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# Platform-Independent Optimization of Pump Power Threshold for Microcomb Generation

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**Abstract**—Kerr optical frequency combs have found various applications in science and technology, and minimizing their pump power has become an important area of research. These combs are generated using a wide variety of platforms, with a size ranging from micrometers to millimeters, and quality factors ranging from millions to billions. It is therefore not trivial to assess the pump power requirements for comb generation when they have such a large diversity in terms of resonator properties and pump configurations. We propose a suitably normalized threshold pump power as a metric to optimize Kerr comb generation independently of the platform. This method allows one to evaluate the minimum threshold power solely based on the properties of the bare resonator, and independently of dispersion, detuning or coupling considerations. In order to confirm the validity of this approach, we experimentally demonstrate Kerr comb generation in a millimeter-size magnesium fluoride whispering-gallery mode resonator with a threshold pump power of only 1.2 mW, which is one of the lowest pump powers reported to date for a mm-size resonator.

**Index Terms**—Whispering-gallery mode resonators, Kerr optical frequency combs.

## I. INTRODUCTION

THE generation of Kerr optical frequency combs has been the focus of numerous theoretical and experimental works in recent years [1]. These optical frequency combs make use of the strong confinement of laser light within a tubular spatial mode of an optical cavity with a significant Kerr nonlinearity and high quality (or  $Q$ ) factor. As a consequence, the intracavity nonlinear effects are enhanced by path recirculation, thereby making it possible design optical frequency combs generators as a battery-operated device [2]. Applications include microwave photonics [3]–[6], mid-infrared optical technology [7], [8], ultrafast optical fiber communications [9]–[11], ranging [12], [13], or spectroscopy [14], [15], just to name a few.

Kerr combs can be generated using a wide variety of platforms, amongst which we can cite crystalline WGM resonators,

silica-based WGM resonators, and chip-scale WGM resonators. Each of these three types of resonator generates frequency combs with repetition rates ranging from a few GHz to more than a THz. With careful control of the material dispersion, microresonators also make possible the generation of octave spanning combs, which are essential for the stabilization of the repetition rate through self-referencing.

Because of the plethora of Kerr comb generation schemes, it is generally difficult to perform a comprehensive comparison of their pump power threshold. It is desirable to excite the comb with the lowest pump power possible in order to achieve high energy efficiency. However, low pump power by itself cannot be a universal indicator of power efficiency because the targeted applications impose vastly different requirements in terms of free-spectral range, or equivalently, resonator size. For example, direct comparison between Kerr comb generators based on a micrometer-size integrated resonator with  $Q$ -factor of one million and a millimeter-size crystalline resonator with  $Q$ -factor of one billion is meaningless. The threshold powers need to be suitably normalized in order to provide a better basis for comparison among Kerr comb generators. In this letter, we propose a figure of merit that makes it possible to evaluate how close one is to the absolute minimum threshold pump power, so that a comparison can be achieved across platforms. We show as well how it can be used to quantify how close a Kerr comb generator is from optimal pump power configuration in experiments.

## II. MODEL

A resonator is pumped by a single-mode laser of power  $P_L$  and wavelength  $\lambda_L = 2\pi c/\omega_L$ , where  $\omega_L$  is the laser angular frequency and  $c$  is the velocity of light in vacuum. Kerr comb generators are characterized by several parameters associated with their geometry, bulk material properties and to the coupling configuration.

The main geometrical parameter is the round-trip path length  $L$  ( $= 2\pi a$  for a ring or disk of radius  $a$ ), which defines the free spectral range (FSR) of the resonator as  $F_R = \Omega_R/2\pi = v_g/L = c/n_g L$ , where  $\Omega_R$  is the angular FSR,  $c$  is the velocity of light in vacuum,  $n_g$  is the group-velocity refraction index at the pump wavelength,  $v_g$  is the corresponding group velocity. The photons are trapped in spatially tubular cavity modes with effective volume  $V_{\text{eff}} = L A_{\text{eff}}$ , with  $A_{\text{eff}}$  being the effective mode area. The bulk material properties of interest in the resonator are its dispersion profile, the losses, and the

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Kerr nonlinearity. Dispersion is measured at  $k$ -th order by the parameter  $\beta_k$ , with  $\beta_2$  being the group-velocity dispersion (or GVD), while  $\beta_k$  for  $k \geq 3$  stands for the higher-order dispersion terms. The parameter  $n_2$  stands for the Kerr nonlinearity of the host material, and it is useful to rescale this nonlinear parameter as  $\gamma = \omega_L n_2 / c A_{\text{eff}}$  (in units of  $\text{W}^{-1} \text{m}^{-1}$ ). The loss properties of the resonators are evaluated through their total (or loaded) quality factors  $Q$ , which typically range from  $10^6$  to  $10^9$  at 1550 nm, or through their half-linewidth for these resonances,  $\kappa = \omega_L / 2Q$ , which equivalently ranges from GHz to MHz. We may express the loaded quality factor as a function of the intrinsic (or cavity) and extrinsic (or coupling) quality factors as  $Q^{-1} = Q_{\text{in}}^{-1} + Q_{\text{ex}}^{-1}$ , which equivalently yields  $\kappa = \kappa_{\text{in}} + \kappa_{\text{ex}}$ .

Accurate description of Kerr comb generation can be obtained using the Lugiato-Lefever equation (or LLE), which is a partial differential equation that governs the dynamics of the intracavity field  $\mathcal{E}(\theta, t)$  as a function of time  $t$  and of the azimuthal angle  $\theta \in [-\pi, \pi]$  along the perimeter of the resonator, and may be written:

$$\frac{\partial \mathcal{E}}{\partial t} = -\kappa \mathcal{E} + i\sigma \mathcal{E} + iv_g \sum_{k=2}^K (i\Omega_R)^k \frac{\beta_k}{k!} \frac{\partial^k \mathcal{E}}{\partial \theta^k} + iv_g \gamma |\mathcal{E}|^2 \mathcal{E} + \sqrt{2\kappa_{\text{ex}}/T_R} \sqrt{P_L}, \quad (1)$$

where  $\sigma = \omega_L - \omega_0$  denotes the detuning between the laser and the pumped resonance frequencies. In (1), the electric field is normalized in such a way that  $|\mathcal{E}|^2$  is in units of watts. This field can also be expanded as  $\mathcal{E}(\theta, t) = \sum_l \mathcal{E}_l(t) e^{il\theta}$ , where  $l = \ell - \ell_0$  with  $\ell$  being an integer standing for the azimuthal eigennumber of the intracavity photons,  $\ell_0 = n_g L / \lambda_L$  is the eigennumber of the pumped mode, and  $l$  is the so-called reduced eigennumber that expands as  $l = 0, \pm 1, \pm 2, \dots$  around the pumped mode.

### III. ABSOLUTE MINIMUM THRESHOLD POWER

The LLE of (1) has been used extensively to investigate the dynamics of Kerr optical frequency combs. It is well-known that the types of combs that can be generated correspond to various intracavity patterns such as Turing rolls, bright solitons, dark solitons, flaticons (or platons), breathers, or spatiotemporal chaos. In general, the threshold power for Kerr comb generation depends on resonator parameters (intrinsic  $Q$ -factor, nonlinearity, group refractive index) as well as external parameters (coupling  $Q$ -factor, laser frequency detuning). However, for any given resonator with fixed internal parameters, it would be useful to define an absolute minimum power below which no Kerr comb can be generated, regardless of the coupling conditions. This absolute minimum would there be equivalent to an *intrinsic* parameter capable to provide the correct order of magnitude to be expected for low-threshold power Kerr comb generation. A detailed analysis of the threshold power needed to excite Kerr combs was developed via a stability analysis of (1), and there are two types of intracavity patterns for which the threshold power can be estimated analytically [16].

The first type of pattern is generally referred to as Turing rolls, and the determination of the threshold power needed to excite

them is amenable to exact analytical calculation. These patterns emerge as a modulation instability of the flat intracavity pattern created by the laser pump. When they feature an integer number  $N$  of azimuthal rolls, their spectral counterparts are the so-called primary combs, featuring optical modes that are spaced by  $N$  times the FSR. For any given resonator the minimum power needed to generate Turing rolls is

$$P_{\text{th,rol}} = \frac{L\omega_L^2}{8\gamma v_g^2} \frac{Q_{\text{ex}}}{Q^3} \left[ 1 + \left( 1 + \frac{\sigma}{\kappa} \right)^2 \right] = \frac{L\omega_L^2}{\gamma v_g^2} \frac{1}{Q_{\text{in}}^2} \frac{1}{8\eta(1-\eta)^2} \left[ 1 + (1-\alpha)^2 \right] \quad (2)$$

with  $\alpha = -\sigma/\kappa$  being the normalized laser detuning frequency while

$$\eta = \frac{\kappa_{\text{ex}}}{\kappa} = \frac{Q_{\text{in}}}{Q_{\text{in}} + Q_{\text{ex}}} \quad (3)$$

is the so-called *escape parameter* that measures the ratio between out-coupling and total losses. One should note that both  $\alpha$  and  $\eta$  are dimensionless parameters. The threshold power defined in (2) effectively leads to Kerr comb generation only in the super-critical case  $\alpha < 41/30$ ; otherwise, the combs are sub-critical and are generated via a mechanism that is more complex.

The second type of intracavity patterns whose threshold can be determined analytically are cavity solitons, which are always subcritical (i. e. they coexist with stable flat-background solutions). However, in this case, the determination of the threshold power can only be achieved approximately. Such solitons can be excited whenever the pump power exceeds

$$P_{\text{th,sol}} \simeq \frac{L\omega_L^2}{\gamma v_g^2} \frac{1}{Q_{\text{in}}^2} \frac{1}{8\eta(1-\eta)^2} \times \frac{2\alpha + \sqrt{\alpha^2 - 3}}{3} \left[ 1 + \left( \frac{\sqrt{\alpha^2 - 3} - \alpha}{3} \right)^2 \right] \quad (4)$$

in the case  $\alpha > \sqrt{3}$ . This approximate formula is valid for both the normal and anomalous dispersion regimes, the only difference being that dark solitons are obtained in the first case, while bright solitons are generated in the latter. It therefore appears that when the pump laser is blue-detuned, the only patterns that can be excited are rolls, while when the laser is red-detuned, we might have both solitons or rolls depending on the pump laser and detuning characteristics.

The analysis above shows that in all cases, the threshold power can be rewritten under a general form as

$$P_{\text{th}} = f(\eta) g(\alpha) \frac{L}{\gamma L_{\text{in}}^2} \quad (5)$$

where  $f(\eta) = [8\eta(1-\eta)^2]^{-1}$  is a coupling-dependent factor,  $g(\alpha)$  is the detuning-dependent factor, while

$$L_{\text{in}} = \frac{v_g Q_{\text{in}}}{\omega_L} = v_g \tau_{\text{in}} \quad (6)$$

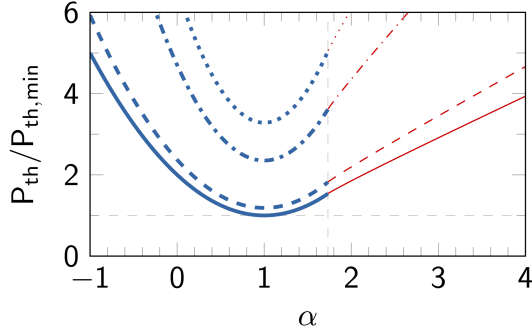


Fig. 1. Normalized threshold power  $P_{th,rol}/P_{th,min}$  and  $P_{th,sol}/P_{th,min}$  as derived in (2) and (4), plotted as a function of the normalized laser detuning frequency  $\alpha = -\sigma/\kappa$ . The various plots correspond to different coupling conditions, namely optimal coupling ( $\eta = 0.33$ , continuous lines), critical coupling ( $\eta = 0.5$ , dashed lines), undercoupling ( $\eta = 0.05$ , dotted lines), and overcoupling ( $\eta = 0.7$ , dashdotted lines). The vertical dashed line is set at  $\alpha = \sqrt{3}$  and delineates the roll solutions on the left [thick blue lines, (2)] from the soliton solutions on the right [thin red lines, (4)]. The horizontal dashed line shows that the absolute minimum threshold power is indeed the roll solution at  $\alpha = 1$  with  $\eta = 1/3$ , as determined in (7).

is the average distance covered by a photon with lifetime  $\tau_{in} = 1/2\kappa_{in}$  when it is only subject to the intracavity losses (interaction length in the bare resonator).

Once a resonator is designed and fabricated, its intrinsic quality factor  $Q_{in}$  is set. However, the extrinsic  $Q$ -factor can be tuned to a wider range, either by moving the waveguide relative to the resonator, or via innovative design such as pulley coupling, which modifies the coupling of laser signals in and out the resonator. For practical purposes, it is interesting to indicate that the resonator is undercoupled when  $0 < \eta < 1/2$  (most losses are intrinsic), overcoupled when  $1/2 < \eta < 1$  (most losses are extrinsic), and critically coupled when  $\eta = 1/2$ . Consequently the free parameters  $\eta$  (controlled via the coupling quality factor  $Q_{ex}$ ) and  $\sigma$  (controlled with the laser detuning frequency) are those to be optimized in order to find the absolute minimum threshold power.

For both rolls and solitons, the extremum condition  $\partial P_{th}/\partial \eta$  yields a minimum for  $P_{th}$  when the escape parameter is such that  $\eta \equiv \eta_{opt} = 1/3$ , equivalent to  $Q_{ex} = 2Q_{in}$  and  $f(1/3) = 27/32$ . It is interesting to note that this absolute minimum power is *not* obtained at critical coupling [with  $\eta = 1/2$  and  $f(1/2) = 1$ ] as one would have intuitively presumed. In fact, critical coupling leads to a threshold power that is about 20% higher.

To determine the optimal detuning, we first have to find  $\partial P_{th}/\partial \alpha$ , using the detuning factor  $g(\alpha)$  which is different for the roll and soliton solutions. From (2), it is straightforward that the threshold power for rolls is minimum when the laser is detuned to the red edge of the resonance with  $\alpha = 1$  (i. e.,  $\sigma \equiv \sigma_{opt} = -\kappa$  so that  $\omega_L = \omega_0 - \kappa$ ), generating a detuning factor of  $g(1) = 1$ . On the other hand, we can deduce from (4) that the threshold power for solitons is minimum when the laser is detuned slightly further the red-edge resonance with  $\alpha = \sqrt{3}$ , i. e.  $\sigma \equiv \sigma_{opt} = -\sqrt{3}\kappa$ , generating a detuning factor  $g(\sqrt{3}) = 8\sqrt{3}/9 \simeq 1.54$ . Fig. 1 summarizes our analysis by displaying the normalized threshold powers as a function of deuning.

Therefore, the absolute minimum for the pump threshold power is found to be the one corresponding to the roll pattern (which has the smallest minimum detuning factor), leading to

$$P_{th,min} = \frac{27}{32} \frac{L}{\gamma L_{in}^2} = \frac{27\pi^2}{8} \frac{n_g^2 L}{\gamma \lambda_L^2 Q_{in}^2} \quad (7)$$

According to (7), the approximate value  $P_{th,min} \simeq L/\gamma L_{in}^2$ , corresponds to the case of critical coupling (with  $\eta = 1/2$  as discussed earlier), which still provides a simple and useful rule of thumb to determine the efficiency of the Kerr comb generation process. It therefore appears that the absolute minimum power to generate a Kerr comb only depends on three key elements: the nonlinearity  $\gamma$  of the resonator, the length  $L$  of the cavity round-trip, and the interaction length  $L_{in}$ . However, it is important to stress that  $P_{th,min}$  is not the power needed to generate Kerr combs [this would be  $P_{th}$  as defined by either (2) or (4)]. Instead, the minimum threshold power determined in (7) provides lower limit below which no Kerr comb can be generated around pump a wavelength  $\lambda_L$  with a resonator of size  $L$ , group refractive index  $n_g$ , nonlinear coefficient  $\gamma$  and intrinsic quality factor  $Q_{in}$ . In that sense, (7) therefore appears as a necessary (but not sufficient) pump power requirement for comb generation. We also note that  $P_{th,min}$  does not depend on the dispersive properties of the resonator.

For a comb that is generated with a laser pump power  $P_L$ , we can now define the power efficiency of the comb generation as

$$\rho = \frac{P_{th,min}}{P_L} \simeq \frac{L}{\gamma P_L L_{in}^2}. \quad (8)$$

The parameter  $\rho$  therefore qualifies as a metric that can unambiguously characterize a low-threshold Kerr comb.

It is interesting to interpret (8) in terms of interaction lengths, as it is routinely done in nonlinear fiber optics. The parameter  $L_{in}$  can be interpreted as the length beyond which losses become dominant. On the other hand, for a given pump power  $P_L$ , a Kerr length (or “nonlinearity length”)  $L_{nl} = 1/\gamma P_L$  can be introduced as the length beyond which the nonlinear effects become significant. As a consequence, the efficiency can be simply rewritten as  $\rho \simeq LL_{nl}/L_{in}^2$ , with a transparent geometrical interpretation.

The theory indicates that regardless of cavity detuning, coupling and dispersion, it is impossible to generate a Kerr comb with a pump power lower than  $P_{th,min}$  as defined in (7). If such a comb is experimentally observed, the potential explanation is generally a faulty definition of  $\gamma$  due to an inaccurate estimate of the effective area  $A_{eff}$ . Indeed, for a spherical resonator of radius  $a$ , one can use the approximation  $V_{eff} = 2\pi a A_{eff} \simeq 3.4 \pi^{3/2} (\lambda_L/2\pi n_g)^3 \ell_0^{1/6}$  which is valid for the limit case of the fundamental mode family. However, for disks resonators of main radius  $a$ , the local curvature at the rim is much smaller than  $a$ : Therefore, the mode confinement is stronger and this formula only yields an upper bound for the mode area.

#### IV. EXPERIMENTAL SETUP

The experimental setup used to generate Kerr combs consists of a tunable 1550 nm continuous-wave fiber laser (Koheras Adjustik) that features a few-kHz linewidth. By applying a



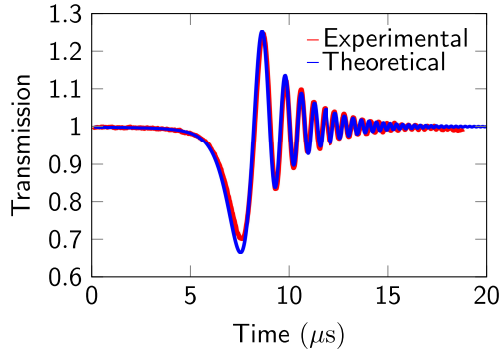


Fig. 2. Experimental cavity ring-down measurement for the determination of the intrinsic and extrinsic quality factors. The measurement yields  $Q_{\text{in}} = 1.82 \times 10^9$  and  $Q_{\text{ext}} = 4.33 \times 10^9$ . The escape ratio is  $\eta = Q_{\text{in}}/(Q_{\text{in}} + Q_{\text{ext}}) = 0.3$ , which is very close to the optimal value  $1/3$ .

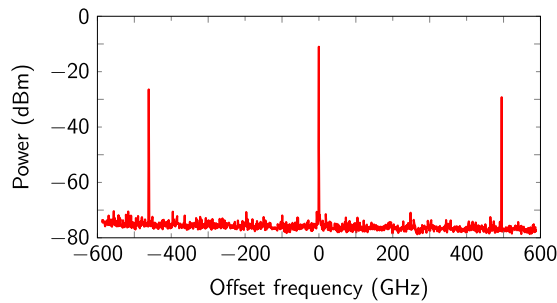


Fig. 3. Experimental spectrum of a Kerr comb generated with a threshold laser power of only 1.2 mW. The magnesium fluoride WGM resonator has a free-spectral range of 6 GHz.

triangular signal to the piezo-actuator of the laser, it is swept over 15 pm, polarization controlled, then coupled to the ultra-High- $Q$  magnesium fluoride WGM resonator. This resonator has a diameter of 12 mm and a group velocity index  $n_g = 1.37$ , corresponding to a free-spectral range around 6 GHz. The evanescent coupling is achieved using a tapered fiber to excite the WGM resonances. One part of the resonator output signal is sent to a photodiode (Thorlabs InGaAs amplified PDA 10CF-EC) connected to an oscilloscope in order to observe these resonances. We then select the mode with the highest  $Q$  factor (leading to the lowest threshold power) to generate Kerr combs. The second part of the signal is sent to an Optical Spectrum Analyzer (OSA) to observe the generated combs. Coupling light within the resonator leads to refractive index change, shift of resonances, and therefore unstable combs. In order to avoid this effect, we rely on a Pound-Drever-Hall loop to lock the laser to the selected resonance before generating the Kerr combs.

Fig. 2 shows that we are able to couple our resonator with an escape ratio  $\eta = 0.3$ , which is very close to the optimal ratio of  $1/3$ . The resulting Kerr comb with spacing of 500 GHz at a threshold power of 1.2 mW is represented in Fig. 3, which is within an order of magnitude of  $P_{\text{th,min}}$ . If we consider  $A_{\text{eff}} \simeq 100 \mu\text{m}^2$  and  $n_2 \sim 10^{-20} \text{ m}^2/\text{W}$ , we are led to a theoretical minimum of  $P_{\text{th,min}} \sim 1 \text{ mW}$ . This value is commensurate with the experimental threshold pump power we have demonstrated, and therefore indicates near-optimal operation from the

energetic viewpoint. We also note that as predicted by the theory, the lowest pump-power pattern we could generate was a primary comb, which is the spectral signature of Turing rolls. From a general perspective, understanding threshold power scales with coupling, detuning or dispersion can be performed through a detailed analysis of the LLE [16], [17].

## V. CONCLUSION

We have developed a theoretical framework for the characterization of low-pump power Kerr optical frequency combs. Since the diversity of bulk materials, size, loss properties and coupling conditions does vary significantly depending on the comb generation platform, the threshold power is not a pertinent indicator of efficiency, appropriate normalization requires to rescale this power with parameters that are intrinsic to the resonator. We have proposed a suitable normalization scheme to that permits to obtain a metric that allows comparison across platforms. Future work will be devoted to the integration of other nonlinear effects into this characterization.

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