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Julia Rogers Prize: Research Strategy
Re: The Fundamental Shared Essence of Dance and Mathematics

As a senior, I have a single room for the first time. Resultantly, I have comfortably done the majority of my homework in my room, as opposed to studying primarily in the library as I did for the first three years of college. However, perhaps in an attempt by the universe to maintain balance, last semester I ended up taking out far more books from the library than ever before. Although I was spending less hours in the library, the stacks of library books in my dorm would have hinted otherwise.

My dance capstone course required me to conduct a semester-long research project, and I chose to explore what I have always believed on an intuitive level – that there are great similarities between mathematics and dance. This belief is what led me to double major in Mathematics and Dance, a combination which always seems to take people by surprise. The capstone project was a chance for me to research in depth the fundamental similarities between the fields and then present my findings.

Goucher's library offered me the chance to consult with the masters of each field and bring them into my internal discussion. I scoured the aisles shelving books on dance pedagogy, dance anthropology, Marius Petipa, group theory, and abstract algebra. Through the online database, I also found relevant and interesting sources, including a 1920s magazine article that called the mathematical area of group theory a manifestation of a "deep human yearning." It was quotes like these that inspired me to dig deeper, to learn more, and to make more connections. The resources I found were written by people who shared my interests and passions, and thus it was these people who made the pursuit of my capstone paper feel not like required schoolwork, but exciting and important research.

The Fundamental Shared Essence of Dance and Mathematics

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Abstract.

This paper seeks to investigate the intricate, inherent relationship between dance and mathematics. Although perhaps not initially obvious to all, mathematics is apparent in all forms of dance. It manifests in a plethora of ways, such as musicality, geometry of the individual body and of bodies in space, and the repetition and patterning that allow differentiation between styles and movements. This paper will utilize the mathematical area known as group theory to analyze the innate dependence of dance on mathematics, looking at classical ballet as evidence and analyzing the work of Marius Petipa and specifically the Lilac Fairy Variation from the Prologue of *The Sleeping Beauty*. The fact that dance has a mathematical character to it is what has allowed it to build upon itself, to expand within and beyond structure, and to become a successful, ever-interesting field and art form. Mathematics and dance inherently share parallels, and these parallels are telling of an innate human need for both. Upon examination, it becomes evident that dance and mathematics, at their truest cores, have a shared fundamental essence of human nature.

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I. INTRODUCTION

Have you ever watched *Barbie in the Nutcracker*? If so, did you notice the ease and grace of Barbie's dancing? Perhaps if you have a trained dance eye you even nodded approvingly, noting the surprising accuracy of Barbie's execution and her technical prowess. What sets Barbie apart from other, less trained animated movers is that she is actually danced by Maria Kowroski, a principal dancer with New York City Ballet. Kowroski's movements were recorded through reflectors and motion capture sensors and then translated into the animation of Barbie. This translation is applied mathematics at work: movement in the three-dimensional space was recorded in two dimensions and then treated with a process which essentially relies on vector normalization, matrix transformations, and linear algebra. Thus, *Barbie in the Nutcracker* is brought to audiences through the cooperation of dance and mathematics.

Barbie in the Nutcracker represents one example of how dance and applied mathematics can be brought together to create a desired product, for it is applied mathematics that allow for motion capture imaging animation. Furthermore, it is the mathematical thinking inside the codified dance form of ballet that allows Maria Kowroski to learn and then execute her movements so precisely that the animated Barbie's dancing is understood universally by those with knowledge of ballet technique. Thus, this example shows the partnership of dance and mathematics in several ways, and while in this example dance and mathematics were deliberately brought together, the two fields can be considered comparable even in settings where they seem most discrete.

Mathematics manifests itself in dance in many ways, and likewise, dance manifests in mathematics. The intersection between the two is not limited to simplistic mathematical concepts like counting, but can also be explored using the lens of abstract mathematics. Specifically in researching the intersection of the mathematical area known as group theory and the form of dance classified as ballet, the assumed distinction between mathematics and dance dissipates and the parallels become evident. Furthermore, these parallels are not arbitrary. In fact, they are telling of a deep human yearning to find order, beauty, and expression of the world. Both mathematics and dance help fulfill this need, and thus at their truest cores they have a shared essence.

II. THOUGHTS ON DANCE AND MATHEMATICS

The pairing of dance and mathematics may seem rather unprecedented to some. Indeed, the two subjects appear quite different; one is studied with the whole body and the other with a narrow focus of the mind, one is performed on stage and the other on a desk. However, a perceived disparity between math and dance could be because most people have never been explicitly asked to think about the relationship between math and dance. To get a basis for what people could see as commonality between mathematics and dance, I conducted a brief survey.

The survey was conducted online, and participation was both anonymous and optional. All participants were selected because they had focused their liberal arts studies on mathematics and/or dance, thus assuring they had interest and some expertise in at least one subject relating to the survey. Potential participants were contacted on social media either directly or through a group identifying them as qualified (i.e. as a student of dance). 24 people chose to participate, all of whom were between the ages of 18 and 24 and were either current students or recent graduates (within the last one or two years) of Goucher College, a liberal arts school in

Baltimore, MD. The complete survey results, as well as the numerical analysis and conclusion can be found in Appendices I and II.

Each of the 24 respondents identified either as a math major or a dancer (of the dancers, 82% were dance majors), with none identifying as both a dancer and a math major. 17 of the respondents were dancers, representing 71% of the population surveyed. Within these respondents, there was a range in wariness to math. Asked to qualitatively describe their relationship to math, 6 respondents used the words “love” or “like,” 5 used the words “hate, “dislike,” or “dread,” and the other 6 fell somewhere around neutral. Despite this variation, when asked to answer in their own words how and if they had thought about mathematics applying to their practice of dance, only two dancers said they had not thought about it (both sophomores). Every other dancer could easily think of ways in which math applied to dance, signifying that the idea of the two subjects’ intersection was not new to them.

All respondents were asked to list any commonalities they saw between mathematics and dance, and there were many common themes through the responses. The most common point brought up was geometry, with 50% of the respondents using the words “geometry,” “angles,” “shape,” or “line.” Dance was thought of by respondents as mathematical in its reliance on shape and form, as one person said, “creating geometrical shapes through movements and formations” (Respondent 7).

The second common point seen as intersection between dance and mathematics was musicality, with 42% of all respondents using the word “music,” “rhythm,” or “counting” (as of music). Interestingly, this was brought up by more than half of the dancers (53%), but only one of the math majors (14%). That music is interpreted as a mathematical element of dance, especially by dancers, is not surprising, since music is referenced using mathematical language

("3/4," "6/8"), and it is counted ("5, 6, 7, 8"). One respondent wrote, "I think about math in relation to dance when it comes to music. I usually don't count while dancing but as a performer it's necessary so I've had to train myself to be more attuned to counting pieces, and staying within the choreographed rhythms of the dance" (Respondent 3).

The third most frequent commonality was patterns; 42% of all respondents explicitly used the word "pattern" in speaking of things math and dance share. They referenced spatial patterns, formations of dancers, and patterns used within choreography. Indeed, both math and dance are woven through with patterning, large and small, deliberate and innate.

Respondents also saw commonality in the process of learning and practicing the two fields. The terms "problem solving," "hard work," "discipline," or "training" were used by 38% of all respondents, showing a general respect for the work required in practicing each field and an understanding that in both critical thinking is required. One respondent wrote of "hard work, memorizing, step-by-step learning, and theories" (Respondent 6), helping to summarize that learning either of the subject requires similar application of the mind,

Finally, several respondents indicated that they found both math and dance to be art or to be beautiful. One person, a math major, wrote, "both include technique and style, and if practiced long enough can produce very beautiful things" (Respondent 20). From this wide range of the parallels that mathematics and dance students could find with minimum effort, it is clear that there is a definitive relationship between the fields. In applying the mind to the work of seeing the many and sundry commonalities between math and dance, one gains greater insight not only into this relationship, but into the fundamental essences of each subject.

III. GROUP THEORY

To go deeper into the relationship, it becomes of interest to examine conceptual commonalities between math and dance on a more abstract level. While the plethora of ways the students from the survey could connect math and dance shows that there is a distinct relationship, these connections tended be on the superficial side, rather than getting at the depths of why the two subjects exist and how they function. What commonalities are there on this level? To discuss, I will start by giving the reader a working knowledge of group theory, an area of mathematics found within abstract algebra.

The mathematical area known as group theory arose around the 1770s, with significant developments in the 19th century. At the time, mathematics was becoming increasingly concerned with abstraction and axiomatic models (that is to say, models which were absolute, unquestionable). In the 1986 *Math Magazine* article "The Evolution of Group Theory: A Brief Survey," Israel Kleiner credits four major mathematical sources with the evolution of group theory: classical algebra, number theory, geometry, and analysis (Kleiner, 1986, p. 196). From all of these established corners of mathematics, concepts were drawn together as mathematicians made new discoveries. Thus, familiar principles from other fields of mathematics transfer into group theory.

Group theory is the study of algebraic structures called groups. Most mathematical studies are built upon pre-established definitions and proven concepts, making explicit definitions important. Accordingly, groups are defined explicitly. A mathematical definition can be found in Appendix III for the interested reader.

To illustrate the definition of a mathematical group (presented in layman's terms), I will utilize the example of a clock. The twelve hours of an analog clock, with the operator addition, form a mathematical group, and this is a group which most people work with constantly and proficiently. For our purposes, we will call the twelfth hour "0" rather than "12" (similar to military time, where the midnight hour is referred to as "00").

A group takes a defined set of elements. This set can be finite or infinite; either way, the elements of the set must be clearly identified. In the clock example, the set contains a finite number of elements (twelve): $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

A group also takes an operator to act on the elements, transforming them in some way. In a group, closure under the operator is critical. This means that when elements of the set are acted upon by the operator, the resulting element is one which already exists in the set. No new elements can be created. They can be combined in different ways, using the given operator, but the resulting elements will always be one from the original set.

In the clock example, the operator is addition. Any of the twelve hours can be added together, and the result will still be an hour from the original set. For example, 2 o'clock plus 8 hours results in 10 o'clock. 11 o'clock plus 5 hours plus 2 more hours results in 6 o'clock. Adding any number of hours to any starting hour will always leave you at one of the hours in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. Thus, the clock group demonstrates closure under addition.

There are three formal properties which must be satisfied for a set of elements and an operator, also known as a binary system, to together form an algebraic group. The first is associativity, which says that adding or moving parentheses (changing the order in which the operator is applied) will not alter the end result. To demonstrate with the example, consider taking 5 o'clock and adding first two hours, then one hour. There are three ways to group this equation with parentheses: $(5 + 2 + 1)$, $(5 + 2) + 1$, and $5 + (2 + 1)$. Since all three equations equal 8, this property is satisfied.

The second property is that in the set there must be an identity element. The essence of an identity element is that when the operator is used to combine it with another element, the other element returns unchanged. In the clock example, the identity element is 0, because adding 0 to any hour will not change the hour. $11 + 0 = 11$, $0 + 4 = 4$, $2 + 1 + 0 = 2 + 1 = 3$, etc.. Here it is evident why 0 works congruently to 12: going around the clock one complete circle, or 12 hours, also does not change the hour.

The third, and final property is that every element must have a pair in the set such that when the two are combined under the operator, the identity element results. These pairs are called inverses, and an element can be its own inverse. In the example of clock addition, the inverse pairs are as follows: $(0,0)$, $(1,11)$, $(2,10)$, $(3,9)$, $(4,8)$, $(5,7)$. and $(6,6)$. These paired hours, added together, each return the clock to hour 0.

This concludes a working definition for the algebraic structure of a group, as well as an example of a group which appears in everyday life. This definition is the premise of a whole field of discrete mathematics; it enables further explorations and applications in abstract algebra. The biggest takeaway, for the purposes of this paper, should be the basic concept of a group: a specified set of elements can be considered and from them, individual elements combined in some way to make a new element, which is already part of the set.

IV. APPLICATION OF GROUP THEORY TO BALLET

Can this concept be translated into dance? Yes, most definitely, and in fact, its principles can already be found in existing forms of dance. In applying the concept to dance, it will be clearest to choose a form of dance which has been explicitly defined. Since ballet is highly codified and spans a wide body of work, it lends itself to this study. Other forms of dance could also be considered, but doing so would arguably require a higher degree of abstraction. Thus, I proceed with the argument that ballet vocabulary forms an algebraic group.

To apply group theory, we must first consider if ballet is based on a set. Luckily for the purposes of this discussion, ballet is highly codified. Its vocabulary has a number of origins and contributors. Arising from the French and Italian courts, ballet began to take its current form during the nineteenth century. Ballet technique was significantly furthered by teachers including Agrippina Vaganova, Enrico Cecchetti, and August Bournonville. Its steps are referred to in French, which is, as Vaganova says, "similar to the use of Latin in medicine and English in sports (Vaganova, 1969, p. 3). In part because of this universality of language, ballet class is quite similar around the world.

To obtain the greatest degree of clarity and simplicity in arguing that ballet forms an algebraic group, I will break ballet vocabulary down into the simple, basic positioning of body parts. To elaborate, there are clear positions of the feet: first, second, third, fourth, fifth, and sixth positions, as well as pointed, flexed, and forced arch. The knees can plié, grand plié, or stretch, either together or in isolation. The legs can extend devant, derrière, or à la seconde, they can be à terre or en l'air, and they can be internally or externally rotated. The arms have their own positions: bras bas, first, second, third, fourth, fifth, allongé. The head can turn as allowed by the neck joint, and the torso can go forward, back, or side to side. These basic capabilities of the human body, when combined with one another, with forces, and with positioning in the room, account for the entirety of classical ballet vocabulary. Thus, they can be taken as the distinct, defined elements which form the set that is the basis for all of ballet.

The next concept to look for is closure under an operator. The operator can loosely be defined as addition; it just needs to take the union of elements. What does this set look like under addition? For a demonstrative example, consider adding together external rotation of the legs, one leg standing in relevé and one bent with a pointed foot touching the opposite knee, and arms in first position. This describes a passé position, which in combination with a rotation of the head and the force taken from arriving at that position, becomes a pirouette. Every step, movement, and combination in ballet can be broken down into a union of the before identified elements that are the basis for ballet. They are put into different combinations and done in different ways, but the result still falls under the umbrella of ballet vocabulary.

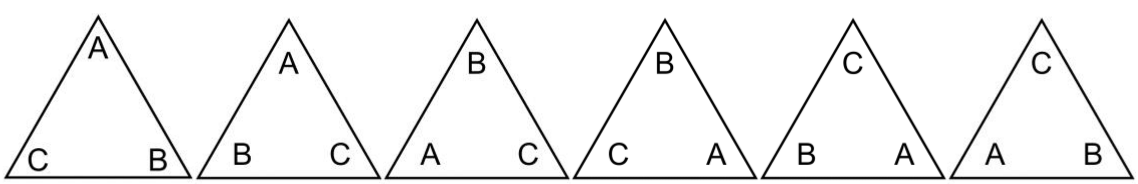
For this set and operator to together form a group, the three properties must hold as true. Satisfying the associative property, it is true that in ballet, changing the way one thinks about a combination will not alter the essence of the combination. For the identity element, there is stillness; adding stillness to a position will not change it. Third to consider is the existence of inverses, which is something that ballet is actually quite concerned about. Ballet is full of pairs and inverses. Movements are done en dehors and en dedans, en avant and en arrière, to the left and to the right. Every up is accompanied by a down (imagine jumps), and overall as the laws of physics demand, every action comes with an equal and opposition reaction. Thus, every element has an inverse which brings the dancing body back towards its neutral state of stillness.

Therefore, ballet vocabulary forms an algebraic group. Its vocabulary has a structure comparable to that of the hours of a clock or any other algebraic group. This gives explanation for the basic truth that ballet steps can be combined in different orders to create combinations and choreographies, which are still ballet. In other words, mathematical closure holds, and ballet is a structured group, a world of its own.

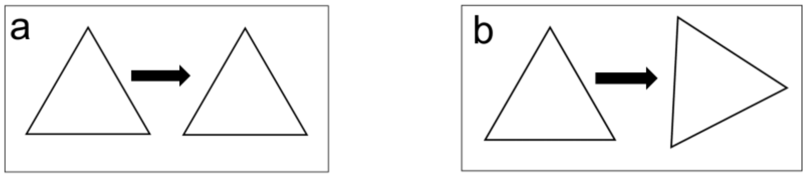
V. AN ALTERNATIVE APPROACH: MOTIONS

Thus far, the discussion has considered the elements of a set as stable, fixed position, and this concept translates to ballet as the elements being static positions of the body. (Of course, a living body is never truly static, and a dancer rarely aims to be static unless per request of a choreographer.) However, this is not the only way to think of elements. Another method that group theorists use, which gives insight into why ballet forms a group so effectively, is looking at elements of a set as motions.

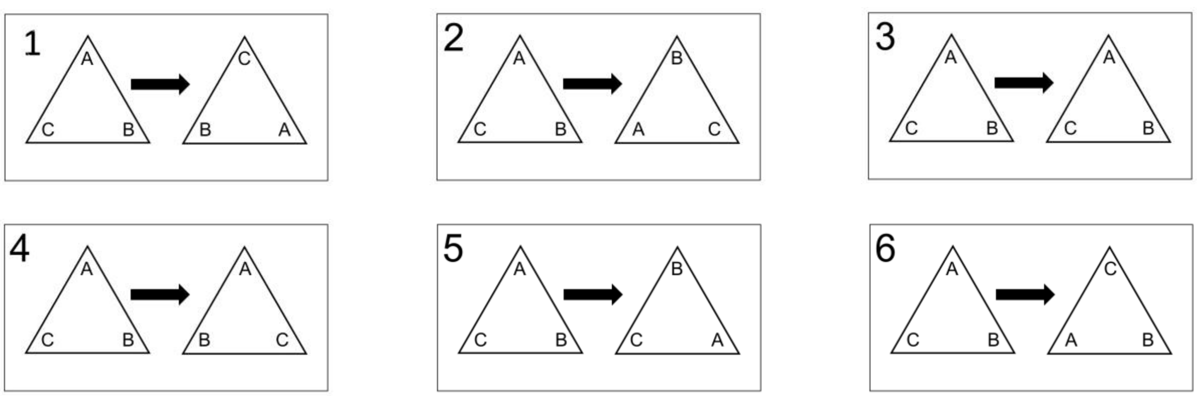
To illustrate this, I will use the example of the cyclic group $\boldsymbol{S_3}$, which can be thought of as the labelings of an equilateral triangle. This group has 6 elements. The first way to think of these elements is as the 6 ways to label the vertices of the same equilateral triangle. They are as follows:



These images represent every possible way to label the vertices of the triangle using the unique indicators ABC each once. Thus, they are the possible positions of the same triangle, as if it was a puzzle piece fitting into a designated negative space. However, there is another way that this same group could be considered: as rigid motions. A rigid motion is considered to be a motion that returns the shape to its original form. In other words, when the motion is done to the triangle, the resulting triangle appears the exact same. So in the images below, (a) is a rigid motion, while (b) is not.



There are 6 possible rigid motions of an equilateral triangle, shown in the diagrams below: rotation 120° counterclockwise (1), rotation 240° counterclockwise (2), rotation 360° counterclockwise (3), flip over the top vertex (4), flip over the top vertex and rotation 120° counterclockwise (5), and flip over the first vertex and rotation 240° counterclockwise (6). Are you thinking, “What about rotating clockwise?” Rotating clockwise $x°$ will give the same result as rotating counter-clockwise by $(360 - x)°$, so that motion is not unique – the two phrases mean the same thing. The six rigid motions can go by other names, but together they give the complete set of possibilities.



In this way, movement manifests in mathematics and specifically in group theory. Thinking of elements as movements gives a different perspective on how ballet forms a group; with this mindset, the building blocks become larger.

Proceeding with this thought, the set of elements that form the basis for the group of ballet vocabulary could be considered as steps, rather than positions of individual body parts. For example, one could take glissade, jeté, pas de chat, and pas de bourrée to be some of the distinct, discrete elements of ballet. Their sum creates a petite allegro combination, a little piece of choreography, which is still in the realm of ballet.

Motions can also be combined to create more intricate steps. For example, a jeté battu is the combination of a simple jeté and a beating of the legs. A fouette turn is the sum of a développé devant, a ronde de jomb to à la seconde, a battu, and a turn. The larger, more complex motions can be broken down into smaller movements. In other words, the larger elements are built through the addition of the simpler movements, which are built through the addition of the basic principles of the body positions. Thus, the elements of ballet vocabulary build on themselves.

This makes the learning and practice of ballet methodical and logical. Students are first taught the principles, like positions of the feet and arms, how to turn out their legs, and how to point their feet. Then they learn to execute the simple movements correctly, and then they proceed to learn more and more complex vocabulary. As Karel Shook writes in his book *Elements of Classical Ballet Technique as Practiced in the School of the Dance Theatre of Harlem,* “Careful adherence to the elementary principles of classical ballet is mandatory because they form the foundation upon which this discipline is built” (Shook, 1977, p. 38). Success in understanding and executing the elementary principles of ballet is critical in order for a dancer to progress.

This logical structure also applies to individual classes of ballet. Ballet classes are structured to methodically activate and warm up the muscles. They begin with barre exercises such as pliés and tendus and then end with large, complex movements like jumps that are built off of pliés, tendus, and the other earlier-accessed building blocks. Summarizing this idea, the ballet master Agrippina Vaganova writes in her *Basic Principles of Classical Ballet*, "The study of any pas in classical ballet is approached gradually from its rough, schematic form to the expressive dance. The same gradation exists also in the mastering of the whole art of the dance, from its first steps to the finished dance on the stage" (Vaganova, 1969, p. 11).

Because closure applies to ballet as a group, no matter which elements, which little pieces of ballet, are put together, the result will still be ballet. As you cannot add two numbers to get a color, you likewise cannot do two ballet steps consecutively and argue that together they are not ballet (although you can certainly argue how good the execution is or the validity of the choices). The closure of ballet vocabulary creates a huge number of possibilities for the makers of balletic dances. Since there are so many steps, there is a huge number of ways to combine them, especially when the choreographer adds musicality, phrasing, and dynamics.

This has allowed for ballet as a field to grow. Each new choreographer uses the basic group elements to make a new work, which is then added to the collection of what ballet is. Choreographers from the nineteenth and twentieth centuries like Arthur St. Leon and Michel Fokine used the same base of vocabulary as current choreographers like Alexei Ratmansky and Justin Peck. Yet, because they used the elements in such different ways, their works are quite different and are representative of their different time periods. Ballet has thus evolved into the contemporary work it includes today. Further, because steps are clearly defined, dancers are able to understand new choreography as a union of familiar pieces, perhaps with unconventional elements added. Audience members, too, are able to understand the dancing they see as either a reference to something that has been done before or something that intends to innovate or break new ground. Thus, the closure of ballet gives it a definitive structure, but within that structure is infinite freedom for dancers and choreographers to explore.

VI. PETIPA AND THE LILAC FAIRY

The concept of group theory can be applied to ballet not only as a wide-reaching vocabulary, but also on a smaller scale. Useful examples for exploring this are Marius Petipa, a critical figure in the shaping of classical ballet, and the Lilac Fairy Variation in the Prologue of *The Sleeping Beauty*, one of Petipa's most successful ballets.

Marius Petipa was a Frenchman, born in 1818 to a father who was a dancer. In his early life, he moved throughout Europe with his family, and as an adult he settled in Russia, signing a contract with the St. Petersburg Imperial Theatre in 1847. There, Petipa became Ballet Master, and produced over sixty full-length ballets in his nearly fifty-year tenure (Moore 1958). Among Petipa's most well-known ballets are *The Nutcracker*, *The Sleeping Beauty, Swan Lake, Don Quixote, La Bayadere*, *Cinderella, and Ramonda*. These ballets are each still performed today; they represent a large portion of what the ballet field considers to be classical ballets.

Petipa's success is evidenced by the plethora of work he created that is still referenced today. As Lillian Moore writes in the introduction to her compilation of Petipa's memoirs, "Marius Petipa was a great choreographer, and more than a choreographer. As virtual dictator of the

Russian ballet, he moulded its course for many years … His renown is undisputed, and his work lives not only in the pages of dance history, but in the active ballet repertoire" (Moore, 1958, p. viii). What was Marius Petipa doing that worked so well and left such a lasting impression? One element was the formulaic, almost mathematical way he approached his ballets.

An algorithm-like plot structure is employed throughout many of Petipa's story ballets. Tim Scholl describes this in *From Petipa to Balanchine: Classical Revival and the Modernization of Ballet*, finding in Petipa's work "a standardized plot structure. The typical Petipa work has a mad scene, a vision scene, and a scene of reconciliation in which the male protagonist and heroine are rejoined – with each scene slightly adapted to the narrative exigencies" (Scholl, 1994, p. 6) He also quotes the ballerina Tamara Karsavina (1885-1978) who said of Petipa, "his productions were all founded on the same formula" (Scholl, 1994, p.9). In comparing the story ballets, this is easily seen as true.

Thus, the plot and structure of Petipa's ballets show a mathematical tendency towards closure: Petipa identified a set of structural elements which were successful and used them as templates, creating an algorithm for which to create new ballets. Ultimately, this created a set of finished ballets which are each unique, but fall into the same group, classical ballets.

Further evidence of group theoretic tendencies can be found in the specific choreography of a piece of one of these ballets: the Lilac Fairy Variation from the Prologue of *The Sleeping Beauty*. To make it accessible to the reader, in Appendix IV a web address is given to a video of this variation performed by the Paris Opera Ballet's Eve Grinsztajn in 2013.

It should be noted that this choreography, which is today seen almost exclusively in productions of *The Sleeping Beauty*, is not the original choreography by Petipa. In fact, there is speculation that the Lilac Fairy was originally not even danced on pointe. Scholl writes, "The choreography for the Lilac Fairy – and her variation in the Prologue in particular – remain central to the debates surrounding the ballet and its choreography. Changes to that variation represent the first known amendments to Petipa's choreography for the ballet" (Scholl, 2004, p. 34). Regardless, the choreography still invites deep investigation and is representative of a Petipa ballet.

The variation is short, only about a minute and forty seconds, with 62 measures of three. It contains an introduction, three main diagonals, and an ending. Each diagonal has one four-measure phrase that is repeated four times, with the exception of the last diagonal, which is repeated three times and then cuts to the ending. Within this highly ordered structure, there are essentially very few positions used, each diagonal phrase is just different variations of arabesques, attitudes, a la second legs, and passes. In mathematical terms, each phrase is just a permutation of the elements in the variation's defining set. The elements are identified, and then they are permuted in different ways to create a dance.

The Lilac Fairy variation is iconic and today seen widely throughout ballet schools and companies. Yet, the choreography is quite simple. This demonstrates the power of mathematics – something simple, when done right, creates something very beautiful.

VII. DISCUSSION: TRIVIAL OR CRITICAL?

Having concluded that ballet vocabulary can be thought of in terms of mathematical group theory, the next logical step is to ask if this is an arbitrary coincidence, or if it is telling of a larger importance in the relationship between mathematics and dance. Is group theory of a particular significance, or could any aspect of the wide breadth of mathematics be randomly chosen to overlay onto the structure of ballet?

In fact, group theory is definitely of a particular significance. Insight into why comes from G.A. Miller's 1921 article "The Group-Theory Element of the History of Mathematics," published in *The Scientific Monthly*. In the article, Miller discusses the evolution of group theory and its importance within mathematics. He also makes interesting points about group theory in relation to the natural world and to human nature, such as the following excerpt:

> *"The idea that a set of distinct elements should have the property that any two of them can be combined into one and that this one is also found in the set was illustrated not only by the natural numbers but also by the movements of figures in space, and hence this idea became firmly fixed in the human mind at an early age. It is in accord with the human yearnings for completeness and it is a natural extension of the notion of cyclic changes which were illustrated by the daily and the seasonal apparent movements of the sun"* (Miller, 1921, p. 77).

In this passage, Miller makes several critical points. The first is that group theory, a seemingly arbitrary study in abstract mathematics, arose from human observations of "the movements of figures in space" (Miller, 1921, p. 77). This has direct application to dance, for what is dance if not the movement of figures in space? Evidencing the idea that movement in space is the foundation of dance, Lynne Anne Blom and L. Tarin Chaplin write in *The Intimate Act of Choreography*, "A body exists in space…moves in space…is contained in space. A dancer's place and design in space…define the image she is creating" (Blom, 1994, p. 31). Of course, dance, being an art form, has much more to it than simply movement in space, but at its core dance is purely just the movement of figures in space.

Miller also quotes E. Borel's 1905 text entitled *Géometrie,* in which Borel said, "elementary geometry is equivalent to the investigation of the group of movement" (Miller, 1921, p. 80). These two mathematicians make the point that group theory is dependent on movement, for it is the study of how elements can be moved into and within groupings. Clearly, dance is likewise dependent on movement, and thus it is conclusive that group theory and dance share a fundamental similarity in their deepest essences.

The second important point from Miller's passage is the quote that group theory "is in accord with the human yearnings for completeness" (Miller, 1921, p. 77). In a similar vein, he also writes, "Few mathematical terms suggest such fundamental human cravings as the term group" (Miller, 1921, p. 75). Together, these quotes provide argument that group theory fulfills an innate human desire, yearning, or craving. Humans have a natural urge to find order, completeness, and groupings. Thus, the study of group theory provides a unique satisfaction.

This concept can be generalized to mathematics as a whole, and then carried over comparatively to dance. Mathematics has long been an integral part of all human societies and studies; it is both critical and universal. In a 1928 article of *The American Mathematical Monthly* called "The Human Significance of Mathematics," Dunham Jackson makes the claim that "A distinguishing characteristic of mathematics is its universality, its independence of time, place, and circumstance. The philosophers have made a great deal of this. They have ascribed to mathematical truth as absolute quality, transcending human experience and human existence" (Jackson, 1928, p. 409). The universality of mathematics and its transcendence of time and space signifies its importance to humans and to society.

Likewise, dance has been an integral part of all human societies. The Polish scholar Roderyk Lange spent his career studying dance anthropology. In his book *The Nature of Dance: An Anthropological Perspective,* he writes, "Dance represents a primeval component of culture and shows many universally human features, being valid throughout the whole history of mankind right down to our own times" (Lange, 1976, p. 90). Similarly, dance anthropologist Joanna Keali'inohomoku is quoted as saying about "dance being a human universal. … Through it we have a sense of history, we have a sense of anticipation, and we have a sense of beauty, we have a sense of why" (Celichowska, 2008, p. 126). To dance anthropologists, it is clear that dance has great importance to humans, regardless of time, culture, and identity.

Like mathematics, dance is universal, and it is critical. Both fulfill a basic human need; else they would not be prevalent in all civilizations. Both enable humans to find beauty and order in the world around them. It is for this reason that mathematics and dance share so many parallels, and that their defining characteristics can be found within one another.

VIII. CONCLUSION

Euripedes, a great tragedian of ancient Greece, once said that "Mighty are numbers, joined with art resistless" (Burton, 2010, p.1). This statement is brief but powerful. Numbers are mighty. Mathematics is found in nature and it plays a critical role in helping people understand and navigate the world. Yet on their own, numbers are not see as beautiful by all (as evidenced by the survey respondents who indicated that they disliked or hated math). Dance, on the other hand, could be considered as a physicalization of beauty, and yet within its depths is a wealth of mathematical principles. Mathematical principles help make dance strong, successful, and beautiful.

The abstract mathematics of group theory applies to ballet so effectively because both subjects capitalize on a human desire for familiarity, consistency, and order. In developing group theory, mathematicians were inadvertently studying many of the principles used by artists like

Marius Petipa. Both focused on establishing distinct sets of elements and then exploring the patterns between them and the many ways they could be permuted to create something new and beautiful.

Mathematics manifests in dance in many superficial ways – the counting of the steps and of the music, the shapes made by bodies, the patterns of sequences and of dancers on the stage. However, on a deeper level, it can also be seen that dance, or at least simplistic motions, are found in math and mathematical principles are found in a conceptual, intellectual analysis of dance. These principles exist because at their cores dance and mathematics are both critical to humans. They serve different purposes, and therefore take very different forms, but both help people find order, beauty, and logic and express that in a way which can be universally understood. Dance and mathematics both transcend time, space, and cultural distinctions, because at their heart is a fundamental shared essence that is of deep importance to mankind.

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APPENDIX I.

Survey Data

The purpose of this survey was to briefly get a sense for what other people, specifically Goucher College students and recent graduates who studied mathematics and/or dance, viewed as the intersection between mathematics and dance. There were 24 respondents and their responses are compiled below, organized by question.

What is (or was) your major?

1. Dance and Business Management
2. Dance and Business
3. Visual and Performance Arts Education
4. Dance
5. Dance
6. Mathematics
7. Mathematics with a Concentration in Secondary Education
8. Dance
9. Dance with Psychology Minor
10. Math & Economics
11. Dance
12. Dance
13. Mathematics
14. International Relations
15. Dance and Psychology
16. Elementary Education
17. Math and secondary Education
18. Economics and Dance
19. Mathematics
20. Mathematics
21. Dance and Spanish
22. Dance and Business
23. Dance
24. Dance and English

What year are you in school?

1. Senior
2. Senior
3. Senior
4. Senior
5. Senior
6. Senior
7. Senior
8. Senior
9. Senior
10. Senior
11. Graduated
12. Graduated

1. Graduate
2. Sophomore
3. Senior
4. Senior

5. Graduated
6. Graduated
7. Graduated
8. Graduated
9. Sophomore
10. Sophomore
11. Senior
12. Junior

Do you identify as a dancer?

1. Yes
2. Yes
3. Yes
4. Yes
5. Yes
6. No
7. No
8. Yes
9. Yes
10. No
11. Yes
12. Yes
13. No
14. Yes
15. Yes
16. Yes
17. No
18. Yes
19. No
20. No
21. Yes
22. Yes
23. Yes
24. Yes

How would you briefly describe your relationship with math?

1. I'm fine with math except with calculus. I was always good at it until college. Don't particularly gravitate towards finances in the business world. I just took accounting and finance for the knowledge but would rather not pursue it.
2. I love math!
3. I feel neutral in general, BUT I have to take a math LER the last semester of my senior year and that makes me dislike it
4. Hate it
5. I like certain kinds of math a lot
6. Love it
7. I love it
8. I hate it- it makes me feel dumb. I should be better at it, I can't even calculate tips accurately):
9. Neutral. I enjoy when I have figured it out, but it's difficult for me to learn and understand concepts.
10. Love it
11. Feel neutral
12. Dread doing any math because I've never been good at it.
13. Love it!!
14. Dislike

15. I love math, it was my strongest subject before college. However, I do not engage in advanced mathematics anymore.
16. I hate math
17. love hate relationship
18. like! 4/5
19. Love love love it
20. Love it!
21. I am good at math and have liked it for the most part.
22. Like it
23. I want to know enough math to do taxes
24. Feel neutral

Briefly, how do you broadly view dance?

1. Mind, body, and soul!! It works me mentally by pushing me past my limits, teaches me perseverance, and challenges me with intricate patterns/combinations and musically. It allows me to be physically active and I love being able to do this with my body that not many regular people can do. And it really inspires me as an artist! It's cheesy but it fuels my soul by allowing me to express myself through movement. It's fun and amazing!!! I don't know what I'd do without it.
2. Physical, challenging, organized chaos, limitless
3. Difficult, challenging, mentally and physically taxing, but also mentally and physically freeing, and also one of my favorite things I've ever done
4. It's a lifestyle, challenging but rewarding
5. Visual music, self-expression, embodied culture
6. Detail- oriented, athletic, cardio, neat to watch
7. I think it's beautiful and expressive.
8. Satisfying, creative, exercise, challenging
9. I love it for its physicality, detail, the way it challenges me to work hard and be artistic and creative.
10. Hard, physical, cool
11. Expressive, detail-oriented, physical
12. Fun, entertaining, a physical and emotional release
13. Beautiful and I admire dancers a lot. It's something I always wished I could do. It seems really difficult!
14. Detail oriented
15. Physical, frustrating, pattern-oriented, cathartic, creative, complex
16. Physical, fun, detail oriented
17. Amazing and hard
18. intricate, challenging, enjoyable
19. Physically demanding, I respect it
20. Fun and cathartic
21. Athletic and artistic
22. Fun and rewarding
23. athletically artistic
24. Extremely strategic but allows me to release things that weigh me down

If you identify as a dancer, in what ways have you thought about mathematic principles applying to your practice?

1. Music is my initial thought. I am huge on counting and being on the music with or without others. For example, counts can be very important choreographically in order for an intricate pattern to form. Other than that I guess I think about math in dance with angles, like getting an arabesque to 90 degrees or flat backs/laterals.
2. Geometry of shape, pattern of combinations, useful in choreographing canons
3. I think about math in relation to dance when it comes to music. I usually don't count while dancing but as a performer it's necessary so I've had to train myself to be more attuned to counting pieces, and staying within the choreographed rhythms of the dance.
4. N/A
5. rhythm, structure, patterns, rotation

6. Not a dancer, but the angles in body movements
7. N/A
8. Just when I count the music.
9. I think the most obvious one is in counting (5,6,7,8...), but also in the musicality and spatial patters that are often key to creating an interesting dance.
10. N/A
11. Mainly in shapes and patterns that can be made physically with the body or with bodies in space
12. Patterns of space and time
13. N/A
14. Musicality, mainly
15. Mostly regarding patterns in large group choreography. I.e. counts, formations, etc. Also, angles of the body. What ratios of space are pleasing to the eye
16. I have not thought about it
17. N/A
18. counts, patterns, shapes
19. N/A
20. N/A
21. I haven't thought about the relationship much.
22. Briefly for a class assignment
23. I think both have method, specificity, and function
24. Gravity, balance, symmetry, force and many other factors are vital for the success of movement vocabulary and although I have not analyzed it to any extent, I find it fascinating

Do you see the two fields (math and dance) as sharing commonalities? What commonalities do you see? List as many things as you can think of

1. Problem solving. Patterns. Angles. Can both be time sensitive.
2. Yes, they both involve logical problem solving, patterns, and testing of limits
3. Dance uses rhythms, which relate to counting like I said above. The physical lines dancers make are geometric shapes. Patterning in space is geometrical. Choreographic processes use elements of math such as the cannon.
4. Not really
5. rhythm, problem solving, patterns, numbers in space
6. Angles, hard work, determination, memorizing, terms, step by step to learn, theories, balancing things, counting steps/numbers
7. The coordination involved in choreography, counting steps and rhythm, displaying mathematical concepts through intricate dance movements, symmetry, creating geometrical shapes through movements and formations
8. Yes! Angles, patterns, counting, it's like that movie Ice Princess where she calculates how to do ice skating jumps and stuff.
9. Yes. Spatially-- making specific shapes in space on different planes, people often use words like "parallel lines", "spiral", "infinite", "energy". The way we train is often shape oriented, energy related, and pattern specific. These are all basic concepts in math (and physics).
10. Yes: angles, patterns, precision
11. Yes I believe there are correlations: shapes, designs, figures, movement principles
12. Patterns, counting, geometry. But in math I feel that these exist and we have to find them, whereas in dance we create them, which makes it less intimidating.
13. Yeah! Maybe a more physics-y side of math, but the precision and shape of movements and patterns in dance could definitely relate to/be modeled by math.
14. Precision (music)
15. Yes. Most classical choreography is based in patterns, as is mathematics. Many people who study dance and/or math are heavily disciplined and detail oriented.
16. Yes- they both have to do with counts and numbers and the different shapes and lines that you can make with your body or even a group of people
17. patterns, symmetry, art, solving problems, one step leads to the next

18. yes! counts, patterns, shapes, logical order/sequencing, similar If/Then structure (If I engage my standing side, then I will stay up, etc.)
19. Beauty, detail-oriented, technical, complex
20. Yes, in some ways. They both involve technique and style, and if practiced long enough can produce very beautiful things.
21. Yes. The movements can be described using numbers and mathematical formations.
22. Shapes, lines, and angles
23. I think both fields require consistent practice to perfect and they both require accuracy.
24. In most cases with highly structured genres of dance, there is only one answer, much like math. The training and repetition are also similar. Must understand one principal before mastering the next.

Anything else you would like to add?

1. I think math is great and I wish I was just a bit better at it!!! Also dancers are smart and are capable of doing math!!! How do people think dance companies/organizations are run?
2. Math and dance are awesome together
3. –
4. –
5. I think about math the most when I do tap dance because of patterns, center of weight, rhythm, etc
6. Angles, hard work, determination, memorizing, terms, step by step to learn, theories, balancing things, counting steps/numbers
7. –
8. –
9. –
10. –
11. –
12. –
13. –
14. –
15. I think that my studies of dance and math enriched each other as I was growing up. Especially in regard to finding patterns and creating “shortcuts” in knowledge and memory
16. –
17. –
18. –
19. –
20. –
21. –
22. –
23. –
24. –

APPENDIX II.

SURVEY ANALYSIS

Methodology:

I directly contacted students and recent graduates who either studied math or dance at Goucher College. The survey was online, optional, and anonymous. Respondents were to answer questions with qualitative responses. My intention was to gather the general thoughts and patterns students of math and/or dance would see between the two fields.

Demographic Information:

Number of Respondents: 24

Year in School:

Sophomores:	12.50%
Juniors:	4.17%
Seniors:	54.17%
Graduated:	29.17%

*Graduates all graduated in the last one or two years
*All students attend(ed) Goucher College in Baltimore, MD
*Students were mostly female and between the ages of 18 and 24

Identified as Dancer:

Yes:	70.83%	(82.24% of these respondents identified as dance majors)
No:	29.17%	(100% of these respondents identified as math majors)

Survey Findings:

When asked to describe their relationship with math in their own words:

- 100% of the math majors used the word “love”
- 35.29% non-math major respondents used the word “love” or “like”
- 29.41% non-math major respondents used the word “hate,” “dislike” or “dread”

When dancers were asked if they had thought about math applying to dance:

- 11.76% had not thought about it (both sophomores)

Commonalities between math and dance most frequently mentioned:

- Geometry (used the word “geometry,” “angles,” “shape,” or “line”)
  - 47.06% of dancers
  - 57.14% of math majors
  - 50% of all respondents
- Music (used the words “music,” “rhythm,” or “counting” (as of music)

  - 52.94% of dancers
  - 14.29% of math majors
  - 41.67% of all respondents
- Patterns (used the word "pattern")
  - 47.06% of dancers
  - 28.57% of math majors
  - 41.67% of all respondents
- Process (used the words "problem solving," "hard work," "discipline," or "training"
  - 35.29% of dancers
  - 42.86% of math majors
  - 37.5% of all respondents

APPENDIX III.

Mathematical Definition of a Group

Definition. Suppose a set G is closed under an operation $*$. That is, suppose $a * b \in G$ for all $a, b \in G$. Then $*$ is called a binary operation of G. We will use the notation $(G, *)$ to represent the set G with this operation. Suppose $(G, *)$ also satisfies the following three properties.

(1) $(a * b) * c = a * (b * c)$ for all $a, b, c \in G$.

(2) There exists an identity element $e \in G$ for which $e * a = a * e = a$ for all a.

(3) For each $a \in G$, there exists an inverse element $b \in G$ for which $a * b = b * a = e$. The inverse of a is usually denoted by a^{-1} if $*$ is a general operation or multiplication, and $-a$ if $*$ is addition.

Then $(G, *)$ is called a group.

This wording was taken from the *Applications of Abstract Algebra with Maple and Matlab* (cited in References).

APPENDIX IV.

URL for Lilac Fairy Variation

https://www.youtube.com/watch?v=DOrO9YIubts