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Quantum stochastic thermodynamic on harmonic networks

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E-mail: sebastian.deffner@gmail.com**Keywords:** stochastic thermodynamics, quantum optics, harmonic networks**Abstract**

Fluctuation theorems are symmetry relations for the probability to observe an amount of entropy production in a finite-time process. In a recent paper Pigeon *et al* (2016 *New J. Phys.* **18** 013009) derived fluctuation theorems for harmonic networks by means of the large deviation theory. Their novel approach is illustrated with various examples of experimentally relevant systems. As a main result, however, Pigeon *et al* provide new insight how to consistently formulate quantum stochastic thermodynamics, and provide new and robust tools for the study of the thermodynamics of quantum harmonic networks.

Classic thermodynamics is the phenomenological theory of the average behavior of heat and work for systems in thermal equilibrium. At the nanoscale, fluctuations dominate and systems generically operate far from thermal equilibrium [1]. The extension of thermodynamics to such situations was achieved only rather recently with the discovery of so-called fluctuations theorems. These theorems are statements of the second law of thermodynamics, and they constitute one of the most important breakthroughs in modern theoretical physics [2]. At their core, they quantify that entropy only increases on average, and that processes with negative entropy production do occur, but they are exponentially unlikely.

The inception of these fluctuation theorems effectively opened a new field of research, which quickly adapted the name ‘stochastic thermodynamics’. In contrast to conventional thermodynamics, in stochastic thermodynamics work, heat, and also entropy production are defined along single trajectories of a classical system [3]. Naturally, the generalization of this new version of thermodynamics to quantum systems has posed a formidable task [4]. In particular, how to define trajectory dependent quantities for a quantum system is complicated, and it even has been proven that quantum work cannot be an observable in the usual sense [5]. Rather, quantum work has to be determined from two projective measurements, and hence work is given by a time-ordered correlation function [6]. Thus, quantum stochastic thermodynamics has been severely limited to isolated systems and has relied on invasive procedures.

In a recent publication Pigeon *et al* [7] managed to overcome such limitations and complications by employing a radically different approach—by focusing on counting processes as they are common in quantum optics [8]. It is important to realize that in quantum optics a quantum trajectory does not refer to the quantum equivalent of a path followed by a classical system in phase space, but is rather taken to be a sequence of outcomes of counting the quanta exchanged by a system with its environment. The statistics of such counting processes can be effectively described by the large deviation theory [9, 10], where the large deviation function is the logarithm of the distribution of counting outcomes in the long-time limit.

In the first part of their work Pigeon *et al* [7] develop a mathematically tractable formalism to compute the large deviation function for harmonic networks. To this end, they represent the dynamics in quantum phase space—a common approach for harmonic systems—and solve for the stationary state with the help of a Gaussian ansatz. It turns out that the large deviation function is fully determined by the covariance matrix of the Gaussian stationary state of the harmonic network. This simple, yet deep result for the large deviation function then allows to analyze the fluctuations in several elucidating case studies. In particular, Pigeon *et al* [7] compute

analytical expressions for harmonic networks with position–position coupling, rotating-wave coupling, and two-mode squeezing coupling.

The large deviation function is intimately connected with the Gallavotti–Cohen type formulation of the fluctuation theorem [11]. In particular, it can be easily seen that there is a fluctuation theorem whenever the large deviation function has a symmetry, and the entropy production is characterized by the symmetric point [11]. As a first and simplest example Pigeon *et al* [7] show that for a single harmonic oscillator coupled to two thermal reservoirs the symmetric point is the typical entropic flux associated with heat conduction. The situation is very similar in the second example, in which a rotating-wave type harmonic chain is studied. It is found again that there is a fluctuation theorem for the entropic flux. For the harmonic chain the analytical prediction is further backed-up by numerically solving the dynamics, and very good agreement between theory and numerics is achieved. The situation is more complicated in the third example—namely two thermal, squeezed modes. Also in this case a fluctuation theorem does exist, but the symmetric point attests that heat is emitted to both reservoirs. This is due to the dissimilarity between the type of inter-oscillator coupling and the one with the baths, and therefore the system cannot thermalize. As a final example Pigeon *et al* [7] analyze two oscillators coupled through the relative distance. In this case it is established that there is a fluctuation theorem in the limits of weak and strong coupling, whereas the intermediate regime is rather inconclusive.

Quantum stochastic thermodynamics is one of the most rapidly growing fields of modern research. Its main purpose is to extend, generalize, and probe thermodynamic relations for small systems, whose dynamics is dominated by fluctuations. However, identifying thermodynamic notions such as work, heat, and entropy production for single realizations is a non-trivial problem. By far the most versatile and important tools originate from the formulation of fluctuation theorems. By formulating the large deviation principle and illustrating its power in deriving fluctuation relations and the corresponding entropy production, Pigeon *et al* [7] opened a new avenue of research and provided a robust tool for the study of the thermodynamics of harmonic networks.

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