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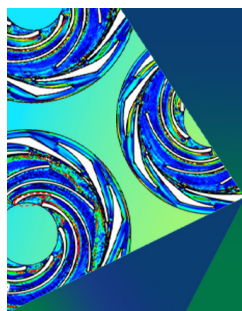


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# Aspect-ratio effects in the driven, flux-core spheromak

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Resistive magnetohydrodynamic simulations are used to evaluate the effects of the aspect ratio  $A$  (length to radius ratio) in a spheromak driven by coaxial helicity injection. The simulations are benchmarked against the Sustained Spheromak Physics Experiment (SSPX) [R. D. Wood *et al.*, Nucl. Fusion **45**, 1582 (2005)]. Amplification of the bias (“gun”) poloidal flux is fitted well by a linear dependence (insensitive to  $A$ ) on the ratio of gun current and bias flux above a threshold dependent on  $A$ . For low flux amplifications in the simulations, the  $n=1$  mode is coherent and the mean-field geometry looks like a tilted spheromak. Because the mode has relatively large amplitude the field lines are open everywhere, allowing helicity penetration. Strongly driven helicity injection at  $A \leq 1.4$  in simulations generates reconnection events which generate cathode-voltage spikes, relaxation of the symmetry-breaking modes, and open, stochastic magnetic field lines; this state is characteristic of SSPX. The time sequences of these events suggest that they are representative of a chaotic process. Near the spheromak tilt-mode limit,  $A \approx 1.67$  for a cylindrical flux conserver, the tilt approaches  $90^\circ$ ; reconnection events are not generated up to the strongest drives simulated. Implications for spheromak experiments are discussed. © 2009 American Institute of Physics. [DOI: 10.1063/1.3134064]

## I. INTRODUCTION

The spheromak<sup>1</sup> is a toroidal, magnetic confinement configuration usually formed in a singly connected volume, with no central post and thus no toroidal magnetic field coil or transformer to drive current. The current and magnetic field are generated by injecting linked toroidal and poloidal magnetic fluxes (and thus magnetic helicity) into a flux-conserving vessel. A magnetic dynamo reconfigures these fluxes into the spheromak configuration. In experiments such as Beta II,<sup>2</sup> the Compact Torus Experiment (CTX),<sup>3</sup> the Flux Amplification Compact Torus (FACT),<sup>4</sup> SPHEX,<sup>5</sup> and the Sustained Spheromak Physics Experiment (SSPX),<sup>6</sup> helicity is injected by a coaxial plasma gun and sometimes called coaxial helicity injection (CHI);<sup>7</sup> the resulting magnetic configuration is a flux-core spheromak.

Helicity injection can be used to form and buildup the spheromak as well as to sustain it. The magnitudes of the generated spheromak poloidal flux and toroidal plasma current need to be high for good confinement (and performance of the experiment) as do the efficiencies of generating and sustaining them. Generally, however, the achieved experimental values are considerably less than desired for a high magnetic field, fusion plasma.<sup>8</sup> In this paper we use resistive magnetohydrodynamic (MHD) simulations, benchmarked against experiment, to examine the role of the spheromak aspect ratio  $A$  (length divided by radius) on these quantities and the physics of their generation. The issue of obtaining high temperatures and good energy confinement<sup>6</sup> once the spheromak is formed will not be addressed here.

In CHI, the injected poloidal flux is provided by an applied (bias) poloidal flux  $\psi_g$  between the gun and flux conserver, and the injected toroidal flux is generated by the gun current  $I_g$  so that the rate of helicity injection into the flux conserver is  $dK/dt = 2V_g\psi_g$ , where  $V_g$  is the voltage across the insulating gap.<sup>9</sup> In SSPX experiments, magnetic reconnection events then reconfigure the injected fluxes into the spheromak geometry.<sup>10</sup> The net result of this dynamo is to generate a poloidal flux difference  $\psi_0$  between the spheromak magnetic axis and the geometric axis at a flux amplification factor  $F_a = \psi_0/\psi_g$  greater than the bias (gun) flux. The value of  $F_a$  is an important measure of the effectiveness of the helicity injection; a large value is needed to generate the high poloidal fluxes needed for good plasma confinement, but it also determines the power losses on the open magnetic field lines relative to those inside the separatrix.<sup>8</sup> This is because the poloidal flux in the open (edge) field lines equals the applied bias in a strongly driven, flux-core spheromak; thus the area of the edge plasma relative to the closed plasma scales essentially as  $\psi_g/\psi_0$ . Ohmic losses on the open edge field lines are high because the electron temperature there is low, and it is estimated that  $F_a \approx 50$  is required for the losses to be acceptable for a spheromak fusion reactor.<sup>11</sup>

Among the factors that can influence the flux amplification is the geometry of the flux conserver. Most experiments have been conducted in approximately cylindrical flux conservers with the length to radius ratio,  $A \sim 1$ , albeit with different helicity injector (“gun”) geometries. A cylindrical aspect ratio of 1 provides a substantial margin of safety against the tilt mode,<sup>12</sup> which is known to occur in spheromaks which are isolated from the gun when  $A \approx 1.67$ .<sup>13,14</sup> It is difficult to change the flux conserver, especially in a high-powered experiment with well conditioned walls, so varying  $A$  in experiments is rare and has not been done systemati-

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cally. CTX made a number of changes of this ratio,<sup>15</sup> but they were small, and the dimensions of the gun entrance region were often changed simultaneously as was the high-voltage power-supply configuration, so no scaling was deduced for the effects of the flux conserver geometry.<sup>1</sup> SSPX made one change in the flux-conserver length<sup>16</sup> which will be used to benchmark the simulations discussed below.

Single-fluid, resistive MHD simulations of spheromaks using the NIMROD code<sup>17</sup> have been quite successful in describing the behavior of SSPX (Ref. 18) with flux amplification especially well predicted,<sup>6,8</sup> so simulations can be used with some confidence to explore the effects of the aspect ratio. Section II begins to address the objective of this paper by considering flux amplification as a function of the aspect ratio. Thus, we present simulations in the SSPX geometry, varying  $L_{fc}$  and determining  $F_a$ . Confidence in the results is strengthened by comparisons with experimental data where possible. As will be seen, there will be a reduction in the threshold value of  $\lambda_g = \mu_0 I_g / \psi_g$  for forming the spheromak as the aspect ratio increases, but  $dF_a/d\lambda_g$  is almost constant.

A variety of fluctuation dynamics (four “classes”) ranging from low-amplitude,  $n=1$  modes to nonsteady, relaxation events were observed in flux-core spheromak simulations done by Sovinec *et al.*<sup>17</sup> primarily in a pillbox geometry. The MHD equations used in the present simulations are extended from those in Ref. 17 by including temperature evolution in the presence of anisotropic thermal conductivity; see Ref. 18 for details. Similar dynamics are seen herein at fixed gun current as in the previous simulations at fixed gun voltage, although we also find that the pinched gun-current column can be unstable to a steady-state  $n=1$  mode below the threshold for flux amplification. At low flux amplification the  $n=1$  mode becomes steady after the amplification saturates, as in Sovinec *et al.* (This behavior was not observed in the SSPX experiment, although low flux-amplification spheromaks were not studied extensively.) At high amplification a nonsteady state occurs, although the nonsteady behavior is chaotic in time at the highest amplifications, extending this class beyond that reported for the earlier simulations. Section III will therefore consider the characteristics of the sustainment of SSPX when strongly driven, including the dynamo associated with relaxation events in which magnetic reconnection results in open magnetic field lines, rearrangement of the fluxes into the spheromak geometry, and gun voltage spikes from the resulting change in the power-supply load inductance.<sup>10</sup> The time history of these events in SSPX simulations (in this regime) and experiment is consistent with a nearly chaotic system.

We will find in Sec. IV that the nature of the dynamo changes as the geometry approaches  $A \sim 1.67$ . At the higher aspect ratio, the dynamo is a large-amplitude, toroidal mode number  $n=1$  mode at nearly constant amplitude, and the mean-field spheromak is highly tilted. Because of the large plasma asymmetry, magnetic field lines are stochastic throughout most of the axisymmetric flux conserver, allowing helicity and current to penetrate the spheromak. When the spheromak is quite strongly driven, the simulation has problems converging and may be evolving toward an almost

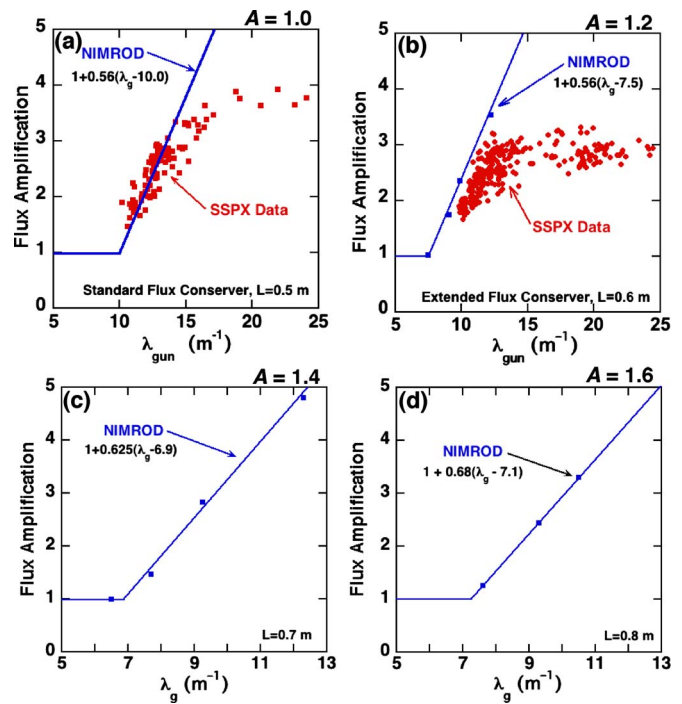


FIG. 1. (Color online) (a) Comparison of flux amplification in SSPX and in NIMROD simulations. These results are in the standard flux conserver with  $R=0.5$  m and  $L=0.5$  m ( $A=1$ ). The red points are from the experiment. (b) Comparison of flux amplification in the SSPX extended flux conserver with NIMROD simulations. These results had  $R=0.5$  m and  $L=0.6$  m ( $A=1.2$ ). The red points scattered about the line in (a) and (b) are from the experiment; the points along the straight line are from simulations. (c) Flux amplification from simulation for  $A=1.4$ . The points along the straight line are from simulations. (d) Flux amplification for  $A=1.6$ ; see the discussion in Sec. IV regarding a simulation at  $\lambda_g=11.9$  m<sup>-1</sup>. The points along the straight line are from simulations.

fully tilted geometry. Relaxation events are not found in any of the simulations conducted in this geometry.

The paper will finish with a discussion exploring whether this behavior can explain some of the differences among spheromak experiments, followed by conclusions and recommendations for further work.

## II. FLUX AMPLIFICATION AND SPHEROMAK ASPECT RATIO

A detailed comparison of the measured flux amplification in SSPX with the predictions of a resistive MHD simulation using the NIMROD code found excellent agreement near the threshold.<sup>8,6</sup> For completeness, the comparison is reproduced in Fig. 1(a). The saturation onsetting at  $\lambda_g > 13$  m<sup>-1</sup> in the experiment was not seen in the simulations. The reason for this is not clear, but apparently lies outside the resistive MHD model. (Saturation was observed by Sovinec *et al.*,<sup>17</sup> but for a gun operated at constant voltage.) It may be a constraint for a reactor, although simulations suggest a means of obtaining the required high flux amplification other than very high  $\lambda_g$ .<sup>8</sup> Additional experimental research will be needed to resolve this issue. In an experimental run in SSPX near the end of its operation the aspect ratio was increased to 1.2,<sup>16</sup> measurements are compared with simulations in Fig. 1(b). The agreement is good,

TABLE I. Flux-amplification parameters.

$A$	$\lambda_{th}$	$\alpha$	$\lambda_{FC}$
1.0	10	0.56	9.9
1.2	7.5	0.56	9.3
1.4	6.9	0.625	8.9
1.6	7.1	0.68	8.6

although less so than seen in Fig. 1(a), perhaps due to relatively poor vacuum conditions; the increased flux-conserver length was obtained by lengthening the diagnostic access slot at the spheromak midplane and the plasma sputtered considerable copper from the vertical bars carrying wall current across the gap. In support of this hypothesis, the plasma quality was observed to decrease throughout the run, and data from early in the run had the best agreement with simulations.

For both aspect ratios, the flux amplification was well fitted above a threshold by

$$F_a = 1 + \alpha(\lambda_{gun} - \lambda_{th}). \quad (1)$$

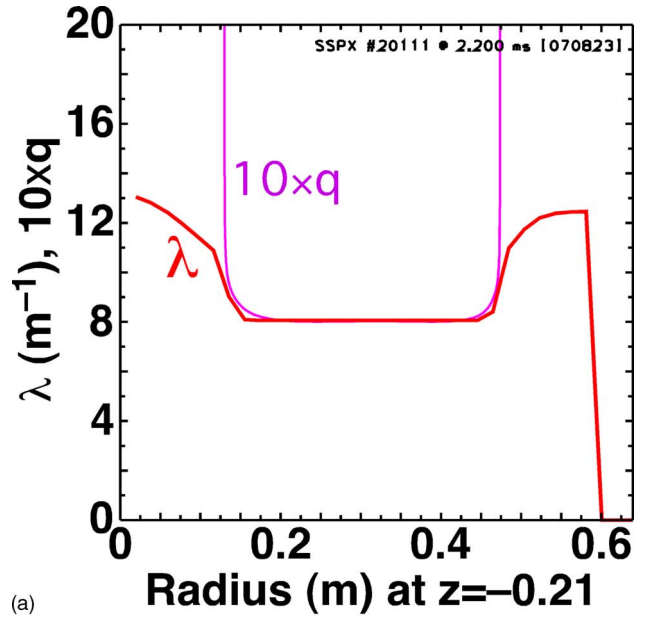
For  $A=1$ ,  $\lambda_{th}=10.0 \text{ m}^{-1}$  and for  $A=1.2$ ,  $\lambda_{th}=7.5 \text{ m}^{-1}$ ; in both cases,  $\alpha=0.56$  m from the fit with the flux-conserver radius fixed at 0.5 m.

Simulations were also run for  $A=1.4$  and  $1.6$ ; see Figs. 1(c) and 1(d). Linear fits as in Eq. (1) were also found, although with slightly larger values of  $\alpha$ . Note that  $F_a$  is found from the  $n=0$  component of the magnetic field. Later in this paper we will see that the spheromak axis is tilted from the vertical, especially when  $A$  is near the tilt-mode limit for the flux conserver. In this case the axisymmetric component of the field is insufficient to fully describe the built-up state.

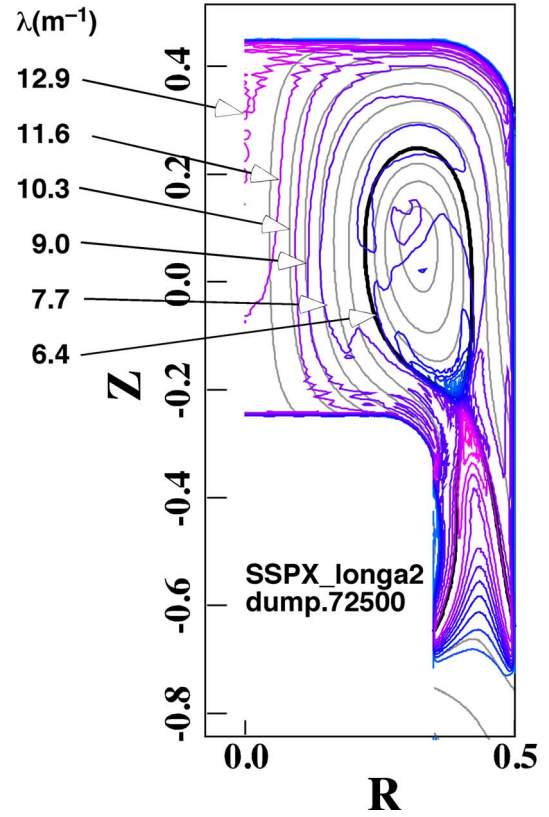
The fits are summarized in Table I. Also shown is the lowest eigenvalue of the flux conserver, estimated from  $\nabla \times \mathbf{B} = \lambda_{FC} \mathbf{B}$  in the cylindrical approximation,<sup>19</sup>  $\lambda_{FC} = \sqrt{(3.83/R_{fc})^2 + (\pi/L_{fc})^2}$ , which is  $>\lambda_{th}$  except at  $A=1$ . Both experimental reconstructions and simulations find that the current peaks strongly along the geometric axis for long (millisecond) discharges, so that the value of  $\lambda = \mu_0 \mathbf{j} \cdot \mathbf{B} / B^2$  on that axis exceeds  $\lambda_{FC}$ , while the value on the separatrix is less. An example from experiment is shown in Fig. 2(a), and one from simulation is shown in Fig. 2(b). In both cases,  $\lambda$  on the separatrix is below  $\lambda_{FC}$ . These profiles thus satisfy the condition that the  $\lambda(r)$  profile must span  $\lambda_{FC}$ .<sup>20</sup> This radial variation makes simple approximations, for example, assuming constant  $\lambda$  in the edge plasma, inadequate for predicting the flux amplification.

### III. DYNAMO CHARACTERISTICS IN A STRONGLY DRIVEN SPHEROMAK AT $A=1$

In many spheromak experiments, the buildup and sustenance of the magnetic field during the formation with  $\lambda_g > \lambda_{th}$  are accompanied by spikes on the electrode voltage. An example from SSPX is shown in Fig. 3. Spikes are also seen in resistive MHD simulations where they are identified as resulting from magnetic reconnection events.<sup>10</sup> An example



(a)



(b)

FIG. 2. (Color online) (a) Profile of  $\lambda$  from SSPX, found by fitting the solution to the Grad-Shafranov equation to magnetic probe measurements on the flux-conserver walls. The separatrix is at  $R=0.13$  m and  $0.475$  m;  $\lambda_g=11.8 \text{ m}^{-1}$ . (b) Contours of  $\lambda$  (axisymmetric part) from the simulation for  $A=1.4$  with  $\lambda_g=7.7 \text{ m}^{-1}$  and  $\lambda_{FC}=8.9 \text{ m}^{-1}$ . Constant poloidal flux is in gray with the separatrix shown as the thick, black line labeled  $\lambda=6.4 \text{ m}^{-1}$ . This example is for a flux amplification of 1.5.

of this behavior from a simulation is shown in Fig. 4 which illustrates an approximately  $100 \mu\text{s}$  history of a strongly driven spheromak.

The behavior between the second and third voltage spikes in Fig. 4 is expanded in Figs. 5 and 6. At each spike



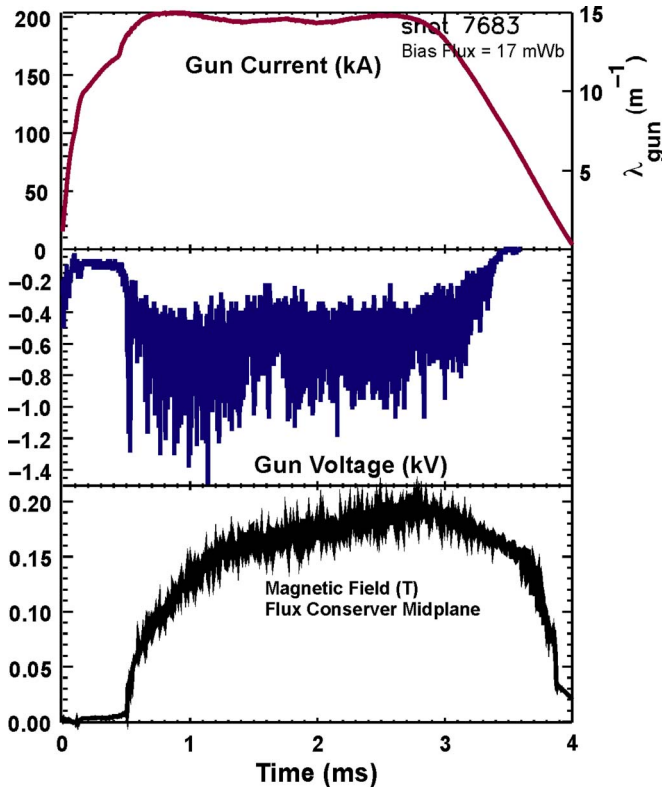


FIG. 3. (Color online) Buildup of magnetic field in a discharge in SSPX illustrating voltage spiking when  $\lambda_g > \lambda_{th}$ .

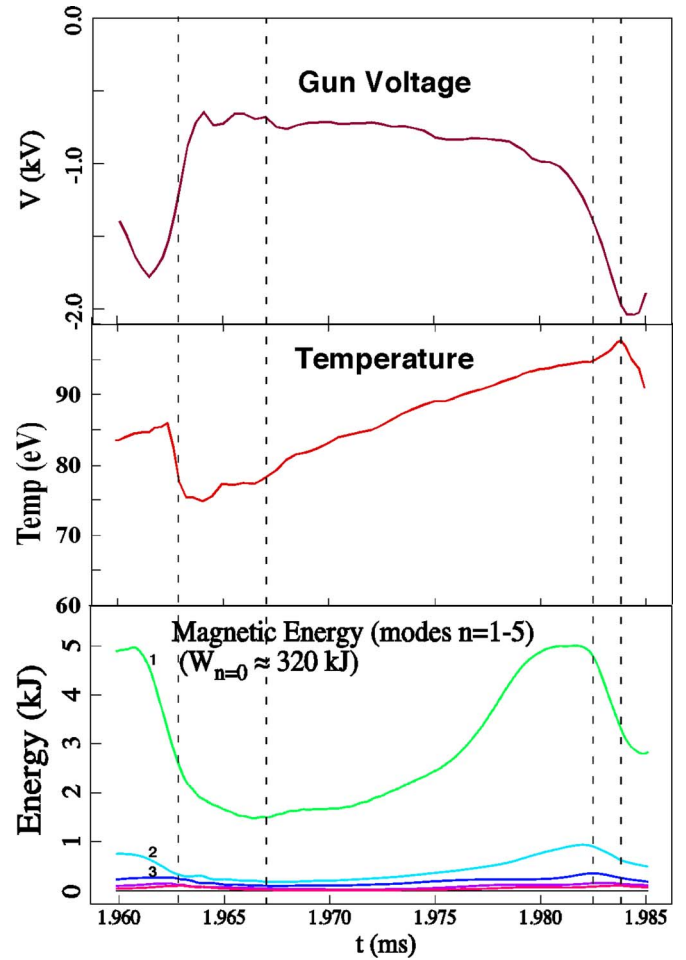


FIG. 5. (Color online) Voltage, temperature, and magnetic energy evolution between two voltage spikes in Fig. 4.

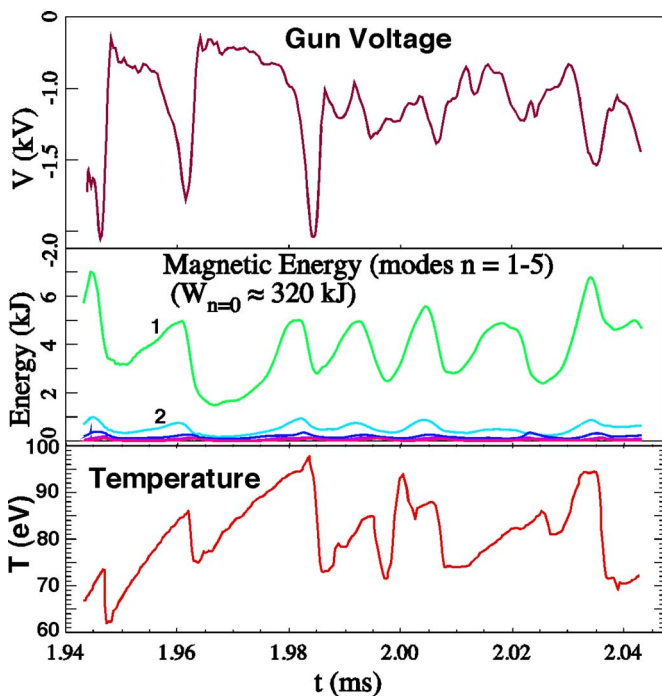


FIG. 4. (Color online) Voltage, magnetic field energy, and temperature during a 100  $\mu$ s history in a simulation. The energy in the axisymmetric ( $n=0$ ) magnetic field is 320 kJ; the relaxation events occur when the energy in the  $n=1$  mode is about 15% of that in  $n=0$ .

the temperature  $T$  and the energy in the  $n=1$  mode both drop rapidly. Magnetic field lines become stochastic throughout the flux-conserver volume, allowing current and flux to penetrate from the edge plasma and heat to flow along field lines to the wall. Following this, flux surfaces heal in the spheromak core while the energy in the  $n=1$  mode increases until the voltage spikes again. Behavior very much like this was found using magnetic probes in the FACT spheromak.<sup>4</sup>

Three-dimensional visualizations during this simulation using the software system VISIT (Ref. 21) are shown in Fig. 7. In these plots the constant- $\lambda$  surface has been chosen to represent the current column along the geometric axis and the constant- $T$  surface has been chosen as a proxy for the flux surfaces which often exist only in a mean-field (approximate) sense.

The constant- $T$  surfaces can take rather more complex shapes than seen in Fig. 7; cf. Fig. 8. Contour plots for the temperature in Fig. 9 often show stellaratorlike surfaces with the  $n=1$ ,  $m=1$  shape corresponding to the dominant mode in the plasma even though magnetic surfaces are not closed. A magnetic field puncture plot for this time is shown in Fig. 10. The field line shown is stochastic, but confined between two Kolmogorov–Arnold–Moser surfaces (see, e.g., Ref. 22) and followed for  $>10^3$  m in our study. The lowest Liapunov exponent for this field line, characteristic of the mean expo-

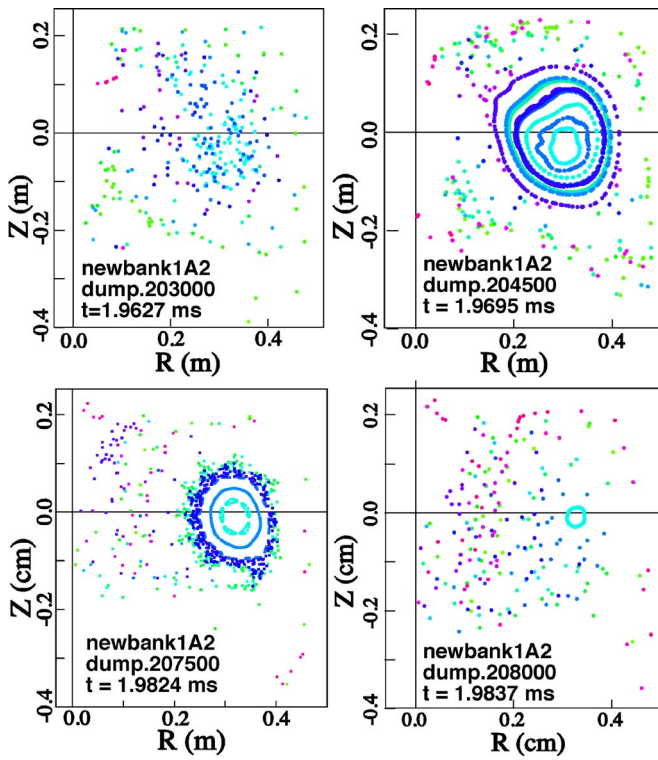


FIG. 6. (Color online) Field line Poincaré (“puncture”) plots at four different times in the simulation shown in Fig. 5, showing the breakup and healing of flux surfaces near the magnetic axis.

nential rate of divergence of nearby field lines, was calculated using the method of Benettin *et al.*<sup>23</sup> and corresponds to a length of 5.0 times around the torus toroidally. Consequently, field lines deviate from an initial, axisymmetric starting surface quite slowly and mean-field surfaces are well defined. Consistent with this conclusion, constant temperature contours in this region are helical ( $m=1$ ,  $n=1$ ) surfaces as seen in Fig. 9, even though there are no true, closed magnetic surfaces in this region. At other times, of course, the deviation of field lines from the initial surface is faster, although the geometry of the *mean* field is still quite close to that of the ideal spheromak. The field-line puncture plots shown in Figs. 6 and 10 show that the magnetic field is often stochastic and can be ergodic in the sense that a line can visit the entire volume.

For the SSPX experiment the generation of stochastic field lines is seen to occur in a relaxation oscillatorlike behavior which itself appears to be chaotic in time. This is meant in the sense that the chaos is characteristic of the full dissipative system rather than just stochastic field lines which can be described by a Hamiltonian. The cathode-voltage autocorrelation function provides a simple, first test of this hypothesis. Thus, for both experimental data and numerical data from the resistive MHD simulations, the autocorrelation function was calculated as

$$C(\tau) = \frac{1}{i_m} \sum_{i=i_0}^{i_0+i_m} (V_i - \bar{V})(V_{i+i_m} - \bar{V}), \quad (2)$$

with  $\tau = i_m \Delta t$  and  $\Delta t$  the time between the voltage samples and  $i_m$  is the sample length and is many voltage pulses long.

Figure 11 compares the autocorrelation function for a SSPX discharge (Fig. 3) and a simulation (Fig. 4).

The high peak at  $\tau=0$  is characteristic of a random process. The width of the experimental result is limited by the sample time of the data,  $1 \mu\text{s}$ , but higher-speed measurements on other discharges indicated that the width of the voltage spikes was close to  $1 \mu\text{s}$ . (Unfortunately, the amplitude resolution was inadequate to calculate the autocorrelation function.) Interestingly, both autocorrelation functions in Fig. 11 show a low-amplitude oscillation with a period of  $6\text{--}7 \mu\text{s}$ . This does not appear to correspond to the average time between the spikes and has not been previously identified in the data.

Sovinec *et al.*<sup>17</sup> identified a nonsteady state which was periodic (their class four), and these are seen in some of our simulations. The low-amplitude oscillation in the autocorrelation function is likely the residual effect of this periodicity.

Thus, for  $A \leq 1.4$  (Fig. 1), simulations at the lowest values of  $\lambda_g$  behave coherently, but at the highest drive the voltage behavior has become chaotic. Such transitions are a well-known characteristic of dissipative systems, consistent with considering the driven spheromak a system which can become chaotic; see, for example, the discussion in Ref. 22. As noted in this reference there are many “paths” to chaos; it is not clear which applies to the spheromak.

#### IV. DYNAMO CHARACTERISTICS AT AN ASPECT RATIO CLOSE TO THE TILT LIMIT

In spheromak simulations in a flux conserver very near the tilt limit the  $n=1$  mode is coherent (approximately a sine wave) at flux amplifications up to  $>3$  and has a large amplitude, as seen for  $A=1.6$  in Fig. 12. A simulation at  $\lambda_g = 11.9 \text{ m}^{-1}$ , however, found the flux amplification increasing steadily toward the fit shown in Fig. 1(d) when the mode amplitude increased abruptly and the  $n=0$  component of the flux decreased abruptly. It appears that the spheromak tilt increased significantly; numerical convergence became difficult and the simulation was discontinued.

At  $\lambda_g < 11.9 \text{ m}^{-1}$ , the constant temperature surfaces (Fig. 13) are stellaratorlike and strongly tilted; the field lines are chaotic throughout most of the volume. Near the magnetic axis some closed, stellaratorlike ( $n=1$ ,  $m=1$ ) surfaces exist as seen in Fig. 14. These surfaces are very similar to ones seen by Sovinec *et al.*<sup>17</sup> and Finn *et al.*<sup>24</sup> in a simulation of a strongly driven spheromak, who observed that the non-axisymmetric configuration is driven by currents in the plasma outside these surfaces and that there is no helicity transport across them; see these publications for further discussion. We also observed them at low flux amplification in lower aspect-ratio simulations.

In the elongated flux conserver, therefore, current and flux can penetrate into the spheromak although not into the stellaratorlike regions. The  $n=0$  component of the magnetic field closely approximates the lowest eigenfunction of the flux conserver, but the plasma is highly tilted, and its axis precesses around the geometric axis. In the present simulations, this precession rate is affected by the boundary condition for the azimuthal velocity at the wall which has been



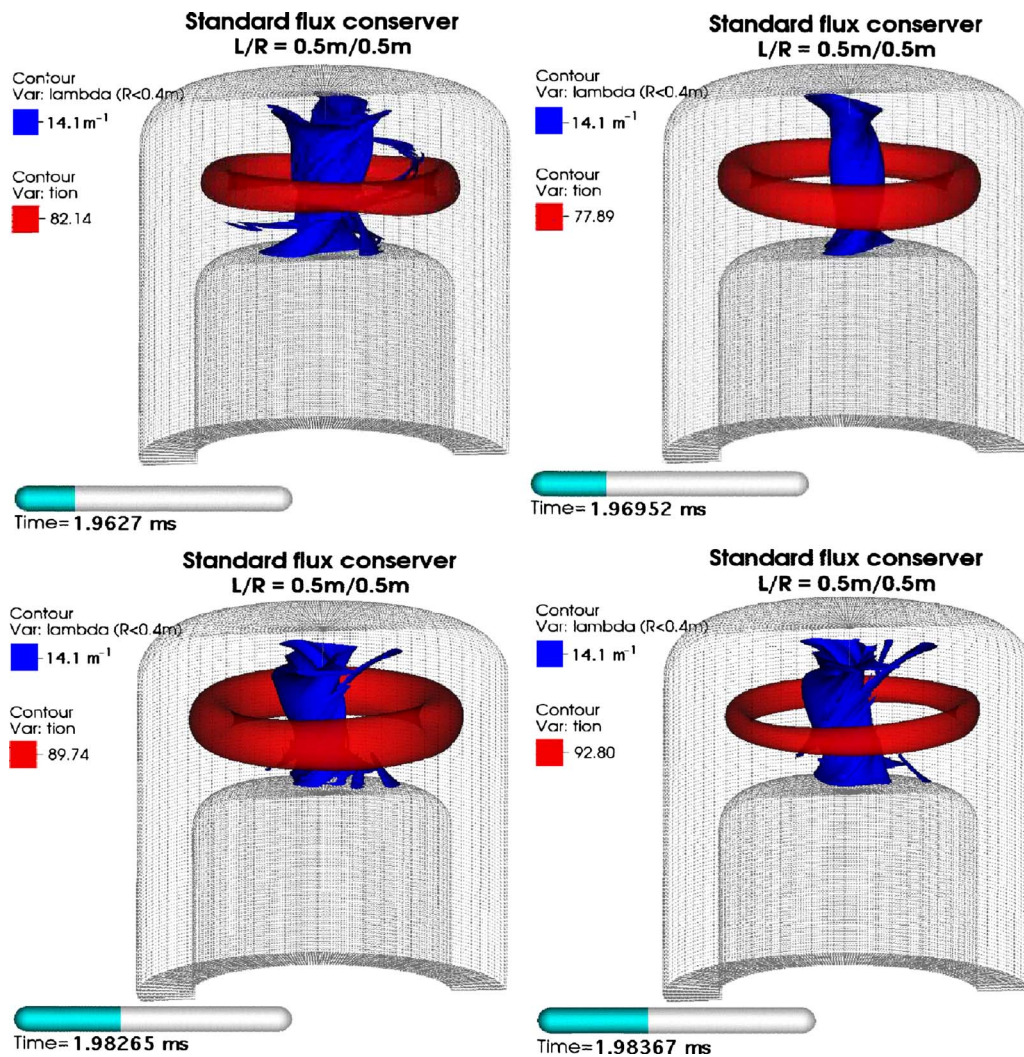


FIG. 7. (Color online) Surfaces of constant  $\lambda = 14 \text{ m}^{-1}$  [blue column between the bottom (cathode) and top (flux conserver)] and constant  $T = 95\%$  of the peak value (red torus) at the times seen in the bars below the flux conserver. In this simulation,  $\lambda_g = 20.3 \text{ m}^{-1}$  (enhanced online). [URL: <http://dx.doi.org/10.1063/1.3134064.1>]

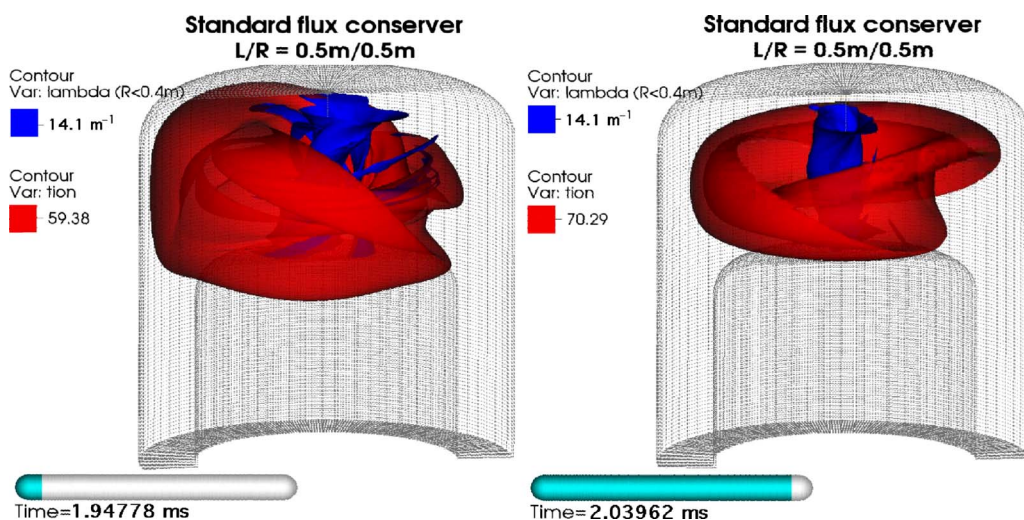


FIG. 8. (Color online) Constant  $\lambda$  and constant  $T$  surfaces as in Fig. 7, showing more complex geometries.



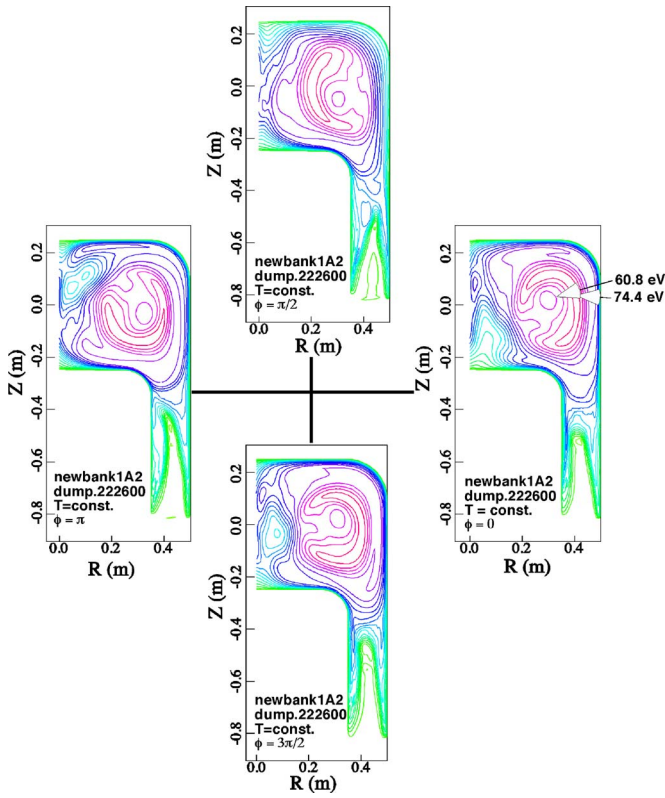


FIG. 9. (Color online) Constant temperature surfaces in the poloidal plane at toroidal angles of 0,  $\pi/2$ ,  $\pi$ , and  $3\pi/2$  (from the simulation in Fig. 7).

chosen to approximate observations in SSPX which presumably are determined by processes outside resistive MHD.<sup>17</sup>

## V. DISCUSSION AND CONCLUSIONS

The simulations yield a simple, offset linear relation between flux amplification and  $\lambda_g$ , with  $dF_a/d\lambda_g$  only weakly dependent on the aspect ratio (shown in Fig. 1 and summarized in Table I). The available experimental data from SSPX track this behavior until  $F_a$  starts to saturate, apparently for

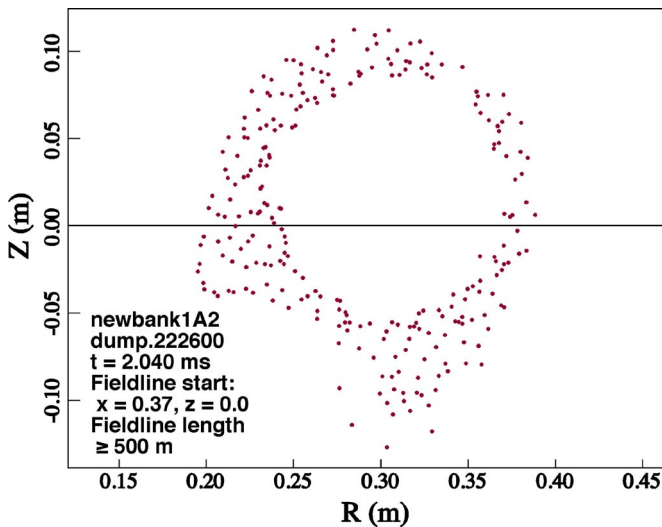


FIG. 10. (Color online) A puncture plot for a single field line in Fig. 8(b). The field line is confined to the volume shown for at least 1000 m.

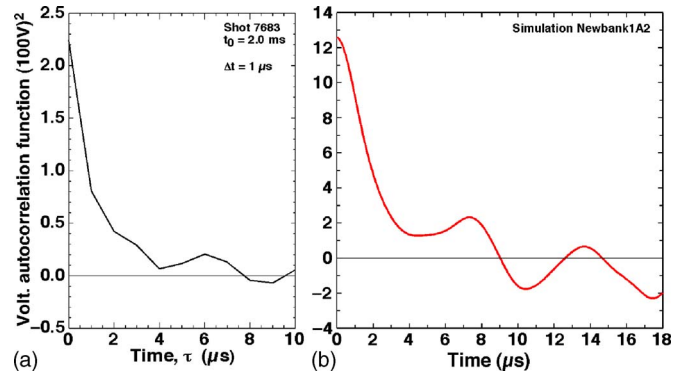


FIG. 11. (Color online) Autocorrelation function for (a) an experimental discharge and (b) a simulation.

reasons that lie outside the single-fluid resistive MHD model. (The saturation seen in Ref. 17 was in a spheromak driven by voltage source, not the current source used here.) The characteristics of the nonlinear steady-state behavior extend the four classes in Sovinec *et al.* by including an oscillating, column-driven mode below the  $\lambda_g$ -threshold for flux amplification and a nearly chaotic state in strongly driven spheromaks.

The transport of helicity into the core of the spheromak requires that magnetic field lines be open at least part of the time, as observed during steady-state, large-amplitude oscillations and during bursty reconnection events: Helicity transport across closed magnetic surfaces is absent in ideal MHD

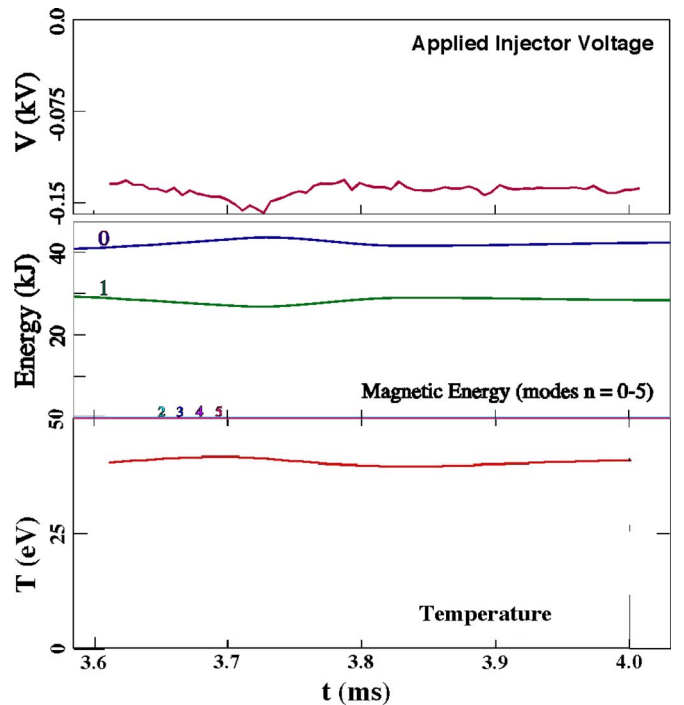


FIG. 12. (Color online) Top to bottom: Voltage, magnetic energy, and temperature for a simulation of the elongated flux conserver,  $A=1.6$ . The behavior is very different from that seen at  $A=1$  (Fig. 4) where there are voltage spikes and other relaxation effects. The energy in the  $n=1$  mode is about 85% of that in  $n=0$ , consistent with the picture of a tilted spheromak rather than an approximately axisymmetric spheromak with an unstable, time-varying  $n=1$  mode.

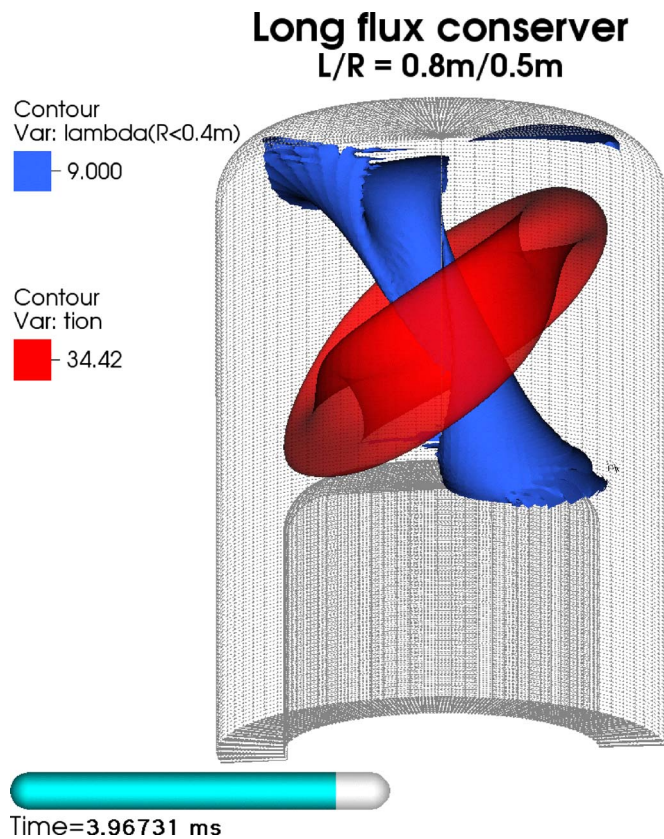


FIG. 13. (Color online) Constant  $\lambda=9.5 \text{ m}^{-1}$  surface (blue column between the cathode and flux conserver) and  $T$  surface at 85% of the peak value (red torus) for a simulation of the  $A=1.6$  flux conserver. Here  $\lambda_g=10.5 \text{ m}^{-1}$  (enhanced online). [URL: <http://dx.doi.org/10.1063/1.3134064.2>]

and is of order the resistivity in resistive MHD.<sup>25–27</sup> Thus, the amplification of magnetic flux found in the SSPX simulations occurs when the field lines are stochastic. (1) For flux conservers far from the tilt-unstable aspect ratio, the system exhibits steady oscillations at low drive, and during strong drive, relaxation oscillations with voltage spiking when a reconnection event occurs. (2) For simulations of the flux conserver close to the tilt limit, there is a large, constant-amplitude column mode ( $n=1$ ); it is an open question whether a strongly driven spheromak will behave chaotically.

These results may explain why some spheromaks behave in a different manner than SSPX. The SPHEX experiment,<sup>5</sup> in particular, typically showed a much more coherent behavior for the  $n=1$  mode<sup>28</sup> than observed in SSPX or CTX. In its standard operation with a positively biased gun, the column carrying the gun current had a structure typified as a steady, rotating “dough-hook,” which looked much like the column shown in Fig. 13. Negative-gun operation was also relatively coherent, although the dough-hook effect was absent.<sup>29</sup> The SPHEX flux conserver had an aspect ratio of unity but was nearly spherical in shape; a spherical flux conserver is marginally stable to the tilt mode.<sup>12</sup> In addition, there was a rather large entrance channel between the gun and the flux conserver. This geometry was thus likely to be closer to the tilt limit than SSPX, allowing the large-amplitude, coherent behavior as observed herein at large aspect ratio. Simulations of SPHEX would be required to verify this conjecture.

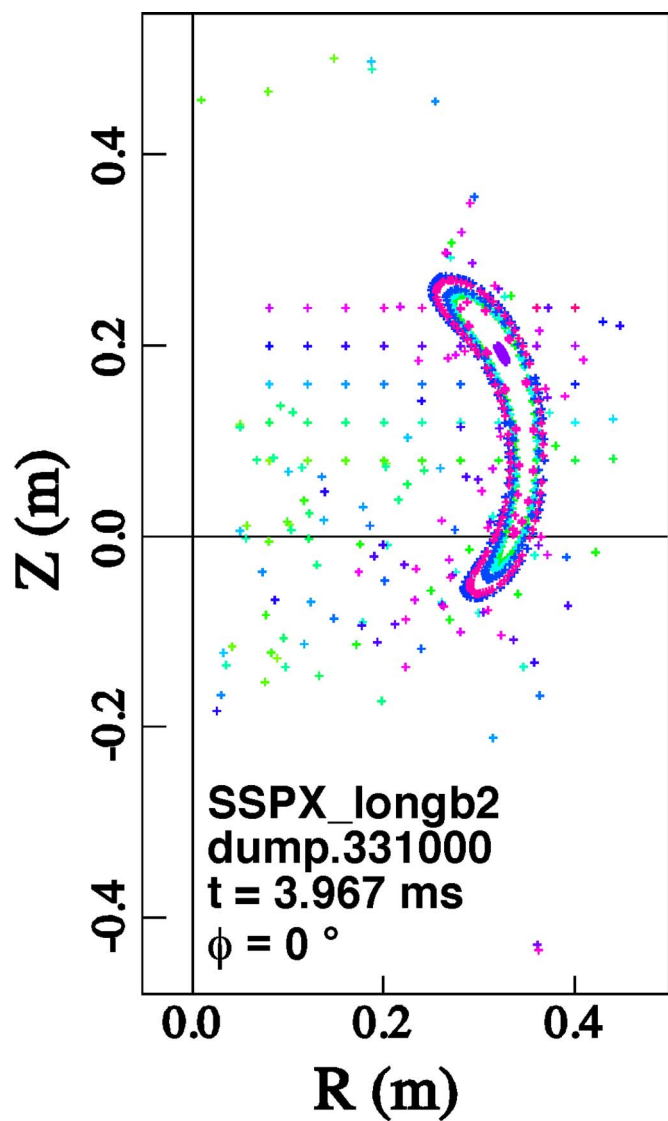


FIG. 14. (Color online) Stellaratorlike magnetic surfaces near the magnetic axis are embedded in a stochastic sea of open field lines (simulation at  $A=1.6$ ).

As discussed in Ref. 16, the efficiency of spheromak formation, e.g., as given by  $B_p/I_g$ , with  $B_p$  the poloidal field measured at the wall, increases as the flux conserver is lengthened. This measure of efficiency is proportional to  $F_a/\lambda_g$ , and thus scales as  $\alpha+(1-\alpha\lambda_{th})/\lambda_g$ . The increase in efficiency near threshold between the aspect ratios of 1 and 1.6 flux conservers is seen to be 1.4 and at large  $\lambda_g$  is 1.2. These factors are not sufficient to reach the efficiencies needed for a fusion-quality plasma. Furthermore, the increased flux amplification is achieved by operating nearer the tilt instability, thereby increasing the strength of the symmetry-breaking,  $n=1$  mode.

This analysis is, of course, only one possible variation in the spheromak geometry. Other shape effects, such as the “bow-tie” flux conserver which is predicted to have higher beta limits than the cylinder<sup>30</sup> or the placement and size of the gun, should be examined for buildup efficiency in future work or in designs for new spheromak experiments.

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