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# Is the supermultiplier stable?

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**Abstract:** Supermultiplier models, which show how autonomous demand can drive both business cycles and long-run GDP growth, are based on a stability assumption. In this paper I look at recent efforts to justify this assumption, and argue that they are not convincing. The supermultiplier literature generally assumes that business investment reacts very slowly to changes in the state of the economy, but faster adjustment speeds are consistent with US data and can generate instability.

## 1. Introduction

Is it possible to regulate a capitalist economy in such a way that it delivers steady, balanced, and egalitarian income growth? And if so, what types of policies can accomplish this in practice? These are obviously important questions for economists.

Clearly, the concept of stability has some relevance here. If an economy is stable, or can be regulated in such a way that it *becomes* stable, then this opens up the possibility of guiding it toward a desired equilibrium growth path. But if an economy is inherently unstable, then it will be much more difficult to manage, and income growth can be expected to exhibit large fluctuations over time.

Marx and Engels' economic writings represent an early formulation of the idea that capitalism is unstable and basically unmanageable. Thus, they argue, capitalist societies are "like the sorcerer who is no longer able to control the powers of the nether world whom he has called up by his spells" (Marx and Engels, 1848, chapter 1). Profit-seeking decisions by firms generate cyclical booms and crises which reflect, at a deeper level, the irrational way in which capitalist production and investment are organized (Engels, 1894, part III, chapter 2).

Today there is a substantial academic literature which also discusses the way business decisions regarding production and investment can generate instability. Skott and Zipperer (2012) use US data to calibrate post-Keynesian models, paying particular attention to the way conditions in the labor market interact with firms' expansion decisions, and show how limit cycle dynamics can arise. Duménil and Lévy (1993), who rely on Marxist crisis theory as well as US economic data, argue that while governments or central banks might stabilize national economies for temporary periods, business decisions related to production, investment, profitability, and technical change will tend to push economies back across the stability threshold as time goes on. On the other hand, in more disaggregated economic models, it is now known that even seemingly innocuous assumptions about agents' behavior can lead to instability (Dupertuis and Sinha, 2009; Toda and Walsh, 2017).

Meanwhile, capitalism's boom and bust cycles continue to pose a challenge for policymakers (Mallaby, 2016). Beginning in the mid-1980s, macroeconomic volatility in advanced capitalist economies did fall significantly, but this period of relative tranquility came crashing down with the global financial crisis of 2008 (Clark, 2009; Duménil and Lévy, 2013). Questions about the stability of capitalism thus remain topical.

Sraffian supermultiplier models offer a distinctive way to think about these issues. Strikingly, they are based on the idea that businesses adjust output and productive investment in response to changes in the demand for their products, and do so *without* destabilizing the economy as a whole. At the same time, these models assume that specific components of aggregate demand—such as exports, government spending and residential investment—are insensitive to changes in current income, and are in this sense autonomous, even in the long run. Under these assumptions, it can be shown that a capitalist economy will converge to an equilibrium path in which the growth rate of GDP is determined by autonomous demand (Serrano, Freitas and Bhering, 2019). In this way, supermultiplier models make it possible to understand how disruptions in the housing market or international trade can lead to a recession, while at the same time downplaying the notion that instability is inherent to the organization of capitalist production and investment.

The purpose of this paper is to take a critical look at supermultiplier theory, with a focus on the stability assumption underpinning that framework's key results. I argue that in the US economy, this stability assumption does not have strong support; empirically plausible parameter values can generate unstable cyclical dynamics, rather than convergence to the long-run equilibrium. The results point to the likelihood that firms' reactions to disequilibria, and in particular the interactions between output and investment decisions, play an important role in driving macroeconomic fluctuations. This casts doubt on the idea that booms and recessions should be understood primarily in terms of changes in autonomous variables.

This paper is not the first to ever question the stability assumptions in supermultiplier-type models. In fact Skott (2017; 2019) has already put forward significant critiques. But in the time since Skott's papers were published, Haluska, Braga, and Summa (2021) have produced an econometric analysis based on the supermultiplier which claims that "the U.S. economy has never approached the dynamic upper stability limit" during the period from 1985 to 2017. I examine this claim and argue that it is untenable, in part because Haluska and co-authors underestimate what their own results imply about the strength of the investment accelerator.

More fundamentally, in this paper I hope to bring greater clarity to an issue that has come up repeatedly in the literature on the supermultiplier and related models. Critics of these models, such as Skott, Santos and da Costa Oreiro (2022), argue that the stability conditions will only hold if business investment decisions adjust extremely slowly to changes in the state of the economy (see also Skott, 2017). Allain (2022), in contrast, defends the supermultiplier approach, and argues that when other model parameters are appropriately specified, stability is compatible with a realistic range of adjustment speeds. But the issue remains unresolved because, as Gallo (2022, p. 1167) notes, little is known about which adjustment speeds are actually supported by economic data. Indeed Skott (2017, p. 190) argues for an adjustment speed nearly three times the one advocated by Alain (2022, p. 96), but neither paper provides evidence to support their preferred values. In what follows, I seek to move the debate forward by proposing a benchmark value based on investment data from the US economy. I show that, if this adjustment speed is accepted, then even under the relatively favorable assumptions about other parameters made by Allain, the supermultiplier will be unstable.

The rest of this paper is organized as follows. Section 2 reviews the supermultiplier framework. Section 3 takes a critical look at recent evidence on stability. Section 4 discusses the implications within the broader literature. Section 5 concludes.

## 2. The Sraffian supermultiplier

### 2.1. The basic framework

In this section, I lay out the basic supermultiplier model developed by Sraffians. I largely follow Serrano, Freitas and Bhering (2019), and Haluska, Braga and Summa (2021). A key idea is the distinction between autonomous and induced components of aggregate demand (Serrano, 1995). A portion of aggregate demand is said to be *autonomous* if it is determined independently of current output, and does not create productive capacity; on the other hand, a portion of aggregate demand is said to be *induced* if it is responsive to changes in current income (induced demand may or may not create productive capacity). The supermultiplier model shows how autonomous demand can determine real GDP in the long run by driving an economy toward an equilibrium growth path.

To demonstrate these ideas, I will now set up the macroeconomic structure for the model. All economic aggregates defined below are measured in real terms. Time is broken into discrete periods, and for a given variable  $a$ , the value of  $a$  during the period  $t$  is denoted by  $a_t$ . Aggregate demand,  $Y_t$ , satisfies the equation

$$Y_t = C_t + I_t + G_t + X_t - M_t, \quad (1)$$

where  $C_t$  is consumption spending,  $I_t$  is gross investment,  $G_t$  is government spending,  $X_t$  is exports and  $M_t$  is imports. All of these variables represent flows of spending per period. Imports are determined by the equation

$$M_t = \mu(C_t + I_t + G_t + X_t), \quad (2)$$

where  $\mu$  denotes the import propensity.

To describe the dynamics of consumer spending, I define households' disposable income,  $Y_t^d$ , to be

$$Y_t^d = (1 - \tau)Y_t + Tr_t, \quad (3)$$

where  $\tau$  is the average tax rate and  $Tr_t$  denotes government transfers. Total consumption spending is then

$$C_t = cY_t^d + C_t^A, \quad (4)$$

where  $c$  is the propensity to consume out of disposable income, and  $C_t^A$  is an autonomous component of consumption spending which may, for instance, be financed by consumer credit.

Similarly, gross investment,  $I_t$ , involves an induced component  $I_t^I$  which responds to changes in output, and an autonomous component  $I_t^A$ :

$$I_t = I_t^I + I_t^A. \quad (5)$$

In this model, residential investment is autonomous, and is therefore represented by the variable  $I_t^A$ , while induced investment is determined by:

$$I_t^I = h_t Y_t. \quad (6)$$

In equation (6),  $h_t$  is a variable that changes with expectations about the future state of the economy, and will be discussed in more detail below. Finally, at each point in time, output is determined by aggregate demand, because businesses adjust capacity

utilization to meet changes in demand for their products, and the available labor supply does not impose a binding constraint on growth.

To establish the relationship between aggregate demand and autonomous demand, I first define  $s$  to be:

$$s = \frac{1}{1 - \mu} - c(1 - \tau). \quad (7)$$

This combines several parameters together, and makes it possible to reduce the relatively complex model developed by Haluska, Braga and Summa (2021) to the simpler system studied by Serrano, Freitas and Bhering (2019). On the other hand, I let  $Z_t$  represent total autonomous demand:

$$Z_t = G_t + X_t + C_t^A + I_t^A + cTr_t. \quad (8)$$

Because  $Z_t$  represents long-run autonomous demand, it is assumed that this variable grows at a constant rate,  $z$ :

$$z = \frac{Z_t - Z_{t-1}}{Z_{t-1}}. \quad (9)$$

After a little bit of algebra, one can see that:

$$Y_t = \left( \frac{1}{s - h_t} \right) Z_t. \quad (10)$$

The factor  $\frac{1}{s - h_t}$  in equation (10) is called the *supermultiplier*. It establishes the relationship between autonomous demand,  $Z_t$ , and aggregate demand,  $Y_t$ .

## 2.2. The capital-stock adjustment process

It is apparent from equation (10) that, if the induced investment share  $h_t$  were constant, then the growth rate of aggregate demand would exactly equal the growth rate of autonomous demand. Thus, given the framework and assumptions described above, if it can be established that  $h_t$  converges to a constant value, then it follows that long-run output growth must be determined by the growth rate of autonomous demand. With this in mind, I will now set up the equations that describe investment decisions in this model.

First, it is necessary to specify how productive capacity, and the utilization of that capacity, are measured. Let  $K_t$  denote the stock of productive capital in use by firms. I let  $Y_t^*$  denote the output level corresponding to full utilization of capacity, and assume  $Y_t^* = K_t / u^d v$ , where  $v$  is a positive constant and  $u^d$  denotes the normal utilization rate. Thus,  $v$  represents the ratio of capital to output when capacity is used at the normal rate:

$$v = \frac{K_t}{u^d Y_t^*}. \quad (11)$$

On the other hand, I define capacity utilization,  $u_t$ , to be:

$$u_t = \frac{Y_t}{Y_t^*}. \quad (12)$$

Note that, when capacity is utilized at the normal rate,  $u = u^d$  and  $v = K_t / Y_t$ .

Now, let  $g_t$  denote the rate of output growth at time  $t$ :

$$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}. \quad (13)$$

On the other hand, let  $g_t^e$  denote the *expected* growth rate during the period  $t$ . Supermultiplier theory assumes that

$$g_t^e = g_{t-1}^e + x(g_{t-1} - g_{t-1}^e), \quad (14)$$

where  $x$  is a parameter that is strictly between zero and one. Thus, firms have expectations about the future rate of growth which are described by the variable  $g_t^e$ , and these expectations adjust over time depending on whether they are above or below the actual growth rate. I will also assume that capital depreciates at a constant rate,  $\delta$ .

The equation describing the induced investment share,  $h_t$ , chosen by firms, is:

$$h_t = v(g_t^e + \delta). \quad (15)$$

To understand the rationale for equation (15), note that it means the capital stock will grow at the rate

$$\frac{K_{t+1} - K_t}{K_t} = \frac{h_t Y_t - \delta K_t}{K_t} = \frac{u_t}{u^d} (g_t^e + \delta) - \delta. \quad (16)$$

Consequently, if capacity utilization is above its normal rate, then  $\frac{u_t}{u^d} > 1$ , and (16) implies that productive capacity will grow faster than the expected growth rate of demand, which should bring capacity utilization back down; similarly, if  $\frac{u_t}{u^d} < 1$ , then capacity will grow more slowly than expected demand, which should bring  $u_t$  up. Thus, equation (15) represents firms' attempts to adjust capacity to the desired level in relation to output; it is a type of investment accelerator. This is the capital-stock adjustment process proposed by Sraffians.

As is the case with any macroeconomic model, the mathematical expressions above are obviously a simplified representation of a very complex process. This is particularly true with respect to the parameter,  $x$ , that governs the speed with which changes in the state of the economy translate into new investment. Of course, businesses have an incentive to act on changes in market conditions as quickly as possible, but their ability to do this will potentially be constrained by how rapidly they can draw conclusions from large volumes of complex information, secure financing, and organize work on a new scale. Thus, the empirical value of  $x$  will likely reflect a variety of different factors, and may change over time based on monetary policy, financial innovation, the progress of information technology, the development of new management techniques, and so on; see Duménil and Lévy (1993) for a discussion of issues related to this.

### 2.3. Sufficient conditions for stability of the supermultiplier

The supermultiplier model has an equilibrium in which the output growth rate,  $g_t$ , and the expected growth rate,  $g_t^e$ , are both equal to  $z$ , which the reader should recall is the growth rate of autonomous demand, defined in equation (9). For a given set of parameters, one says the supermultiplier is stable if solution trajectories in the model gravitate toward this equilibrium.

To derive the conditions for stability, let us first note that, after some manipulation of equation (10), one obtains the equation

$$g_t = z + \frac{(1+z)vx(g_{t-1} - g_{t-1}^e)}{s - v(g_t^e + \delta)}. \quad (17)$$

This equation shows how the growth rate of output,  $g_t$ , is determined in each period. Together, equations (17) and (14) represent a two-dimensional discrete-time dynamical system with the state variables  $g_t$  and  $g_t^e$ . Note that, in view of equation (10), economic output will only be non-negative if  $s - h_t > 0$ . Therefore, since  $h_t = v(z + \delta)$  along the equilibrium growth path, I assume that

$$s > v(z + \delta). \quad (18)$$

With these preliminaries in hand, I will now establish the conditions for the stability of the supermultiplier model.

After differentiating both sides of equation (17) with respect to  $g_{t-1}$ , and then plugging in equilibrium values, one obtains:

$$\frac{\partial g_t}{\partial g_{t-1}} = \frac{(1+z)xv}{s - v(z + \delta)}. \quad (19)$$

It follows that, if the output growth rate is perturbed by a small amount  $\Delta g$  from its equilibrium value, then in the next period the perturbation from the equilibrium will be, approximately,

$$\frac{(1+z)xv}{s - v(z + \delta)} \Delta g. \quad (20)$$

Therefore, if  $\frac{(1+z)xv}{s - v(z + \delta)} < 1$ , then the perturbation will tend to dissipate over time, and the equilibrium will attract points that are sufficiently close. On the other hand, if  $\frac{(1+z)xv}{s - v(z + \delta)} > 1$ , then small shocks to the equilibrium will tend to become amplified with the passage of each subsequent period.

These observations suggest that the supermultiplier will be locally stable if

$$xv(1+z) < s - v(z + \delta). \quad (21)$$

Indeed one can formally justify this stability condition by calculating a Jacobian matrix at the equilibrium point for the full system consisting of equations (17) and (14) (see Serrano, Freitas and Bhering (2019) for more details). Thus, if a solution trajectory begins at an initial condition that is sufficiently close to the equilibrium point, and  $xv(1+z)$  is not too large in relation to the equilibrium autonomous demand share  $s - v(z + \delta)$ , then the trajectory will move toward that equilibrium.

To better understand the stability condition (21), it is useful to note that the value  $xv(1+z)$  is related to the strength of the investment accelerator. The parameter  $v$ , which represents the ratio of productive capital to output when capacity is utilized at the normal rate, is especially important here. Larger values of  $v$  imply larger increases in investment, relative to GDP, in response to a given increase in the previous period's output growth rate. This fact can be seen via inspection of equations (14) and (15). Thus, larger values of  $v$  mean a stronger accelerator effect. Similarly, larger values for  $x$  mean that firms react more quickly to changes in the previous period's output growth rate, and this strengthens the accelerator effect as well. On the other hand, equation (9) means that autonomous demand is unresponsive to the previous period's output growth rate, so a larger autonomous demand share translates into a larger dampening effect in equation (19). Putting this all together, one can see that the model will be stable if the investment accelerator is not too strong in relation to the size of the autonomous demand share.

The stability condition (21) also creates a partial overlap with ideas in Marxist crisis theory. Duménil and Lévy (1993) look at changes in the profit rate in the US

economy, which they argue are driven largely by changes in the ratio of output to capital, and show how a decline in the profit rate can lead to macroeconomic instability. In the supermultiplier framework, a similar effect can arise, because long-run decreases in the output-capital ratio translate into *higher* values for  $\nu$ , which make the stability condition harder to satisfy. In these two perspectives, the underlying economic mechanism is different: for Duménil and Lévy, changes in the profit rate influence stability through their effects on firms' liquidity, while in the supermultiplier model, the issue is that increases in  $\nu$  will create a stronger accelerator effect. Nonetheless, these frameworks both predict that increases in the long-run ratio of capital to output will make it harder to maintain stability. These issues are relevant to the US economy, where it appears that, due to a slowdown in the rate of technical progress, fixed capital has been rising in relation to output (Duménil and Lévy, 2016).

All that aside, advocates of the Sraffian supermultiplier framework argue that the stability condition tends to be satisfied in practice. For example, see Serrano, Freitas and Bhering (2019, p. 280), Haluska, Braga and Summa (2021), and Summa, Petrini and Teixeira (2023, Section 5). On this basis, they expect GDP growth rates of capitalist economies to gravitate toward the growth rates of autonomous demand. For this reason, the stability assumption should be seen as a key part of the supermultiplier approach.

#### 2.4. Notes on (semi-)autonomous demand

In addition to the stability condition just described, the concept of autonomous demand is also clearly an important part of the supermultiplier framework. In the standard formulation given above, significant components of aggregate demand grow at a rate that is unaffected by changes in income. But some economists, including Skott (2019) and Nikiforos (2018), have criticized this assumption.

To get a better sense of what is at stake here, one can consider a slightly modified version of the supermultiplier model, in which  $z$ , the growth rate of  $Z_t$ , is partly influenced by income. To this end, let

$$z_t = \alpha g_{t-1} + (1 - \alpha)\gamma, \quad (22)$$

where  $\alpha$  is a positive constant less than or equal to one, and  $\gamma$  is another constant representing factors other than income which influence the growth rate of  $Z_t$ . Thus equation (22) means that  $z$  is now determined by a weighted average of income growth, represented by  $g_{t-1}$ , and other factors which are represented by  $\gamma$ ; the higher the value of  $\alpha$ , the more of a role income will play in determining  $z_t$ . Following Fiebiger and Lavoie (2019), I refer to this as *semi-autonomous* demand. The equilibrium is now characterized by  $g_t = g_t^e = z_t = \gamma$ . It is straightforward (see the appendix) to show that higher values of  $\alpha$  will make the stability condition harder to satisfy. In other words, the more  $Z_t$  is influenced by income flows, the less likely it is that the supermultiplier will be stable.

It should be acknowledged here that equation (22) is—like equation (9), which assumes that  $Z$  grows at a constant rate—a simplification of reality. The truth is that a variety of different variables must jointly determine the value of  $Z_t$ , and the strength of these influences can fluctuate over time. However, to the extent that some components of semi-autonomous demand tend to be influenced by income, equation (22) provides a



simple way to quantify the typical size of that effect by drawing on existing econometric studies. For example, economists working in the supermultiplier tradition sometimes treat consumer durables spending as a type of autonomous demand, but Baghestani and Fatima (2021) estimate that in the US, a 1% increase in real income tends to generate a 0.5% increase in consumer durables spending in the next quarter. Equation (22) provides a straightforward way to estimate whether these sorts of effects are strong enough to be destabilizing.

Related points can be made regarding government spending. Heimberger (2023) estimates that in developed countries, the average elasticity of government spending growth with respect to GDP growth is not statistically different from zero, while for developing countries it is 0.4, suggesting that, depending on the context, an equation like (9) or (22) might provide a useful tool for investigating the way fiscal policies in use by different governments affect macroeconomic stability. It would also be possible to incorporate counter-cyclical policies into this framework, which can stabilize supermultiplier-based models that would otherwise be unstable (Skott, Santos and da Costa Oreiro, 2022). In this respect, nonlinearities in fiscal policy may be especially important (Skott, 2023, p.68). In short, equations (9) and (22) may be useful for assessing stability in particular contexts, but they represent simplifying assumptions, and results based on these assumptions must be interpreted with some caution.

There has also been significant debate about these issues in relation to residential investment. In defending the assumption that residential investment is autonomous, Summa, Petrini and Teixeira (2023) point out that the real estate market is influenced by institutions, and argue that, in contrast to non-residential investment, there is no a priori reason why the demand for new homes should be systematically related to GDP. But Nikiforos, Santetti and von Arnim (2023) counter that residential investment is largely undertaken by businesses, rather than households, and will therefore be influenced by firms' expectations about future demand; based on this, they challenge the idea that residential investment is autonomous. In a similar vein, Skott (2019) questions the notion that decisions to purchase new homes could be unrelated to income. Moreover, as Petrini and Teixeira (2023, p. 707-709) discuss in their literature review, there is some evidence that income positively influences residential investment. To the extent that this is the case, it will make stability in supermultiplier models harder to ensure.

An interesting and related issue has to do with the way that crises are represented in the supermultiplier framework. Ultimately, the point of the stability results above is that autonomous demand tends to play a decisive role in setting the rate of growth for capitalist economies, and so if major downturns *do* occur, then fluctuations in the growth of autonomous demand must be the reason (Pariboni, 2016, p. 222). In this spirit, Dejuán and Dejuán-Bitriá (2022) build a supermultiplier model which focuses on the role of the housing market in the 2008 crisis. Their analysis assumes that the supermultiplier is stable, and based on this, they treat the non-residential investment share as a constant, while assuming that residential investment is autonomous. Yet their analysis also emphasizes the way that a wave of layoffs can cause disruptions in the financial system which, in turn, cause residential investment to decline; this calls into question the idea that residential investment is autonomous in the first place. In short, to

the extent that semi-autonomous expenditures interact with other variables, this may complicate the analysis of crises in the supermultiplier framework.

Finally, it should be noted that there is evidence to support the idea that significant components of aggregate demand *are* essentially autonomous, and that they drive long-run economic growth. For example, Perez-Montiel and Manera (2022), looking at data for the US, define autonomous demand as the sum of exports, government spending and residential investment; they find unidirectional Granger causality going from autonomous demand to output in both the short run and the long run. Pérez-Montiel and Pariboni (2022) look at residential investment in the US and obtain a similar result. Girardi and Pariboni (2020) find evidence that autonomous demand drives the investment share in OECD countries. Thus, despite facing criticism, concepts related to autonomous demand do have some empirical support. With these preliminaries established, it is now possible to examine the plausibility of the stability assumption that underpins supermultiplier theory.

### 3. Is the supermultiplier stable in the US economy?

#### 3.1. Evidence for stability

Just how plausible are the stability conditions laid out in the previous section? Haluska, Braga and Summa (2021, henceforth HBS) try to answer this question by looking at quarterly data for the US economy covering the period from 1985 to 2017. To assess stability, they rearrange the condition (21), to obtain

$$z < \frac{s - v\delta - vx}{v(1 + x)}. \quad (23)$$

The condition (23) shows that the supermultiplier will be stable if the long-run rate of growth,  $z$ , is beneath the threshold value  $\frac{s - v\delta - vx}{v(1 + x)}$ . Therefore, if estimated parameter values cause the right-hand side of (23) to be well above an economy's actual long-run rate of growth, then this supports the idea that the supermultiplier is stable.

To estimate the relevant parameters, HBS observe that, after combining equations (14) and (15), one obtains

$$h_t = vx\delta + (1 - x)h_{t-1} + vxg_{t-1}. \quad (24)$$

In the above equation, all variables and parameters have the usual meanings from the previous section, so (given quarterly data)  $h_t$  is the investment share at time  $t$ ,  $g_t$  is the quarterly growth rate for the period which begins at time  $t - 1$  and ends at time  $t$ , etc.

Based on this, HBS estimate an equation of the form

$$h_t = \theta_0 + \theta_1 h_{t-1} + \theta_2 g_{t-1}^a \quad (25)$$

where  $h_t$  is the ratio of private non-residential fixed investment in equipment and structures to GDP in the US economy at time  $t$ ,  $g_{t-1}^a$  represents the *annual* growth rate of real GDP for the year ending at time  $t - 1$ , and  $\theta_0, \theta_1$  and  $\theta_2$  are parameters. Based on their econometric analysis, they infer that the value of  $x$  for the US economy is 0.091.

For the purposes of their stability analysis, HBS assume that exports, government spending, residential investment, and non-residential investment in intellectual property products are all autonomous, but household consumption (including consumer durables spending; see HBS's footnote 18) is induced. Based on this, they find a consumption propensity of  $c = 0.701$ , a tax rate of  $\tau = 0.171$ , an import propensity

of  $\mu = 0.131$ , a normal capital-output ratio of  $v = 0.826$ , and a depreciation rate of  $\delta = 8.4\%$  per year.<sup>1</sup> It follows (from equation (7)) that  $s$  is 0.57. When these values are plugged into the inequality (23), the right-hand side becomes 47.2%, which is the value reported by HBS. They conclude that, because the actual GDP growth rate in the US has remained far below 47.2% per year, the supermultiplier is stable.

However, there are problems with this argument. They can be grouped into three categories: the adjustment speed for the investment function calibrated by HBS, the classification of different types of investment, and feedback effects between the US and the rest of the world. Once these issues are accounted for, it becomes plausible for the supermultiplier to be unstable.

### 3.2. The adjustment speed for the investment function

To begin, it is important to note that HBS's mix of quarterly data and annual growth rates leads to a model which has stability properties that are not the same as the one derived in Section 2. As a result, one cannot determine the stability of the supermultiplier simply by plugging HBS's values into the condition (23). When this issue is accounted for, the maximum stable growth rate for the model falls significantly below the 47.2% per year value obtained by HBS.

Before going further, it is necessary to derive a formal model consistent with equation (25). To this end, note that  $g_t^a$ , which denotes the annual growth rate during the year ending at time  $t$ , is related to the per-quarter growth rates  $g_t, g_{t-1}, g_{t-2}$  and  $g_{t-3}$  as follows:

$$1 + g_t^a = 1 + \frac{Y_t - Y_{t-4}}{Y_{t-4}} = (1 + g_t)(1 + g_{t-1})(1 + g_{t-2})(1 + g_{t-3}). \quad (26)$$

HBS establish a relationship between the annual growth rate  $g^a$  and the expected annual growth rate  $g^{e,a}$  of the form:

$$g_t^{e,a} = g_{t-1}^{e,a} + x(g_{t-1}^a - g_{t-1}^{e,a}). \quad (27)$$

They also set:

$$h_t = v(g_t^{e,a} + \delta). \quad (28)$$

It follows that the per-period (i.e., quarterly) growth rate of real GDP is:

$$g_t = z + \frac{(1 + z)vx(g_{t-1}^a - g_{t-1}^{e,a})}{s - v(g_t^{e,a} + \delta)}. \quad (29)$$

Although equations (27) and (29) superficially resemble equations (14) and (17) from Section 2, there is an important difference:  $g_{t-1}^a$  is not a per-period growth rate, but instead an annual growth rate which depends on the values of the quarterly growth rates during the previous three periods. As a result, equations (27) and (29) must be considered as part of a five-dimensional dynamical system involving the state variables  $g_t, g_{t-1}, g_{t-2}, g_{t-3}$  and  $g_t^{e,a}$ .

Although it is relatively difficult to comprehensively analyze the stability properties of a five-dimensional dynamical system, one can use a computer to easily determine

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<sup>1</sup> HBS's tax parameter  $\tau$  corresponds to federal taxes in the US. If state and local taxes were also included, then this would increase the value of  $\tau$ , but would also make it necessary to re-calculate the induced consumption share (Haluska, Braga and Summa, 2021, p. 353 footnote). For the purposes of the stability analysis, these two changes would essentially cancel each other out, so I leave  $\tau$  as is.

whether a given set of parameters makes the model stable or unstable. Indeed it is straightforward to check the stability of the model for a range of different growth rates, taking the other parameters to have the values given by HBS. I report results from this exercise in the appendix. The highest growth rate consistent with stability shrinks from 47% per year, the value reported by HBS, to about 20% per year.

The underlying issue can be understood as follows. To find the maximum annual growth rate consistent with stability, HBS plug annual values<sup>2</sup> for the normal capital-output ratio and the depreciation rate into the condition (23), together with their estimated value for  $x$  and the other parameters. In effect, because they use annual values, they are checking stability conditions for a model in which expectations only update once per year. But their equation (25) iterates every *quarter*, and therefore the cumulative change in expectations over the course of a year is not fully captured by the size of the parameter  $x$ . As a result, in using the inequality (23) to check stability, HBS *understate* the speed with which expectations adjust, and thus *overstate* the stability of the supermultiplier.

### 3.3. Intellectual property investment

A second, and related, issue concerns the classification of different types of investment spending. HBS assume that non-residential spending on equipment and structures is determined by an investment accelerator (equations (27) and (28)), but argue that investment in intellectual property products (IPP) should be considered autonomous. This is important because if IPP investment is *not* autonomous, and is instead induced like the other two categories of non-residential fixed investment, then this will increase the value of the induced capital-output ratio,  $v$ . As discussed in Section 2, higher values of  $v$  would mean a stronger accelerator effect relative to GDP, and this would make it harder to ensure stability.

IPP investment includes research and development (R&D) spending and investment in software, as well as the creation of artistic, literary and entertainment products. Scholars working in the supermultiplier tradition sometimes classify R&D as a type of autonomous demand, on the grounds that it is a discretionary expenditure (Serrano, 1995, p. 71). HBS argue that, in the US, firms' investment in software can be considered autonomous as well, because "this segment of investment presents a growth rate far above the average GDP growth and its share on output increases practically uninterruptedly from 0.6% in 1985 to 1.8% in 2017, which might represent some sort of structural change rather than an investment induced by demand" (Haluska, Braga and Summa, 2021, p. 352).

However, there are theoretical and empirical reasons to believe that firms' IPP spending is linked with expectations regarding future output. First of all, as with machinery and other fixed-capital equipment, R&D expenditures increase potential output through their effects on productivity (Zachariadis, 2004). And R&D spending is particularly critical for creating the output of the technology sector (Miller, 2022). Investments in software, on the other hand, are necessary for managing production

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<sup>2</sup> Notice that because these parameters relate stocks with flows, their values depend on the size of the period; the annual value for  $v$  is smaller than the quarterly value of that parameter. Of course, parameters such as  $\mu$ , which relate flows with flows, do not depend on the length of the period.

when the scale of output increases (Beniger, 2009). Software and related investments have important impacts on productivity as well (Relich, 2017).

Thus, IPP spending co-determines productive capacity along with other types of fixed investment, which suggests that, when changes in market conditions prompt firms to adjust their spending on equipment and structures, similar motivations should lead them to also adjust the level of IPP investment. This is corroborated by McGrattan and Prescott's (2014, p. 181) finding that microeconomic-level IPP spending is "highly correlated" with other non-residential fixed investment in the United States. Indeed, they note, after dividing investment data for computer and information sectors by a trend, that: "Spending on software and R&D grew rapidly in the 1990s during the technology boom. This peaked in 2000 and has subsequently fallen, then risen, then fallen again in the 2008–2009 downturn. The series for equipment is very similar" (ibid.). With all this in mind, I take total non-residential fixed investment (i.e., firms' equipment, structures and IPP spending) to be induced in the US.

For the US economy during the past four decades, the average value for the ratio of non-residential fixed assets to GDP has been about 1.2 (Fazzari, Ferri and Variato, 2020, Supplementary Appendix). If one sets  $\nu = 1.2$ , assumes a steady-state growth rate of 2% per year, and retains HBS's other parameter values discussed above, the model described by equations (26) through (29) will still be stable. But it will now be close to the stability limit: as I show in the appendix, a 5% annual growth rate is sufficient to generate instability.

The category of induced investment could also potentially be expanded even further. Maccini and Rossana (1981) present evidence that inventory investment in the US is governed by a flexible accelerator with a rapid adjustment speed. Incorporating this into the supermultiplier model would mean further expanding the GDP share of induced investment, and reducing the maximum stable growth rate a bit more.

### *3.4. Interactions with the rest of the world*

A third issue concerns the dynamics of export demand for the US economy. Recall that HBS classify export demand as an autonomous expenditure, and their estimates regarding stability are based on the assumption that autonomous demand is completely unaffected by other variables. Thus, their calculations implicitly treat the US as a small open economy.

But in fact, due to its role in global trade, investment, and finance, the US economy has an important impact on the rest of the world. Arora and Vamvakidis (2006), seeking to quantify this impact, estimate that changes in the growth rate for the US economy are eventually associated with one-for-one changes in the growth rate of the rest of the world. Similarly, Kose, Lakatos, Ohnsorge and Stocker (2017) estimate that a 1% increase in US GDP is associated with a 0.7% increase in the rest of world's GDP after four quarters.

To represent these interactions in the model, let  $Y^\#$  denote GDP in the rest of the world, and let  $\mu^\#$  denote the rest of the world's import propensity, defined in analogy with equation (2) above. I will assume that relative prices do not change. Demand for US exports is then:

$$X = \frac{\mu^\#}{1 - \mu^\#} Y^\#. \quad (30)$$

On the other hand, to account for the issues raised in the previous paragraph, one can assume:

$$\widehat{Y}_t^\# = \epsilon_0 + \epsilon_1 g_{t-4} \quad (31)$$

where  $\epsilon_0$  and  $\epsilon_1$  are parameters,  $g_{t-4}$  is the US GDP growth rate four quarters ago, and  $\widehat{Y}_t^\#$  is the growth rate of the rest of the world during quarter ending at time  $t$ . I set

$$\epsilon_1 = 0.7, \quad (32)$$

based on Kose, Lakatos, Ohnsorge and Stocker (2017, p. 14). Although these equations are admittedly a crude representation of the global economy, they are in keeping with the limited purpose of this exercise, which is to capture the typical size of the feedback effects between income in the US and the rest of the world.

Now, consider what all this means for  $Z$ , the variable representing total semi-autonomous demand. Taking growth rates of both sides of equation (30) yields  $\widehat{X} = \widehat{Y}^\#$ , and combining this with (31) gives the equation

$$\widehat{X}_t = \epsilon_0 + \epsilon_1 g_{t-4}. \quad (33)$$

Let  $Z^{\text{home}}$  denote the sum of all semi-autonomous expenditures except for exports, so  $Z = Z^{\text{home}} + X$ , and let  $\widehat{Z}_t$  and  $\widehat{Z}_t^{\text{home}}$  denote the growth rates of  $Z_t$  and  $Z_t^{\text{home}}$ , respectively, during the period ending at time  $t$ . The equation for the growth rate of  $Z$  becomes:

$$\widehat{Z}_t = \frac{Z_{t-1}^{\text{home}}}{Z_{t-1}} \widehat{Z}_t^{\text{home}} + \frac{X_{t-1}}{Z_{t-1}} \widehat{X}_t. \quad (34)$$

Finally, I will continue to assume that  $Z^{\text{home}}$  is completely autonomous, and therefore set

$$\widehat{Z}_t^{\text{home}} = \gamma, \quad (35)$$

where  $\gamma$  is a constant.<sup>3</sup> Thus equation (34) becomes

$$\widehat{Z}_t = \frac{Z_{t-1}^{\text{home}}}{Z_{t-1}} \gamma + \frac{X_{t-1}}{Z_{t-1}} (\epsilon_0 + \epsilon_1 g_{t-4}). \quad (36)$$

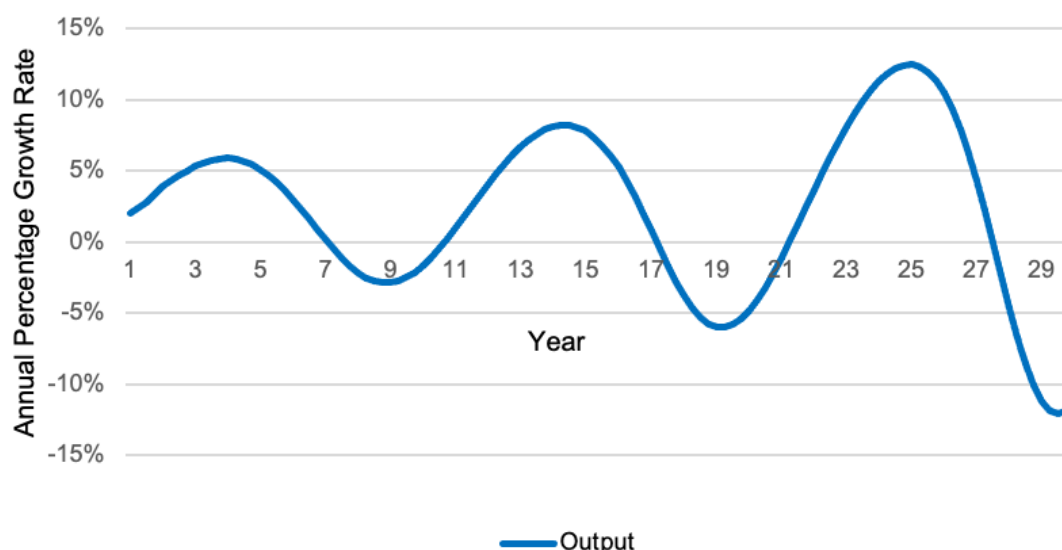
This is essentially a more complex version of equation (22), with  $\frac{X_{t-1}\epsilon_1}{Z_{t-1}}$  taking the place of  $\alpha$ .

It is now possible to see the implications of all this for the stability of the supermultiplier in the US. Figure 1 shows a simulation of the model based on equations (26) through (29), in which  $v = 1.2$ , the growth rate of semi-autonomous demand is determined by equation (36), steady-state output growth is 2% per year, and the other parameters are equal to the values chosen by HBS. In this specification of the model, changes in output have a relatively modest impact on  $Z$ ; the ratio of  $X$  to  $Z$  stays at about 1/3, and this fact in combination with equation (32) means that a one percent increase in GDP will generate a roughly 0.23% increase in  $Z$  after four quarters. Nonetheless, this is sufficient to destabilize the model. As one can see in Figure 1, after

<sup>3</sup> Equation (35) implicitly assumes that the sum of government spending and residential investment is independent of GDP. But as discussed in Section 2, residential investment may respond to changes in income. In this sense, equation (36) represents a conservative estimate for the effect that output changes have on total semiautonomous demand.

the initial conditions are disturbed from the equilibrium, the model generates business cycle fluctuations that become increasingly amplified over time.

**Figure 1.** A simulation of the supermultiplier model, using equations (26) through (29). All non-residential fixed investment is induced ( $v = 1.2$ ), and US GDP influences GDP in the rest of the world (equation (36) with  $\epsilon_1 = 0.7$ ,  $\gamma = 0.005$ ,  $\epsilon_0 = (1 - \epsilon_1)\gamma$ ).



It is notable that, although exports only depend on output values from several periods in the past, this still affects stability in a significant way. These sorts of effects could be important for other autonomous expenditure components as well. For example, Serrano (1995, p. 73) discusses the idea that some components of autonomous demand are paid for using stocks of existing financial assets. But these stocks of financial assets themselves will depend, partly, on income from previous periods. To the extent that home loans require a significant initial down payment, these issues could be especially relevant for residential investment.

### 3.5. Summary

Economists have argued that semi-autonomous expenditures, when interacting with an investment accelerator, can stabilize the US economy along an equilibrium growth path. But the evidence is not convincing. This is due to a combination of issues, which are summarized in Table 1. As the bottom row of the table shows, if past income has a modest impact on total semi-autonomous demand (equation (36)), and all of non-residential fixed investment is induced ( $v = 1.2$ ), then this is sufficient to destabilize the model. Other factors that have been left out of the model, such as inventory investment, and the possible effects of income on residential investment, could further reduce the stable parameter range.

These results come with caveats. First, because there is a lack of certainty regarding some key parameter values, it is not possible to say definitively that the supermultiplier is unstable in the US. Second, counter-cyclical policies, and feedback

effects between output and employment, could potentially change the stability properties of the model in important ways (Skott 2023). And obviously there are floors and ceilings for the level of economic activity which would generally prevent macroeconomic fluctuations in the US from being as large as the ones shown in Figure 1. But based on the existing evidence, it is difficult to feel confident that the interactions between income, the investment accelerator, and semi-autonomous expenditures, on their own, can guide the US economy toward an equilibrium growth path.

**Table 1.** Specifications of the model that generate instability. Except where noted, all parameters are set to the values chosen by Haluska, Braga and Summa (2021).

Capital-Output Ratio	Annual Steady-State Rate of GDP Growth	Dependence of (Semi-) Autonomous Demand on GDP
$v = 0.83$	21%	Not dependent on GDP
$v = 1.2$	5%	Not dependent on GDP
$v = 1.2$	2%	Equation (36) with $\epsilon_1 = 0.7, \gamma = 0.005, \epsilon_0 = (1 - \epsilon_1)\gamma$

## 4. Discussion

### 4.1. Adjustment speeds for investment functions

Economists working in the Sraffian and Post Keynesian traditions have formulated a series of models that combine an investment accelerator effect and an autonomous expenditure component. The assumptions vary, but, as Skott, Santos and da Costa Oreiro (2022) point out, the stability conditions across different models tend to be very similar to the one for the basic supermultiplier model discussed in Section 2. Since the adjustment speed for business expectations plays such a critical role in these stability conditions, it would be useful to have a generally accepted benchmark value.

Motivated by this, I will now draw upon Haluska, Braga and Summa's (2021) econometric results to estimate the annual adjustment speed for business expectations in the US economy. Recall from the previous section that Haluska et al. calibrate a model based on equation (27), with quarter-year periods, and obtain the per-period adjustment parameter  $x = 0.091$ . To interpret this, and following Skott (2017), one can consider the hypothetical situation in which GDP grows at a constant rate  $\bar{g}$ , and then ask how quickly firms' expected growth rate would catch up to the actual one. In this situation, equation (27) implies that:

$$g_t^{e,a} - \bar{g} = (1 - x)^k (g_{t-k}^{e,a} - \bar{g}) \quad (37)$$

for each positive integer  $k$ . Setting  $k = 4$  and  $x = 0.091$ , a simple calculation then shows:

$$g_t^{e,a} - \bar{g} = (1 - 0.32)(g_{t-4}^{e,a} - \bar{g}). \quad (38)$$



It follows that firms close 32% of the gap between actual and expected growth per year.<sup>4</sup> After two years, firms would close just over half the gap, which is in line with Skott's (2017) suggestion.

Interestingly, Haluska, Summa and Serrano (2023) show that, if a nearly identical adjustment speed ( $x = 0.316$ ) is used, then the capital-stock adjustment process described by equations (14) through (16) can approximately replicate the dynamics of industrial capacity in the US economy. More specifically, they use a model which has one-year periods and is equivalent to the following system:

$$u_t = u_{t-1} \left( \frac{1 + g_t}{1 + g_t^{Y^*}} \right) \quad (39)$$

$$g_{t+1}^{Y^*} = \frac{u_{t-1}}{u^d} (g_{t-1}^e + \delta) - \delta \quad (40)$$

and

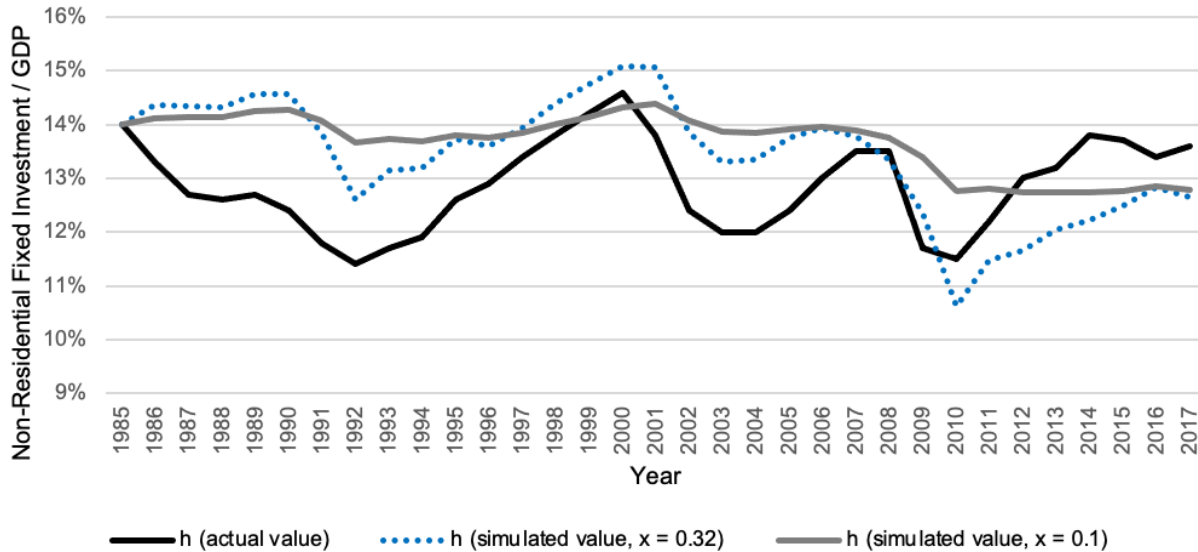
$$g_t^e = g_{t-1}^e + x(g_{t-1} - g_{t-1}^e). \quad (41)$$

In the above equations, the variables have the same meanings as before, with  $g_t^{Y^*}$  representing the annual growth rate of productive capacity. During each period, Haluska, Summa and Serrano plug the actual US GDP growth rate into the model, and find that the dynamics of the resulting utilization rate are similar to the actual utilization rate for industrial capacity in the US economy. This adds further support to the idea that

$$x = 0.32 \quad (42)$$

is a reasonable estimate of the adjustment speed for expectations.

**Figure 2.** Simulated and actual investment shares for the US economy. Source: US Bureau of Economic Analysis and author's calculations.



<sup>4</sup> Unsurprisingly, if the annual value  $x = 0.32$  is plugged into the stability conditions for the two-dimensional system presented in Section 2, with periods specified to last one year, then the results will closely approximate the properties of the more complex five-dimensional model discussed in Section 3 with quarterly periods. For example, as discussed in Section 3, the simulation shown in Figure 1 implies an  $\alpha$  value of about 0.23. For the model in Section 2, if periods last one year, with  $v = 1.2$ ,  $\delta = 0.083$ ,  $x = 0.32$  and  $\alpha = 0.23$ , then the maximum stable annual growth rate is slightly below 1% per year, which is consistent with what occurs in the simulations of the more complex model.

Finally, Figure 2 provides a way to visualize all this. Along with the actual non-residential fixed investment share (equipment, structures and IPP) for the US economy from 1985 to 2017, the figure shows the results from two simulations of the capital stock adjustment process (equations (14) and (15)), both of which assume  $\nu = 1.2$  and  $\delta = 0.084$ . The approach is analogous to the one used by Haluska, Summa and Serrano (2023): the model is set up with one-year periods, and each year, the previous year's GDP growth rate for the US economy is plugged into the equations to calculate the predicted investment share.

Although the fit is far from perfect, the figure shows that if the adjustment speed is set to  $x = 0.32$ , there is a clear correspondence between the actual and simulated fluctuations of the investment share. On the other hand, when the adjustment speed is set to  $x = 0.1$ , the investment share is largely unresponsive to changes in the state of the economy; if firms only closed 10% of the gap between actual and expected growth per year, the length and depth of the typical recession simply would not be sufficient to provoke a significant change in firms' investment policies. One can also see that there is a long-run increase in the actual investment share relative to the simulated ones, which may be due, at least in part, to increases in the actual values of  $\nu$  and/or  $\delta$ .<sup>5</sup> But while technical change and other factors undoubtedly play a role in the actual dynamics of investment, the observed cyclical fluctuations are consistent with the idea that firms close around 32% of the gap between actual and expected growth per year.

#### *4.2. Stability in other supermultiplier models*

In recent years, multiple papers have set up stable supermultiplier-type models with parameters based on the US economy. However, these papers rely on the idea that investment reacts very slowly to changes in demand. Here, I discuss two examples which illustrate the relevance of the points made above to the wider literature.

First, consider Allain's (2022) supermultiplier model. In his formulation, the growth rate of the capital stock is a function of the capacity utilization rate and firms' long-run expectations, giving rise to an investment equation which is similar to equation (16) but has a different functional form. Allain assumes that firms close 12% of the gap between the actual and expected growth rates per year, and sets the other parameters to benchmark values for the US economy. These assumptions imply that the model is stable.

To justify the slow adjustment speed, Allain relies on a theoretical argument by Freitas and Serrano (2015). But, although Freitas and Serrano's argument says that investment will react gradually to changes in demand, it does not follow that the adjustment speed has to be as low as the 12% per year value chosen by Allain. Moreover, based on the discussion above, it may be more realistic to assume that firms

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<sup>5</sup> Duménil and Lévy (2016) find that the long-run ratio of net domestic product to non-residential fixed assets for the US economy started falling in 2004. This suggests that the actual values for  $\nu$  and/or  $\delta$  increased at around that time, which would cause the actual investment share to rise in relation to simulated values that assume  $\nu$  and  $\delta$  are constant. This is consistent with what occurs in Figure 2. Similar considerations suggest that the true values of  $\nu$  and/or  $\delta$  were lower from 1986 to 2004.

close around 32% of the expectations gap per year. This faster adjustment speed would violate Allain's stability condition.<sup>6</sup>

As a second example, consider the framework developed by Fazzari, Ferri and Variato (2020). They calibrate a relatively complex supermultiplier model for the US economy, and find that it is stable. Their model is set up in discrete time, with one-year periods and an expectations adjustment process like equation (14). Their estimated parameter values imply that each year, firms close 10% of the gap between actual and expected growth.

To estimate the adjustment speed for expectations, Fazzari and co-authors use data from surveys of professional forecasters' estimates of future GDP growth in the US. But it is not clear how closely aggregate forecasts by economists will correspond to firms' micro-level expectations regarding the demand for their own products. And if the adjustment speed is increased to 32% per year, this brings the model close to the stability threshold.<sup>7</sup> Additionally, the authors do not consider the possible lagged effects of output on autonomous spending which, as shown in the previous section, can make supermultiplier models unstable.

In summary, the stability question should be regarded as an unresolved problem for supermultiplier theory. Based on the existing evidence, instability is not a guaranteed outcome, but it is certainly a plausible one when supermultiplier models are calibrated with US data.

#### *4.3. What if the supermultiplier is unstable?*

What does it mean for supermultiplier theory if the stability assumptions are dropped? Fazzari, Ferri and Variato (2020) show that, when their supermultiplier model is unstable, autonomous demand can create a floor for the level of economic activity, while supply-side limits can impose a ceiling. This forms the basis for a theory of cyclical growth in which, although aggregate demand plays a key role in the dynamics, the long-run rate of expansion is determined by the maximum feasible growth rates for productivity and the labor force.

However, it is also possible that aggregate demand problems initiate recessions before supply-side limits are reached. This raises the prospect of an unstable, cyclical growth process in which the long-run trend is determined on the demand side of the economy. Godley's (1999) work is relevant here. He argues that, for given levels of government spending, exports, the tax rate and the import share, there is an upper limit on how much it is financially sustainable for the domestic private sector of a country to spend. This limit, in turn, imposes a constraint on the growth of aggregate demand, and Godley provides data from the US economy to illustrate the way this works in practice. Although credit-driven spending binges can push GDP beyond this constraint for temporary periods, the implied levels of indebtedness ultimately lead to a "sensational day of reckoning" if the process continues for too long (Godley, 1999, p. 230).

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<sup>6</sup> Condition (12) in Allain (2022), together with the model's other parameter values, imply that if the adjustment speed  $\psi$  is set to 0.32, then the maximum stable growth rate will be about 1% per year.

<sup>7</sup> Table 2 in Fazzari, Ferri and Variato (2020) shows that when parameters are set to benchmark values, the model becomes unstable if firms close 36% of the gap between expected and actual growth each year (note that their  $\alpha$  parameter is equal to  $1 - x$  in the present paper).

To understand the logic of Godley's argument, it is necessary to establish some additional definitions. While continuing to use the notation from the rest of the paper, I also introduce a new variable,  $\bar{\tau}_t$ , to represent the tax rate net of transfers, and a new parameter,  $\bar{\mu}$ , which represents the import share of GDP.<sup>8</sup> Following Godley (1999, p. 223), I define the *combined fiscal and trade ratio* (CFTR) to be:

$$\frac{G_t + X_t}{\bar{\tau}_t + \bar{\mu}}. \quad (43)$$

Now, equation (1) can be rewritten as  $Y_t = C_t + I_t + G_t + X_t - \bar{\mu}Y_t$ , which, after some algebraic manipulation, becomes:

$$[\bar{\tau}_t Y_t - G_t] + [\bar{\mu} Y_t - X_t] = C_t + I_t - (1 - \bar{\tau}_t) Y_t. \quad (44)$$

Equation (44), of course, describes a well-known relationship between the balances of the public sector ( $\bar{\tau}_t Y_t - G_t$ ), the foreign sector ( $\bar{\mu} Y_t - X_t$ ), and the domestic private sector ( $(1 - \bar{\tau}_t) Y_t - C_t - I_t$ ). The key thing to notice is that whenever  $Y_t$  exceeds the CFTR, the expression  $[\bar{\tau}_t Y_t - G_t] + [\bar{\mu} Y_t - X_t]$  will be positive, and this, when combined with equation (44), will imply:

$$(1 - \bar{\tau}_t) Y_t < C_t + I_t. \quad (45)$$

In other words: GDP can only exceed the CFTR if the domestic private sector's spending on goods and services exceeds the share that it receives of GDP. Godley argues that this not financially sustainable, and that as a result, GDP will not *permanently* grow more quickly than the CFTR. Thus, demand-side issues can impose a ceiling on the rate of long-run growth.

Some further insight can be gained by representing these interactions as a dynamical system. In the simplest case, consumer spending *instantaneously* adjusts to keep net financial wealth at a constant level in relation to income, while exports and government spending grow at a constant rate  $\gamma$ . The resulting model is globally stable, and the output growth rate converges to  $\gamma$ , which connects with Luxemburg's (1913) idea that exports and government spending serve as "external markets" which sustain the long-run expansion of capitalism (Thompson, 2020). It is also possible to consider a more general case, in which exports and government expenditures may or may not be completely autonomous, and consumer spending decisions may react slowly to changes in financial stock-flow ratios. In this more general scenario, the model will be unstable if the investment accelerator effect is sufficiently strong, but the mechanism described in the previous paragraph still impose a ceiling on the level of economic activity, and as a result, semi-autonomous external markets will determine the long-run trend rate of output expansion (Thompson 2022). Business investment decisions create perpetual booms and crashes along this trend, but the trend itself can be understood in terms of simple formulas that determine the long-run average behavior of the model (ibid.).

In short, if conditions for local stability do not hold, then long-run growth will depend on the floors and ceilings for the level of economic activity. In models that combine an investment accelerator with a semi-autonomous expenditure component, either supply-side ceilings or demand-side ceilings could end up being more important.

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<sup>8</sup> In terms of the notation established already, this means  $\bar{\tau}_t = \tau - \frac{Tr_t}{Y_t}$  and  $\bar{\mu} = \frac{\mu}{1-\mu}$ . Note that, since  $\bar{\tau}_t$  is the tax rate net of transfers,  $(1 - \bar{\tau}_t) Y_t$  is equal to disposable income as defined in equation (3).

But in either case, the absence of local stability means that firms' investment decisions can play a non-negligible role in driving cycles.

## 5. Conclusion

The supermultiplier framework explains economic expansions and crashes in terms of changes in autonomous variables. However, to the extent that these variables are actually influenced by the state of the economy, their own dynamics may be jointly determined with other processes. This raises the possibility that business investment decisions, GDP, and semi-autonomous demand, by interacting with each other, can create unstable cyclical dynamics.

This possibility could be ruled out if business investment reacted sufficiently slowly to changes in GDP. And indeed this type of stability assumption has been defended in the supermultiplier literature. But in this paper, I argued that the evidence for the stability assumption is unconvincing. Investment dynamics in the US are consistent with an expectations adjustment process in which firms close about 32% of the gap between actual and expected growth per year; this entails a robust investment accelerator effect, and it follows that, under plausible assumptions, the supermultiplier can be unstable.

These results do not definitively rule out the stability of the supermultiplier, because there is still uncertainty about the strength of the interactions between various components of the aggregate demand. But if the slow adjustment speeds found in the literature are considered essential to the supermultiplier framework, then they need to be given firmer empirical support. Alternatively, if the stability assumptions are dropped, then there should be more research into which floors and ceilings are most important for containing macroeconomic fluctuations, since these will play a potentially decisive role in setting long-run rates of growth.

## Appendix

### A.1. Basic equations

This paper sets up several different variants of the supermultiplier framework, but all of them can be written in the form

$$Y_t = (1 - s)Y_t + h_t Y_t + Z_t,$$

where  $s$  is some positive constant. Obviously, no matter what assumptions are made regarding  $h_t$  or  $Z_t$ , as long as  $h_t \neq s$  one has

$$Y_t = \frac{Z_t}{s - h_t}.$$

To avoid the possibility of output becoming negative, one must assume that parameters are such that, along the equilibrium growth path,  $s > h_t$ .

Now, setting  $g_t = (Y_t - Y_{t-1})/Y_{t-1}$ , and  $z_t = (Z_t - Z_{t-1})/Z_{t-1}$ , and then doing a little algebra, one has

$$g_t = z_t + \frac{(1 + z_t)(h_t - h_{t-1})}{s - h_t}.$$

This equation is valid for all of the different variants of the supermultiplier model presented in this paper. Thus, it will be used as a basis for deriving results concerning the different special cases.

### A.2. Results from Section 2

Let us now consider the version of the model in which  $z_t$  is determined by equation (22). The system is characterized by the two equations

$$g_t = \alpha g_{t-1} + (1 - \alpha)\gamma + \frac{(1 + \alpha g_{t-1} + (1 - \alpha)\gamma)vx(g_{t-1} - g_{t-1}^e)}{s - v(g_{t-1}^e + \beta(g_{t-1} - g_{t-1}^e) + \delta)}$$

and

$$g_t^e = g_{t-1}^e + x(g_{t-1} - g_{t-1}^e).$$

At the equilibrium point, which is characterized by  $g_t = g_t^e = \gamma$ , the Jacobian matrix is

$$J = \begin{bmatrix} \alpha + \frac{(1 + \gamma)vx}{s - v(\gamma + \delta)} & -\frac{(1 + \gamma)vx}{s - v(\gamma + \delta)} \\ x & 1 - x \end{bmatrix},$$

and the stability conditions are:

$$\text{Det}(J) < 1$$

$$\text{Tr}(J) - \text{Det}(J) < 1$$

$$\text{Tr}(J) + \text{Det}(J) > -1$$

One has  $\text{Det}(J) = \alpha(1 - x) + \frac{(1 + \gamma)vx}{s - v(\gamma + \delta)}$  and  $\text{Tr}(J) = \alpha + \frac{(1 + \gamma)vx}{s - v(\gamma + \delta)} + 1 - x$ . The second two stability conditions are satisfied automatically (keep in mind that  $s > v(z + \delta)$ ,  $0 < x < 1$ , and  $0 \leq \alpha < 1$ ). Finally,  $\text{Det}(J) < 1$  is equivalent to the condition

$$\alpha(1 - x) + \frac{(1 + \gamma)vx}{s - v(\gamma + \delta)} < 1.$$

If  $\alpha = 0$ , this reduces to the original stability condition (21). More generally, the larger the value of  $\alpha$ , the harder it will be to satisfy the above inequality. Thus, the more  $z$  depends on income flows, the less likely it is that the model is stable.

### A.3. Results from Section 3

As discussed in Section 3, Haluska, Braga and Summa (2021) calibrate a supermultiplier model in which time is subdivided into quarter-year periods, and the induced investment share  $h_t$  is determined by the equation

$$h_t = v(g_t^{e,a} + \delta).$$

In this equation,  $h_t$  denotes the induced investment share during the quarter beginning at time  $t$ ,  $g_t^{e,a}$  denotes the expected *annual* growth rate,  $v$  is the targeted ratio of productive capacity to *annual* GDP, and  $\delta$  is the *annual* depreciation rate. As before, I let  $g_t$  denote the quarterly GDP growth rate for the quarter beginning at time  $t - 1$  (and ending at time  $t$ ), and I let  $g_t^a$  denote the actual annual GDP growth rate for the year beginning at time  $t - 4$  (and ending at time  $t$ ). One obtains the system of equations

$$g_t = z + \frac{(1 + z)vx(g_{t-1}^a - g_{t-1}^{e,a})}{s - v(g_t^{e,a} + \delta)}$$

$$g_t^{e,a} = g_{t-1}^{e,a} + x(g_{t-1}^a - g_{t-1}^{e,a})$$

$$a_t = g_{t-1}$$

$$b_t = a_{t-1}$$

$$c_t = b_{t-1}$$

where  $g_{t-1}^a = (1 + g_{t-1})(1 + a_{t-1})(1 + b_{t-1})(1 + c_{t-1}) - 1$ . This system has an equilibrium in which, for all  $t$ ,  $g_t = a_t = b_t = c_t = z$  and  $g_t^{e,a} = (1 + z)^4 - 1$ . The Jacobian matrix, when evaluated at the equilibrium, is:

$$J = \begin{bmatrix} \frac{(1+z)^4 vx}{s-h^*} & -\frac{(1+z)vx}{s-h^*} & \frac{(1+z)^4 vx}{s-h^*} & \frac{(1+z)^4 vx}{s-h^*} & \frac{(1+z)^4 vx}{s-h^*} \\ x(1+z)^3 & 1-x & x(1+z)^3 & x(1+z)^3 & x(1+z)^3 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

where  $h^* = v((1+z)^4 - 1 + \delta)$ .

For a given set of parameters, one can check the stability for this system by computing the Jacobian matrix and finding the absolute value of the dominant eigenvalue(s). The model is locally stable if the modulus of a dominant eigenvalue is less than one, and unstable if the modulus is greater than one. Table 2 shows the relevant numbers for various equilibrium growth rates, given the parameter values used by HBS ( $x = 0.091$ ,  $v = 0.826$ ,  $\delta = 0.084$ ,  $\mu = 0.131$ ,  $\tau = 0.171$  and  $c = 0.701$ , with  $s = 0.57$  calculated using formula (7)). One can see that as the annual GDP growth rate increases to 21% per year, the supermultiplier becomes unstable.

**Table 2:** Eigenvalues corresponding to different annual GDP growth rates.

<b>Equilibrium growth rate</b>	15%	20%	21%	22%
<b>Dominant eigenvalue(s)</b>	0.947849 $\pm 0.183231i$	0.979153 $\pm 0.186456i$	0.986174 $\pm 0.186487i$	0.993463 $\pm 0.186257i$
<b>Absolute value</b>	0.9653978	0.99674793	1.00365161	1.01077218

If one sets  $v = 1.2$ , but keeps the other parameter values the same as before, and chooses the equilibrium annual GDP growth rate to be 5%, then one obtains the pair of dominant eigenvalues  $0.987809 \pm 0.186459i$ , with a modulus of 1.005, indicating instability. If  $Z$  is determined by equation (36), and the steady-state growth rate is set to 2% per year, but the other parameter values are kept the same, then the instability of the model is demonstrated by the simulation shown in Figure 1.

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