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A Study of Dual-Pumped Microresonator Solitons using 3-Wave Equations

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Abstract: We show that 3-wave equations can be used to accurately model the multi-color solitons that appear in dual-pumped microresonators. These equations can be used to analyze the soliton properties. © 2023 The Author(s)

1. Introduction

The Lugiato-Lefever Equation (LLE) has played a key role in the theoretical and computational study of optical solitons in microresonators [1]. Recent investigations have demonstrated that by employing dual laser pumping in a microresonator, broadband microcombs can be produced with over an octave of bandwidth. The dual laser pumping also unlocks new nonlinear interactions in the phase velocity domain resulting in the presence of a second repetition rate in the carrier envelope offset [2]. These microcombs are generated by microresonator solitons that have distinct components, which propagate at different phase velocities. We refer to these distinct components as “colors” because their central frequencies differ by large fractions of the total optical bandwidth. Although theoretical studies have utilized a modified LLE (MLLE) that incorporates two pumping terms [3,4], the resulting multi-color solitons are non-stationary, hindering study of their properties and, in particular, their stability. In this study, we introduce a set of three coupled wave equations, termed 3-wave equations, to model the three primary colors of the solitons. Our analysis reveals excellent agreement between the solutions derived from the 3-wave equations and those obtained from the MLLE. Moreover, the solutions derived from the 3-wave equations are stationary, thereby facilitating future analyses of the stability and other properties of the multi-color solitons.

2. The 3-Wave Equations

This study investigates the modeling of dual-pumped microresonators, as shown in Fig. 1(a), using the modified Lugiato-Lefever Equation (MLLE) and the 3-wave equations. The normalized MLLE is employed to describe the behavior of a microresonator with quadratic and cubic dispersion components, and is written as

$$\frac{\partial \psi}{\partial t} = -(1 + i\alpha_0)\psi + i|\psi|^2\psi - i\frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial \theta^2} - i\frac{\beta_3}{6}\frac{\partial^3 \psi}{\partial \theta^3} + F_0 + F_+ \exp[-i\mu_+ \theta + i(\alpha_+ - \alpha_0)t + iD_{\text{int}}(\mu_+)t] \quad (1)$$

where t is time, θ is the azimuthal coordinate, ψ is the normalized field envelope of a single transverse optical mode, α_0 (α_+) and F_0 (F_+) represent the detuning from the pumped mode and power of the primary (secondary) pump respectively, β_2 (β_3) is the second (third) order dispersion coefficient, μ_+ is the mode closest to the secondary pump. $D_{\text{int}}(\mu)$ is the integrated dispersion of the microresonator calculated at the primary pump, and $D_{\text{int}}(\mu_+)$ is the phase shift at the secondary pump. Solving the MLLE produces a soliton solution characterized by a spectrum consisting of four prominent peaks associated with the primary pump, secondary pump, a dispersive wave, and an idler generated through four-wave mixing, therefore introducing a new color in the cavity, as depicted in Fig. 1(b).

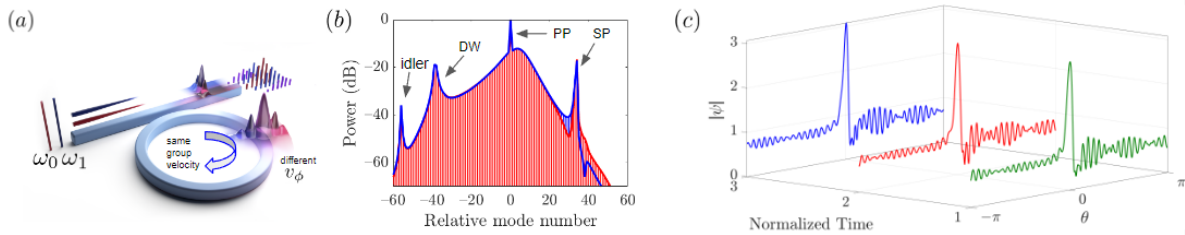


Fig. 1: (a) A dual pumped microresonator with a multi-color soliton. (b) Comparison of the spectra obtained from the MLLE (blue) and the 3-wave equations (red) containing prominent peaks due to the dispersive wave (DW), primary pump (PP), secondary pump (SP), and an idler. (c) The soliton solutions to the MLLE at three different times showing the non-stationary evolution.

The three dominant colors of the soliton, contributed by the primary pump, secondary pump, and idler frequencies, are effectively modeled using the 3-wave equations, which may be written,

$$\begin{aligned}\frac{\partial \psi_+}{\partial t} &= -(1 + i\alpha_+) + i\mathcal{D}_+ \psi_+ + i(|\psi_+|^2 + 2|\psi_0|^2 + 2|\psi_-|^2)\psi_+ + i\psi_0^2 \psi_-^* + F_+, \\ \frac{\partial \psi_0}{\partial t} &= -(1 + i\alpha_0) + i\mathcal{D}_0 \psi_0 + i(2|\psi_+|^2 + |\psi_0|^2 + 2|\psi_-|^2)\psi_0 + 2i\psi_- \psi_+ \psi_0^* + F_0, \\ \frac{\partial \psi_-}{\partial t} &= -(1 + i(2\alpha_0 - \alpha_+)) + i\mathcal{D}_- \psi_- + i(2|\psi_+|^2 + 2|\psi_0|^2 + |\psi_-|^2)\psi_- + i\psi_0^2 \psi_+^*.\end{aligned}\quad (2)$$

where ψ_0 , ψ_+ , and ψ_- , correspond respectively to primary pump, secondary pump, and idler amplitudes. \mathcal{D}_0 , \mathcal{D}_+ , and \mathcal{D}_- represent the integrated dispersion operators, centered at μ_0 , μ_+ , and μ_- respectively. The integrated dispersion at the secondary pump and idler frequencies are related to the integrated dispersion at the primary pump as $D_{+,-}(\mu) = D_0(\mu) \pm \Omega$ where $\Omega = D_0(\mu_+) + \alpha_+$. The 3-wave equations are not accurate in the modulation instability state that is encountered when sweeping the detuning to find a soliton state. Therefore, we initialize the 3-wave equations with a solution ansatz by choosing a hyperbolic secant profile for ψ_0 and small random initial conditions for ψ_+ and ψ_- . The solutions obtained by integrating the 3-wave equations using the split-step Fourier method converge to stationary quantities, shown in Fig. 2. The ripples that we observe in $|\psi_0|$ and in $|\psi_-|$ are due to dispersive wave components.

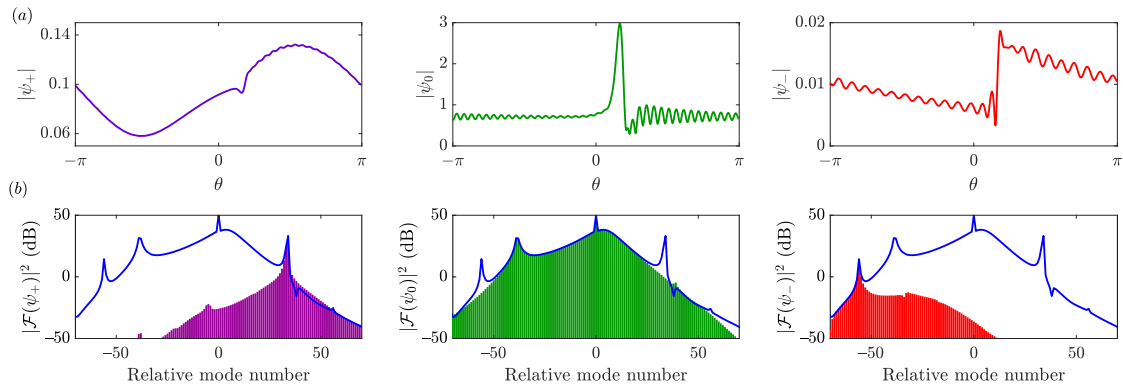


Fig. 2: **(a)** Stationary solutions obtained by solving the 3-wave equations using the same parameters as in Fig. 1, and **(b)** their spectra. The blue envelope represents the spectrum obtained from the MLLE

To validate the accuracy of the 3-wave equations, we perform a discrete Fourier transform on the combined solutions of Eq. (2) and compare the spectrum to the spectrum that we obtain from the solution of Eq. (1). In Fig. 1(b), we observe excellent agreement between the spectra generated by the two equations, providing strong evidence for the validity of the 3-wave equations in modeling the multi-color soliton dynamics.

3. Conclusion

We have demonstrated that the 3-wave equations can accurately model soliton dynamics in dual-pumped microresonators. Future work will focus on using the 3-wave equations to calculate the stability boundaries of multi-color solitons, which is not possible using the MLLE, and finding approximate solutions for the soliton components.

References

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