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Turbulence-free Interference Induced by the Turbulence Itself

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SUPPLEMENTARY MATERIAL

The following supplementary material provides a calculation of the second-order coherence function in the Heisenberg picture.

Using Glauber’s theory of quantum coherence, a laser beam with TEM$_{00}$ spatial mode can be approximated as being in a single spatial-mode and multifrequency coherent state

$$|\Psi\rangle = \prod_\omega |\alpha(\omega)\rangle,$$

which is a pure state representing the state of a group of identical photons. When optical turbulence is introduced, a random phase shift is introduced to each path. The field operator at coordinate $(x_j, t_j)$ of the $j$th photodetector

$$\hat{E}^{(+)}(x_j, t_j) = \int d\omega \hat{a}(\omega) g_A(\omega; x_j, t_j) e^{-i\phi_A(t_j)}$$

$$+ \int d\omega \hat{a}(\omega) g_B(\omega; x_j, t_j) e^{-i\phi_B(t_j)}$$

$$= \hat{E}_A^{(+)}(x_j, t_j) + \hat{E}_B^{(+)}(x_j, t_j)$$

where $g_A(\omega; x_j, t_j)$ and $g_B(\omega; x_j, t_j)$ are the Green’s functions (or propagators) which propagate the $\omega$ mode of the state from slit-A and slit-B to the photodetector $D_j$ at space-time coordinate $(x_j, t_j)$. With the help of the quantum state and the field operators, the second-order coherence function $G^{(2)}(x_1, t_1; x_2, t_2)$ is calculated as follows

$$G^{(2)}(x_1, t_1; x_2, t_2)$$

$$= \langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En}$$

$$= \langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En}$$

which results in sixteen expectations to evaluate. In this equation, $\langle \langle \rangle \rangle_{En}$ means ensemble average. For brevity, we will drop terms that have no contribution to the measurement of $(n(x_1)n(x_2))$ or cannot survive the ensemble average by taking into account all possible turbulence introduced random phases along the $A$-path and the $B$-path,

$$G^{(2)}(x_1, t_1; x_2, t_2)$$

$$= \langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En}$$

$$= \langle \langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En},$$

where $\psi_i(x_j, t_j)$ is the effective wavefunction of the group-$i$ identical photons,

$$\psi_i(x_j, t_j) = \langle \Psi | \hat{E}_{i}^{(+)}(x_j, t_j) |\Psi\rangle$$

$$= \int d\omega \alpha(\omega) g_i(\omega; x_j, t_j) e^{-i\phi_i(t_j)}.$$

In this notation, the photon number fluctuation correlation

$$\langle \Delta n(x_1) \Delta n(x_2) \rangle$$

$$= \langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En}$$

$$+ \langle \langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En}$$

is the cross terms of the following superposition of pairs of two-photon effective wavefunctions

$$\langle \langle |\Psi| \hat{E}^{(-)}(x_1, t_1) \hat{E}^{(-)}(x_2, t_2)$$

$$\times \hat{E}^{(+)}(x_2, t_2) \hat{E}^{(+)}(x_1, t_1) |\Psi\rangle \rangle_{En},$$

which corresponds to two different yet indistinguishable alternatives for the two distinguishable groups of identical photons to produce a joint photodetection event of $D_1$ and $D_2$: (1) group-$A$ of identical photons propagate to $D_1$ and group-$B$ of identical photons propagate to $D_2$; and (2) group-$A$ of identical photons propagate to $D_2$ and group-$B$ of identical photons propagate to $D_1$; indicating the interference of two distinguishable groups of identical photons; which we can label as two-photon interference.


$^6$M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge University Press, 1997).