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# Stability and deterministic generation of single solitons and soliton crystals in microresonators with avoided crossings

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**Abstract:** We determine the stable regions in the parameter space of single soliton and soliton crystal frequency combs in microresonators with avoided crossings. We show that solitons are obtained deterministically when the stable region is enlarged. © 2020 The Author(s)

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High- $Q$  optical microresonators with a Kerr nonlinearity that are pumped by continuous-wave (CW) laser light can produce frequency combs that have important potential applications to metrology and high-resolution spectroscopy [1, 2]. However, these frequency combs, which are typically generated by a single soliton or soliton crystal pulses in the microresonator, are difficult to obtain deterministically. They are usually obtained through a quasi-random process in which the pump frequency detuning with respect to a cavity resonance is repeatedly swept until a soliton appears at the end of one of the sweeps. Single solitons, soliton crystals, and more generalized periodic structures—cnoidal waves—have been extensively studied [3–6]. A cnoidal wave of periodicity  $N$  is a waveform that repeats periodically  $N$  times in one round trip in the microresonator. In contrast to single solitons, soliton crystals use the pump more efficiently and produce higher-power comb lines. Understanding the conditions under which single solitons and soliton crystals can be obtained is thus important for microresonator design.

It has been found that avoided crossings due to the interaction of transverse modes can facilitate the generation of single solitons or soliton crystals [7, 8]. However, the parameters of the avoided crossings that lead to deterministic generation of a single soliton or soliton crystal of a given periodicity  $N$  have not been systematically explored to date. Design rules to generate solitons or soliton crystals with a periodicity  $N$  remain to be found.

In this work, we study theoretically the stability of cnoidal waves, including both single solitons and soliton crystals with avoided crossings. We calculate the stability chart in the (frequency detuning)  $\times$  (pump amplitude) parameter space. We study two different microresonators in which single solitons [7] and soliton crystals can be generated. Our result indicates that with avoided crossings at some strengths and frequency separations from the pump frequency the stable regions of single solitons or soliton crystals are enlarged. In these cases, we find that the corresponding single solitons or soliton crystals can be accessed deterministically. Since these stable regions can be rapidly and accurately calculated using dynamical methods [5], this result makes it possible to know which parameters of the avoided crossings will deterministically yield cnoidal waves with a desired periodicity.

We start with the generalized Lugiato-Lefever equation (LLE), which after normalization becomes [7]

$$\frac{\partial \psi}{\partial t} = i \frac{\partial^2 \psi}{\partial x^2} + i |\psi|^2 \psi - (i\alpha + 1) \psi + F + i \sum_{\mu=-\infty}^{\infty} \frac{a}{b - \mu} \tilde{\psi}_{\mu} \exp(-2\pi i \mu x / L), \quad (1)$$

where  $\psi$  is the slowly varying envelope of the electric field,  $t$  is time,  $x$  is the azimuthal coordinate,  $\alpha$  is the detuning of the stimulated cavity resonance from the pump laser at the ambient temperature,  $F$  is the pump amplitude,  $\mu$  is the mode number with respect to the central mode. The parameter  $a$  denotes the strength of the avoided crossings, and  $b$  denotes its frequency offset with respect to the pump frequency normalized to the free spectral range. We have  $-L/2 \leq x \leq L/2$ , where  $L$  is the mode circumference normalized with respect to the dispersive scale length. In Eq. (1), the function  $\tilde{\psi}_{\mu}(t) = (1/L) \int_{-L/2}^{L/2} \psi(x, t) \exp(i2\pi \mu x / L) dx$  is the Fourier transform of  $\psi(x, t)$ .

We calculated the stable regions in the  $\alpha$ - $F$  parameter space for the single soliton and the periodicity-2 cnoidal wave with the avoided crossing strength  $a = 36$ , frequency offset  $b = 13.8$ , and  $L = 32.5$ , which corresponds approximately to the Bao et al. experiments [7]. We used the dynamical methods that are described in detail in [5]. Figure 1(a) shows the stable regions of the single soliton in blue and the periodicity-2 cnoidal wave in red. The red-dashed curves in Figs. 1(a) and (b) indicate the limit above which continuous waves are unstable. Figure 1(c)

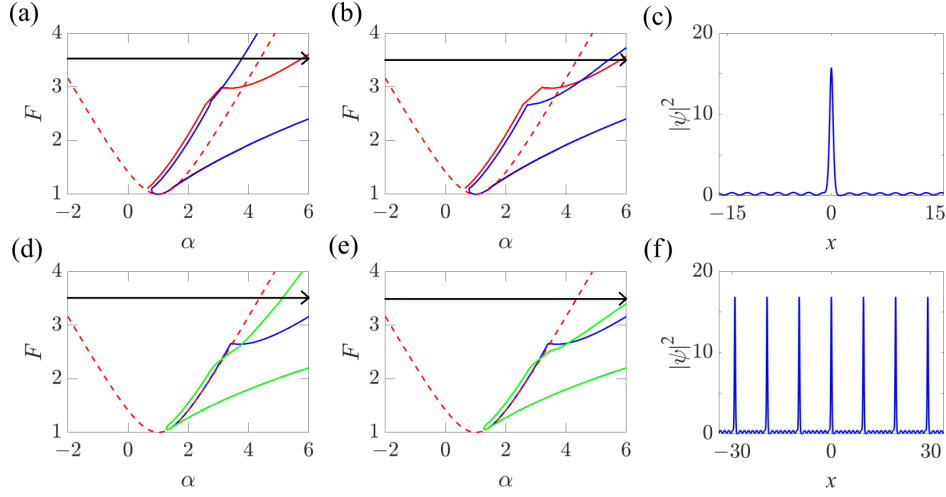


Fig. 1. Stable regions of the single soliton (blue) and the periodicity-2 cnoidal wave (red) at  $b = 13.8$  with (a)  $a = 36$  and (b)  $a = 5.7$ , respectively. The red-dashed curves show the limit above which continuous waves are unstable. The black arrow line shows the tuning scan direction at  $F = 3.5$ . (c) The profile of the amplitude-squared for the single soliton vs.  $x$  at  $F = 3.5$  and  $\alpha = 6$ . Stable regions of the single soliton (blue) and the periodicity-7 soliton crystal (green) at  $b = 13.8$  with (d)  $a = 36$  and (e)  $a = 5.7$ , respectively. (f) The profile of the amplitude-squared for the periodicity-7 soliton crystal vs.  $x$  at  $F = 3.5$  and  $\alpha = 6$ .

shows the profile of the amplitude-squared for the single soliton vs.  $x$ . Our previous results have shown that without avoided crossings the stable regions of single solitons and small periodicity cnoidal waves almost overlap with each other [5]. We also plot the stable regions of the single soliton and the periodicity-2 cnoidal wave with a weaker avoided crossing strength  $a = 5.7$  and frequency offset  $b = 13.8$  in Fig. 1(b). With a strong avoided crossing, the stable region of the single soliton is dramatically enlarged. As a result, the stable regions of the single soliton and the periodicity-2 cnoidal wave do not overlap anymore. We solved the LLE computationally by sweeping  $\alpha$  from  $-3$  to  $6$  100 times at  $F = 3.5$  as shown by the arrows in Fig. 1(a) and 1(b). When  $a = 5.7$ , we only obtained a soliton 21/100 times, while when  $a = 13.8$ , we obtained a soliton in all cases. This result is consistent with [7].

We also studied a case in which soliton crystals can be obtained deterministically by using avoided crossings. We calculated the stable regions in the  $\alpha$ - $F$  parameter space for the single soliton and the periodicity-7 cnoidal wave with the avoided crossing strength  $a = 42$  and frequency offset  $b = 51.7$  with  $L = 68$ . Fig. 1(d) shows the stable regions of the single soliton in blue and the periodicity-7 cnoidal wave in green. We also plot the stable regions of the single soliton and the periodicity-7 cnoidal wave at a weaker avoided crossing strength  $a = 6$  with the same frequency offset  $b = 51.7$  in Fig. 1(e). With a strong avoided crossing, the stable region of the periodicity-7 cnoidal wave is dramatically enlarged. Hence, stable regions for single solitons and periodicity-7 cnoidal wave do not overlap. The stable region of the periodicity-7 cnoidal wave will be accessed by scanning the frequency detuning. We ran 100 simulation for both cases and we obtain periodicity-7 cnoidal wave in all cases for  $a = 42$  and 18/100 times for  $a = 6$ . In this case we set  $F = 3.5$  and swept  $\alpha$  from  $-3$  to  $6$  in  $900 \mu s$  along the paths that we show in Fig. 1(d) and (e). We have observed that for both the single solitons and soliton crystals, a sizable pedestal oscillates azimuthally with a period approximately equal to  $L/b$  and this period approximately equals to the full width at half maximum of the solitons.

In conclusion, we have studied the stability of cnoidal waves by means of the dynamical method in two special cases that correspond respectively to cases in which single solitons [7] or soliton crystals [8] have been deterministically generated in the presence of avoided crossings. We have found that the stable region for the single soliton or the soliton crystal that appears is greatly enhanced and separates from the stable regions for other cnoidal waves and, in the case of single solitons, from the stable region for continuous waves. This result suggest that it should be possible to determine which cnoidal waves will appear deterministically in the presence of an avoided crossing by plotting the stability chart. This result also suggests that it should be possible to find design rules for generating soliton crystals with a desired periodicity by appropriately choosing the parameters  $(a, b)$ .

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