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Improving schedule reliability based on copulas: An application to five of the most congested US airports☆

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A B S T R A C T

This study, based Newark Liberty International, New York John F. Kennedy, New York LaGuardia, Chicago O'Hare International and San Francisco International in the US, provides a perspective on the predictability of on-time performance evaluation by applying an actuarial and financial method called copula. It is common for the percentage of on-time gate arrivals to be portrayed in daily news as an indicator of schedule reliability. However, the degree of interdependency between gate arrival delays and block delays may provide some indication of schedule reliability.

1. Introduction

Although scheduled arrivals and departures at the US Operational Evolution Plan 35 airports[1] declined from 1.364 million in 2004 to 1.350 million in 2009, the percent of flights arriving on-time at the gates decreased from 78.2% in 2004 to 73.2% in 2007. In the next two years, on-time performance improved from 75.6% to 78.9%[2] at the same group of airports. Block delays followed an identical trend as they increased on average from 4.00 to 4.55 min in the 2004–2007 period before going down from 4.27 to 3.77 min in 2008–2009.[3] Similar trends in gate arrival and block delays suggest that there is some degree of concordance between the two types of delay, which may eventually be a predictor of schedule reliability and robustness.

In a context of constrained airport capacity, congestion and delays, airline practitioners have turned their attention to the issue of schedule reliability and robustness (Wu, 2010). Janic (2007) defined reliability as "the percentage of realized flights as compared with the number of planned ones over a given period of time (day, week, year)." Robustness represents a measure of how well a schedule can cope with a delay to a specific flight. Reliability and robustness are all the more significant as they represent the cornerstones of airline planning that usually involves schedule design, fleet assignment, maintenance routine and crew scheduling.

We examine a sample of five airports ranked among the most congested and delayed facilities in the US during June to August 2009 between the local hours of 07.00–21.59. The airports are Newark Liberty International (EWR), New York John F. Kennedy (JFK), New York LaGuardia (LGA), Chicago O'Hare International (ORD), and San Francisco International (SFO). We determine whether the degree of concordance between late arrivals and block delays may predict the risk of schedule unreliability. In other words, are the minutes of gate arrival delays – as a measure of airline performance – contingent upon the minutes of block delays– as a measure of national airspace system performance? Gate arrival delay represents the difference in minutes between the scheduled and actual arrival time of an aircraft at the gate. Block delay accounts for the difference in minutes between actual and scheduled 'gate-out' at the departure airport to 'gate-in' times at the arrival airport[4].

Since the distribution of block and gate arrival delays cannot be assumed to be normal, correlation statistics such as the Pearson's product-moment coefficient is not appropriate to measure the linear dependence between both gate arrival and block delays. Therefore, this paper applies a methodology used in financial risk

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[1] The Operational Evolution Plan/Partnership airports include the largest 35 U.S airports that account for about 75% of the annual US passengers.

[2] The Official Airline Guide (http://www.oag.com) and Innovata (http://www.innovata-llc.com) are the sources for the schedule and ASPM (Aviation Systems Performance Metrics) provides the airline and airport on-time statistics.

[3] The source is ASPM (http://aspm.faa.gov).

[4] The actual arrival time is recorded by ARINC's OOOI data (**O**ut of the gate-**O**ff the ground-**O**n the ground-**I**nto the gate) collected through the Aircraft Communications Addressing and Reporting System (ACARS). The homepage is http://www.ARINC.com.

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modeling (Embrechts et al., 2002), economics (Dardoni and Lambert, 2001) and econometrics (Miller and Liu, 2002).

2. Methodology

2.1. The Concepts of concordance, dependence, correlation and copula

The relationship between two or more variables can be qualified as concordant, dependent or correlated. Concordance exists when pairs of observations vary together. A pair of observations with the larger value of $\times_1$ has also the larger value for X_2. This can be expressed as

$$\Pr[\text{Concordance}] = \Pr\left[(X_1 - X_1')\,(X_2 - X_2') > 0\right] \quad (1)$$

Two random variables X and Y are independent if the knowledge of X does not imply the knowledge of Y. This can be expressed in mathematical terms as:

$$\Pr(X \le x;\ Y \le y) = \Pr(X \le x).\Pr(Y \le y) \quad (2)$$

Correlation measures the degree of association between two variables. The analysis is based on Kendall's Tau b; a non-parametric statistic based on rank correlation, that is different from Tau a in that the former makes adjustments for ties. Tau b is a non-parametric measure of association based on the number of pairs of concordances and discordances in a sample of data that accounts for ties in the ranks. The correlation coefficient provides information on the strength (from 0 to 1) and direction (positive or negative) of the association between two variables. When variables are not normally distributed and linear correlation is not a satisfactory measure of the dependence between two variables, copulas can be used to model correlation structures. While correlation is a scalar measure of dependence, copulas define the dependence relationship by joining the marginal distributions together to form a joint distribution.

For any multivariate distribution, the univariate marginal distribution and the dependence structure can be separated, that is, any joint distribution can be written in copula form based on Sklar's theorem (1959):

Let F be a joint distribution function with margins $F_1, F_2, \ldots, F_d$, then there exists a copula C such that for all $(x_1, x_2, \ldots, x_d) \in R^d$, $F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F2(x_2)\ldots Fd(x_d))$. If $F_1, F_2, \ldots, F_d$ are continuous, then the copula function is unique and it combines the marginal to form the multivariate distribution.

In the case of multivariate distributions, copulas allow the separation of the dependence structure from the behavior of the univariate margins. As Nelsen (1999) explained, "copulas are functions that join or 'couple' multivariate distribution functions to their one-dimensional distribution functions. Alternatively, copulas are multivariate distribution functions whose one-dimensional margins are uniform in the interval [0, 1]." Erdman and Sinko (2008) maintain that "copula theory is the formalization of the separation of the correlation structure of a multivariate distribution from the marginal distributions that make up the multivariate distribution."

2.2. Estimation of copulas

A non-parametric approach based on kernel density estimation was selected to derive the probability density function of gate arrival and block delays. Such a methodology does not require any a-priori model specification. According to SAS,[5] "kernel density estimation is a non-parametric technique for density estimation in which a known density function (the kernel) is averaged across the observed data points to create a smooth approximation." Cherubini et al. (2004) defined kernel as "a functional form, usually chosen for its smooth properties, that is used as the building block to get the desired estimator."

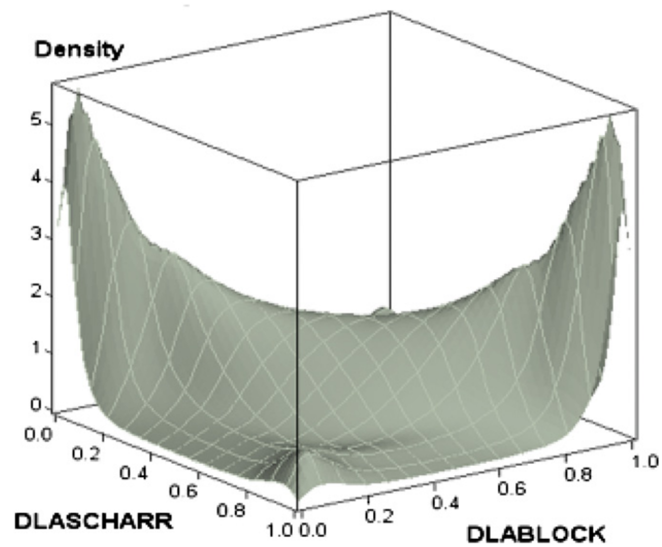

Fig. 1. Example of surface plot of the kernel density for gate arrival delays and block delays for SFO.

The first step in the estimation of the copulas consists in determining the kernel.[6] In this study, Kendall's Tau b represented the kernel. The CORR procedure in SAS provided Kendall's Tau b correlation coefficients that can be expressed as

$$Rk(X_1,\ X_2) = \Pr[\text{Concordance}] - \Pr[\text{Discordance}] \quad (3)$$

If the marginal of X_1 and X_2 are continuous, then R_k can be written as

$$Rk = 2\Pr\left[(X_1 - X_1')(X_2 - X_2') > 0\right] - 1 \quad (4)$$

In the second step, Kendall's Tau b is expressed in terms of a copula such that

$$Rk = 4\int_0^1\int_0^1 C(u_1,\ u_2)d\,C(u_1,\ u_2) - 1 \quad (5)$$

Kendall's Tau b correlation coefficients between gate arrival and block delays are 0.53, 0.58, 0.56, 0.79 and 0.46 respectively for EWR, JFK, LGA, ORD and SFO. In all cases, the probability that Tau b is zero is less than 0.0001. Remarkably, the Tau b coefficients for the three New York airports have about the same magnitude.

The random couple (X_1, X_2) represents continuous random variables with copula C. The copula captures the dependence between the selected variables and it serves to model the marginal

[5] SAS 9.22 Online Documentation: http://support.sas.com/documentation/cdl_main/.

[6] The kernel density estimator in one dimension is defined by $f(x) = 1/na\Sigma^n{}_{j=1} K(x - X_i/a_n)$ where K(.) is a known density function and a_n is a constant called the bandwidth or window width. The bandwidth depends upon n since $a_n \to 0$ as $n \to \infty$.

Table 1
Kernel density and degree of concordance for EWR, JFK, LGA, ORD and SFO.

EWR						LGA					
T Copula						T Copula					
		Gate Arrival Delays		Block Delays				Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper	Percent	Density	Lower	Upper	Lower	Upper
99	5.1204	0.017	0.98	0.017	0.98	99	5.4541	0.017	0.98	0.017	0.98
100	5.5442	0.034	0.034	0.034	0.034	100	5.9066	0.034	0.034	0.034	0.034
Gaussian						Gaussian					
		Gate Arrival Delays		Block Delays				Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper	Percent	Density	Lower	Upper	Lower	Upper
99	4.2954	0.017	0.98	0.017	0.98	99	4.705	0.017	0.98	0.017	0.98
100	4.6259	0.97	0.97	0.97	0.97	100	5.0148	0.034	0.034	0.034	0.034
Normal Mixture						Normal Mixture					
		Gate Arrival Delays		Block Delays				Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper	Percent	Density	Lower	Upper	Lower	Upper
99	5.7871	0.017	0.98	0.017	0.98	99	6.1132	0.017	0.98	0.017	0.98
100	6.1967	0.034	0.034	0.034	0.034	100	6.5084	0.034	0.034	0.034	0.034
0.5											
JFK						ORD					
T Copula						T Copula					
		Gate Arrival Delays		Block Delays				Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper	Percent	Density	Lower	Upper	Lower	Upper
99	5.6543	0.017	0.98	0.017	0.98	99	4.6085	0.017	0.98	0.017	0.98
100	6.0688	0.034	0.034	0.034	0.034	100	4.9921	0.034	0.034	0.034	0.034
Gaussian						Gaussian					
		Gate Arrival Delays		Block Delays				Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper	Percent	Density	Lower	Upper	Lower	Upper
99	4.8636	0.017	0.98	0.017	0.98	99	3.7797	0.017	0.98	0.017	0.98
100	5.1908	0.97	0.97	0.97	0.97	100	4.0454	0.97	0.97	0.97	0.97
Normal Mixture						Normal Mixture					
		Gate Arrival Delays		Block Delays				Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper	Percent	Density	Lower	Upper	Lower	Upper
99	6.2606	0.017	0.98	0.017	0.98	99	5.2691	0.017	0.98	0.017	0.98
100	6.6476	0.034	0.034	0.034	0.034	100	5.7187	0.034	0.034	0.034	0.034

SFO					
T Copula					
		Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper
99	4.54	0.017	0.98	0.017	0.98
100	4.91	0.034	0.034	0.034	0.034
Gaussian					
		Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper
99	3.71	0.017	0.98	0.017	0.98
100	3.96	0.97	0.97	0.97	0.97
Normal Mixture					
		Gate Arrival Delays		Block Delays	
Percent	Density	Lower	Upper	Lower	Upper
99	5.03	0.017	0.98	0.017	0.98
100	5.65	0.034	0.034	0.034	0.034

distribution.[7] The KDE procedure in SAS enabled to compute the bivariate kernel densities of the minutes of gate arrival delays and the minutes of block delays. It also generated the estimates of the percentiles of the probability density function under consideration, based on 10,000 random observations. The data set for the simulation included 18,301,830 observations for each of the five airport samples.[8] In this study, three types of copulas have been considered: the Gaussian,[9] the t or Student, and the normal mixture copula.

Finally, the MODEL procedure allowed the combination and simulation of multivariate distributions with different marginal using Monte Carlo simulation. It also made it possible to combine and simulate multivariate distributions with different marginal and to compute the Gaussian, t (Student) and normal mixture copulas.

[7] The marginal probability mass function can be expressed as $\Pr(X = x)$: $\Pr(X = x) = \Sigma_y \Pr(X = x, Y = y) = \Sigma_y \Pr(X = x|Y = y)\Pr(Y = y)$ where $\Pr(X = x, Y = y)$ represents the joint distribution of X and Y and $\Pr(X = x|Y = y)\Pr(Y = y)$ is the conditional distribution of X given Y. The marginal probability density function can be written as $p_x(x)$: $p_x(x) = \int_y px,y(x,y)dy = \int_y px|y(x|y)p_y(y)dy$ where $px,y(x,y)$ is the joint distribution of X and Y and $px|y(x|y)$ is the conditional distribution of X given Y.

[8] A grid of 60 X 60 observations was used to fit the entire data range and a simple normal reference rule was used to bivariate smoothing.

[9] Li (2000) made the Gaussian copula popular in the financial world "to map the approximate correlation between two variables. In the financial world it was used to express the relationship between two assets in a simple form" ("In Defense of the Gaussian Copula", *The Economist*, April 29, 2009). The Gaussian copula has been used to price collateralized debt obligations.

A Gaussian copula[10] is derived by creating a correlation matrix (i.e., Pearson in the present case) and then deriving the marginal of the distribution. While the normal copula is derived from the bivariate normal distribution, the Student or t copula is obtained from the bivariate Student's distribution. The parameters make it easier to determine the tail dependence and the overall correlation coefficient simultaneously. Furthermore, even for low or zero overall correlation, the t copula provides positive dependence at the tail. Finally, the t copula tends to the Gaussian copula as the degrees of freedom increase.[11] The t copula "presents more observations in the tails than the Gaussian one" (Cherubini et al., 2004). The normal mixture copula relaxes the assumption of multivariate normality for gate arrival and block delays.

3. Findings

Fig. 1 shows the surface plot of gate arrival delays by block delays. It is a tridimensional representation of the concordance between the two variables. The kernel density is more significant at extreme locations compared with the other types of copulas. This implies that the risk that scheduled arrivals are unreliable may depend on the degree of concordance between gate arrival delays and block delays at the highest percentiles of the joint density function. In other words, the risk of schedule unreliability increases with the probability that the density of gate arrival delays is concordant with block delays. This may provide some evidence that the 'padding' of schedules minimizes the risks of unreliability at lower percentiles of the kernel density during the period under investigation.

The SAS output in Table 1 shows that arrival schedule reliability is more likely to depend on the degree of concordance between gate arrival delays and block delays at the higher rather than the lower bound. Despite a lower density at the 99th and 100th percentiles, the Gaussian copula features higher coefficients at the 100th percentile, which provides some explanation as to why it is selected in the present case.

In the case of EWR, JFK, ORD and SFO, the Gaussian copula is preferable to the other types because of the higher kernel density at the lower and upper bounds of the 90th and 100th percentile. There is no difference among the copulas in the case of LGA.

4. Conclusions

Schedule reliability is often thought in terms of how airlines' on-time gate arrivals match published schedules. This study argues that reliability should instead be measured as the degree of concordance between gate arrival delays and block delays. Block delays measure the capacity of airlines to anticipate any en route and/or on-the-ground impediments to the on-time completion of a flight. As a result, the joint density function of gate arrival delays and block delays provides some indications of how unreliable schedules are likely to be at a given kernel density percentile. This study has some important ramifications for airline practitioners who evaluate schedule reliability and airline schedulers who need to assess an acceptable level of 'padding' to minimize the possibility that block delays will be related to gate arrival delays. The concordance of gate arrival and block delays is an indicator of schedule reliability and, as a result, copulas can be used as an instrument to measure the predictability of on-time gate arrivals.

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[10] If the latent variables X have a normal distribution with correlation matrix R, then the copula of X can be expressed as: $C_R = (u_1, \ldots u_m) = \Phi_R(\Phi^{-1}(u_1), \ldots \Phi^{-1}(u_m)$ where Φ_R is the joint distribution function of a normal random vector with correlation vector with correlation matrix R and Φ as the distribution function of univariate standard normal.

[11] As Malvergne and Sornette (2006) explained, "by construction, the [Gaussian and the Student's] copulas are close to each other. In their central part [the shape of the matrix ρ], and become closer and closer in their tail only when the number of degrees of freedom of the Student's copula increases…These two copulas may have drastically different behaviors with respect to the dependence between extremes." In other words, "when the number m of degrees of freedom tends to $+\infty$ then the t copula converges to the normal copula. For a limited number of degrees of freedom, however, the two copulas have quite different shapes" (Denuit et al., 2005).