B T \ln \frac{1}{\omega T} \), in which case the frequency dependence and the temperature dependence are linked as
\[ \gamma \equiv -\frac{\partial \ln S}{\partial \ln \omega} = 1 - \frac{1}{\ln \omega T} \left( \frac{\partial \ln S}{\partial \ln T} - 1 \right). \] (14)

Modifying Eq. (13) to the case of a fluctuating sub-bath temperature (more precisely, starting from Eq. 3 and assuming \( F \) is broad, with a width much larger than \( k_B T \) such that the sech function can be approximated as a delta function),
\[ S = \frac{\sqrt{2\pi k_B}}{\omega \sigma_{sb}} \int_{-\infty}^{\infty} d\tau e^{T_{sb} F(kT_{sb})} \ln \left( \frac{1}{\omega T} \right) \exp \left( -\frac{(T - T_{sb})^2}{2\sigma_{sb}^2} \right). \] (15)

Since the distribution of activation energies is assumed narrow compared to the distribution of sub-bath temperatures, we can do the integration in Eq. (15) by approximating
\[ T_{sb} F(kT_{sb}) \ln \left( \frac{1}{\omega T} \right) \approx TF(E_\omega) \]
+ \( F(E_\omega) + E_\omega F'(E_\omega) (T_{sb} - T) \)
+ \( \frac{E_\omega}{T} F'(E_\omega) + \frac{E_\omega^2}{2T^2} F''(E_\omega) (T_{sb} - T)^2 \). (16)

Plugging this into Eq. (15) yields
\[ S(\omega) = \frac{2\pi k_B T}{\omega} F(E_\omega) \]
+ \( \frac{\pi k_B^2 \sigma_{sb}^2}{\omega} \left[ 2 \ln \frac{1}{\omega T} F'(E_\omega) + E_\omega \ln \frac{1}{\omega T} F''(E_\omega) \right]. \] (17)

Then it is straightforward to obtain the corresponding approximate modified Dutta-Horn relationship:
\[
\gamma = 1 - \frac{1}{\ln \omega \tau} \left( \frac{\partial \ln S}{\partial \ln T} - 1 \right) \left( 1 + \frac{\sigma^2_{sb}/T^2}{1 - \sigma^2_{sb}/T^2} \right) \left( \frac{2\sigma^2_{sb} (2F'(E_\omega) + E_\omega F''(E_\omega)) + E_\omega F'''(E_\omega)}{2(T^2 - \sigma^2_{sb}) F'(E_\omega) + E_\omega \sigma^2_{sb} (2F''(E_\omega) + E_\omega F'''(E_\omega))} \right).
\]

The main point here is that including temperature fluctuations destroys the key feature of the Dutta-Horn result that the relationship between the frequency dependence and temperature dependence is independent of the details of the activation energy distribution. Only in the restricted case of negligible second- and higher-order derivatives of \( F \) does the relationship become independent of the form of \( F \):

\[
\gamma = 1 - \frac{1}{\ln (\omega \tau)} \left( \frac{\partial \ln S}{\partial \ln T} - 1 \right) \left( 1 + \frac{\sigma^2_{sb}/T^2}{1 - \sigma^2_{sb}/T^2} \right).
\]

Generally, the frequency and temperature dependences are now decoupled in the sense that one cannot predict one from the other without knowing the underlying distribution. In such a scenario, the details of the noise would matter a great deal leading possibly to nonuniversal experimental behavior.

We now turn to the experimental data of Ref. [1]. There the charge noise in several silicon double quantum dots was measured as a function of temperature at 1 Hz, as well as the local frequency dependence exponent, \( \gamma \). We find that the data can be described reasonably well with our theory using as few as two discrete TLFs,

\[
\sum_{i=1}^{2} \frac{\Delta_i^2 \sqrt{2mAk_B}}{\hbar \sqrt{3T}} \int_{-\infty}^{\infty} dT_{sb} e^{-\left(\frac{E_i - \omega T_{sb}}{\hbar k_B T_{sb}}\right)^2} \text{sech} \left( \frac{E_i - \omega T_{sb}}{k_B T_{sb}} + \ln(\omega \tau_i) \right)
\]

by fitting over the switching times (\( \tau_i \)), activation energies (\( E_i \)), and fluctuator strengths (\( \Delta_i^2 \)), as well as a common 2D sub-bath area (\( A \)), where we have made the physically reasonable assumption that the heat capacity of the thermal bath is dominated by the electronic contribution and used Eq. (2). The objective function simultaneously minimizes net deviations from the moving average of the noisy \( S \) and \( \gamma \) data with equal weighting, and the minimization is carried out via a local gradient search using the *fmincon* function in Matlab. The fitting parameters are constrained to lie within \( 0 - 10 \) s for \( \tau \), \( 0 - 100 \) meV for \( E_i \), \( 0 - 10^4 \) meV^2 for \( \Delta_i^2 \), and 2nm–100μm for \( \sqrt{A}/\pi \) when it is finite. In the infinite sub-bath case, \( A \to \infty \), the temperature distribution corresponds to a delta function.

In Fig. 3 we show an example of the results of the fitting with and without taking a microscopic sub-bath area. (Fits to the complete data set are included in the Supplementary Information.) Even the fit using an infinite thermal sub-bath appears better than the standard Dutta-Horn results, although this happens because we are just fitting \( S \) (a function of \( T \) and \( \omega \)) over a cut at constant \( \omega \) while also fitting the derivative \( \partial_{\omega} S \) perpendicular to the cut. As we showed at the outset, incorporating a microscopic sub-bath area in principle allows for a 1/\( \omega \)-type frequency dependence over the whole plane. Even restricting ourselves to the data in hand, there is a noticeable improvement in the fit when including the effects of a microscopic sub-bath. It is interesting that the sub-bath sizes that emerge from the fit are quite consistent across different dots and correspond to disks with radii of about 100 nm, which is physically reasonable for these devices. The data can be fit more closely with three or four TLFs (see Supplemental Information), but given how much variance the data displays, it does not make sense to strive for too much precision in fitting the average.

One ramification of having only a few relevant TLFs

![Graph](image-url)
would be that increasing temperature may not be as dele-
terious to coherence as it is for typical $1/f$ noise, as sug-
gested in Ref. [6]. If it is furthermore true that these
TLFs are indeed coupled to a microscopic sub-bath with
appreciable temperature fluctuations, it could have some
other surprising but testable ramifications. For instance,
it is natural to wonder what happens if the sub-bath
is small enough that the effective temperature distribu-
tion has a long tail leading to fluctuations larger than
the mean temperature. Indeed, for a simple planar bath
where there is no other relevant length scale, the sub-
bath area should go like the square of the distance, $d$,
between the TLF and the bath, so the critical distance
at which $\sigma_{sb} \sim T$ is $d_c \sim \hbar/\sqrt{mk_BT}$, which is around
200nm for $T \sim 100$mK in Si. The dependence of the
low-frequency PSD amplitude on the distance goes like
$\sim d \exp (-d^2 k_B T m/\hbar^2)$ (cf. Eqs. (8) and (2)), which
diminishes linearly with decreasing distance below $d_c$
before saturating at the Lorentzian floor. Thus, one has the
counterintuitive possibility of suppressing low-frequency
noise by bringing the thermal electronic bath (presum-
ably the capacitively coupled surrounding 2DEG, or the
metal gates) in closer contact with the TLFs (perhaps
charged defects at the oxide interface, or near the semi-
conductor surface).

In conclusion, we have shown how a $1/f$ noise power
spectral density (PSD) with nonlinear temperature de-
pendence, often modeled as arising from a broad distri-
bution of two-level fluctuators (TLFs) via the Dutta-Horn
relation, can in fact emerge from even one or two TLFs
when coupled to a microscopic thermal sub-bath due to
effective temperature fluctuations. If a broad distribu-
tion of TLFs is coupled to such a bath, the strict connec-
tion between local frequency and temperature scalings
enforced by the Dutta-Horn relation is relaxed. Finally,
we noted that recent experimental measurements of both
the local frequency scaling and a nonlinear temperature
dependence in silicon quantum dots can be reasonably
explained as arising from as few as two TLFs.

The authors thank Elliot Connors for providing the
data sets measured in Ref. [1]. SA and SDS acknowl-
dedge support by the Laboratory for Physical Sciences.
JPK acknowledges support by the Army Research Office
(ARO) under Grant Number W911NF-17-1-0287.

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B. Hensen, T. Tanttu, F. E. Hudson, K. M. Itoh,
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<table>
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<th>Parameter</th>
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<th>Fluctuator 2</th>
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<tbody>
<tr>
<td>$A \to \infty$</td>
<td>(ms)</td>
<td>8.397</td>
</tr>
<tr>
<td>$E$ (meV)</td>
<td>0.216</td>
<td>5.414 x 10(^{-11})</td>
</tr>
<tr>
<td>$\Delta_i^2$ (meV(^2))</td>
<td>2.688</td>
<td>1.644</td>
</tr>
<tr>
<td>$A &lt; \infty$</td>
<td>(ms)</td>
<td>3.843 x 10(^{-5})</td>
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<td>$E$ (meV)</td>
<td>1.518</td>
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<tr>
<td>$\Delta_i^2$ (meV(^2))</td>
<td>21.899</td>
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</tr>
<tr>
<td>$\sqrt{A/\pi}$ (nm)</td>
<td>68.855 nm</td>
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Supplemental Material:
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I. FITTING RESULTS

Device 1 | QD R1 | Left
Device 1 | QD R1 | Right
Device 2 | QD R1 | Left
Device 2 | QD R1 | Right
Device 1 | QD L1 | Left
Device 1 | QD L1 | Right
Device 2 | QD L1 | Left
Device 2 | QD L1 | Right
Device 1 | QD L2 | Left
Device 1 | QD L2 | Right
Device 3 | QD R1 | Left
Device 3 | QD R1 | Right

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FIG. 1: Fitting results using two fluctuators for $S$ and $\gamma$ along with the DH lines and measured data from Supplementary Fig. 2 of Ref. [1]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fluctuator1</th>
<th>Fluctuator2</th>
<th>Fluctuator1</th>
<th>Fluctuator2</th>
<th>Fluctuator1</th>
<th>Fluctuator2</th>
<th>Fluctuator1</th>
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<td>$A \rightarrow \infty$</td>
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<td>$E$ (meV)</td>
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<td>A</td>
<td>$\tau$ (ms)</td>
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<td>4.104 $\times 10^{-3}$</td>
<td>0.055</td>
<td>0.018</td>
<td>2.443</td>
<td>0.803</td>
<td>9.176 $\times 10^{-3}$</td>
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<tr>
<td>$\Delta^2$ (meV$^2$)</td>
<td>1.559</td>
<td>7.999</td>
<td>1.529</td>
<td>21.899</td>
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<td>29.722</td>
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<td>3.909</td>
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<tr>
<td>$\sqrt{A/\pi}$ (nm)</td>
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<td>68.855</td>
<td>51.741</td>
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Table I: Fitting parameter values for Fig. 1
FIG. 2: Fitting results using three fluctuators for $S$ and $\gamma$ along with the DH lines and measured data from Supplementary Fig. 2 of Ref. [1]
### Table II: Fitting parameter values for Fig. 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Device 1 QD R1 Left</th>
<th>Device 1 QD R1 Right</th>
<th>Device 2 QD R1 Left</th>
<th>Device 2 QD R1 Right</th>
<th>Device 1 QD L1 Left</th>
<th>Device 1 QD L1 Right</th>
<th>Device 2 QD L1 Left</th>
<th>Device 2 QD L1 Right</th>
<th>Device 1 QD L2 Left</th>
<th>Device 1 QD L2 Right</th>
<th>Device 3 QD R1 Left</th>
<th>Device 3 QD R1 Right</th>
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<tr>
<td>$\tau$ (ms)</td>
<td>1.414</td>
<td>15.557</td>
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<td>0.747</td>
<td>4.466</td>
<td>1.223×10^{-3}</td>
<td>0.426</td>
<td>0.182</td>
<td>1.144</td>
<td>2.768</td>
<td>1.217</td>
</tr>
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<td>$E$ (meV)</td>
<td>0.371</td>
<td>0.115</td>
<td>7.108×10^{-4}</td>
<td>1.223×10^{-3}</td>
<td>0.426</td>
<td>0.182</td>
<td>1.144</td>
<td>2.768</td>
<td>1.217</td>
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<td>1.398</td>
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<td>2.768</td>
<td>1.217</td>
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<td>0.041</td>
<td>6.543×10^{-3}</td>
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<td>2.954</td>
<td>0.881</td>
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<tr>
<td>$\tau$ (ms)</td>
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<td>8.652</td>
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<td>$E$ (meV)</td>
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<td>1.719×10^{-3}</td>
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<td>$\tau$ (ms)</td>
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<td>90.957</td>
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<td>$E$ (meV)</td>
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<td>$\Delta^2$ (meV^2)</td>
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<td>$\tau$ (ms)</td>
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<td>$\Delta^2$ (meV^2)</td>
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</table>

TABLE II: Fitting parameter values for Fig. 2
FIG. 3: Fitting results using four fluctuators for $S$ and $\gamma$ along with the DH lines and measured data from Supplementary Fig. 2 of Ref. [1]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$A \to \infty$</td>
<td>12.955</td>
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<td>$E$ (meV)</td>
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<td>$\Delta^2$ (meV$^2$)</td>
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<td>$E$ (meV)</td>
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<td>$\Delta^2$ (meV$^2$)</td>
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<td>$E$ (meV)</td>
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<td>$\Delta^2$ (meV$^2$)</td>
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<td>$E$ (meV)</td>
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<td>$\Delta^2$ (meV$^2$)</td>
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<td>$A/\pi$ (nm)</td>
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<td>$A \to \infty$</td>
<td>15.052</td>
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<td>$E$ (meV)</td>
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<td>$\Delta^2$ (meV$^2$)</td>
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<td>$A/\pi$ (nm)</td>
<td>0.146</td>
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</table>

**Table III:** Fitting parameter values for Fig. 3
II. TRUNCATED TEMPERATURE DISTRIBUTION

We now rederive a modified version of Eq. 8 of the main text, starting from Eq. 4, using a truncated gaussian distribution to avoid including negative temperatures.

\[
S(\omega) = \frac{2\Delta^2}{\omega \sqrt{\pi/2} \sigma_{sb}} \left( 1 + \text{erf} \left( \frac{T_{sb} - T}{\sqrt{2} \sigma_{sb}} \right) \right) \int_0^\infty dT_{sb} e^{-\frac{(T_{sb} - T)^2}{2\sigma_{sb}^2}} \text{sech} \left( \frac{E}{k_B} \left( \frac{1}{T_{sb}} - \frac{1}{T} \right) \right)
\]

\[
\approx \frac{2\Delta^2}{\omega} \text{sech} \left( \frac{E}{k_B} \left( \frac{1}{T} - \frac{1}{T_{sb}} \right) \right) + \frac{2\Delta^2}{\omega} \frac{\sigma_{sb}^{2}T_{sb} + \sigma_{\omega}^{2}T_{\omega}}{\sqrt{2} \sigma_{sb} \sigma_{\omega} \sqrt{\sigma_{sb}^{2} + \sigma_{\omega}^{2}}} \frac{1 + \text{erf} \left( \frac{\sigma_{sb}^{2}T_{sb} + \sigma_{\omega}^{2}T_{\omega}}{\sqrt{2} \sigma_{sb} \sigma_{\omega} \sqrt{\sigma_{sb}^{2} + \sigma_{\omega}^{2}}} \right) e^{-\frac{(T - T_{sb})^2}{2\sigma_{sb}^2}}}{1 + \text{erf} \left( \frac{T}{\sqrt{2} \sigma_{sb}} \right)},
\]

assuming well-separated peaks at \( T \) and \( T_{\omega} \) as before. In the low-frequency limit \( T_{\omega} \ll T \) and \( \sigma_{\omega} \ll \sigma_{sb} \),

\[
S(\omega) \approx \frac{4\tau \Delta^2}{1 + \omega^2 \tau^2} + \frac{2E\Delta^2}{k_B \sigma_{sb} \omega \ln^2 \frac{1}{\omega \tau}} + \frac{2e^{-\frac{(T - T_{\omega})^2}{2\sigma_{sw}^2}}}{1 + \text{erf} \left( \frac{T}{\sqrt{2} \sigma_{sw}} \right)}.
\]

This reduces to Eq. 8 of the main text when the erf function is unity.