On the possibility of measuring the Lense–Thirring effect with a LAGEOS–LAGEOS II–OPTIS–mission

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Abstract. A space mission, OPTIS, has been proposed for testing the foundations of Special Relativity and Post–Newtonian gravitation in the field of Earth. The constraints posed on the original OPTIS orbital geometry would allow for a rather wide range of possibilities for the final OPTIS orbital parameters. This freedom could be exploited for further tests of Post-Newtonian gravity. In this paper we wish to preliminarily investigate if it would be possible to use the orbital data from OPTIS together with those from the existing geodetic passive laser ranged LAGEOS and LAGEOS II satellites in order to perform precise measurements of the Lense–Thirring effect. With regard to this possibility, it is important to notice that the drag–free technology which should be adopted for the OPTIS mission would yield a lifetime of many years for this satellite. It turns out that the best choice would probably be to adopt the same orbital configuration of the proposed LAGEOS–like LARES satellite and, for testing, select a linear combination including the nodes of LAGEOS, LAGEOS II and OPTIS and the perigee of OPTIS. The total systematic error should be of the order of 1%. The LARES orbital geometry should not be too in conflict with the original specifications of the OPTIS mission. However, a compromise solution could be adopted as well. A comparison with the new perspectives of measuring the Lense–Thirring effect with the existing laser–tracked satellites opened by the new gravity models from CHAMP and, especially, GRACE is made. It turns out that an OPTIS/LARES mission would still be of great significance because the obtainable accuracy would be better than that offered by a reanalysis of the currently existing satellites.

1. Introduction

Since gravity is the by far most weak interaction, tests of relativistic gravity always go to the limits of experimental capabilities. One way to increase the accuracy

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of results is to go to space where much larger distances, velocities, gravitational potential differences, and, most important, free fall for an, in principle, infinitely long time are available. These conditions are the experimental basis for the gravity– and relativity–related space missions GP-A (Vessot *et al* 1980), GP-B (Everitt *et al* 2001), GG (Nobili *et al* 2000), MICROSCOPE (Touboul 2001b), STEP (Lockerbie *et al* 2001), SPACETIME (Maleki and Prestage 2001), HYPER (see on the WEB http://www.esa.int/export/esaSC/SEM056WO4HD_index_0_m.html), ASTROD (Huang *et al* 2002), LATOR (Turyshev *et al* 2003a; 2003b), SEE (Sanders *et al* 2000), and OPTIS (Lämmerzahl *et al* 2001) and the observations based on the LAGEOS and LAGEOS II system (Ciufolini 2002). (See also (Lämmerzahl and Dittus 2002) for a review).

In this paper we consider the mission OPTIS (Lämmerzahl *et al* 2001) which was designed for much improved tests of (i) the independence of the velocity of light from the velocity of the laboratory, and (ii) of the universality of the gravitational redshift. We claim that this mission might furthermore be used for an improvement of the tests of the Post–Newtonian gravitomagnetic Lense–Thirring effect (Lense and Thirring 1918) which until now has been experimentally checked with an accuracy of the order of‡ 20%–30% (Ciufolini 2002) by analyzing the laser data to the existing geodetic satellites LAGEOS and LAGEOS II. Therefore, there is a tempting requirement to improve the quality of this test which OPTIS may contribute to. The main reason why the OPTIS mission may contribute to an improvement of the test of the Lense–Thirring effect is the drag–free motion of the satellite which is made possible due to very accurate inertial sensors and very fine–tunable thrusters.

A test of gravitomagnetic effects is the subject of many space missions: the LAGEOS–LAGEOS II system already observed the Lense–Thirring effect in its original form, namely through the precession of the satellite's node Ω and pericentre ω. GP-B aims at a test of the Lense–Thirring effect in the form of the frame–dagging of inertial systems represented by gyroscopes, also called Schiff effect (Schiff 1960) (see (Schäfer 2003) for a short review). This is a local version of gravitomagnetism since in this case the phenomenon is not related to a whole orbit but to a small region of space. In both cases the effect is related to the g_{0i} part of the spacetime metric. This also extends to the usually applied PPN parameterization. In this sense, also the HYPER mission is planned to test the local frame dragging while OPTIS and ASTROD are sensitive to the global effect. Another realization of a global version of gravitomagnetic effects is based on clocks in counter–rotating satellites (Mashhoon *et al* 2001) which, however, is beyond today's technical capabilities.

In this paper we first briefly review the basic features of the OPTIS mission and the possible tests of relativistic gravity using satellites. Then we consider the features of a combined LAGEOS–LAGEOS II–OPTIS scheme in order to observe the Lense–Thirring effect. The expected accuracy of the observation of the Lense–Thirring effect for various

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‡ Other scientists in (Ries *et al* 2003) propose a different error budget.

scenarios for the OPTIS mission are calculated and compared.

2. OPTIS - a satellite mission for testing basic aspects of Special and General Relativity

OPTIS (Lämmerzahl *et al* 2001) is a recently proposed satellite–based mission§ which would allow for precise tests of basic principles underlying Special and General Relativity. This mission is based on the use of a spinning drag–free satellite in an eccentric, high–altitude orbit which should allow to perform a three orders of magnitude improved Michelson–Morley test and a two orders of magnitude improved Kennedy–Thorndike test. Moreover, it should also be possible to improve by two orders of magnitude the tests of the universality of the gravitational redshift by comparison of an atomic clock with an optical clock. The proposed experiments are based on ultrastable optical cavities, lasers, an atomic clock and a frequency comb generator. Since it is not particularly important for the present version of the mission the final orbital configuration of OPTIS has not yet been fixed; in (Lämmerzahl *et al* 2001) a perigee height of 10000 km and apogee height of 36000 km, with respect to Earth's surface, are provisionally proposed assuming a launch with Ariane 5.

The requirements posed by the drag–free technology to be used, based on the field emission electrical propulsion (FEEP) concept, yield orbital altitudes not less than 1000 km. On the other hand, the eccentricity should not be too high in order to prevent passage in the Van Allen belts which could affect the on–board capacitive reference sensor. Moreover, the orbital period $P_{\rm OPT}$ should be shorter than the Earth's daily rotation of 24 hours. The orbital configuration proposed in (Lämmerzahl *et al* 2001) would imply a semimajor axis $a_{\rm OPT}=29300$ km and an eccentricity $e_{\rm OPT}=0.478$. With such values the difference of the gravitational potential U, which is relevant for the gravitational redshift test, would amount to

$$\frac{\Delta U}{c^2}=\frac{GM}{c^2a}\left[\frac{1}{(1-e)}-\frac{1}{(1+e)}\right]\sim 1.8\times 10^{-10}, \tag{1}$$

where G is the Newtonian gravitational constant, M the mass of Earth and c the speed of light in vacuum. The result of eq.(1) is about three orders of magnitude better than that obtainable in an Earth–based experiment.

An essential feature of OPTIS is the drag–free control of the orbit. Drag–free motion is required for the SR and GR tests which are carried through using optical resonators. Even very small residual accelerations of $10^{-7}\,g$ may distort the resonators leading to error signals. As a by–product, this drag–free control also guarantees a very high quality geodesic motion which may be used, when being tracked, as probe of orbital relativistic gravitational effects.

For a drag–free motion of the satellite a sensor measuring the actual acceleration and thrusters counteracting any acceleration to the required precision are needed. The

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§ See also http://www.exphy.uni-duesseldorf.de/OPTIS/optis.html.

sensor, which is based on a capacitive determination of the position of a test mass, has a sensitivity of up to $10^{-12}\text{cm s}^{-2}\text{Hz}^{-\frac{1}{2}}$ (Touboul 2001a). This means that for one orbit of about 12 h the difference of the real position from the position achieved by ideal drag–free motion is of the order of 2 mm. Similar drag–free systems of similar accuracy and with mission adapted modifications will be used in MICROSCOPE, STEP and LISA. These systems have a lifetime of many years.

3. Review of possible satellite tests of Post–Newtonian gravitation

As stated above, the observation of the motion of freely falling test bodies, i.e. satellites, is an important and feasible way to test Post–Newtonian properties of the gravitational field created by Earth. These are tests of the gravitational redshift and of general relativistic gravitoelectromagnetic effects such as the Lense–Thirring effect, all of order $\mathcal{O}(c^{-2})$.

3.1. *The gravitational redshift*

The gravitational redshift relates the frequencies f of clocks located at different gravitational potentials to the potential difference

$$\frac{\Delta f}{f} = \frac{f(\boldsymbol{r}) - f(\boldsymbol{r}_0)}{f(\boldsymbol{r}_0)} = \psi_{\text{clock}} \frac{U(\boldsymbol{r}) - U(\boldsymbol{r}_0)}{c^2} \, . \tag{2}$$

In General Relativity $\psi_{\text{clock}} = 1$. This has been tested, e.g., by Pound and Rebka (Pound and Rebka 1960) and at best by the first fundamental physics space mission GP-A (Vessot *et al* 1980) with an accuracy $|\psi_{\text{clock}} - 1| \leq 1.4 \times 10^{-4}$.

OPTIS aims at a test of this gravitational redshift with up to three orders of improvement, that is up to $|\psi_{\text{clock}} - 1| \leq 10^{-7}$. In comparison, the planned experiments ACES-PHARAO, SUMO, and PARCS to be carried through onboard of the International Space Station (ISS) (Lämmerzahl *et al* 2004) are supposed to reach the 10^{-5} level, while the ISS project RACE is projected to approach the 10^{-7} level. As compared to the ISS, OPTIS has the advantage to fly on a high elliptic orbit, to have a long mission time and to have verious clocks onboard (H–maser, optical resonators, ion clocks). The OPTIS mission also aims at a test of the universality of this gravitational redshift, that is the equality of ψ_{clock} for different clocks, $|\psi_{\text{clock2}} - \psi_{\text{clock1}}|$. Due to the various clocks onboard of the OPTIS satellite also various combinations can be tested. Again, the high elliptic orbit, the mission time as well as the number of clocks will lead to an improvement of previous results by up to three orders.

3.2. *The Lense–Thirring effect*

One of the most interesting Post–Newtonian gravitational effects is the general relativistic gravitomagnetic Lense–Thirring effect or dragging of inertial frames whose source is the proper angular momentum $\boldsymbol{J}$ of the central mass which acts as source of the gravitational field. Its effect on the orientations of the spins $\boldsymbol{s}$ of four freely orbiting superconducting gyroscopes should be tested, among other things, by the important GP–B mission (Everitt *et al* 2001) at a claimed accuracy level of the order of 1% or better. Another possible way to measure such relativistic effect is the analysis of the laser–ranged data of some existing, or proposed, geodetic satellites of LAGEOS–type, such as LAGEOS, LAGEOS II (Ciufolini 1996) and the proposed LAGEOS III–LARES (Ciufolini 1986; 1998). In this case the whole orbit of the satellite is to be thought of as a giant gyroscope whose node and perigee undergo the Lense–Thirring secular precessions

$$\dot{\Omega}_{\rm LT} = \frac{2GJ}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \tag{3}$$

$$\dot{\omega}_{\rm LT} = -\frac{6GJ\cos i}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \tag{4}$$

where i is the inclination of the orbital plane to the Earth's equator. Note that in the original paper by Lense and Thirring the longitude of the pericentre $\varpi = \Omega + \omega$ is used instead of ω.

Since 1996 measurements of the Lense-Thirring dragging of the orbits of the existing LAGEOS and LAGEOS II satellites at a claimed accuracy of the order of 20%–30% (Ciufolini *et al* 1998; Ciufolini 2002) have been reported. Based on the original proposal of a laser ranging mission LARES (LAser RElativity Satellite), recently an improved version of this mission has been proposed (Iorio *et al* 2002; Iorio 2003a) which should allow to reach an accuracy level of the order of 1%. Below we are going to discuss possible measurement of the Lense–Thirring effect by tracking the OPTIS satellite in the LARES orbital configuration.

4. A joint LAGEOS–LAGEOS II–OPTIS relativity measurement

In this paper we wish to investigate the possibility to use the orbital data of OPTIS for performing precise tests of general relativistic gravitoelectromagnetism as well. The rather free choice of the orbital parameters of OPTIS and the use of a new drag–free technology open up the possibility to extend its scientific significance with new important general relativistic gravitoelectromagnetic tests. Indeed, it would be of great impact and scientific significance to concentrate as many relativistic tests as possible in a single mission, including also measurements in geodesy, geodynamics. Another important point is that OPTIS is currently under serious examination by a national space agency-the German DLR. Then, even if it turns out that OPTIS would yield little or no advantages for the measurement of the Lense–Thirring effect with respect to the originally proposed LARES, if it will be finally approved and launched it will nevertheless be a great chance for detecting, among other things, the Lense-Thirring effect.

In Table 1 we report the orbital parameters of the existing or proposed LAGEOS–type satellites and of the originally proposed OPTIS configuration.

The main characteristics of such a mission are the already mentioned drag–free technique for OPTIS and the Satellite Laser Ranging (SLR) technique for tracking.

Table 1. Orbital parameters of LAGEOS, LAGEOS II, LARES and OPTIS.

Orbital parameter	LAGEOS	LAGEOS II	LARES	OPTIS
a (km)	12270	12163	12270	29300
e	0.0045	0.014	0.04	0.478
i (deg)	110	52.65	70	63.4
n (s^{-1})	4.643×10^{-4}	4.710×10^{-4}	4.643×10^{-4}	1.258×10^{-4}

Today it is possible to track satellites to an accuracy as low as a few mm. This may be further improved in the next years.

4.1. A modified OPTIS scenario

It seems that an orbital configuration of OPTIS identical to that of LARES of Table 1 would not be in dramatic contrast with the requirements for the other originally planned tests of Special and General Relativity. For example, the perigee height of LARES would amount to 5400 km while the apogee height would be 6382 km, with respect to Earth's surface. The difference in the gravitational potential $\frac{\Delta U}{c^2}$ would be of the order of 3×10^{-11}, which is only one order of magnitude smaller than the one that could be obtained with the originally proposed OPTIS configuration. In the case of a satellite with spherically symmetric shape and a small ratio cross sectional area to mass, such as the LAGEOS satellites, the orbital perturbations due to the non-gravitational perturbations are small and can be modelled with high accuracy. Indeed, the orbits of these laser ranged satellites can be modelled with root mean square of the orbital residuals, i.e. the difference between the observed and the calculated orbital elements, as little as about 1 cm over periods of about two weeks. However, in the case of a satellite of complex shape such as OPTIS we must rely on the drag-free system to reduce the effect of the non-gravitational perturbations. For a satellite with relatively small orbital eccentricity, the non-gravitational perturbations are more effective on the perigee rate than on the nodal rate. Indeed, over one orbital period, the total torque on the orbit due to an acceleration $\boldsymbol{A}$, constant in magnitude and direction is proportional to the eccentricity and the corresponding nodal precession is proportional to $\frac{Ae}{na}$. A perturbing acceleration $\boldsymbol{A}$, constant in magnitude and direction (in the along-track direction), for example the nearly constant along-track particle drag or a similar disturbing acceleration $\boldsymbol{A}$, would produce a perigee precession proportional to $\frac{A}{na}$. However a time–varying periodical perturbation with period equal to the orbital period would produce a nodal precession proportional to $\frac{A}{na}$ and a perigee precession proportional to $\frac{A}{nae}$. Let us then estimate the order of magnitude of the perigee and node perturbations of OPTIS–in the LARES orbital configuration–due to the residual accelerations not eliminated by the drag-free system. For OPTIS, in the frequency range around 10^{-4} Hz and 10^{-3} Hz, corresponding to the orbital period of OPTIS/LARES, the drag free system will reduce the spurious accelerations down to 10^{-12} cm s^{-2}. Let us then calculate the perigee and nodal rates induced by a periodical acceleration of magnitude 10^{-12} cm s^{-2} with period about equal to the OPTIS orbital period. By integrating the Gauss equation for perigee and node over one orbital period, we find that a perturbing acceleration of 10^{-12} cm s^{-2} with frequency $\frac{2\pi}{P}$, where P is the orbital period of OPTIS, would produce a perigee precession of the order of 0.2 mas yr^{-1} and a nodal precession of about 0.02 mas yr^{-1}. Perturbing accelerations with frequencies $\frac{2\pi}{P^*}$, where the period P^* is near the orbital period P, would also produce a perigee precession with a comparable order of magnitude. Since also sub-harmonics and higher harmonics would generate perigee and nodal precessions and since there are various non-gravitational perturbations with components near the orbital frequency, i.e. direct solar and Earth albedo radiation pressure, solar Yarkovsky, Yarkovsky-Rubincam effect, etc., we may assume that the total effect of all the non-gravitational perturbations with amplitude of the order of 10^{-12} cm s^{-2}, or less, would at most induce a perturbation of a few percent of the Lense-Thirring effect on the OPTIS perigee and of a fraction of percent of the Lense-Thirring effect on the OPTIS node.

4.2. A scenario without the perigee of LAGEOS II

In (Iorio *et al* 2002; Iorio 2003a) a multisatellite combination of the orbital residuals of the nodes of LAGEOS, LAGEOS II and LARES and the perigees of LAGEOS II and LARES has been proposed in order to improve the obtainable accuracy of the measurement of the Lense–Thiring effect. Such kind of combinations are motivated by the need of reducing the impact of the systematic error induced by the classical secular precessions on the node and the perigee due to the mismodelling in the even zonal harmonics of the geopotential (Iorio 2003b). Indeed, using the combination of (Iorio *et al* 2002) it would be possible to cancel out the contributions of the first four even zonal harmonics of the geopotential.

However, a possible weak point of this strategy is that it forces to include in the combination the data of the perigee of LAGEOS II, although with a weighing coefficient of the order of 10^{-3}. It is well known that the perigee of LAGEOS II is severly affected by many non–gravitational perturbations (Lucchesi 2002; 2002). Some of them, like the asymmetric reflectivity, the solar Yarkovsky-Schach and the terrestrial Yarkovsky-Rubincam effects, are of thermal origin and depend on the temporal evolution of the LAGEOS II spin axis (Lucchesi 2002). The key point is that, when (and if) OPTIS and/or LARES will be finally launched, the spin may be chaotic and unpredictable. For this purpose measurements of the LAGEOS II spin axis are and will be performed using different techniques. Moreover, it may happen that the averaging period necessary to overcome the large variations in the perigee of LAGEOS II will exceed the lifetime of the drag–free satellite which is related to the amount of fuel and, consequently, to the mission budget.

In view of these considerations it would be meaningful to explore the possibility of adopting a combination of orbital residuals which does not include the perigee of

LAGEOS II, even if it would yield a systematic error due to the geopotential slightly less favorable than the previous results. Indeed, when new, more accurate Earth gravity models from the CHAMP (Pavlis 2000) and, especially, GRACE missions (Ries *et al* 2002) will be available, the impact of the geopotential in the total error budget should be dramatically reduced and should fall below negligible levels.

By assuming for OPTIS the same orbital configuration of LARES the following combination yields high accuracy

$$\delta\dot{\Omega}^{\rm LAGEOS}+c_1\delta\dot{\Omega}^{\rm LAGEOS\ II}+c_2\delta\dot{\Omega}^{\rm OPTIS}+c_3\delta\dot{\omega}^{\rm OPTIS}\ =61.3\mu_{\rm LT}, \tag{5}$$

with

$$c_1\sim 3\times 10^{-3},\ c_2\sim 9.9\times 10^{-1},\ c_3\sim 1\times 10^{-3}. \tag{6}$$

In eq.(5) the quantity $\mu_{\rm LT}$, which is 0 in Galileo–Newton mechanics and 1 in General Relativity[‖], is the solved–for least square parameter which accounts for the Lense–Thirring effect. The orbital residuals $\delta\dot{\Omega}$ and $\delta\dot{\omega}$ would entirely absorb the Lense–Thirring effect because the gravitomagnetic force would be purposely set equal to zero in the force models of the orbital processors, contrary to all the other classical and relativistic accelerations which, instead, would be included in them. The resulting gravitomagnetic signal would be a linear trend with a slope of 61.3 mas yr^{-1}.

Note that, even if eq.(5) only cancels out the first three even zonal harmonics, the systematic error due to the remaining harmonics of higher degree amounts to

$$\left(\frac{\delta\mu_{\rm LT}}{\mu_{\rm LT}}\right)_{\rm even\ zonals}=4\times 10^{-4}. \tag{7}$$

The full covariance matrix of EGM96 (Lemoine *et al* 1998) up to degree $l=20$ has been used. It can be shown that this result is also insensitive to orbital injection errors in the inclination of OPTIS. Indeed, for $i_{\rm OPT}$ ranging from 69 deg to 71 deg the corresponding error varies from 0.04% to 0.06%. It becomes 0.2%–0.3% according to just the variance matrix of EGM96 used up to degree $l=20$ in a Root–Sum–Square (RSS) fashion. A very pessimistic upper bound can be obtained by simply summing up the absolute values of the individual errors induced by the various even zonal harmonics. For EGM96 it amounts to 0.4%–0.6%. It may be interesting to get some insights about the possible improvements which could be reached with the new Earth gravity models from CHAMP and GRACE by using the data from the recently released preliminary GGMC01C model which combines the TEG-4 gravity model (Tapley *et al* 2000) with the first data from GRACE. It can be retrieved on the WEB at http://www.csr.utexas.edu/grace/gravity/. According to a RSS calculation with the variance matrix, the systematic relative error due to the remaining harmonics of higher degree amounts to[¶] 3×10^{-5}, with a pessimistic

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[‖] As explained in (Ciufolini 1996), $\mu_{\rm LT}$ is also affected by the remaining, non–cancelled even zonal harmonics of higher degree and by the small residuals of the inclination.

[¶] Note that, contrary to EGM96 in which the recovered even zonal harmonics are highly correlated, the covariance matrices of the GGM01C/S models are almost diagonal; then, in this case, a RSS calculation should give a reliable estimate of the error due to the mismodelling in the even zonal harmonics of geopotential. Moreover, the released sigmas of the J_l coefficients in the GGM01 models, although they are preliminary, are not the mere formal, statistical errors but are tentatively calibrated.

upper bound of 6×10^{-5} obtained by summing up the absolute values of the individual errors. Moreover, also the secular variations of the even zonal harmonic coefficients of geopotential do not affect the proposed combination. Indeed, it turns out that they can be accounted for by an effective time rate (Eanes and Bettadpur 1996)

$$\dot{J}_2^{\rm eff} \sim \dot{J}_2 + 0.371\dot{J}_4 + 0.079\dot{J}_6 + 0.006\dot{J}_8 - 0.003\dot{J}_{10}... \qquad (8)$$

whose magnitude is of the order of $(-2.6 \pm 0.3) \times 10^{-11}$ yr^{-1}, and eq.(5) is designed in order to cancel out the effects of just the first three even zonal harmonic coefficient of geopotential. Finally, eq.(5) is affected neither by the problem of the systematic bias of the even zonal harmonics due to the Lense–Thirring signature (Ciufolini 1996). It consists of the fact that that in the solutions of the various Earth gravity models General Relativity-and the Lense-Thirring effect itself- is assumed to be true, so that the recovered J_l are biased by this a priori assumption. However, it turns out that, at least for the LAGEOS satellites, such feature is mainly concentrated in the first two–three even zonal harmonics.

With regard to the non-gravitational perturbations on the LAGEOS satellites, only the contribution of the nodes of LAGEOS and LAGEOS II, weighted by the small coefficients of eq.(6), have to be considered. This is quite relevant in the final error budget because the nodes of the LAGEOS satellites, contrary to the perigees of these laser-ranged satellites, are orbital elements much less sensitive to the action of the non–gravitational perturbations. With regard to the effect on the non–gravitational perturbations on the OPTIS satellite we have already estimated the impact of the residual accelerations, thus also according to the evaluations of Table 2 and Table 3 of (Iorio *et al* 2002), over $T_{\rm obs} = 7$ years

$$\left(\frac{\delta\mu_{\rm LT}}{\mu_{\rm LT}}\right)_{\rm NGP} \sim 3 \times 10^{-3}. \qquad (9)$$

It should be noted that the estimate of eq.(9) is probably pessimistic. Indeed, the periods of many time–dependent perturbations of the nodes of LAGEOS and LAGEOS II, contrary to the perigee of LAGEOS II, are far shorter than 7 years^{+}, so that would be possible to adopt a $T_{\rm obs}$ of just a few years during which it should be possible to save fuel and fit and remove the small time–varying non-gravitational signals affecting the nodes of the LAGEOS satellites or average them out. Moreover, the perigee of OPTIS would have an impact of the order of 10^{-4} for a residual, unbalanced acceleration $\delta A = 10^{-12}$ cm s^{-2}. Last but not least, the impact of the perigee of LAGEOS II, difficult to be modeled at a high level of accuracy, is absent. So, the total final systematic error budget in measuring the Lense–Thirring effect with eq.(5) should be of the order of 1%.

When more robust and confident Earth gravity solutions will be available in the near future, the need for canceling out as many even zonal harmonics as possible will

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[+] For example, the period of the tesseral ($m = 1$) tidal perturbation K_1, which is one of the most powerful perturbations affecting the node of LAGEOS, amounts to 2.85 years (Iorio 2001). Of course, the semisecular orbital tidal perturbations induced by the 9–year and 18.6–year tides do not affect eq.(5) because they are $l = 2$, $m = 0$ perturbtions and, consequently, are cancelled out.

be less stringent than now and, then, it could be possible to discard both the perigee of LAGEOS II and of OPTIS as well. So, a three–nodes combination could be considered. Indeed, by using the nodes of LAGEOS, LAGEOS II (with a coefficient of 3×10^{-3}) and OPTIS (with a coefficient of 9.9×10^{-1}) the relative error due to the static part of geopotential, according to the variance matrix of GGM01C (RSS calculation) would be 3×10^{-5}, with an upper bound of 6×10^{-5}. The slope of the gravitomagnetic signal would be 61.4 mas yr^{-1}. In this case, since the nodes are insensitive to the larger Post–Newtonian gravitoelectric force which, instead, affects the perigee, the result of such test would be independent of the inclusion of it into the force models*. With the three–nodes combination it should not be too optimistic to predict a total systematic error of the order of, or less than 1% over a time span of a few years.

4.3. The impact of the observational errors

Of crucial importance for the presented scenario would be, of course, the quality of the OPTIS tracking and orbital data reduction which should be, if possible, of the same level as that of the LAGEOS satellites. At present, the technique for the OPTIS orbital reconstruction has not yet been established, according to (Lämmerzahl *et al* 2001). If it will be finally decided to adopt the SLR (Satellite Laser Ranging) approach, a too high altitude of OPTIS might reduce the quality of the recovered orbit due to calibration problems and to the variable number of photons reflected back to the ranging stations. However, within the suggested project ASTROD (Huang *et al* 2002) research is under way with the goal to make precise phase coupling to very weak laser beams. Therefore, the accuracy of laser ranging might improve in the near future also for larger distances. Moreover, NASA has successfully tested laser ranging from Earth to Mars some years ago. The concept is based on a slightly modified system from what SLR community now uses on LAGEOS and the other existing geodetic satellites, but it could be applied to OPTIS without any problem. The fact that the satellite is rotating should not present a problem, as long as there is some ahead of launch planning to deal with this. Nearly all satellites spin, and not all of them are spherical such as LAGEOS, but they are still tracked. It will simply be considered when the CCR arrays to account for that spinning will be designed.

Another point to consider is that the large eccentricity of the originally proposed OPTIS configuration, contrary to the other existing geodetic satellites of LAGEOS–type, would not allow for a uniform coverage of the laser–ranged data in the sense that certain portions of the orbit would remain poorly tracked.

In conclusion, the scenario of eq.(5), with OPTIS instead of LARES in its orbital configuration, would yield a very accurate measurement of the Lense–Thirring effect at a level of the order of, or better than 1%. Instead, the peculiar originally proposed

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* However, it should be pointed out that the gravitoelectric pericentre advance has already been tested in the gravitational field of Sun with interplanetary ranging at a $10^{-2} - 10^{-3}$ precision level (Will 1993) ; this level of accuracy in its knowledge would have a negligible impact in a measurement of the Lense–Thirring effect with an observable like that of eq.(5) with the coefficient c_3 given by eq.(6).

orbital configuration of OPTIS may pose some problems for the orbital reconstruction with the currently available SLR technique.

5. The problem of the eccentricity

Perhaps the major point of conflict between the original designs of the OPTIS and LARES missions is represented by the eccentricity e of the orbit of the spacecraft. Indeed, while for the gravitational redshift test, given by eq.(1), a relatively large value of e is highly desirable, the originally proposed LARES orbit has a smaller eccentricity. The point is twofold: on one hand, it is easier and cheaper, in terms of requirements on the performances of the rocket launcher, to insert a satellite in a nearly circular orbit, and, on the other, the present status of the ground segment of SLR would assure a uniform tracking of good quality for such kind of orbits.

However, the originally proposed OPTIS mission implies the use of a rocket ARIANE 5 to insert the spacecraft into a GTO orbit and, then, the use of a kick motor. Moreover, it may be reasonable to assume that, when OPTIS/LARES will be launched, the network of SLR ground stations will have reached a status which will allow to overcome the problem of reconstructing rather eccentric orbits to a good level of accuracy.

Then, a reasonable compromise between the OPTIS and LARES requirements could be an eccentricity of, say, $e = 0.1$. In that case eq.(1) yields a gravitational redshift of $\frac{\Delta U}{c^2} = 7.3 \times 10^{-11}$ about 2 - 3 times smaller than in the original OPTIS proposal. Accordingly, the accuracy of the tests concerning the gravitational red shift will be worse be a factor 2 to 3. With regard to the Lense–Thirring effect, it turns out that, for the combination without the perigee of LAGEOS II of eq.(5), the error due to the even zonal harmonics of geopotential would amount to 1.5%, according to the diagonal part only of the covariance matrix of EGM96 up to degree $l = 20$ (RSS calculation). The sum of the absolute values of the individual terms yields an upper bound of the order of 3%. However, the forthcoming Earth gravity models from CHAMP and GRACE will greatly improve also such estimates. Indeed, the variance matrix of the very preliminary GGM01C model yields an error of 0.02% (RSS calculation) and a pessimistic upper bound of the order of 0.04% from the sum of the absolute values of the individual errors.

It may also be interesting to note that the originally proposed observable of the LAGEOS–LARES mission, i.e. the sum of their nodes, would be affected by such change in the eccentricity of the orbit of LARES at a 5%–7% level, according to diagonal part only of the covariance matrix of EGM96 up to degree $l = 20$ (RSS calculation), with an upper bound of the order of 12%–16% from the sum of the absolute values of the individual errors. A RSS calculation with the variance matrix of GGM01C yields a 0.7%–1.5% level of percent error and an upper bound of 1%–2.2% from the sum of the absolute values of the individual errors.

With a larger eccentricity the impact of the non–gravitational perturbations would be reduced and, on the other hand, the accuracy of the measurement of the Lense-Thirring effect on the OPTIS/LARES perigee would be increased. For example, the amplitude of the non–gravitational perturbations would reduce to 0.1 mas yr^{-1} and, for $\delta r^{\mathrm{exp}} \sim 1$ cm over a certain time span, the observational error in the perigee would amount to just 1 mas.

6. Would an OPTIS/LARES mission still be useful in measuring the Lense–Thirring effect?

In view of the expected improvements of the even zonal harmonics of geopotential by the new forthcoming Earth gravity models from CHAMP and GRACE, which should ameliorate our knowledge of, especially, the mid–high degree part of the even zonal harmonics spectrum, it seems legitimate to ask if the corresponding improvements in the obtainable accuracy of measurements of the Lense–Thirring effect with the currently existing laser–tracked satellites would make unnecessary a dedicated mission to this goal like LARES.

In (Iorio and Morea 2004) preliminary estimates with the recently released EIGEN-2 CHAMP–only and GGM01C GRACE–based models have been carried out. It turned out that the systematic error due to the mismodelling in the even zonal harmonics of geoptential of the combination proposed in (Ciufolini 1996), which involves the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II, would be $\leq 3\%$, while the error of the combination put forth in (Iorio and Morea 2004), based on the nodes of LAGEOS and LAGEOS II only, would be $\leq 18\%$, according to the variance matrix of GGM01C. The use of the nodes of other existing SLR satellites (Ajisai, Starlette, Stella) would induce a systematic error $\leq 123\%$. Note that such figures represent a conservative, pessimistic upper bound obtained by adding the sum of the absolute values of the individual errors.

If further improvements in our knowledge of the terrestrial gravitational field will come from new, more robust and reliable solutions from GRACE, as it is expected, it seems reasonable to suppose that the systematic errors due to geopotential will reduce to a some percent level for the nodes–only combination of (Iorio and Morea 2004) and below the 1% level for the three-elements combination involving also the perigee of LAGEOS II. Much will depend on the magnitude of the improvements of the low–degree even zonal harmonics $J_4,\ J_6, J_8, ...$ which will be obtained. The LAGEOS satellites are not particularly sensitive to the even zonal harmonics of degree higher than $l = 12$, so that an increased accuracy in knowing them would be of relatively little usefulness for the measurement of the Lense–Thirring effect with LAGEOS and LAGEOS II.

In view of this considerations, and by noting that,of course, also the proposed measurement with OPTIS/LARES would benefit from the improved knowledge of the terrestrial gravitational field, it is possible to conclude that the level of accuracy in measuring the gravitomagnetic force obtainable by implementing the OPTIS mission will remain far higher than that could be reached by simply reanalyzing the data of the existing SLR satellites. Needless to say that the latter approach would not allow to perform all the other tests of Special Relativity and Post–Newtonian gravitation which, in turn, could be implemented by the originally proposed LARES with a lower accuracy or could not be implemented at all.

7. Conclusions

In this paper we have shown that it would be possible to perform a very accurate measurement of the Lense-Thirring effect with the orbital data of the proposed OPTIS drag–free satellite, in addition to the other previously proposed tests of Special Relativity and Post–Newtonian gravitation. OPTIS is currently under serious examination by the German Aerospace Agency. In order to use the orbital data of OPTIS for precise tests of relativistic gravity it would be necessary, first of all, to carry onboard some SLR passive retroreflectors in order to reconstruct with great accuracy its orbit. To this aim, it turns out that the originally proposed orbital configuration of OPTIS, based on a highly eccentric orbit with a perigee of 10000 km, would not probably be well suited for, e.g., adequate SLR tracking. It would be better to adopt a LAGEOS–like orbit; it turns out that the orbital configuration of, e.g., the proposed LARES would not be in dramatic contrast with the requirements of the other relativistic experiments originally planned for OPTIS. With such a choice it would be possible to adopt a linear combination of the orbital residuals of the nodes of LAGEOS, LAGEOS II and OPTIS in order to measure the Lense–Thirring effect with a total systematic error that should be of the order of 1% or, perhaps, better. Such orbital test of Post–Newtonian gravitomagnetism require observational temporal intervals of some years in order to average out or fit and remove various time–dependent perturbations of gravitational and, especially, non–gravitational origin acting on the Keplerian orbital elements to be adopted in the analysis. So, it is of the utmost importance that the lifetime of the drag–free apparatus of OPTIS, which would not be a passive, spherical, geodetic satellite of LAGEOS–type, would not be shorter than the time span of the data analysis. However, on one hand the technology to be adopted should meet such requirements yielding lifetimes of the order of 10 years, on the other, the exclusion of the perigee of LAGEOS II, which is affected by some gravitational and non–gravitational perturbations with long periods, assures that not too long observational time spans would be needed. Finally, a comparison between the obtainable accuracy in measuring the Lense–Thirring effect with OPTIS and the one that could be obtained by simply reanalyzing the data of the existing SLR satellites with the new Earth gravity models shows that the former approach would yield unrivalled results.

In conclusion, the use of OPTIS for measuring the Lense–Thirring effect is feasible: in regard to this goal, the best choice would, probably, be to adopt the orbital configuration of LARES. It would not too seriously affect the obtainable accuracy in the gravitational red-shift test which is particularly sensitive to the orbital eccentricity. However, a compromise solution could also be adopted. The same observables as LARES could be employed with better results thanks to the active drag–free apparatus to be employed on OPTIS; indeed, LARES would be a totally passive satellite. Moreover, in addition to the Lense–Thirring effect, OPTIS would allow to perform many other tests of Special Relativity and Post–Newtonian gravitation.

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