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Index Futures and the Implied Volatility of Options

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Abstract

This paper examines the relation between the pricing of stock index futures contracts and the implied volatilities of index and individual equity options. For data from the 1986-1988 period, the difference between the implied volatility of calls and puts on the Standard and Poor's 100 index appears to be closely related to variation in the premium or discount of the major U.S.-traded futures contracts. We construct an arbitrage argument to support this observation. Reasons are also offered for the finding that index put volatilities exceed those for calls during the entire sample period.

I. Introduction

This paper examines futures pricing and its interaction with options pricing. Because forward and reverse conversions involve the difficult endeavor of buying and selling an index portfolio, there may be violations of put-call parity as well as different prices for traded futures contracts and synthetic futures formed from options. Rubinstein [1987] reviews this issue, while noting that academic studies of index options are sparse. Brenner [1987] addresses the effects of interest rate risk, cash settlement, and discrete dividend flows on index option prices. Whaley [1986] finds that the implied volatility of puts was higher than that of calls in the market for options on index futures, as did Evnine and Rudd [1985] for S&P 100 index options. Rubinstein [1985] discovers maturity and moneyness biases using transactions data on equity options from the Chicago Board Options Exchange. Figlewski [1985] shows that the pricing of New York Composite index options is closer to the futures contract than to the cash index.

In theory, the equilibrium return from selling index futures short while simultaneously buying an index-mimicking portfolio of stocks should be equal to the riskless rate of interest. Various features of execution, such as bid-ask spreads and the market impact of actual orders, create an indifference band around the theoretical cost-of-carry value of the futures contract. Violations of this pricing relation can be analyzed and related to options markets that function as parallel or synthetic futures markets.

Investors may find that synthetic futures contracts constructed of index options occasionally offer superior pricing and executions. A long futures position is emulated by buying an at-the-money call and shorting an at-the-money put. The equivalent of a short futures position is attained by buying a put and selling an uncovered call option. Some traders favor placing orders on the Chicago Board Options Exchange rather than on the Chicago Mercantile Exchange because the former market promulgates bid-ask spreads, keeps more complete time and sales records, and, according to some, has fewer trading abuses. Also, the options vehicle allows "legging"—working the put or the call side separately for possible execution gains.

By a straightforward arbitrage argument, if a real futures contract is mispriced, then it is reasonable to ask if the synthetic is also mispriced. For example, when the real futures contract is at a premium, the synthetic futures must be composed of an overpriced call and an underpriced put (i.e., traders are buying calls and selling puts). If the futures are selling at a discount, the synthetic consists of an underpriced call and an overpriced put.

This paper attempts to document such a relation. That is, changes in the futures premium or discount are correlated with changes in the magnitude of the violation of index put-call parity. Another way of saying this is that as the futures premium goes up (down), call implied volatility (CIV) minus put implied volatility (PIV) goes up (down), since by arbitrage CIV goes up (down) and PIV goes down (up). Also, just as arbitrageurs exploit price differences between real futures and synthetic futures, it is reasonable to assume that there should be some correlation between the implied volatilities of calls and puts on indices and those of the securities that make up the index. This potential relation does not appear with the present data set.

Section II below describes the data. Section III reviews the institutional environment. Section IV analyzes the empirical findings on the hypothesized relation between index futures and options.

II. Data

The study period is from May 17, 1986, through August 1, 1986, and from May 1, 1987, through June 3, 1988. A two-week period from October 19, 1987, through November 2, 1987, is excluded because bad quotations, uncoordinated market closings, uneven opening times for individual stocks, and extremely high bid-ask spreads make the data for this period surrounding Black Monday untrustworthy. The 314 days of data were checked for obvious errors and inconsistencies. Implied volatilities (IV) were computed from closing prices of the Standard and Poor's 100 index (OEX) options as well as those of the equities that comprise that index. CIV and PIV are those volatilities that force the Cox-Ross-Rubinstein binomial prices to the observed market value for each at-the-money (or closest-to-the-money) OEX option. Futures premia were calculated from closing prices on the three major traded futures contracts—the Standard and Poor's 500 index (SPX), the New York Stock Exchange Composite index (NYA), and the American Stock Exchange Major Market Index (XMI).

In computing IVs, the yield used was that of the U.S. Treasury bill (converted to a bond-equivalent basis) that matured closest to the expiration of each option. The dividends used were the estimates that were circulated, as of each quote date, by a major institutional brokerage firm. The expiration month of each OEX option is matched to that of the nearest expiring futures contract. Estimation of IVs may be distorted by the potential for nonsynchronous pricing, model misspecification, and the bid-ask spread in both futures and options markets (see Whaley [1986]). With the present sample of 314 days, it is reasonable to assume that any errors introduced by these features are unbiased.

For SPX and NYA, the near contract (i.e., March, June, September, or December) is used. For XMI, which trades the interim months, the second-nearest contract was used when necessary for consistency. Hedged returns, defined as the return from selling the futures contract while simultaneously buying the underlying index stocks, are used to quantify discounts and premia. When the futures are at a discount to theoretical value, the hedged return as used in this paper is negative. This figure is then divided by the time-matched T-bill rate to create a standardized measure of the premium (SPXHEDG/BILL, NYAHEDG/BILL, and XMIHEDG/BILL). Thus, it is possible to perform analyses of possible links between option volatilities and futures premia.

III. Institutional Features of U.S. Index Trading

SPX futures and options on futures trade on the Chicago Mercantile Exchange with cash-settled options on the Chicago Board Options Exchange. NYA futures trade on the New York Futures Exchange, as do the options on futures; the cash-settled options are handled by a special area of the New York Stock Exchange trading floor.

This paper examines OEX options because their volume dwarfs that of all the other available index options. OEX options are traded on the floor of the Chicago Board Options Exchange in a pit of several hundred people. Unlike in the SPX pit at the Chicago Mercantile Exchange, signaling and shouting from the edge of the OEX pit are discouraged, so it takes somewhat longer to change OEX limit orders. There is no specialist in OEX trading, but, in the crowd of competitive market-makers, groups form to specialize in certain strikes and expiration months. The limit order book plays an active role. The huge number of public limit orders holds the bid-ask spread to one sixteenth for most actively traded contracts. At the time of writing, OEX options are the most actively traded index options in the world. XMI is the second most active; SPX and NYA are considerably smaller markets.

The XMI futures contract trades in an environment where related products facilitate cash-futures-options arbitrage. In New York and London, firms routinely buy, sell, and quote the basket of 20 stocks that comprise XMI. This eliminates the execution uncertainty of trading a basket stock by stock.

At the American Stock Exchange, a specialist system operates in the presence of competing market-makers, which allows XMI option trading to begin almost as quickly as the stock market opens. At the Chicago Board Options Exchange, the pure competitive market-maker system, combined with the huge volume of OEX options, causes opening rotations to last as long as 20 minutes.

In Amsterdam, further depth is provided by XMI trading facilities provided by the European Options Exchange. XMI options are traded under Chicago Board Options Exchange rules; they can be exercised or offset on the American Stock Exchange options floor or in Amsterdam. Tight XMI arbitrage bounds are also facilitated by the Chicago Board of Trade futures contract. OEX lacks an active futures contract, although one did briefly exist in the past, and an OEX look-alike index futures contract is in the planning stages as of 1988.

Although OEX options, unlike individual equity options, settle in cash (see Rubinstein [1987] and Evnine and Rudd [1985]), index funds are natural covered sellers of OEX calls. These institutions reap income from their outstanding equity long positions by selling calls or by engaging in index arbitrage. If they attempt to take advantage of mispricing of both options and futures, then there exists a market mechanism that prices real futures and synthetic futures simultaneously.

The put seller may hedge using any one of several index futures contracts or may even choose to remain naked, or uncovered. From the observation of persistently higher IVs to be demonstrated below for index puts, it appears that there is a finite supply of these sellers, as suggested by Rubinstein [1987].

During the market crash of October 19, 1987, there was a marketwide liquidity crisis. Spreads widened and order-execution channels narrowed. These conditions probably would not be as prevalent in a violent upward movement of the market. The cost of selling puts should include the expected value of these distress costs, which implies that higher put IV is not an anomaly.

IV. Empirical Analysis of Option Volatilities and Futures Discounts

A. Behavior of Indices, Futures, and Hedged Returns

The analysis proceeds in the domain of four indices whose characteristics are described in Table 1. Predictably, the least volatility is displayed by NYA (over 1,500 stocks) and SPX (500 stocks). OEX (100 stocks) and XMI (20 stocks) are more volatile. The difference may be due to infrequent trading and lagged price adjustment of small-capitalization stocks.

Table 1. Characteristics of four stock market indices, related futures, and hedged returns

	May 17-August 1, 1986 (n = 50) Mean (standard deviation)	May 1, 1987-June 3, 1988* (n = 264) Mean (standard deviation)
OEX	230.56 (5.66)	270.88 (32.46)
SPX	242.52 (5.32)	279.17 (30.35)
NYA	139.44 (2.85)	156.91 (16.82)
XMI	349.88 (8.26)	435.51 (50.83)
SPX futures	243.10 (5.63)	280.22 (30.92)
NYA futures	139.81 (3.14)	157.54 (17.18)
XMI futures	350.67 (8.41)	436.34 (51.47)
T-bill rate	6.16 (0.53)	5.77 (.573)
SPX hedged return	5.17 (2.65)	5.72 (3.23)
NYA hedged return	5.60 (3.67)	5.94 (3.83)
XMI hedged return	4.94 (2.46)	5.83 (2.89)

*Excluding October 19, 1987, through November 2, 1988

For the futures contracts, the variability is larger, but mirrors that of the spot indices. This may reflect the lack of nontrading effects; sentiment or market expectations may also be evident. Although XMI is more volatile than the other two futures contracts, its hedged return is less volatile. Perhaps this is because index arbitrage can be performed with tighter bounds when the basket of index securities is small. Thus, mispricing of XMI futures is less frequent than those of SPX and NYA. Note that the hedged return data behave like a traditional capital market line. The mean hedged return ascends from XMI to SPX to NYA, as does the corresponding risk. The high variability of 1987-88 futures prices reflects the turmoil in the markets following the crash of October 19, 1987. Hedged returns and implied volatility of options were also higher than in 1986.

Table 2. Average futures discount (theoretical value − futures price)

	1986	1987-88
SPX	0.46*	−.065
NYA	2.26*	.04
XMI	0.39*	.155*

*Significant at the .05 level

Discounts and premia are defined with reference to the futures contract's fair value. Fair value is equal to the (cash) index price, plus the T-bill yield, less the estimated dividend flow on the index stocks. Table 2 shows that for all three indices there is a significant discount during 1986 and mixed results for 1987-88.[1] In the presence of arbitrageurs, the synthetic futures contract should also trade at a discount. To mimic the selling that has driven the real futures contract down, the call will be sold as the put is bought, thus driving the implied volatility of puts up and that of calls down. As in Table 3, this is exactly what prevailed. Confirming results suggested by Whaley [1986] for options on Standard and Poor's 500 futures contracts, PIV is greater than CIV in the cash-settled index options under study here. Index calls had lower implied volatilities and slightly less variability of that measure. In both sample periods, CIV − PIV was approximately the same percentage of the index, despite the fact that there was no demonstrable futures discount in the latter period.

Table 3. Implied volatilities of OEX index calls and puts

	1986	1987-88
CIV	16.50 (1.51)	25.27 (7.69)
PIV	17.21 (1.26)	26.45 (9.14)
CIV − PIV	−.709 (2.18)*	−1.15 (3.35)*

*CIV − PIV (unpooled data) is significant at the .05 level

B. Correlations of Hedged Futures Returns and OEX Implied Volatilities

In synthetic futures arbitrage as described earlier, the variable of interest is the difference between CIV and PIV, and that quantity's relation to the standardized hedged returns on the three futures contracts. An alternative formulation treated here is to use the first differences of CIV − PIV, as well as those of the standardized hedged futures returns.[2]

In Table 4, CIV − PIV correlates most strongly with the standardized hedged return on XMI during both sample periods, rather than with either of the broad-based futures contracts. Note that SPXHEDG/BILL and NYAHEDG/BILL have an extremely high correlation coefficient, which indicates their shared broad-based composition. In 1987-88, the correlations were uniformly higher than in 1986, which may be an artifact of the huge prevailing equity volatility. This is not true, however, for the second panel of Table 4, which portrays the correlations of first differences.

[1] A tendency to trade at a discount may be a function of the uptick rule, which makes it difficult for an arbitrageur to sell a basket of stocks short while buying futures in a falling market (see Goldman Sachs and Co. [1988]). While bullish investors can buy stocks or futures, bearish investors obtain immediacy by using futures to circumvent the uptick rule. Anticipatory buying of stocks (and selling futures) is encouraged when a small premium occurs, since this position facilitates unwinding if a discount emerges subsequently. When a small discount is present, selling stocks and buying futures is sluggish because arbitrageurs who can sell long expect those retarded by the uptick rule to drive the futures contract down more.

[2] I am grateful to an anonymous referee who suggested that this formulation would be more straightforward than the original.

Table 4. Correlations of standardized hedged futures returns and implied volatilities

	SPXHEDG/ BILL	NYAHEDG/ BILL	XMIHEDG/ BILL	CIV − PIV
SPXHEDG/BILL	1			
NYAHEDG/BILL	.95/.92	1		
XMIHEDG/BILL	.33/.62	.15/.54	1	
CIV − PIV	.40/.53	.27/.44	.52/.57	1
Correlations—first-difference format				
SPXHEDG/BILL	1			
NYAHEDG/BILL	.95/.96	1		
XMIHEDG/BILL	.60/.71	.59/.66	1	
CIV − PIV	.46/.49	.38/.47	.34/.39	1

*The format of each entry is "1986/1987-88."

C. Regression Analysis of Hedged Returns and OEX Implied Volatilities

Table 5 strengthens the above results, which are central to the paper. Regressing CIV − PIV on the futures' standardized hedged return as in (1) confirms the relation with all three futures contacts. The regressions are performed seriatim, to avoid a multicollinearity problem. Consistent with the higher 1987-88 correlations observed above, the R^2 is higher for the latter period. R^2 is also higher, in general, for the first-difference format. This is precisely the point that stimulated the paper, namely that arbitrage against the synthetic futures caused the change in the real futures premium to vary contemporaneously with the change in the put-call IV relation.

$$CIV - PIV = a + b1\,(x_i),\ \text{for } i=1,2,3 \qquad (1)$$

where

1= SPXHEDG/BILL
2= NYAHEDG/BILL
3= XMIHEDG/BILL

Table 5. Regression analysis of CIV − PIV on standardized hedged futures returns

	1986		1987-88	
	Slope	R^2	Slope	R^2
SPXHEDG/BILL	1.98*	.16	2.65*	.29
NYAHEDG/BILL	.97	.07	1.85*	.19
XMIHEDG/BILL	2.90*	.28	3.60*	.32
Regressions—first-difference format				
SPXHEDG/BILL	2.30**	.21	2.70**	.52
NYAHEDG/BILL	1.50**	.14	2.40**	.47
XMIHEDG/BILL	1.89**	.12	3.50**	.46

* p=.05
** p=.01

D. The Relation Between Index Options and Options on Index Components

Arbitrage should enforce a strong relation between the implied volatility of index options and that of options on the market capitalization-weighted components of the index. As shown in Cox and Rubinstein [1985], the sum of the premia on the securities in a portfolio will always be greater than the premium on an option on the whole portfolio. There will be a spread between the two that will be cheap or rich relative to its own history. When the implied volatility of OEX calls goes up, potential index call buyers should consider whether an index emulating a portfolio of calls on the individual equities in the index would be relatively cheaper. Although at least one major U.S.-based brokerage firm actually provides such information to institutional traders, it is possible that trading delays, thinness, and large (in percent) bid-ask spreads prevent traders from choosing between index options and mimicking portfolios of equity options (denoted here as CSTIV for calls and PSTIV for puts). In Table 6, the difference between CIV and PIV is regressed on the difference between CSTIV and PSTIV. The insignificant coefficient and the near-zero R^2 reveal the lack of an obvious linkage between put-call parity for index options and the pricing of index-mimicking portfolios of options. This is confirmed when the regression is repeated using the first differences of both time series.

Table 6. Regression analysis of link between OEX and individual equity options

$$CIV - PIV = a + b_1 (CSTIV - PSTIV)$$

Slope	R^2
.06	.001
Regressions—first-difference format	
.40	.04

The lack of an easily traceable link between OEX options and component equity options may be due to the nonsynchronous nature of closing quotations. For at-the-money OEX options, trading occurs every few seconds right up to the close of the Chicago options markets. During the study period, that close was 15 minutes after stock trading closed. During this quarter hour, the holder of an OEX index option may elect to exercise, although the writer of that contract will not be informed until the following morning. This "wild card" option distinguishes index options from options on the component stocks in the index.

The last trade in options of the component equities may occur hours before the market close, due to thinness in some issues. In addition, when an order for over 10 contracts arrives on the floor, bid-ask spreads change unfavorably for many issues. Thus the mechanics of arbitrage between OEX and those of component equities is extremely difficult.

The "market" component of individual equity premia may dominate only after a large market move, when these options are catching up to changes in the index IVs. In more stable periods, the stock-specific component may overcome any tendency of equity options premia as a group to move with index premia. A subject of future research is to test for lagging or nonlinear relation between index IVs and component IVs. It may turn out that settlement in kind (rather than in cash) for equity options dampens correlation with the futures market. After all, the existence of futures premia and the related index options phenomena may occur precisely because arbitrage against physicals is so difficult.

There are also differences in the tax treatment of option writers. The seller of index options is governed by Section 1256 of the Internal Revenue Code, which allows a premium earned to be treated as 60 percent long-term gain and 40 percent short-term gain. In addition, unlike losses on stocks, bonds, or equity options, those on index option positions may be carried back three years against like gains.

Within the 1986 data for individual equity options, there is a high correlation (.81) between CSTIV and PSTIV, which is confirmed as significant through regression. Unlike the present results for index options, where the CIV/PIV correlation is −.23 and insignificant under regression, there does appear to be a strong linkage between call and put IVs. This is reminiscent of Klemkosky and Resnick [1980], who found no economically significant violations of put-call parity for Chicago Board Options Exchange equity options transactions data. Numerous option pricing and analytical services readily portray the implied volatilities of calls and puts. For equity options, settlement is in stock, and there is no wild-card option. The mechanics of conversions and reverse conversions are easier; put-call parity is more likely than in index options.

V. Summary and Conclusions

This paper set out to study the implied volatilities of Standard and Poor's 100 index (OEX) options as well as their relations with related futures contracts and individual equity options. Three findings emerged:

1. The prices of OEX options (viewed as implied volatilities) vary systematically with those of the three major traded index futures contracts. Specifically, the difference between call implied volatility (CIV) and put implied volatility (PIV) varies with the premium on the futures contract. As the futures premium goes up, CIV − PIV goes up; that is, the call-implied volatility increases relative to that of the put.

2. As found in two previous studies of index options, CIV is lower than PIV. Distress costs and other institutional features of index markets have been suggested to explain this.

3. Implied volatilities of OEX options do not relate systematically to those of market capitalization-weighted baskets of options on individual equities that comprise the index. This may be due to the easier mechanics of conversions and reverse conversions in the equity options. Additional considerations of thinness, nonsynchronous trading, and taxation may be relevant. Unlike OEX index options, implied volatilities of equity calls and puts do rise and fall together, which reinforces this paper's view that index option markets are characterized by unique features worthy of continued study.

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