Journal of Quantitative Spectroscopy & Radiative Transfer 113 (2012) 1981–1984

Contents lists available at SciVerse ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Notes

Comment on the transmission matrix for a dielectric interface

Peng-Wang Zhai [a,*], George W. Kattawar [b], Yongxiang Hu [c]

[a] SSAI MS 475 NASA Langley Research Center, Hampton, VA 23681-2199, USA
[b] Department of Physics & Astronomy, Texas A&M University, College Station, TX 77843, USA
[c] MS 475 NASA Langley Research Center, Hampton, VA 23681-2199, USA

ARTICLE INFO

Article history:
Received 10 May 2012
Received in revised form
27 June 2012
Accepted 2 July 2012
Available online 7 July 2012

Keywords:
Atmosphere and ocean optics
Propagation, transmission, attenuation, and radiative transfer
Scattering, polarization

ABSTRACT

In a recent paper the reflection and transmission matrices for a dielectric interface based on Fresnel formulas are derived [Garcia RDM. Fresnel boundary and interface conditions for polarized radiative transfer in a multilayer medium. J Quant Spectrosc Radiat Transfer 2012;113:306–17]. Although the final formulas appear to be correct, we found that there are some significant conceptual and logical flaws in the derivation. Here we explain that the misunderstanding is due to the different physical significances of the Stokes parameters for the coherent and diffuse radiation field and that the so-called transmission factor directly originates from the physical definition of the Stokes parameters. We also clarify a few incorrect interpretations in the aforementioned paper about previously published works.

1. Background

Garcia [1] derived the reflection and transmission matrices for a dielectric interface using the Fresnel formulas and energy conservation. In Ref. [1] the Stokes (radiance) vector is defined as

$$\mathbf{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_l E_l^* + E_r E_r^* \\ E_l E_l^* - E_r E_r^* \\ E_l E_r^* + E_r E_l^* \\ i(E_l E_r^* - E_r E_l^*) \end{pmatrix}, \tag{1}$$

where E_l and E_r are the complex amplitudes of the electric field along the two vectors which are parallel and perpendicular to a reference plane; and $*$ denotes the complex conjugate. The transmission matrix is then defined by (Eq. (11) in Ref. [1])

$$\mathbf{I}^T = \mathbf{T}_{ab}\mathbf{I}^I, \tag{2}$$

where $\mathbf{I}^I$ and $\mathbf{I}^T$ are the incident and transmitted radiance vectors, respectively; $\mathbf{T}_{ab}$ is the transmission matrix for the radiance vector from medium a to b. The transmission matrix is then given by (Eq. (12) in Ref. [1])

$$\mathbf{T}_{ab} = f_T \begin{pmatrix} \frac{1}{2}(T_l T_l^* + T_r T_r^*) & \frac{1}{2}(T_l T_l^* - T_r T_r^*) & 0 & 0 \\ \frac{1}{2}(T_l T_l^* - T_r T_r^*) & \frac{1}{2}(T_l T_l^* + T_r T_r^*) & 0 & 0 \\ 0 & 0 & \Re\{T_l T_r^*\} & \Im\{T_l T_r^*\} \\ 0 & 0 & -\Im\{T_l T_r^*\} & \Re\{T_l T_r^*\} \end{pmatrix}, \tag{3}$$

where T_l and T_r are the amplitude transmission coefficients given by the Fresnel formulas; the superscript $*$ denotes complex conjugate; $\Re$ and $\Im$ denote the real and imaginary parts of the enclosed complex number. The factor f_T is derived from energy conservation relations (Eq. (16) in Ref. [1]):

$$I_x^I d\Omega_a dA \cos\theta_a = I_x^R\, d\Omega_a dA \cos\theta_a + f_T I_x^T\, d\Omega_b\, dA \cos\theta_b, \tag{4}$$

for both $x=l$ and $x=r$, where the superscripts R and T indicate "reflected" and "transmitted", respectively; $I_x^R = R_x R_x^* I_x^I$ and $I_x^T = T_x T_x^* I_x^I$. Finally the factor f_T is (Eq.(21) in Ref. [1])

$$f_T = n_{ba}^3 \left(\frac{\cos\theta_b}{\cos\theta_a}\right), \tag{5}$$

* Corresponding author. Tel.: +1 757 864 6288.
E-mail address: Pengwang.zhai-1@nasa.gov (P.-W. Zhai).

0022-4073/$ - see front matter
http://dx.doi.org/10.1016/j.jqsrt.2012.07.001

where θ_a and θ_b are the incident and transmission angles, respectively; $n_{ba} = n_b/n_a$, n_a and n_b are the indices of refraction for media a and b.

In Ref. [1] it is emphasized that the transmission factor of Eq. (5) should be used with the strict form of the Stokes (radiance) vector as shown in Eq. (1). If the definition of the Stokes vector depends on the medium properties, the transmission factor f_T should be changed accordingly. One specific comment is made on the work by Tsang et al. [2,3], in which they have an extra factor of $1/\eta$ for the Stokes (radiance) vector:

$$\mathbf{I}' = \begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \frac{1}{\eta} \begin{pmatrix} E_l E_l^* + E_r E_r^* \\ E_l E_l^* - E_r E_r^* \\ E_l E_r^* + E_r E_l^* \\ i(E_l E_r^* - E_r E_l^*) \end{pmatrix}, \tag{6}$$

where $\eta = (\mu_m/\epsilon)^{1/2}$; μ_m is the magnetic permeability and ϵ is the electric permittivity of the medium. In this situation, Ref. [1] claims that the transmission matrix will have a different multiplicative factor of n_2/n_1, a consequence from Eq. (4).

2. The physical interpretation of Stokes parameters for a coherent plane wave

It is important to recall the physical meaning of radiance (the first element of the Stokes vector) at this point, which is the monochromatic electromagnetic power flow through a unit area element perpendicular to its propagation direction per unit steradian of a solid angle. Therefore radiance should be proportional to the flow of electromagnetic energy, which is in turn proportional to $(\epsilon/\mu_m)^{1/2}$ [4]. This directly contradicts Garcia's strict definition of Stokes vector of Eq. (1), which does not include the factor of $(\epsilon/\mu_m)^{1/2}$. Note that the usual interpretation of Eq. (1) is that $(\epsilon/\mu_m)^{1/2}$ is omitted because normally only relative intensity is measured [5]. Besides, if we simply take the exact definition form of Stokes vector shown in Eq. (1), the transmission matrix for Stokes (radiance) vector will be obtained immediately from the Fresnel formulas, for example the transmitted radiance $I_l^T = T_l T_l^* I_l^I$ would lead to $\mathbf{T}_{ab,l} = T_l T_l^*$, in which $\mathbf{T}_{ab,l}$ is the transmission matrix for the parallel component of radiance. This makes his derivations and arguments not self-consistent and leaves no room for the transmission factor f_T to be introduced.

In order to resolve this apparent contradiction, it is necessary to reiterate that Eq. (6) is the correct physical definition of the Stokes vector and Eq. (4) is not applicable for incident plane waves. The reason is that the coherent and diffuse Stokes vectors have different physical meanings. Mishchenko has recently developed the phenomenological radiative transfer equation from Maxwell's equations [6] in which it has been made clear that the coherent Stokes vector defined by Eq. (6) for a plane wave has the dimensions of monochromatic electromagnetic power per unit area of a small surface element perpendicular to the incidence direction, while the Stokes vector for diffuse light in the radiative transfer equation has the dimensions of radiance, which is the monochromatic electromagnetic power per unit area per unit steradian of a small solid angle. Therefore, energy conservation across the Fresnel interface should be written as

$$\left(\frac{\epsilon_a}{\mu_m}\right)^{1/2} E_x^I E_x^{*I}\, dA \cos\theta_a = \left(\frac{\epsilon_a}{\mu_m}\right)^{1/2} E_x^R E_x^{*R}\, dA \cos\theta_a + \left(\frac{\epsilon_b}{\mu_m}\right)^{1/2} E_x^T E_x^{*T}\, dA \cos\theta_b, \tag{7}$$

where x denotes the parallel or perpendicular component. The use of Fresnel formulas leads to

$$1 = R_x R_x^* + \frac{n_b}{n_a}\frac{\cos\theta_b}{\cos\theta_a} T_x T_x^*, \tag{8}$$

where $n_b/n_a = (\epsilon_b/\epsilon_a)^{1/2}$ is used.

The physical interpretation of Eq. (8) is that the reflectivity and transmissivity are $R_x R_x^*$ and $(n_b \cos\theta_b)/(n_a \cos\theta_a) T_x T_x^* = 1 - R_x R_x^*$, respectively, for a Fresnel surface. In other words, if the incident plane wave has the first Stokes parameter of I_x, the incident irradiance is then $I_x \cos\theta_a$. The reflected and transmitted irradiances are $I_x^R \cos\theta_a = R_x R_x^* I_x \cos\theta_a$ and $I_x^T \cos\theta_b = (n_b \cos\theta_b)/(n_a \cos\theta_a) T_x T_x^* I_x \cos\theta_a$, respectively. It is observed that the factor of $(n_b \cos\theta_b)/(n_a \cos\theta_a)$ is the direct consequence of the factor $1/\eta$ in Eq. (6). One can further check that Eq. (8) holds for any angle of incidence, another hint that Eq. (6) is the true correct definition of the Stokes parameters. By definition, $I_x^T = n_b/n_a T_x T_x^* I_x$, which means that $n_b/n_a T_x T_x^*$ has to be the transformation factor to relate the incident and transmitted Stokes parameters. However, the factor of $(n_b \cos\theta_b)/(n_a \cos\theta_a) T_x T_x^*$ is kept in the following, keeping in mind that this is the transmissivity which relates the incident and transmitted irradiance. The reason is that the transmissivity is more convenient in the use of boundary conditions for radiative transfer in coupled atmosphere and ocean systems.

Similar to the scalar case of Eq. (8), the transmissivity matrix for the Stokes vector for a plane wave should include $(n_b \cos\theta_b)/(n_a \cos\theta_a)$ as well

$$\mathbf{t}_{ab} = \frac{n_b}{n_a}\frac{\cos\theta_b}{\cos\theta_a} \begin{pmatrix} \frac{1}{2}(T_l T_l^* + T_r T_r^*) & \frac{1}{2}(T_l T_l^* - T_r T_r^*) & 0 & 0 \\ \frac{1}{2}(T_l T_l^* - T_r T_r^*) & \frac{1}{2}(T_l T_l^* + T_r T_r^*) & 0 & 0 \\ 0 & 0 & \Re\{T_l T_r^*\} & \Im\{T_l T_r^*\} \\ 0 & 0 & -\Im\{T_l T_r^*\} & \Re\{T_l T_r^*\} \end{pmatrix}, \tag{9}$$

which has been shown previously in different notations in [3,7,8], though Ref. [7] names it as "transmission matrix" which is not appropriate strictly speaking.

3. The n^2 law for radiance

Eq. (9) shows the transmissivity matrix for the coherent Stokes vector for a plane wave. In practical application a more important problem is to find the transmission matrix for the diffuse Stokes vector. It differs from the coherent Stokes vector because the diffuse Stokes vector is a continuous function of the viewing angle and has the dimensions of radiance. Assuming a beam of light with solid angle of $d\Omega_a$ is incident at a dielectric interface, then the beam of light is reflected by and transmitted through the interface. Eq. (9) provides the relationship between the electromagnetic power of transmission Φ_T and incidence Φ_I. Energy conservation is now in the form of

$$\Phi_T = \mathbf{t}_{ab}\Phi_I, \tag{10a}$$

$$\Phi_T = \mathbf{I}_d^T \, d\Omega_b \, dA \cos\theta_b, \tag{10b}$$

$$\Phi_I = \mathbf{I}_d^I \, d\Omega_a \, dA \cos\theta_a, \tag{10c}$$

where the symbols are the same as Eq. (4) except that a subscript d is used to denote the diffuse light. Note that $d\Omega_a = \sin\theta_a \, d\theta_a \, d\phi$, $d\Omega_b = \sin\theta_b \, d\theta_b \, d\phi$, where ϕ is the azimuth angle of the beam of light, one can find that

$$\mathbf{I}_d^T = \frac{n_b^2}{n_a^2}\mathbf{t}_{ab}\mathbf{I}_d^I, \tag{11}$$

where $n_a \sin\theta_a = n_b \sin\theta_b$ and $n_a \cos\theta_a d\theta_a = n_b \cos\theta_b d\theta_b$ have been used. Eq. (11) is the vector form of the n^2 law, a familiar phenomenon in ocean optics [9,10]. It is also well known that I_d/n^2 is a fundamental invariant of radiation transfer. It is worthy to emphasize that the factor of n_b^2/n_a^2 in combination with $(n_b \cos\theta_b)/(n_a \cos\theta_a)$ yields the identical factor of f_T as derived in Ref. [1]. However, the correct apparent form of the factor f_T is obtained based on improper physics and assumptions, which leads to the incorrect conclusion that $\mathbf{T}_{ab}$ should be dependent on the exact form of Eq. (1), which is unphysical since electromagnetic energy flow always depends on $(\epsilon/\mu_m)^{1/2}$.

4. Discussion

Nevertheless, Eqs. (3) and (5) form the correct transmission matrix for diffuse light, although the Stokes vector should be defined by Eq. (6). A comparison is then made in Ref. [1] between Eq. (3) and Eq. (48) in Zhai et al. [7] and claimed that "we have found that the transmission matrix used by Zhai et al. [40] needs to be multiplied by an $(n_t/n_i)^2$ factor to agree with our result. In the notation of these authors, n_i denotes the refractive index of the medium of incidence and n_t that of the medium to where radiation is transmitted …" (see page 311, the paragraph below Eq. (25b) in Ref. [1]). Indeed, the transmission matrix $\mathbf{T}_{ab}$ defined in Ref. [1] has a different physical explanation from the matrix $\mathbf{t}$ in Ref. [7]. The matrix $\mathbf{t}$ defined by Eq. (48) in Ref. [7] is equivalent to Eq. (9) in this paper, which is part of the bidirectional transmission matrix for a rough dielectric surface and the factor of $(n_t/n_i)^2$ is already considered outside of $\mathbf{t}$ (see Eq. (34) of Ref. [7]). To make the comparison clearer, we refer readers to Ref. [11] in which the same topic of the radiance vector boundary conditions for a flat interface is considered. Note that in the boundary conditions (Eqs. (8c) and (8e), Ref. [11]) for diffuse radiance vector across a flat dielectric surface the matrix $\mathbf{t}$ is always accompanied by a factor of n_w^2 or $1/n_w^2$, depending on whether the incident light is on the air or water side, where n_w is the refractive index of water. This factor of n_w^2 or $1/n_w^2$ is exactly the factor of $(n_t/n_i)^2$ difference mentioned in Ref. [1] and the discrepancy pointed out in Ref. [1] is merely an equation rearrangement. We also found that Ref. [15] also used a similar logical separation of $(n_b/n_a)^2$ from $(n_b \cos\theta_b)/(n_a \cos\theta_a)$. Based on the same reason, Garcia's misinterpretation of Ref. [15] in terms of the transmission factor f_t is also incorrect.

It is also stated in Ref. [1] that Refs. [12,13] did not mention the factor of f_T to ensure energy conservation. Note that Kattawar and Adams [12] use the Monte Carlo method to simulate radiation propagation in the medium. When a radiation packet, often termed "photon" in the Monte Carlo method for convenience, encounters the atmosphere–ocean interface, a random number is generated to be compared with the reflectivity, which is the $(1,1)$ element of reflection matrix $\mathbf{r}_{11} = 0.5(R_l R_l^* + R_r R_r^*)$. If the random number is smaller than $\mathbf{r}_{11}$, the radiation packet is reflected. Otherwise, the radiation packet is transmitted. Then the normalized or reduced Mueller matrix (Mueller matrix divided by the $(1,1)$ element) is applied to the Stokes parameters to include the polarization. This way energy conservation is automatically ensured because the transmissivity $\mathbf{t}_{11} = 1 - \mathbf{r}_{11}$ as suggested in Eq. (8). The n^2 law is also considered when the radiation packet goes directly from the interface to the detector. The readers are referred to the paragraph just below Fig. 11 in Page 1465 of Kattawar and Adams [12] for original descriptions. A more recent work [14] also uses the reduced form of the transmission matrix and employs $\mathbf{t}_{11} = 1 - \mathbf{r}_{11}$ to ensure energy conservation. The n^2 law is also taken into account. The procedure is emphasized in the paragraph which encloses Eq. (13) of Ref. [14] and the paragraph next to it. Ref. [13] employs the same technique to deal with the atmosphere–ocean interface, though it is not stressed. It is very important to emphasize that the Monte Carlo computer code in Ref. [13] is well validated and all the published the numerical results are correct.

5. Conclusion

In this note we show that a recent derivation of the transmission matrix for the Stokes vector across a dielectric interface in Ref. [1] is physically incorrect which has led to an imprecise claim that Eq. (1) has to be assumed for Eq. (3) to be valid. We reiterate that Eq. (6) is the only correct definition of the Stokes vector and the subtle difference between the coherent and diffuse Stokes parameters has to be recognized to obtain the transmission matrix correctly. The coherent Stokes vector is defined in terms of a transverse plane wave, which is the electromagnetic power per unit area perpendicular to the propagation direction. The diffuse Stokes vector is interpreted as the electromagnetic power per unit area perpendicular to the propagation direction per unit solid angle around it. One does not need to use energy conservation to "find" the transmission factor. On the contrary, energy conservation is automatically satisfied using the correct definition of the Stokes vector. The n^2 law is then a consequence of the projected area and solid angle change across the dielectric interface for diffuse light. We also clarify that the use of the transmission matrix in our Monte Carlo and successive order of scattering codes for the atmosphere and ocean system, energy conservation, and the n^2 law are in full compliance. The numerical results published in Refs. [7,11,13] are physically correct and numerically accurate to the limit of the methodologies.

References

[1] Garcia RDM. Fresnel boundary and interface conditions for polarized radiative transfer in a multilayer medium. J Quant Spectrosc Radiat Transfer 2012;113:306–17.
[2] Tsang L, Kong JA. Radiative transfer theory for active remote sensing of half-space random media. Radio Sci 1978;13:763–73.
[3] Tsang L, Kong JA, Shin RT. Theory of microwave remote sensing. New York: Wiley; 1985.
[4] Jackson JD. Classical electrodynamics. 3rd ed.John Wiley & Sons, Inc.; 1999.
[5] Bohren CF, Huffman DR. Absorption and scattering of light by small particles.New York: Wiley; 1983.
[6] Mishchenko MI. Multiple scattering, radiative transfer, and weak localization in discrete random media: unified microphysical approach. Rev Geophys 2008;46:RG2003.
[7] Zhai P, Hu Y, Chowdhary J, Trepte CR, Lucker PL, Josset DB. A vector radiative transfer model for coupled atmosphere and ocean systems with a rough interface. J Quant Spectrosc Radiat Transfer 2010;111:1025–40.
[8] Sommersten ER, Lotsberg JK, Stamnes K, Stamnes JJ. Discrete ordinate and Monte Carlo simulations for polarized radiative transfer in a coupled system consisting of two media with different refractive indices. J Quant Spectrosc Radiat Transfer 2010;111:616–33.
[9] Preisendorfer RW. Radiative transfer on discrete spaces.Oxford: Pergamon Press; 1965.
[10] Mobley CD. Light and water: radiative transfer in natural waters.San Diego: Academic; 1994.
[11] Zhai P, Hu Y, Trepte CR, Lucker PL. A vector radiative transfer model for coupled atmosphere and ocean systems based on successive order of scattering method. Opt Express 2009;17:2057–79.
[12] Kattawar GW, Adams CN. Stokes vector calculations of the submarine light field in an atmosphere–ocean with scattering according to a Rayleigh phase matrix: effect of interface refractive index on radiance and polarization. Limnol Oceanogr 1989;34:1453–72.
[13] Zhai P, Kattawar GW, Yang P. Impulse response solution to the three-dimensional vector radiative transfer equation in atmosphere–ocean systems. I. Monte Carlo method. Appl Opt 2008;47:1037–47.
[14] Tynes HH, Kattawar GW, Zege EP, Katsev IL, Prikhach AS, Chaikovskaya LI. Monte Carlo and multicomponent approximation methods for vector radiative transfer by use of effective Mueller matrix calculations. Appl Opt 2001;40:400–12.
[15] Lam CM, Ishimaru A. Calculation of Mueller matrices and polarization signatures for a slab of random medium using vector radiative transfer. IEEE Trans Antennas Propag 1993;41:851–62.