Gaia Early Data Release 3

Acceleration of the solar system from Gaia astrometry

(Affiliations can be found after the references)

ABSTRACT

Context. Gaia Early Data Release 3 (Gaia EDR3) provides accurate astrometry for about 1.6 million compact (QSO-like) extragalactic sources, 1.2 million of which have the best-quality five-parameter astrometric solutions.

Aims. The proper motions of QSO-like sources are used to reveal a systematic pattern due to the acceleration of the solar system barycentre with respect to the rest frame of the Universe. Apart from being an important scientific result by itself, the acceleration measured in this way is a good quality indicator of the Gaia astrometric solution.

Methods. The effect of the acceleration is obtained as a part of the general expansion of the vector field of proper motions in Vector Spherical Harmonics (VSH). Various versions of the VSH fit and various subsets of the sources are tried and compared to get the most consistent result and a realistic estimate of its uncertainty. Additional tests with the Gaia astrometric solution are used to get a better idea on possible systematic errors in the estimate.

Results. Our best estimate of the acceleration based on Gaia EDR3 is \( \frac{2.32 \pm 0.16}{10^{-10}} \text{ m s}^{-2} \) (or \( 7.33 \pm 0.51 \text{ km s}^{-1} \text{ Myr}^{-1} \)) towards \( \alpha = 269.1^{\circ} \pm 5.4^{\circ}, \delta = -31.6^{\circ} \pm 4.1^{\circ} \), corresponding to a proper motion amplitude of \( 5.05 \pm 0.35 \text{ \mu as yr}^{-1} \). This is in good agreement with the acceleration expected from current models of the Galactic gravitational potential. We expect that future Gaia data releases will provide estimates of the acceleration with uncertainties substantially below 0.1 \text{ \mu as yr}^{-1}.

Key words. astrometry – proper motions – reference systems – Galaxy: kinematics and dynamics – methods: data analysis

1. Introduction

It is well known that the velocity of an observer causes the apparent positions of all celestial bodies to be displaced in the direction of the velocity, an effect referred to as the aberration of light. If the velocity is changing with time, that is if the observer is accelerated, the displacements are also changing, giving the impression of a pattern of proper motions in the direction of the acceleration. We exploit this effect to detect the imprint in the Gaia data of the acceleration of the solar system with respect to the rest-frame of remote extragalactic sources.

1.1. Historical considerations

In 1833 John Pond, the Astronomer Royal at that time, sent to print the Catalogue of 1112 stars, reduced from observations made at the Royal Observatory at Greenwich (Pond 1833), the happy conclusion of a standard and tedious observational work, and a catalogue much praised for its accuracy (Grant 1852). At the end of his short introduction he added a note discussing Causes of Disturbance of the proper Motion of Stars, in which he considered the secular aberration resulting from the motion of the solar system in free space, stating that,

"...[t]he effects of either of these suppositions would be, to produce uniform motion in every star according to its position, and might in time be discoverable by our observations, if the stars had no proper motions of their own [...] But it is needless to enter into further speculation on questions that appear at present not likely to lead to the least practical utility, though it may become a subject of interest to future ages.

This was a simple, but clever, realisation of the consequences of aberration, really new at that time and totally outside the technical capabilities of the time. The idea gained more visibility through the successful textbooks of the renowned English astronomer John Herschel, first in his Treatise of Astronomy (Her-
Lalande 1833, §612) and later in the expanded version Outlines of Astronomy (Herschel 1849, §862), both of which went through numerous editions. In the former he referred directly to John Pond as the original source of this ‘very ingenious idea’, whereas in the latter the reference to Pond was dropped and the description of the effect looks unpromising:

This displacement, however, is permanent, and therefore unrecognizable by any phenomenon, so long as the solar motion remains invariable; but should it, in the course of ages, alter its direction and velocity, both the direction and amount of the displacement in question would alter with it. The change, however, would become mixed up with other changes in the apparent proper motions of the stars, and it would seem hopeless to attempt disentangling them.

John Pond in 1833 wrote that the idea came to him ‘many years ago’ but did not hint at borrowing it from someone else. For such an idea to emerge, at least three devices had to be present in the tool kit of a practising astronomer: a deep understanding of aberration, well known since James Bradley’s discovery in 1728; the secure proof that stars have proper motion, provided by the Catalogue of Tobias Mayer in 1760; and the notion of the secular motion of the Sun towards the apex, established by William Herschel in 1783. Therefore Pond was probably the first, to our knowledge, who combined the aberration and the free motion of the Sun among the stars to draw the important observable consequence in terms of systematic proper motions. We have found no earlier mention, and had it been commonly known by astronomers much earlier we would have found a mention in Lalande’s Astronomie (Lalande 1792), the most encyclopaedic treatise on the subject at the time.

References to the constant aberration due to the secular motion of the solar system as a whole appear over the course of years in some astronomical textbooks (e.g. Ball 1908), but not in all with the hint that only a change in the apex would make it visible in the form of a proper motion. While the bold foresight of these forerunners was by necessity limited by their conception of the Milky Way and the Universe as a whole, both Pond and Herschel recognised that even with a curved motion of the solar system, the effect on the stars from the change in aberration would be very difficult to separate from other sources of proper motion. This would remain true today if the stars of the Milky Way had been our only means to study the effect.

However, our current view of the hierarchical structure of the Universe puts the issue in a different and more favourable guise. The whole solar system is in motion within the Milky Way and there are star-like sources, very far away from us, that do not share this motion. For them the only source of apparent proper motion could be precisely that resulting from the change in the secular aberration. We are happily back to the world without proper motions contemplated by Pond, and we show in this paper that Gaia’s observations of extragalactic sources enable us to discern, for the first time in the optical domain, the signature of this systematic proper motion.

1.2. Recent works

Coming to the modern era, the earliest mention we have found of the effect on extragalactic sources is by Fanselow (1983) in the description of the JPL software package MASTERFIT for reducing Very Long Baseline Interferometry (VLBI) observations. There is a passing remark that the change in the apparent position of the sources from the solar system motion would be that of a proper motion of $6\mu$as yr$^{-1}$, nearly two orders of magnitude smaller than the effect of source structure, but systematic. There is no detailed modelling of the effect, but at least this was clearly shown to be a consequence of the change in the direction of the solar system velocity vector in the aberration factor, worthy of further consideration. The description of the effect is given in later descriptions of MASTERFIT and also in some other publications of the JPL VLBI group (e.g. Sovers & Jacobs 1996; Sovers et al. 1998).

Eubanks et al. (1995) have a contribution in IAU Symposium 166 with the title Secular motions of the extragalactic radio-sources and the stability of the radio reference frame. This contains the first claim of seeing statistically significant proper motions in many sources at the level of $30\mu$as yr$^{-1}$, about an order of magnitude larger than expected. This was unfortunately limited to an abstract, but the idea behind was to search for the effect discussed here. Proper motions of quasars were also investigated by Gwinn et al. (1997) in the context of search for low-frequency gravitational waves. The technique relied heavily on a decomposition on VSH (Vector Spherical Harmonics), very similar to what is reported in the core of this paper.

Bastian (1995) rediscovered the effect in the context of the Gaia mission as it was planned at the time. He describes the effect as a variable aberration and stated clearly how it could be measured with Gaia using 60 bright quasars, with the unambiguous conclusion that ‘it seems quite possible that Gaia can significantly measure the galactocentric acceleration of the solar system’. This was then included as an important science objective of Gaia in the mission proposal submitted to ESA in 2000 and in most early presentations of the mission and its expected science results (Perryman et al. 2001; Mignard 2002). Several theoretical discussions followed in relation to VLBI or space astrometry (Sovers et al. 1998; Kopeikin & Makarov 2006). Kovalovsky (2003) considered the effect on the observed motions of stars in our Galaxy, while Mignard & Kilioner (2012) showed how the systematic use of the VSH on a large data sample like Gaia would permit a blind search of the acceleration without ad hoc model fitting. They also stressed the importance of solving simultaneously for the acceleration and the spin to avoid signal leakage from correlations.

With the VLBI data gradually covering longer periods of time, detection of the systematic patterns in the proper motions of quasars became a definite possibility, and in the last decade there have been several works claiming positive detections at different levels of significance. But even with 20 years of data, the systematic displacement of the best-placed quasars is only $0.1\mas$, not much larger than the noise floor of individual VLBI positions until very recently. So the actual detection was, and remains, challenging.

The first published solution by Gwinn et al. (1997), based on 323 sources, resulted in an acceleration estimate of $(g_x, g_y, g_z) = (1.9\pm6.1, 5.4\pm6.2, 7.5\pm5.6)\mu$as yr$^{-1}$, not really above the noise level. Then a first detection claim was by Titov et al. (2011), using 555 sources and 20 years of VLBI data. From the proper motions of these sources they found $|g| = g_x = 6.4\pm1.5\mu$as yr$^{-1}$ for the amplitude of the systematic signal, compatible with the expected magnitude and direction. Two years later they published an improved solution from 34 years of VLBI data, yielding $g = 6.4\pm1.1\mu$as yr$^{-1}$ (Titov & Lambert 2013). A new solution by Titov & Krásná (2018) with a global fit of the dipole
on more than 4000 sources and 36 years of VLBI delays yielded
\(g = 5.2 \pm 0.2\ \mu\text{as yr}^{-1}\), the best formal error so far, and a
direction a few degrees off the Galactic centre. Xu et al. (2012)
also made a direct fit of the acceleration vector as a global pa-
parameter to the VLBI delay observations, and found a modulus of
\(g = 5.82 \pm 0.32\ \mu\text{as yr}^{-1}\) but with a strong component perpen-
dicular to the Galactic plane.

The most recent review by MacMillan et al. (2019) is a re-
port of the Working Group on Galactic Aberration of the In-
ternational VLBI Service (IVS). This group was established to
incorporate the effect of the galactocentric aberration into the
VLBI analysis with a unique recommended value. They make
a clear distinction between the galactocentric component that may
be estimated from Galactic kinematics, and the additional
contributions due to the accelerated motion of the Milky Way
in the intergalactic space or the peculiar acceleration of the so-
lar system in the Galaxy. They use the term ‘aberration drift’
for the total effect. Clearly the observations cannot separate the
different contributions, neither in VLBI nor in the optical do-
main with Gaia. Based on their considerations, the working group’s recommendation is to use \(g = 5.8\ \mu\text{as yr}^{-1}\) for the
galactocentric component of the aberration drift. This value,
estimated directly in a global solution of the ICRF3 solution
data set, is slightly larger than the value deduced from Galac-
tic astronomy. This recommendation has been finally adopted
in the ICRF3 catalogue, although an additional dedicated anal-
ysis of almost 40 years of VLBI observations gave the acceler-
gation \(g = 5.83 \pm 0.23\ \mu\text{as yr}^{-1}\) towards \(\alpha = 270.2^\circ \pm 2.3^\circ\),
\(\delta = -20.2^\circ \pm 3.6^\circ\) (Charlot et al. 2020).

To conclude this overview of related works, a totally different
approach by Chakrabarti et al. (2020) was recently put forward,
relying on highly accurate spectroscopy. With the performances of
spectrographs reached in the search for extra-solar planets, on
the level of 10 cm s\(^{-1}\), it is conceivable to detect the variation
of the line-of-sight velocity of stars over a time baseline of at
least ten years. This would be a direct detection of the Galactic
acceleration and a way to probe the gravitational potential at ∼
kpc distances. Such a result would be totally independent of the
acceleration derived from the aberration drift of the extragalactic
sources and of great interest.

Here we report on the first determination of the solar system
acceleration in the optical domain, from Gaia observations. The
paper is organised as follows. Section 2 summarises the astro-
metric signatures of an acceleration of the solar system barycen-
tre with respect to the rest frame of extragalactic sources. The-
oretical expectations of the acceleration of the solar system are
presented in Sect. 3. The selection of Gaia sources for the deter-
mination of the effect is discussed in Sect. 4. Section 5 presents
the method, and the analysis of the data and a discussion of ran-
don and systematic errors are given in Sect. 6. Conclusions of
this study as well as the perspectives for the future determination
with Gaia astrometry are presented in Sect. 7. In Appendix A we
discuss the general problem of estimating the length of a vector
from the estimates of its Cartesian components.

### 2. The astrometric effect of an acceleration

In the Introduction we described aberration as an effect changing
the ‘apparent position’ of a source. More accurately, it should be
described in terms of the ‘proper direction’ to the source: this is
the direction from which photons are seen to arrive, as measured
in a physically adequate proper reference system of the observer
(see, e.g. Klioner 2004; 2013). The proper direction which we
designate with the unit vector \(\mathbf{u}\), is what an astrometric in-
strument in space ideally measures.

The aberration of light is the displacement \(\delta u\) obtained when
comparing the proper directions to the same source, as measured
by two co-located observers moving with velocity \(v\) relative to
each other. According to the theory of relativity (both special and
general), the proper directions as seen by the two observers are
related by a Lorentz transformation depending on the velocity \(v\)
of one observer as measured by the other. If \(\delta u\) is relatively large,
as for the annual aberration, a rigorous approach to the com-
putation is needed and also used, for example in the Gaia data
processing (Klioner 2003). Here we are however concerned with
small differential effects, for which first-order formulae (equiva-
letal to first-order classical aberration) is sufficient. To first order
in \(|v|/c\), where \(c\) is the speed of light, the aberrational effect is
linear in \(v\),

\[
\delta u = \frac{v}{c} - \frac{v - u}{c} u. \tag{1}
\]

Equation (1) is accurate to \(< 0.001\ \mu\text{as} \text{ for } |v| < 0.02\ \text{km s}\(^{-1}\)\text{, and to } < 1'' \text{ for } |v| < 600\ \text{km s}\(^{-1}\) \text{ see, however, below).}

If \(v\) is changing with time, there is a corresponding time-
dependent variation of \(\delta u\), which affects all sources on the sky
in a particular systematic way. A familiar example is the annual
aberration, where the apparent positions seen from the Earth
are compared with those of a hypothetical observer at the same lo-
cation, but at rest with respect to the solar system barycentre.
The annual variation of \(v/c\) results in the aberrational effect that
outlines a curve that is close to an ellipse with semi-major axis
about 20’ (the curve is not exactly an ellipse since the barycen-
tric orbit of the Earth is not exactly Keplerian).

The motion with respect to the solar system barycentre is not
the only conceivable source of aberrational effects. It is well
known that the whole solar system (that is, its barycentre) is
in motion in the Galaxy with a velocity of about 248 km s\(^{-1}\)
(Reid & Brunthaler 2020), and that its velocity with respect to
the Cosmic Microwave Background Radiation (CMBR) is about
370 km s\(^{-1}\) (Planck Collaboration et al. 2020). Therefore, if one
compares the apparent positions of the celestial sources as seen
by an observer at the barycentre of the solar system with those
seen by another observer at rest with respect to the Galaxy or
the CMBR, one would see aberrational differences up to ∼171’
or ∼255’, respectively — effects that are so big that they could
be recognized by the naked eye (see Fig. 1 for an illustration of
this effect). The first of these effects is sometimes called secu-
lar aberration. In most applications, however, there is no reason
to consider an observer that is ‘even more at rest’ than the solar
system barycentre. The reason is that this large velocity — for
the purpose of astrometric observations and for their accuracies —
can usually be considered as constant; and if the velocity is con-
stant in size and direction, the principle of relativity imposes that
the aberrational shift cannot be detected. In other words, without
knowledge of the ‘true’ positions of the sources, one cannot re-
veal the constant aberrational effect on their positions.

However, the velocity of the solar system is not exactly con-
stant. The motion of the solar system follows a curved orbit in
the Galaxy, so its velocity vector is slowly changing with time.
The secular aberration is therefore also slowly changing with
time. Considering sources that do not participate in the galactic
rotation (such as distant extragalactic sources), we will see their
apparent motions tracing out aberration ‘ellipses’ whose period
is the galactic ‘year’ of ∼213 million years — they are of course
not ellipses owing to the epicyclic orbit of the solar system (see
Fig. 1). Over a few years, and even thousands of years, the tiny
arcs described by the sources cannot be distinguished from the
tangent of the aberration ellipse, and for the observer this is seen
as a proper motion that can be called additional, apparent, or
spurious:
\[ \frac{d(\delta u)}{dt} = \frac{a - a \cdot u}{c}u. \]  
(2)

Here \(a = dv/dt\) is the acceleration of the solar system barycen-
tre with respect to the extragalactic sources. For a given source,
this slow drift of the observed position is indistinguishable from
its true proper motion. However, the apparent proper motion as
given by Eq. (2) has a global dipolar structure with axial sym-
metry along the acceleration: it is maximal for sources in the di-
rection perpendicular to the acceleration and zero for directions
along the acceleration. This pattern is shown as a vector field in
Fig. 2 in the case of the centripetal acceleration directed towards
the galactic centre.

Because only the change in aberration can be observed, not
the aberration itself, the underlying reference frame in Eq. (1) is
irrelevant for the discussion. One could have considered another
reference for the velocity, leading to a smaller or larger aber-
ration, but the aberration drift would be the same and given by
Eq. (2). Although this equation was derived by reference to the
galactic motion of the solar system, it is fully general and tells
us that any accelerated motion of the solar system with respect
to the distant sources translates into a systematic proper-motion
pattern of those sources, when the astrometric parameters are
referenced to the solar system barycentre, as it is the case for
Gaia. Using a rough estimate of the centripetal acceleration of
the solar system in its motion around the galactic centre, one gets
the approximate amplitude of the spurious proper motions to be
\(\sim 5\ \mu\text{as yr}^{-1}\). A detailed discussion of the expected acceleration
is given in Sect. 3.

It is important to realize that the discussion in this form is
possible only when the first-order approximation given by Eq.
(1) is used. It is the linearity of Eq. (1) in \(v\) that allows one,
in this approximation, to decompose the velocity \(v\) in various
parts and simply add individual aberrational effects from those
components (e.g. annual and diurnal aberration in classical as-
tronomy or also a constant part and a linear variation). In the
general case of a complete relativistic description of aberration
via Lorentz transformations, the second-order aberrational ef-
fects depend also on the velocity with respect to the underlying
reference frame and can become large. However, when the ast-
rometric parameters are referenced to the solar system barycen-
tre, the underlying reference frame is at rest with respect to the
barycentre and Eq. (2) is correct to a fractional accuracy of about
\(\nu_{\text{obs}}/c \sim 10^{-4}\), where \(\nu_{\text{obs}}\) is the barycentric velocity of the
observer. While this is fully sufficient for the present and antici-
pated future determinations with Gaia, a more sophisticated
modelling is needed, if a determination of the acceleration to be
better than \(\sim 0.01\%\) is discussed in the future.

An alternative form of Eq. (2) is
\[ \mu = \mathbf{g} - (\mathbf{g} \cdot \mathbf{u}) \mathbf{u}, \]  
(3)

where \(\mu = d(\delta u)/dt\) is the proper motion vector due to the aber-
ration drift and \(\mathbf{g} = a/c\) may be expressed in proper motion
units, for example \(\mu\text{as yr}^{-1}\). Both vectors \(\mathbf{a}\) and \(\mathbf{g}\) are called ‘ac-
celeration’ in the context of this study. Depending on the con-
text, the acceleration may be given in different units, for example
\(\text{m s}^{-2}, \mu\text{as yr}^{-1}\), or \(\text{km s}^{-1} \text{Myr}^{-1}\). (1 \(\mu\text{as yr}^{-1}\) corre-
sponds to \(1.45343 \text{km s}^{-1} \text{Myr}^{-1} = 4.60566 \times 10^{-11}\ \text{m s}^{-2}\).)

Equation (3) can be written in component form, using Carte-
sian coordinates in any suitable reference system and the asso-
ciated spherical angles. For example, in equatorial (ICRS) ref-
ence system \((x,y,z)\) the associated angles are right ascension
and declination \((\alpha, \delta)\). The components of the proper motion,
\(\mu_x \equiv \mu_x \cos \delta \) and \(\mu_y \equiv \mu_y \sin \delta \), are obtained by projecting \(\mu\) on the unit
vectors \(e_x\) and \(e_y\) in the directions of increasing \(\alpha\) and \(\delta\) at the
position of the source (see Mignard & Klioner 2012, Fig. 1 and
their Eqs. 64 and 65). The result is
\[ \mu_x = -g_x \sin \alpha + g_z \cos \alpha, \]
\[ \mu_y = -g_y \sin \delta \cos \alpha - g_z \sin \delta \sin \alpha + g_x \cos \delta, \]  
(4)

where \((g_x, g_y, g_z)\) are the corresponding components of \(\mathbf{g}\). A cor-
responding representation is valid in arbitrary coordinate system.
In this work, we will use either equatorial (ICRS) coordinates
\((x, y, z)\) or galactic coordinates \((X, Y, Z)\) and the correspon-
ding associated angles \((\alpha, \delta)\) and \((l, b)\), respectively (see Sect. 3.4).
Effects of the form in Eq. (4) are often dubbed ‘glide’ for the
reasons explained in Sect. 5.

3. Theoretical expectations for the acceleration

This Section is devoted to a detailed discussion of the expected
gravitational acceleration of the solar system. We stress, how-
ever, that the measurement of the solar system acceleration as
motion is almost entirely a reflex of the motion of the Sun around the Galactic centre. Its distance (Gravity Collaboration et al. 2019) is 
\[ d_{\odot-GC} = 8.178 \pm 0.013 \text{ (statistical)} \pm 0.022 \text{ (systematic)} \text{kpc}, \]
and its proper motion along the Galactic plane is \(-6.411 \pm 0.008 \text{ mas yr}^{-1}\) (Reid & Brunthaler 2020). The Sun is not on a circular orbit, so we cannot directly translate the corresponding velocity into a centripetal acceleration. To compensate for this, we can correct the velocity to the ‘local standard of rest’ – the velocity that a circular orbit at \(d_{\odot-GC}\) would have. This correction is \(12.24 \pm 2 \text{ km s}^{-1}\) (Schönrich et al. 2010), in the sense that the Sun is moving faster than a circular orbit at its position. Considered together this gives an acceleration of \(-6.98 \pm 0.12 \text{ km s}^{-1} \text{ Myr}^{-1}\) in the \(R'\) direction. This corresponds to the centripetal acceleration of \(4.80 \pm 0.08 \mu \text{as yr}^{-1}\) which is compatible with the values based on measurements of Galactic rotation, discussed for example by Reid et al. (2014) and Malkin (2014).

3.2. Acceleration from non-axisymmetric components

The Milky Way is a barred spiral galaxy. The gravitational force from the bar and spiral have important effects on the velocities of stars in the Milky Way, as has been seen in numerous studies using \textit{Gaia} DR2 data (e.g. Gaia Collaboration et al. 2018a). We separately consider acceleration from the bar and the spiral. Table 1 in Hunt et al. (2019) summarises models for the bar potential taken from the literature. From this, assuming that the Sun lies \(30^\circ\) away from the major axis of the bar (Wegg et al. 2015), most models give an acceleration in the negative \(d'\) direction of \(0.04 \text{ km s}^{-1} \text{ Myr}^{-1}\), with one differing model attributed to Pérez-Villegas et al. (2017) which has a \(d'\) acceleration of \(0.09 \text{ km s}^{-1} \text{ Myr}^{-1}\). The Portail et al. (2017) bar model, the potential from which is illustrated in Figure 2 of Monari et al. (2019), is not included in the Hunt et al. (2019) table, but is consistent with the lower value.

The recent study by Eilers et al. (2020) found an acceleration from the spiral structure in the \(d'\) direction of \(0.10 \text{ km s}^{-1} \text{ Myr}^{-1}\) in the opposite sense to the acceleration from the bar. Statistical uncertainties on this value are small, with systematic errors relating to the modelling choices dominating. This spiral strength is within the broad range considered by Monari et al. (2016), and we estimate the systematic uncertainty to be of order \(\pm 0.05 \text{ km s}^{-1} \text{ Myr}^{-1}\).

3.3. Acceleration towards the Galactic plane

The baryonic component of the Milky Way is flattened, with a stellar disc which has an axis ratio of ~1:10 and a gas disc, with both H\(_{\text{II}}\) and H\(_{\text{2}}\) components, which is even flatter. The Sun is slightly above the Galactic plane, with estimates of the height above the plane typically of the order \(z'_{\odot} = 25 \pm 5 \text{ pc}\) (Bland-Hawthorn & Gerhard 2016).

We use the Milky Way gravitational potential from McMillan (2017), which has stellar discs and gas discs based on literature results, to estimate this component of acceleration. We find an acceleration of \(0.15 \pm 0.03 \text{ km s}^{-1} \text{ Myr}^{-1}\) in the negative \(z'\) direction, i.e. towards the Galactic plane. This uncertainty is found using only the uncertainty in \(d_{\odot-GC}\) and \(z'_{\odot}\). We can estimate the systematic uncertainty by comparison to the model from McMillan (2011), which, among other differences, has no gas discs. In this case we find an acceleration of \(0.13 \pm 0.02 \text{ km s}^{-1} \text{ Myr}^{-1}\), suggesting that the uncertainty associated with the potential is...
comparable to that from the distance to the Galactic plane. For reference, if the acceleration were directed exactly at the Galactic centre we would expect an acceleration in the negative $z'$ direction of $-0.02 \text{ km s}^{-1} \text{ Myr}^{-1}$ due to the mentioned elevation of the Sun above the plane by 25 pc, see next subsection.

Combined, this converts into an acceleration of $(-6.98 \pm 0.12, +0.06 \pm 0.05, -0.15 \pm 0.03) \text{ km s}^{-1} \text{ Myr}^{-1}$ in the $(R', \phi', z')$ directions.

3.4. Transformation to standard galactic coordinates

For the comparison of this model expectation with the EDR3 observations we have to convert both into standard galactic coordinates $(X, Y, Z)$ associated with galactic longitude and latitude $(l, b)$.

The standard galactic coordinates are defined by the transformation between the equatorial (ICRS) and galactic coordinates given in Sect. 1.5.3, Vol. 1 of ESA (1997) using three angles to be taken as exact quantities. In particular, the equatorial plane of the galactic coordinates is defined by its pole at ICRS coordinates $(\alpha = 192.85948^\circ, \delta = +27.12825^\circ)$, and the origin of galactic longitude is defined by the galactic longitude of the ascending node of the equatorial plane of the galactic coordinates on the ICRS equator, which is taken to be $l_0 = 32.93192^\circ$. This means that the point with galactic coordinates $(l = 0, b = 0)$, that is the direction to the centre, is at $(\alpha \approx 266.40499^\circ, \delta = -28.93617^\circ)$.

The conversion of the model expectation takes into account the above-mentioned elevation of the Sun, leading to a rotation of the $Z$ axis with respect to the $z'$ axis by $(10.5 \pm 2) \text{ arcmin}$, plus sign flips of the axes’ directions. This leaves us with the final predicted value of $(a_X, a_Y, a_Z) = (+6.98 \pm 0.12, -0.06 \pm 0.05, -0.13 \pm 0.03) \text{ km s}^{-1} \text{ Myr}^{-1}$. Note that the rotation of the vertical axis is uncertain by about $2'$, due to the uncertain values of $d_{G-GC}$ and $Z_G$. This, however, gives an uncertainty of only $0.004 \text{ km s}^{-1} \text{ Myr}^{-1}$ in the predicted $a_Z$.

We should emphasize that these transformations are purely formal ones. They should not be considered as strict in the sense that they refer the two vectors to the true attractive center of the real galaxy. On the one hand, they assume that the standard galactic coordinates $(X, Y, Z)$ represent perfect knowledge of the true orientation of the Galactic plane and the true location of the Galactic barycentre. On the other hand, they assume that the disk is completely flat, and that the inner part of the Galactic potential is symmetric (apart from the effects of the bar and local spiral structure discussed above). Both assumptions can easily be violated by a few arcmin. This can easily be illustrated by the position of the central black hole, Sgr A*. It undoubtedly sits very close in the bottom of the Galactic potential trough, by dynamical necessity. But that bottom needs not coincide with the barycentre of the Milky Way, nor with the precise direction of the inner galaxy’s force on the Sun. In fact, the position of Sgr A* is off galactic longitude zero by $-3.3'$, and off galactic latitude zero by $-2.7'$. This latitude offset is only about a quarter of the 10.5' correction derived from the Sun’s altitude above the plane.

Given the present uncertainty of the measured acceleration vector by a few degrees (see Table 2), these considerations about a few arcmin are irrelevant for the present paper. We mention them here as a matter of principle, to be taken into account in case the measured vector would ever attain a precision at the arcminute level.

3.5. Specific objects

Bachchan et al. (2016) provide in their Table 2 an estimate of the acceleration due to various extragalactic objects. We can use this table as an initial guide to which objects are likely to be important, however mass estimates of some of these objects (particularly the Large Magellanic Cloud) have changed significantly from the values quoted there.

We note first that individual objects in the Milky Way have a negligible effect. The acceleration from $\alpha$ Cen AB is $\sim 0.004 \text{ km s}^{-1} \text{ Myr}^{-1}$, and that from any nearby giant molecular clouds is comparable or smaller. In the local group, the largest effect is from the Large Magellanic Cloud (LMC). A number of lines of evidence now suggest that it has a mass of $(1-2.5) \times 10^{11} M_\odot$ (see Erkal et al. 2019 and references therein), which at a distance of $49.5 \pm 0.5$ kpc (Pietrzynski et al. 2019) gives an acceleration of 0.18 to 0.45 $\text{ km s}^{-1} \text{ Myr}^{-1}$ with components $(a_X, a_Y, a_Z)$ between $(+0.025, -0.148, -0.098)$ and $(+0.063, -0.371, -0.244) \text{ km s}^{-1} \text{ Myr}^{-1}$. We note therefore that the acceleration from the LMC is significantly larger than that from either the Galactic plane or non-axisymmetric structure.

The Small Magellanic Cloud is slightly more distant $(62.8 \pm 2.5 \text{ kpc};$ Cioni et al. 2000), and significantly less massive. It is thought that it has been significantly tidally stripped by the LMC (e.g. De Leo et al. 2020), so its mass is likely to be substantially lower than its estimated peak mass of $\sim 7 \times 10^{10} M_\odot$ (e.g. Read & Erkal 2019), but is hard to determine based on dynamical modelling. We follow Patel et al. (2020) and consider the range of possible masses $(0.5-3) \times 10^{10} M_\odot$ which gives an acceleration of 0.005 to 0.037 $\text{ km s}^{-1} \text{ Myr}^{-1}$. Other local group galaxies have a negligible effect. M31, at a distance of 752 $\pm 27$ kpc (Riess et al. 2012), with mass estimates in the range $(0.7-2) \times 10^{12} M_\odot$ (Fardal et al. 2013) imparts an acceleration of 0.005 to 0.016 $\text{ km s}^{-1} \text{ Myr}^{-1}$. The Sagittarius dwarf galaxy is relatively nearby, and was once relatively massive, but has been dramatically tidally stripped to a mass $\lesssim 4 \times 10^5 M_\odot$ (Vasiliev & Belokurov 2020; Law & Majewski 2010), so provides an acceleration $\lesssim 0.003 \text{ km s}^{-1} \text{ Myr}^{-1}$. We note that this discussion only includes the direct acceleration that these local group bodies apply to the Solar system. They are expected to deform the Milky Way’s dark matter halo in a way that may also apply an acceleration (e.g., Garavito-Camargo et al. 2020).

We can, like Bachchan et al. (2016), estimate the acceleration due to nearby galaxy clusters from their estimated masses and distances. The Virgo cluster at a distance $16.5 \text{ Mpc}$ (Mei et al. 2007) and a mass $(1.4-6.3) \times 10^{14} M_\odot$ (Ferrarese et al. 2002; Kashibadze et al. 2020) is the most significant single influence of 0.002 to 0.010 $\text{ km s}^{-1} \text{ Myr}^{-1}$. However, we recognise that the peculiar velocity of the Sun with respect to the Hubble flow has a component away from the Local Void, one towards the centre of the Laniakea supercluster, and others on larger scales that are not yet mapped (Tully et al. 2008; Tully et al. 2014), and that this is probably reflected in the acceleration felt on the solar system barycentre from large scale structure.

For simplicity we only add the effect of the LMC to the value given at the end of Sect. 3.2 to give an overall estimate of the expected range of, adding our estimated 1$\sigma$ uncertainties from the Galactic models to our full range of possible accelerations from the LMC to give $(a_X, a_Y, a_Z)$ as $(+6.89, -0.20, -0.20)$ to $(+7.17, -0.48, -0.40) \text{ km s}^{-1} \text{ Myr}^{-1}$.

2 To take the solar system as an illustrative analogue: the bottom of the potential trough is always very close to the centre of the Sun, but the barycentre can be off by more than one solar radius, i.e. the attraction felt by a Kuiper belt object at, say, 30 au can be off by more than 0.5'.

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Gaia Collaboration et al.: Gaia Early Data Release 3 – Acceleration of the solar system
4. Selection of Gaia sources

4.1. QSO-like sources

Gaia Early Data Release 3 (EDR3; Gaia Collaboration et al. 2020a) provides high-accuracy astrometry for over 1.5 billion sources, mainly galactic stars. However, there are good reasons to believe that a few million sources are QSOs and other extra-galactic sources that are compact enough for Gaia to obtain good astrometric solutions. These sources are hereafter referred to as ‘QSO-like sources’. As explained in Sect. 4.2 it is only the QSO-like sources that can be used to estimate the acceleration of the solar system.

Eventually, in later releases of Gaia data, we will be able to provide astrophysical classification of the sources and thus find all QSO-like sources based only on Gaia’s own data. EDR3 may be the last Gaia data release that needs to rely on external information to identify the QSO-like sources in the main catalogue of the release. To this end, a cross-match of the full EDR3 catalogue was performed with 17 external QSO and AGN catalogues. The matched sources were then further filtered to select astrometric solutions of sufficient quality in EDR3 and to have parallaxes and proper motions compatible with zero within five times the respective uncertainty. In this way, the contamination of the sample by stars is reduced, even though it may also exclude some genuine QSOs. It is important to recognise that the rejection based on significant proper motions does not interfere with the systematic proper motions expected from the acceleration, the latter being about two orders of magnitude smaller than the former. Various additional tests were performed to avoid stellar contamination as much as possible. As a result, EDR3 includes 1 614 173 sources that were identified as QSO-like; these are available in the Gaia Archive as the table agn_cross_id. The full details of the selection procedure, together with a detailed description of the resulting Gaia-CRF3, will be published elsewhere (Gaia Collaboration et al. 2020b).

In Gaia EDR3 the astrometric solutions for the individual sources are of three different types (Lindegren & al. 2020a):

- two-parameter solutions, for which only a mean position is provided;
- five-parameter solutions, for which the position (two coordinates), parallax, and proper motion (two components) are provided;
- six-parameter solutions, for which an astrometric estimate (the ‘pseudocolour’) of the effective wavenumber is provided together with the five astrometric parameters.

Because of the astrometric filtering mentioned above, the Gaia-CRF3 sources only belong to the last two types of solutions: more precisely the selection comprises 1 215 942 sources with five-parameter solutions and 398 231 sources with six-parameter solutions. Table 1 gives the main characteristics of these sources. The Gaia-CRF3 sources with six-parameter solutions are typically fainter, redder, and have somewhat lower astrometric quality (as measured by the re-normalised unit weight error, RUWE) than those with five-parameter solutions. Moreover, various studies of the astrometric quality of EDR3 (e.g. Fabricius et al. 2020; Lindegren & al. 2020a,b) have demonstrated that the five-parameter solutions generally have smaller systematic errors, at least for $G > 16$, that is for most QSO-like sources. In the following analysis we include only the 1 215 942 Gaia-CRF3 sources with five-parameter solutions.

Important features of these sources are displayed in Figs. 3 and 5. The distribution of the sources is not homogeneous on the sky, with densities ranging from 0 in the galactic plane to 85 sources per square degree, and an average density of 30 deg$^{-2}$. The distribution of Gaia-CRF3 sources primarily reflects the sky inhomogeneities of the external QSO/AGN catalogues used to select the sources. In addition, to reduce the risk of source confusion in crowded areas, the only cross-matching made in the galactic zone ($|\sin b| < 0.1$, with $b$ the galactic latitude) was with the VLBI quasars, for which the risk of confusion is negligible thanks to their accurate VLBI positions. One can hope that the future Releases of Gaia-CRF will substantially improve the homogeneity and remove this selection bias (although a reduced source density at the galactic plane may persist due to the extinction in the galactic plane).

As discussed below, our method for estimating the solar system acceleration from proper motions of the Gaia-CRF3 sources involves an expansion of the vector field of proper motions in a set of functions that are orthogonal on the sphere. It is then advantageous if the data points are distributed homogeneously on the sky. However, as shown in Sect. 7.3 of (Mignard & Klioner 2012), what is important is not the ‘kinematical homogeneity’ of the sources on the sky (how many per unit area), but the ‘dynamical homogeneity’: the distribution of the statistical weight of the data points over the sky (how much weight per unit area). This distribution is shown on Fig. 4.

For a reliable measurement of the solar system acceleration it is important to have the cleanest possible set of QSO-like sources. A significant stellar contamination may result in a systematic bias in the estimated acceleration (see Sect. 4.2). In this context the histograms of the normalised parallaxes and proper motions in Fig. 6 are a useful diagnostic. For a clean sample of extragalactic QSO-like sources one expects that the distributions of the normalised parallaxes and proper motions are normal distributions with (almost) zero mean and standard deviation (almost) unity. Considering the typical uncertainties of the proper motions of over 400 µas yr$^{-1}$ as given in Table 1 it is clear that the small effect of the solar system acceleration can be ignored in this discussion. The best-fit normal distributions for the normalised parallaxes and proper motions shown by red lines on Fig. 6 indeed agree remarkably well with the actual distribution of the data. The best-fit Gaussian distributions have standard deviations of 1.052, 1.055 and 1.063, respectively for the parallaxes ($\alpha$), proper motions in right ascension ($\mu_{\alpha}$), and proper motions in declination ($\mu_{\delta}$). Small deviations from normal distributions (note the logarithmic scale of the histograms) can result both from statistical fluctuations in the sample and some stellar contaminations. One can conclude that the level of contaminations is probably very low.

4.2. Stars of our Galaxy

The acceleration of the solar system affects also the observed proper motions of stars, albeit in a more complicated way than...

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3 The effective wavenumber $v_{\text{eff}}$ is the mean value of the inverse wave-length $\lambda^{-1}$, weighted by the detected photon flux in the Gaia passband $G$. This quantity is extensively used to model colour-dependent image shifts in the astrometric instrument of Gaia. An approximate relation between $v_{\text{eff}}$ and the colour index $G_{\text{BP}} - G_{\text{RP}}$ is given in Lindegren & al. (2020a). The values $v_{\text{eff}} = 1.3$, 1.6, and 1.9 roughly correspond to, respectively, $G_{\text{BP}} - G_{\text{RP}} = 2.4$, 0.6, and $-0.5$.

4 The RUWE (Lindegren & al. 2020a) is a measure of the goodness-of-fit of the five- or six-parameter model to the observations of the source.
Table 1. Characteristics of the Gaia-CRF3 sources.

<table>
<thead>
<tr>
<th>type of solution</th>
<th>number of sources</th>
<th>$G$ [mag]</th>
<th>BP–RP [mag]</th>
<th>$v_{\text{eff}}$ [$\mu$m yr$^{-1}$]</th>
<th>RUWE</th>
<th>$\sigma_{\mu_\alpha}$ [\muas yr$^{-1}$]</th>
<th>$\sigma_{\mu_\delta}$ [\muas yr$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>five-parameter</td>
<td>1 215 942</td>
<td>19.92</td>
<td>0.64</td>
<td>1.589</td>
<td>1.013</td>
<td>457</td>
<td>423</td>
</tr>
<tr>
<td>six-parameter</td>
<td>398 231</td>
<td>20.46</td>
<td>0.92</td>
<td>–</td>
<td>1.044</td>
<td>892</td>
<td>832</td>
</tr>
<tr>
<td>all</td>
<td>1 614 173</td>
<td>20.06</td>
<td>0.68</td>
<td>–</td>
<td>1.019</td>
<td>531</td>
<td>493</td>
</tr>
</tbody>
</table>

Notes. Columns 3–8 give median values of the $G$ magnitude, the BP–RP colour index, the effective wavenumber $v_{\text{eff}}$ (see footnote 3; only available for the five-parameter solutions), the astrometric quality indicator RUWE (see footnote 4), and the uncertainties of the equatorial proper motion components in $\alpha$ and $\delta$. The last line (‘all’) is for the whole set of Gaia-CRF3 sources. In this study only the sources with five-parameters solutions are used.

Fig. 3. Distribution of the Gaia-CRF3 sources with five-parameter solutions. The plot shows the density of sources per square degree computed from the source counts per pixel using HEALPix of level 6 (pixel size $\sim 0.84$ deg$^2$). This and following full-sky maps use a Hammer–Aitoff projection in galactic coordinates with $l = b = 0$ at the centre, north up, and $l$ increasing from right to left.

Fig. 4. Distribution of the statistical weights of the proper motions of the Gaia-CRF3 sources with five-parameter solutions. Statistical weight is calculated as the sum of $\sigma_{\mu_\alpha}^2 + \sigma_{\mu_\delta}^2$ in pixels at HEALPix level 6.

Fig. 5. Histograms of some important characteristics of the Gaia-CRF3 sources with five-parameter solutions. From top to bottom: $G$ magnitudes, colours represented by the effective wavenumber $v_{\text{eff}}$ (see footnote 3), and the astrometric quality indicator RUWE (see footnote 4).

for the distant extragalactic sources.\(^5\) Here it is however masked by other, much larger effects, and this section is meant to explain why it is not useful to look for the effect in the motions of galactic objects.

The expected size of the galactocentric acceleration term is of the order of $5 \mu$as yr$^{-1}$ (Sect. 3). The galactic rotation and shear effects are of the order of $5–10$ mas yr$^{-1}$, i.e. over a thousand times bigger. In the Oort approximation they do not contain a glide-like component, but any systematic difference between the solar motion and the bulk motion of some stellar popula-

\(^5\) For the proper motion of a star it is only the differential (tidal) acceleration between the solar system and the star that matters.
vicinity, the tip of the long galactic bar, the motion of the disk stars through a spiral wave crest, and so on.

For all these reasons it is quite obvious that there is no hope to discern an effect of $5 \, \text{mas yr}^{-1}$ amongst chaotic structures of the order of $10 \, \text{mas yr}^{-1}$ in stellar kinematics. In other words, we cannot use galactic objects to determine the glide due to the acceleration of the solar system.

As a side remark we mention that there is a very big ($\approx 6 \, \text{mas yr}^{-1}$) direct effect of the galactocentric acceleration in the proper-motion pattern of stars on the galactic scale: it is not a glide but the global rotation which is represented by the minima in the well-known textbook double wave of the proper motions $\mu_\alpha$, in galactic longitude $l$ as function of $l$. But this is of no relevance in connection with the present study.

5. Method

One can think of a number of ways to estimate the acceleration from a set of observed proper motions. For example, one could directly estimate the components of the acceleration vector by a least-squares fit to the proper motion components using Eq. (4). However, if there are other large-scale patterns present in the proper motions, such as from a global rotation, these other effects could bias the acceleration estimate, because they are in general not orthogonal to the acceleration effect for the actual weight distribution on the sky (Fig. 4).

We prefer to use a more general and more flexible mathematical approach with Vector Spherical Harmonics (VSH). For a given set of sources, the use of VSH allows us to mitigate the biases produced by various large-scale patterns, thus bringing a reasonable control over the systematic errors. The theory of VSH expansions of arbitrary vector fields on the sphere and its applications to the analysis of astrometric data were discussed in detail by Mignard & Klioner (2012). We use the notations and definitions given in that work. In particular, to the vector field of proper motions $\mu(\alpha, \delta) = \mu_\alpha, e_\alpha + \mu_\delta e_\delta$ (where $e_\alpha$ and $e_\delta$ are unit vectors in the local triad as in Fig. 1 of Mignard & Klioner 2012) we fit the following VSH representation:

$$\mu(\alpha, \delta) = \sum_{l,m} t_{lm} T_{lm} + s_{lm} S_{lm}$$

+ \sum_{l=1}^3 \left( t_{lm} T_{lm} - s_{lm} S_{lm} + t_{lm} S_{lm} - s_{lm} T_{lm} \right). \quad (5)

Here $T_{lm}(\alpha, \delta)$ and $S_{lm}(\alpha, \delta)$ are the toroidal and spheroidal vector spherical harmonics of degree $l$ and order $m$, $t_{lm}$ and $s_{lm}$ are the corresponding coefficients of the expansion (to be fitted to the data), and the superscripts $\mathcal{R}$ and $\mathcal{I}$ denote the real and imaginary parts of the corresponding complex quantities, respectively. In general, the VSHs are defined as complex functions and can represent complex-valued vector fields, but the field of proper motions is real-valued and the expansion in Eq. (5) readily uses the symmetry properties of the expansion, so that all quantities in Eq. (5) are real. The definitions and various properties of $T_{lm}(\alpha, \delta)$ and $S_{lm}(\alpha, \delta)$, as well as an efficient algorithm for their computation, can be found in Mignard & Klioner (2012).

The main goal of this work is to estimate the solar system acceleration described by Eq. (4). As explained in Mignard & Klioner (2012), a nice property of the VSH expansion is that the first-order harmonics with $l = 1$ represent a global rotation (the toroidal harmonics $T_{lm}$) and an effect called ‘glide’

Fig. 6. Distributions of the normalized parallaxes $\pi/\sigma_\pi$ (upper pane), proper motions in right ascension $\mu_\alpha/\sigma_{\mu_\alpha}$ (middle pane) and proper motions in declination $\mu_\delta/\sigma_{\mu_\delta}$ (lower pane) for the Gaia-CRF3 sources with five-parameter. The red lines show the corresponding best-fit Gaussian distributions.

of rotation in Cygnus, $\alpha \approx 318^\circ$, $\delta \approx 48^\circ$). Since these two directions – by pure chance – are only $\sim 40^\circ$ apart on the sky, the sum of their effects will be in the same general direction.

But both are distance dependent, i.e. the size of the glide strongly depends on the stellar sample used. The asymmetric drift is, in addition, age dependent. Both effects attain the same order of magnitude as the Oort terms at a distance of the order of 1 kpc. That is, like the Oort terms they are of the order of a thousand times bigger than the acceleration glide. Because of this huge difference in size, and the strong dependence on the stellar sample, it is in practice impossible to separate the tiny acceleration effect from the kinematic patterns.

Some post-Oort terms in the global galactic kinematics (e.g. a non-zero second derivative of the rotation curve) can produce a big glide component, too. And, more importantly, any asymmetries of the galactic kinematics at the level of 0.1% can create glides in more or less random directions and with sizes far above the acceleration term. Examples are halo streams in the solar vicinity.
In principle, therefore, one could restrict the model to a spherical form as the effect by higher-order systematics.

of the data over the sphere, which is illustrated by Fig. 4 for the other. This means that the coefficients for increasing values of \( l_{\text{max}} \) in (5) is the maximal order of the VSHs that are taken into account in the model and is an important instrument for analysing systematic signals in the data: by calculating a series of solutions for increasing values of \( l_{\text{max}} \), one probes how much the lower-order terms (and in particular the glide terms) are affected by higher-order systematics.

With the \( L^2 \) norm, the VSHs \( T_{lm}(\alpha, \delta) \) and \( S_{lm}(\alpha, \delta) \) form an orthonormal set of basis functions for a vector field on a sphere. It is also known that the infinite set of these basis functions is complete on \( S^2 \). The VSHs can therefore represent arbitrary vector fields. Just as in the case of scalar spherical harmonics, the VSHs with increasing order \( l \) represent signals of higher spatial frequency on the sphere. VSHs of different orders and degrees are orthogonal only if one has infinite number of data points homogeneously distributed over the sphere. For a finite number of points and/or an inhomogeneous distribution the VSHs are not strictly orthogonal and have a non-zero projection onto each other. This means that the coefficients \( T_{lm}^\alpha, T_{lm}^\delta, S_{lm}^\alpha \) and \( S_{lm}^\delta \) are correlated when working with observational data. The level of correlation depends on the distribution of the statistical weight of the data over the sphere, which is illustrated by Fig. 4 for the source selection used in this study. For a given weight distribution there is a upper limit on the \( l_{\text{max}} \) that can be profitably used in practical calculations. Beyond that limit the correlations between the parameters become too high and the fit gets useless. Numerical tests show that for our data selection it is reasonable to have \( l_{\text{max}} \leq 10 \), for which correlations are less than about 0.6 in absolute values.

Projecting Eq. (5) on the vectors \( e_\alpha \) and \( e_\delta \) of the local triad one gets two scalar equations for each celestial source with proper motions \( \mu_\alpha \) and \( \mu_\delta \). For \( k \) sources this gives \( 2k \) observation equations for \( 2l_{\text{max}}(l_{\text{max}} + 2) \) unknowns to be solved for using a standard least-squares estimator. The equations should be weighted using the uncertainties of the proper motions \( \sigma_\mu_\alpha \) and \( \sigma_\mu_\delta \). It is also advantageous to take into account, in the weight matrix of the least-squares estimator, the correlation \( \rho_{\mu_\alpha,\mu_\delta} \) between \( \mu_\alpha \) and \( \mu_\delta \) of a source. This correlation comes from the Gaia astrometric solution and is published in the Gaia catalogue for each source. The correlations between astrometric parameters of different sources are not exactly known and no attempt to account for these inter-source correlations was undertaken in this study.

It is important that the fit is robust against outliers, that is sources that have proper motions significantly deviating from the model in Eq. (5). Peculiar proper motions can be caused by time-dependent structure variation of certain sources (some but not all such sources have been rejected by the astrometric tests at the selection level). Outlier elimination also makes the estimates robust against potentially bad, systematically biased astrometric solutions of some sources. The outlier detection is implemented (Lindegren 2018) as an iterative elimination of all sources for which a measure of the post-fit residuals of the corresponding two equations exceed the median value of that measure computed for all sources by some chosen factor \( k \geq 1 \), called clip limit. As the measure \( X \) of the weighted residuals for a source we choose the post-fit residuals \( \Delta \mu_\alpha \) and \( \Delta \mu_\delta \) of the corresponding two equations for \( \mu_\alpha \) and \( \mu_\delta \) for the source, weighted by the full covariance matrix of the proper motion components:

\[
X^2 = \left[ \Delta \mu_\alpha, \Delta \mu_\delta \right] \left[ \begin{array}{cc} \sigma^2_{\mu_\alpha} & \rho_{\mu_\alpha,\mu_\delta} \sigma_{\mu_\alpha} \sigma_{\mu_\delta} \\ \rho_{\mu_\alpha,\mu_\delta} \sigma_{\mu_\alpha} \sigma_{\mu_\delta} & \sigma^2_{\mu_\delta} \end{array} \right]^{-1} \left[ \Delta \mu_\alpha, \Delta \mu_\delta \right] = \frac{1}{1 - \rho^2_{\mu_\alpha,\mu_\delta}} \left( \Delta \mu_\alpha \sigma_{\mu_\alpha} + \Delta \mu_\delta \sigma_{\mu_\delta} \right)^2.
\]

At each iteration the least-squares fit is computed using only the sources that were not detected as outliers in the previous iterations; the median of \( X \) is however always computed over the whole set of sources. Iteration stops when the set of sources identified as outliers is stable.\(^6\) Identification of a whole source as an outlier and not just a single component of its proper motion (for example, accepting \( \mu_\alpha \), and rejecting \( \mu_\delta \)) makes more sense from the physical point of view and also makes the procedure independent of the coordinate system.

It is worth recording here that the angular covariance function \( V_\psi(\hat{\theta}) \), defined by Eq. (17) of Lindegren et al. (2018), also contains information on the glide, albeit only on its magnitude \( |g| \), not the direction. \( V_\psi(\hat{\theta}) \) quantifies the covariance of the proper motion vectors \( \mu \) as a function of the angular separation \( \hat{\theta} \) on the sky. Figure 14 of Lindegren & al. (2020a) shows this function for Gaia EDR3, computed using the same sample of QSO-like sources with five-parameter solutions as used in the present study (but without weighting the data according to their uncertainties). Analogous to the case of scalar fields on a sphere (see Sect. 5.5 of Lindegren & al. 2020a), \( V_\psi(\hat{\theta}) \) is related to the VSH expansion of the vector field \( \mu(\alpha, \delta) \). In particular, the glide vector \( g \) gives a contribution of the form

\[
V_\psi^{\text{glide}}(\hat{\theta}) = |g|^2 \frac{1}{6} \left( \cos^2 \theta + 1 \right).
\]

Using this expression and the \( V_\psi(\hat{\theta}) \) of Gaia EDR3 we obtain an estimate of \( |g| \) in reasonable agreement with the results from the VSH fit discussed in the next section. However, it is obvious from the plot in Lindegren & al. (2020a) that the angular covariance function contains other large-scale components that could bias this estimate as they are not included in the fit. This reinforces the argument made earlier in this section, namely that the estimation of the glide components from the proper motion data should not be done in isolation, but simultaneously with the estimation of other large-scale patterns. This is exactly what is achieved by means of the VSH expansion.

6. Analysis

The results for the three components of the glide vector are shown in Fig. 7. They have been obtained by fitting the VSH expansion in Eq. (5) for different \( l_{\text{max}} \) to the proper motions of the 1 215 942 Gaia-CRF3 sources with five-parameter solutions. The corresponding spheroidal VSH parameters with \( l = 1 \)...
were transformed into the Cartesian components of the glide using Eq. (6). Figure 7 displays both the equatorial components \((g_x, g_y, g_z)\) and the galactic components \((g_x, g_y, g_z)\) of the glide vector. The equatorial components were derived directly using the equatorial proper motions published in the \textit{Gaia} Archive. The galactic components can be derived either by transforming the equatorial components of the glide and their covariance matrix to galactic coordinates, or from a direct VSH fits using the proper motions and covariances in galactic coordinates. We have verified that the two procedures give strictly identical results.

![Figure 7](image_url)

**Fig. 7.** Equatorial (upper pane) and galactic (lower pane) components of the solar system acceleration for fits with different maximal VSH order \(l_{\text{max}}\) (alone means that the three glide components were fitted with no other VSH terms). The error bars represent \(\pm 1\sigma\) uncertainties.

One can see that starting from \(l_{\text{max}} = 3\) the estimates are stable and generally deviate from each other by less than the corresponding uncertainties. The deviation of the results for \(l_{\text{max}} < 3\) from those of higher \(l_{\text{max}}\) shows that the higher-order systematics in the data need to be taken into account, although their effect on the glide is relatively mild. We conclude that it is reasonable to use the results for \(l_{\text{max}} = 3\) as the best estimates of the acceleration components.

The unit weight error (square root of the reduced chi-square) of all these fits, and of all those described below, is about 1.048. The unit weight error calculated with all VSH terms set to zero is also 1.048 (after applying the same outlier rejection procedure as for the fits), which merely reflects the fact that the fitted VSH terms are much smaller than the uncertainties of the individual proper motions. The unit weight error is routinely used to scale up the uncertainties of the fit. However, a more robust method of bootstrap resampling was used to estimate the uncertainties (see below).

To further investigate the influence of various aspects of the data and estimation procedure, the following tests were done.

- Fits including VSH components of degree up to \(l_{\text{max}} = 40\) were made. They show that the variations of the estimated acceleration components remain at the level of a fraction of the corresponding uncertainties, which agrees with random variations expected for the fits with high \(l_{\text{max}}\).
- The fits in Fig. 7 used the clip limit \(\kappa = 3\), which rejected about 3800 of the 1 215 942 sources as outliers (the exact number depends on \(l_{\text{max}}\)). Fits with different clip limits \(\kappa\) (including fits without outlier rejection, corresponding to \(\kappa = \infty\)) were tried, showing that the result for the acceleration depends on \(\kappa\) only at a level of a quarter of the uncertainties.
- The use of the correlations \(\rho_p\) between the proper motion components for each source in the weight matrix of the fit influences the acceleration estimates at a level of \(0.1\) of the uncertainties. This should be expected since the correlations \(\rho_p\) for the 1 215 942 \textit{Gaia}-CRF3 sources are relatively small (the distribution of \(\rho_p\) is reasonably close to normal with zero mean and standard deviation 0.28).

Analysis of the \textit{Gaia} DR3 astrometry has revealed systematic errors depending on the magnitude and colour of the sources (Lindegren et al. 2020a,b). To check how these factors influence the estimates, fits using \(l_{\text{max}} = 3\) were made for sources split by magnitude and colour:

- Figure 8 shows the acceleration components estimated for subsets of different mean \(G\) magnitude. The variation of the components with \(G\) is mild and the estimates are compatible with the estimates from the full data set (shown as horizontal colour bands) within their uncertainties.

![Figure 8](image_url)

**Fig. 8.** Equatorial components of the acceleration and their uncertainties for four intervals of \(G\) magnitude: \(G \leq 18\) mag (29 200 sources), \(18 < G \leq 19\) mag (146 614 sources), \(19 < G \leq 20\) mag (490 161 sources), and \(G > 20\) mag (549 967 sources). The horizontal colour bands visualize the values and uncertainties (the height corresponds to twice the uncertainty) of the corresponding components computed from the whole data set.
Figure 9 is a corresponding plot for the split by colour, as represented by the effective wavenumber $v_{\text{eff}}$. Again one can conclude that the estimates from the data selections in colour agree with those from the full data set within their corresponding uncertainties.

It should be noted that the magnitude and colour selections are not completely independent since the bluer QSO-like sources tend to be fainter than the redder ones. Moreover, the magnitude and colour selections are less homogeneous on the sky than the full set of sources (for example owing to the Galactic extinction and reddening). However, we conclude that the biases in the acceleration estimates, due to magnitude- and colour-dependent effects in the Gaia DR3 astrometry, are below the formal uncertainties for the full sample.

Another possible cause of biases in the Gaia data is charge transfer inefficiency (CTI) in the CCDs (e.g. Crowley et al. 2016). A detailed simulation of plausible CTI effects unaccounted for in the Gaia data processing for Gaia DR3 showed that the estimated glide is remarkably resilient to the CTI and may be affected only at a level below $0.1 \mu\text{as yr}^{-1}$ – at most a quarter of the quoted uncertainty.

Our selection of Gaia sources cannot be absolutely free from stellar contaminants. As discussed in Sect. 4.2, stars in our Galaxy have very large glide components in the vector field of their proper motions. This means that even a small stellar contamination could bias our estimate of the solar system acceleration. One can hope that the mechanism of outlier elimination used in the VSH fit in this work (see Sect. 5) helps to eliminate at least some of the most disturbing stellar-contamination sources. It is, however, worth to investigate the possible biases by direct simulation. By construction, the stellar contaminants in our list of QSO-like sources must have five-parameter solutions in Gaia DR3 that satisfy the selection criteria discussed in Sect. 4.1 and (Gaia Collaboration et al. 2020b). It is therefore of interest to investigate the sample of sources obtained by making exactly the same selection of Gaia DR3 sources, but without the cross-match to the external QSO/AGN catalogues. There are a total of 23.6 million such sources in Gaia DR3, including the 1.2 million (5.2%) included in Gaia-CRF3. Most of them are stars in our Galaxy, but one also sees stars in nearby dwarf galaxies, globular clusters, and bright stars in other galaxies. Applying the VSH method to this sample gives a glide of about $360 \mu\text{as yr}^{-1}$ in a direction within a few degrees of ($l, b$) = $(270^\circ, 0^\circ)$, that is roughly opposite to the direction of motion of the Sun in the Galaxy. This glide has obviously nothing to do with the acceleration of the solar system (see Sect. 4.2) and its precise value is irrelevant. However, it is very relevant that it is practically perpendicular to the glide obtained from the QSO-like sample, for it means that a (small) stellar contamination will not significantly alter the magnitude of the glide $|g|$. It could however bias the direction of the observed glide towards ($l, b$) = $(270^\circ, 0^\circ)$, that is mainly in galactic longitude. We do not see a clear sign of this in our estimates (the estimated direction is within one $\sigma$ from the Galactic centre) and we therefore conclude that the effect of a possible stellar contamination in Gaia-CRF3 is negligible for the claimed estimate of the solar system acceleration.

Finally, it should be remembered that systematic errors in the Gaia ephemeris may also bias the estimate of the solar system acceleration. The standard astrometric parameters in the Gaia astrometric solution are defined for a fictitious observer located in the ‘solar system barycentre’. The latter is effectively defined by the Gaia ephemeris in the Barycentric Celestial Reference Frame (BCRS; Soffel et al. 2003; Klioner 2003) that is used in the data processing. In particular, the Gaia’s barycentric velocity is used to transform the observations from the proper frame of Gaia to the reference frame at rest with respect to the solar system barycentre (Klioner 2004). Systematic errors in the Gaia ephemeris may result in systematic errors in the astrometric parameters. In particular, a systematic error in the Gaia velocity, corresponding to a non-zero average acceleration error over the time interval of the observations (about 33 months for Gaia EDR3), will produce the same systematic error in the measured solar system acceleration.

The barycentric ephemeris of Gaia is obtained by combining the geocentric orbit determination, made by the Mission Operations Centre at ESOC (Darmstadt, Germany) using various Doppler and ranging techniques, with a barycentric ephemeris of
the Earth. For the latter, the INPOP10e planetary ephemerides (Fienga et al. 2016) was used in Gaia EDR3. The errors in the geocentric orbit have very different characteristics from those of the planetary ephemerides, and the two contributions need to be considered separately. For the geocentric part, one can rule out an acceleration bias greater than about $2 \times 10^{-13}$ m s$^{-2}$ persisting over the 33 months, because it would produce an offset in the position of Gaia of the order of a km, well above the accuracy obtained by the ranging. For the barycentric ephemeris of the Earth, we can obtain an order-of-magnitude estimate of possible systematics by comparing the INPOP10e planetary ephemerides with the Earth, we can obtain an order-of-magnitude estimate of possible systematic errors in the Earth ephemeris come from the improvements in the Earth ephemeris due to the Gaia astrometry does not require any dedicated astrometric solution. The astrometric data used in this work to detect the 7 The ‘geocentric’ orbit of Gaia is also defined in the BCRS and represents the difference of the BCRS coordinates of Gaia and those of the geocentre.

Table 2. Principal results of this work: equatorial and galactic components of the estimated acceleration of the solar system, with uncertainties and correlations.

<table>
<thead>
<tr>
<th>quantity</th>
<th>value</th>
<th>uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>equatorial components</td>
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</tr>
<tr>
<td>$g_x$ [μas yr$^{-1}$]</td>
<td>$-0.07$</td>
<td>0.41</td>
</tr>
<tr>
<td>$g_y$ [μas yr$^{-1}$]</td>
<td>$-4.30$</td>
<td>0.35</td>
</tr>
<tr>
<td>$g_z$ [μas yr$^{-1}$]</td>
<td>$-2.64$</td>
<td>0.36</td>
</tr>
<tr>
<td>$a$</td>
<td>$269.1^\circ$</td>
<td>5.4$^\circ$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$-31.6^\circ$</td>
<td>4.1$^\circ$</td>
</tr>
<tr>
<td>correlations</td>
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</tr>
<tr>
<td>$P_{g_x,g_y}$</td>
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<td>$a_x$ [km s$^{-1}$ Myr$^{-1}$]</td>
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<tr>
<td>$a_y$ [km s$^{-1}$ Myr$^{-1}$]</td>
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<td>$b$</td>
<td>$-3.3^\circ$</td>
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<tr>
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<td>g</td>
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<tr>
<td>$</td>
<td>a</td>
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</tr>
<tr>
<td>$</td>
<td>10^{-10}$ m s$^{-2}$]</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Notes. All uncertainties are ±1σ estimates obtained using bootstrap resampling. The absolute values of the acceleration are computed as the Euclidean norm of the estimated vector, and may be biased as discussed in Appendix A.

7. Conclusions and prospects

The exquisite quality of the Gaia DR3 astrometry together with a careful selection of the Gaia-CRF3 sources (Sect. 4.1) have allowed us to detect the acceleration of the solar system with respect to the rest-frame of the remote extragalactic sources, with a relative precision better than 10%. The stability of the derived estimates was extensively checked by numerous experiments as discussed in Sect. 6. The consistency of the results support the overall claim of a significant detection. We note that our estimate of the solar system acceleration agrees with the theoretical expectations from galactic dynamics (Sect. 3) within the corresponding uncertainties.

We stress that the detection of the solar system acceleration in the Gaia astrometry does not require any dedicated astrometric solution. The astrometric data used in this work to detect the acceleration and analyze its properties are those of the astrometric solution published in Gaia EDR3.

Although the relative accuracy obtained in the estimate is very satisfactory for this data release, it is at this stage impossible to tell whether there are acceleration contributions from other components than the motion of the solar system in the Milky Way. As discussed in Sect. 3, even this contribution is complex and cannot be modelled with sufficient certainty to disentangle the different contributions.

We can ask ourselves what should be expected from Gaia in the future. The astrometric results in Gaia EDR3 are based only on 33 months of data, while the future Gaia DR4 will be based on about 66 months of data and the final Gaia DR5 may use up to 120 months of data. Since the effect of the acceleration is equivalent to proper motions, the random uncertainty of its measurement improves with observational time $T$ as $T^{-3/2}$. Therefore, we can expect that the random errors of the acceleration estimated in Gaia DR4 and Gaia DR5 could go down by factors of about 0.35 and 0.15, respectively.

But random error is just one side of the story. What has made this solution possible with Gaia EDR3, while it was not possible...
with the Gaia DR2 data, is the spectacular decrease of the systematic errors in the astrometry. To illustrate this point, the glide determined from the Gaia-CRF2 data ( Sect. 3.3 in Gaia Collaboration et al. 2018b) was at the level of 10 μas yr⁻¹ per component, much higher than a solution strictly limited by random errors. With the Gaia EDR3 we have a random error on each proper motion of about ≈ 400 μas yr⁻¹ and just over 1 million sources. So one could hope to reach 0.4 μas yr⁻¹ in the formal uncertainty of the glide components, essentially what is now achieved. In future releases, improvement for the solar system acceleration will come both from the better random errors and the reduced systematic errors, although only the random part can be quantified with some certainty. In the transition from Gaia DR2 to Gaia EDR3 a major part of the gain came from the diminishing of systematic effects.

The number of QSO-like sources that can become available in future Gaia data releases is another interesting aspect. In general, a reliable answer is not known. Two attempts (Shu et al. 2019; Bauer-Jones et al. 2019) to find QSO-like sources in Gaia DR2 data ended up with about 2.7 million sources each (and even more together). Although an important part of those catalogues did not show the level of reliability we require for Gaia-CRF3, one can hope that the number of QSO-like sources with Gaia astrometry will be doubled in the future compared to Gaia DR3. Taking all these aspects into account, it is reasonable to hope the uncertainty of the acceleration to reach the level of well below 0.1 μas yr⁻¹ in the future Gaia releases.

Considering the expected accuracy, an interesting question here is if we could think of any other effects that would give systematic patterns in the proper motions of QSO-like sources at the level of expected accuracy. Such effects are indeed known (a good overview of these effects can be found e.g. in Bachchan et al. 2016). One such effect is the ‘cosmological proper motion’ (Kardashev 1986), or ‘secular extragalactic parallax’ (Paine et al. 2020), caused by the motion of the solar system with respect to the rest frame of the CMB at a speed of 370 km s⁻¹ ≈ 78 au yr⁻¹ towards the point with galactic coordinates l = 264.02°, b = 48.25° (Planck Collaboration et al. 2020; see also Sect. 2). This gives a reflex proper motion of 78 μas yr⁻¹ × (1 Mpc / d) sin β, where d is the distance to the object and β is the angle between the object and the direction of motion (Bachchan et al. 2016). The effect is analogous to the systematic proper motions of nearby stars caused by the apex motion of the Sun (Sect. 4.2), and like it decreases with the inverse distance to the sources. At a redshift of 0.2 the systematic proper motion should be about 0.1 μas yr⁻¹ at right angle to the solar motion. However, only a few thousand QSO-like objects can be expected at such small redshifts, and, as discussed e.g. by Paine et al. (2020), the effect is muddled by the peculiar velocities of the objects and deviations of their bulk motions from the Hubble flow due to the gravitational interactions with large-scale structures. It therefore remains questionable if this systematic proper motion will become accessible to Gaia in the future.

Another secular shift of the positions of extragalactic sources comes from the light bending in the gravitational field of the Galaxy, which depends (among other things) on the angle between the object and the Galactic centre. The motion of the solar system in the Galaxy results in a slow variation of this angle, which causes a variation of the light bending. This will be seen as a proper motion of the extragalactic source. The effect is independent of the distance to the source (as long as it is far away from the Milky Way), but depends on its position on the sky according to the details of the Galactic potential. The VSH technique used in this work seems to be very well suited for disentangling this effect from that of the solar system acceleration.

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Appendix A: Spherical coordinates and transformation bias

In Sect. 6 the solar system acceleration vector was estimated in the equatorial and galactic reference systems. The main result was given in the form of the three Cartesian components of the vector and their covariance matrix. We also gave the result in the form of the modulus (length) of the acceleration vector and the spherical coordinates \((\alpha, \delta)\) or \((l, b)\) of its direction, the latter to facilitate a direct comparison with the expected pointing roughly towards the Galactic centre.

While the least-squares solution for the Cartesian components of the vector naturally yields unbiased estimates, it does not automatically imply that transformed estimates, such as the modulus and spherical coordinates, are unbiased. If the transformation is non-linear, as is clearly the case here, the transformed quantities are in general biased. Because the discussion has more general applications than the specific problem in this paper, we use generic notations in the following.

Consider the multivariate distribution of a vector \(\mathbf{x}\) in \(\mathbb{R}^n\) with modulus \(r = (x^\top x)^{1/2}\). We use \(x_0 = \mathbb{E}(\mathbf{x})\) for the true value of the vector, and \(r_0 = (x_0^\top x_0)^{1/2}\) for the true value of its modulus. The covariance matrix of \(\mathbf{x}\) is \(C = \mathbb{E}(\xi^\top \xi)\), where \(\xi = \mathbf{x} - x_0\) is the deviation from the true vector. We take \(\mathbf{x}\) to represent our (unbiased) estimate of \(x_0\) and assume that \(C\) is exactly known. Making the arbitrary transformation \(y = f(\mathbf{x})\) of the estimate, the bias in \(y\) can be understood as \(\mathbb{E}(f(\mathbf{x})) - \mathbb{E}(f(\mathbf{x}_0))\). This is zero if \(f\) is linear, but in general non-zero for non-linear \(f\). It should be noted that the bias in general depends on the true vector \(x_0\), and therefore may not be (exactly) computable in terms of the known quantities \(x\) and \(C\).

Let us first consider the square of the modulus, that is \(r^2 = x^\top x\). Putting \(x = x_0 + \xi\) we have

\[
\mathbb{E}(r^2) = \mathbb{E}[(x_0 + \xi)^\top (x_0 + \xi)] = \mathbb{E}[x_0^\top x_0 + x_0^\top \xi + \xi^\top x_0 + \xi^\top \xi] = r_0^2 + \mathbb{E}(\xi^\top \xi),
\]

since \(\mathbb{E}(\xi) = \mathbf{0}\) and \(\mathbb{E}(\xi^\top \xi) = \mathbb{E}(\xi^2) = \mathbb{E}(\xi^\top \xi) = \mathbb{E}(\xi^2)\). In this case the bias is exactly computable: an unbiased estimate of \(r^2\) is \(r^2 = \mathbb{E}(\xi^2)\). Note, however, that this estimate will sometimes be negative: not always a convenient result!

Considering now the modulus \(r = (x^\top x)^{1/2}\), we have to second order in the deviations \(\xi\),

\[
r = (x_0^\top x_0 + x_0^\top \xi + \xi^\top x_0 + \xi^\top \xi)^{1/2} \\
= r_0 + \frac{1}{2} \frac{(x_0^\top \xi + \xi^\top x_0)}{r_0} + \frac{1}{2} \frac{\xi^\top \xi}{r_0^3} - \frac{1}{8} \frac{(x_0^\top \xi + \xi^\top x_0)^2}{r_0^5} + O(\xi^3),
\]

\[
= r_0 + \frac{1}{2} \frac{x_0^\top \xi}{r_0} + \frac{1}{2} \frac{\xi^\top x_0}{r_0^3} - \frac{1}{2} \frac{x_0^\top \xi}{r_0^3} + O(\xi^3),
\]

where in the last equality we used the general properties of scalar products, \(v^\top w = w^\top v\) and \((v^\top w)^2 = v^\top (w^\top w)w = w^\top (v^\top v)w\). Taking now the expectation of Eq. (A.2) gives

\[
\mathbb{E}(r) = r_0 + \frac{1}{2} \frac{\mathbb{E}(C)}{r_0} - \frac{1}{2} \frac{x_0^\top C x_0}{r_0^3} + O(\xi^3).
\]

In contrast to Eq. (A.1), the truncated expression in Eq. (A.3) is only approximate, and moreover depends on the unknown quantities \(r_0\) and \(x_0\). A useful correction for the bias may nevertheless be computed by inserting the estimated quantities \(r\) and \(x\) for \(r_0\) and \(x_0\): thus

\[
r_0 \approx r - \frac{\mathbb{E}(C)}{2r} + \frac{x_0^\top C x_0}{2r^3}.
\]