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A simple formula for estimating the magnetic fields generated by tsunami flow

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[1] Ocean flow is known to generate magnetic fields which reach land and satellite observatories with detectable amplitudes. But because the expected signal-to-noise ratio is poor, one must typically know in advance of the data analyses much about the expected behavior in order to identify these small oceanic contributions. We show that in the case of ocean flow associated with tsunamis, a very simple formula (relating both amplitudes and phase) allows the expected magnetic fields to be directly related to the sea surface displacement. Using sea surface data from Jason 1, the formula gives maximum magnetic amplitudes of about 2 nT for the recent Dec. 26, 2004 tsunami which occurred near the magnetic equator. But near the magnetic poles, the maximum amplitudes reach about 20 nT per meter sea surface displacement. Determining the feasibility for extraction of tsunami generated magnetic signals is of interest in designing early-warning systems and for remotely tracking tsunami development. **Citation:** Tyler, R. H. (2005), A simple formula for estimating the magnetic fields generated by tsunami flow, *Geophys. Res. Lett.*, 32, L09608, doi:10.1029/2005GL022429.

1. Introduction

[2] Magnetohydrodynamic interaction of ocean flow with the Earth's main magnetic field is known to generate weak secondary magnetic fields which have also been identified in land and satellite observatory data—at least in the case of the ocean tides which have a distinct periodic behavior that can be used to control the search [e.g., *Malin*, 1970; *McKnight*, 1995; *Tyler et al.*, 2003]. In principle, ocean flow associated with tsunamis will also generate magnetic fields which may conceivably be used one day in an early-warning system and/or to map the amplitudes and characteristics of a tsunami as it progresses. Previous discussion of the feasibility of using electromagnetic field sensing to detect tsunamis favored in situ measurements of the electric field [*Larsen*, 1971] which, because it requires installation of dedicated seafloor electrometers, will likely be curbed at limited spatial resolution. But given the recent expansion of high-resolution satellite-borne magnetic surveys (rsted, CHAMP, SAC-C, SWARM) and modern developments in SQUID (Superconducting QUantum Interference Device) technologies, it is useful to examine the feasibility of tsunami detection using remote and in situ magnetic field measurements. While the expected magnetic signals are of detectable amplitudes they will occur with time and space scales (≈ 1 h and ≈ 100 km) which overlap appreciably

with other incompletely resolved magnetic sources thereby requiring development and demonstration of a scheme for retrieving these signals which have a low signal-to-noise ratio. The goal of this paper is limited to providing a simple formula to aid in such endeavors.

[3] While it is possible to numerically model the magnetic fields given a realistic simulation of tsunami flow, such calculations are computationally expensive and unnecessary as a first step. Indeed, the dynamics of open-ocean tsunamis are relatively simple and these flow constraints are used below to derive a very simple expected relationship (formula) between the magnetic fields at or above sea level and the sea surface displacement. The formula is sought in terms of sea surface displacement (rather than flow velocity, say) because this parameter is the one most readily available in observations and simulations. While the formula is an approximation, its convenience should allow easy assessment of opportunities for sensing tsunamis through the magnetic fields they generate. More general solutions which apply to intermediate and long-wave flows and also allow for a parameterization of mantle electrical conductivity are described by *Larsen* [1971].

2. Formulation

2.1. Induction Equation

[4] The starting point for this analysis will be the magnetic induction equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B} - K \nabla \times \mathbf{B}) \quad (1)$$

which is constructed by combining three of the Maxwell's equations and Ohm's Law for a moving conductor, which under suitable approximations for this study are

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \quad (2)$$

$$\nabla \times \mathbf{B} = \mu \mathbf{J} \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4)$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{u} \times \mathbf{B}). \quad (5)$$

Above, \mathbf{u} is the velocity of the fluid, $K = (\mu\sigma)^{-1}$ is the magnetic diffusion coefficient, σ , is the electrical conductivity, μ is the magnetic permeability (assumed to be the free-space value in this paper), and other notation is conventional.

[5] For our analysis we wish to adopt a reference frame rotating with the Earth. Although Maxwell's equations and the constitutive relationships are not formally invariant to translation to a rotating frame (even for non-relativistic rotation velocities), for the application we will consider the forms shown above are approximately valid in the rotating frame [Tyler and Mysak, 1995].

2.2. The Induction Equation on Geopotential Surfaces

[6] In pursuit of a coordinate system suitable for simplifying (1), let us define a vector $\mathbf{g} = -\nabla\Phi$ where Φ is some scalar field. Taking the scalar product of \mathbf{g} with (1) and using standard vector identities we write

$$\mathbf{g} \cdot \partial_t \mathbf{B} = \nabla \cdot (\{\mathbf{u} \times \mathbf{B}\} \times \mathbf{g} - K\{\nabla \times \mathbf{B}\} \times \mathbf{g}). \quad (6)$$

which is an expression describing generation of the magnetic component perpendicular to constant Φ surfaces. If we were considering flow in a perfect spherically symmetric shell, for example, it would be useful to take $\Phi = \text{radius}$, in which case this equation becomes an equation describing the generation of the radial magnetic component. For the purpose of directly using flow constraints it will be more formal, however, to take Φ to be the effective gravitational potential (gravitational potential plus potential due to centrifugal accelerations due to rotation). The vector \mathbf{g} is then the effective gravity. When we assume \mathbf{g} is relatively constant and uniform in the ocean layer, (6) can be written as

$$\partial_t B_z + \nabla_H \cdot (B_z \mathbf{u}_H) = \nabla_H \cdot (u_z \mathbf{B}_H) + \nabla_H \cdot \{K \nabla_H B_z - K \partial_z \mathbf{B}_H\} \quad (7)$$

where the subscript H refers to the “horizontal” components along constant Φ surfaces, while z refers to the outward “vertical” component perpendicular to the Φ surfaces. Hence, (7) describes generation of the magnetic field component perpendicular to the geopotential surfaces.

[7] Now let us include flow constraints. Typical large-scale geophysical flows are in hydrostatic balance to first order, meaning the fluid momentum equations with only the most important terms retained would express simply a balance between the hydrostatic pressure gradient forces and effective gravity. The flow (then a higher-order effect) is predominantly along geopotential surfaces, hence while B_z and \mathbf{B}_H may be of comparable magnitude, we will assume that $|\mathbf{u}_H| \gg |u_z|$ and therefore neglect the first term on the right of (7). Strictly, neglect of this term is not appropriate very close (within a couple of degrees) to the magnetic equator. The appropriate condition is that sine of magnetic latitude be much greater than the ratio of water depth to horizontal flow length scales which must already be $\ll 1$ for consistency with the assumptions above. Also, because $|u_z|$ is small, the primary result is that sources for b_z near the magnetic equator are negligible. Discussion of the organization of large-scale flow with respect to the geopotential surfaces can be found in standard references of geophysical fluid dynamics [e.g., Pedlosky, 1979]. We will also decompose the magnetic field $\mathbf{B} = \mathbf{F} + \mathbf{b}$ into a steady main field part \mathbf{F} (obeying $\nabla \times \mathbf{F} = 0$, $\nabla^2 \mathbf{F} = 0$) and an ocean generated part \mathbf{b} for which we initially assume both

amplitudes and gradients are much smaller than that of \mathbf{F} . Equation (7) then becomes

$$\partial_t b_z = -\nabla_H \cdot (F_z \mathbf{u}_H) + \nabla_H \cdot (K \nabla_H b_z - K \partial_z \mathbf{b}_H) \quad (8)$$

which, when K is uniform along the Φ surfaces (i.e. $K = K(z)$), and using (4) can be written as

$$\partial_t b_z = -\nabla_H \cdot (F_z \mathbf{u}_H) + K \nabla^2 b_z \quad (9)$$

which involves only one variable b_z .

2.3. Fourier Solution

[8] The basic flow dynamics of the tsunami are described by the class of long surface-gravity waves. For this case we can assume that \mathbf{u}_H is independent of ocean depth and that \mathbf{u}_H (and concomitantly \mathbf{b}) vary more rapidly in the horizontal than do other parameters. This amounts to assuming that the ocean wavelength, while much larger than the ocean depth, is much smaller than the scale for variations in F_z , ocean bathymetry or depth-averaged conductivity. This assumption should be reasonable in the deep ocean much further than a wavelength from the coastlines. Furthermore, let us first assume that the regions outside of the ocean are electrically insulating (hence, outside the ocean $K \rightarrow \infty$ and $\nabla^2 b_z = 0$) and that K inside the ocean is represented by its depth averaged value which may vary slowly in the horizontal. In this case, we can apply (9) within the ocean, $\nabla^2 b_z = 0$ outside the ocean and match solutions at the interfaces through requirement of continuous magnetic fields—which translate here to continuity in b_z as well as $\partial_z b_z$ which is required in consideration of continuity of \mathbf{b}_H and (4). Requiring boundedness above and below the approximately planar thin ocean layer, we then solve for a space/time Fourier component by assuming $b_z, F_z \mathbf{u}_H \propto e^{-i(\omega t - \kappa \cdot \mathbf{x})}$ where ω is the frequency, κ is the wave number vector describing the horizontal variation, and \mathbf{x} is the horizontal position vector. This yields a solution which evaluated at and above the sea surface ($0 \leq z$) is

$$b_z = -\frac{\sinh(\alpha h/2)}{\alpha K(\alpha \sinh(\alpha h/2) + \kappa \cosh(\alpha h/2))} \nabla \cdot (F_z \mathbf{u}_H) e^{-\kappa z} \quad (10)$$

where $\alpha = (\kappa^2 - i\omega/K)^{1/2}$, $\kappa = |\kappa|$, and h is the ocean layer thickness which we will also refer to as depth.

2.4. A Simple Conversion Formula (Response Map)

[9] For the present application, this solution is most interesting in the simultaneous limit of $\kappa h/2 \ll 1$ (layer half thickness $h/2$ much smaller than horizontal scales) and $h/\delta \ll 1$ (thickness h much smaller than an electrical skin depth $\delta \equiv (2K/\omega)^{1/2}$). With these assumptions, $\alpha h/2$ is a small parameter, b_z is approximately uniform through the ocean layer and has the value described by (10) which to leading order in $\alpha h/2$ is

$$b_z \approx \left(1 - \frac{i\omega h}{2K\kappa}\right)^{-1} \frac{h}{2K} [\kappa^{-1} \nabla \cdot (F_z \mathbf{u}_H)]. \quad (11)$$

For typical parameters and wave periods greater than or equal to ten minutes, the differences between the

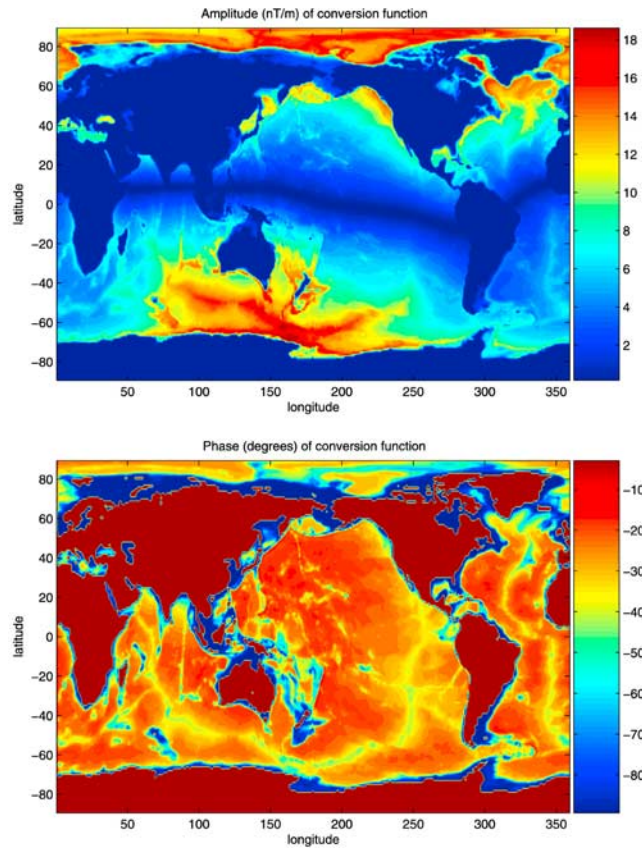


Figure 1. (top) Amplitude of conversion formula ($\frac{F_z c}{h c_s}$) which gives the amplitude (nT) at sea level of the vertical magnetic field b_z per unit meter of sea surface displacement η associated with a long surface gravity wave (tsunami). (bottom) Phase of conversion formula (degrees) indicating phase lag between b_z and η .

approximate expression (11) and (10) is less than six percent.

[10] We further note that under our assumption that \mathbf{u}_H varies more rapidly in the horizontal than either F_z or h , the continuity equation for the flow allows us to rewrite $\nabla \cdot (F_z \mathbf{u}_H) = i\omega\eta F_z/h$ where η is the sea surface displacement. Also, for long surface gravity waves $\omega/\kappa = c$ (i.e. ratios of frequency and wave number can be replaced by a parameter, the long wavelength non-dispersive surface gravity wave speed, $c = (gh)^{1/2}$, which depends only on gravity g and water depth h). Combining these simplifications and defining the complex scaling speed

$$c_s = c + ic_d \quad (12)$$

introduced by Tyler [2001], where $c_d = 2K/h$ is the lateral diffusion speed, we can write (11) as simply

$$\frac{b_z}{\eta} = \left(\frac{F_z c}{h c_s} \right) e^{-\kappa z} \quad (13)$$

[11] The right side of (13) is then a simple conversion formula which allows us to estimate b_z from η or vice-versa. The solutions for the horizontal components $\mathbf{b}_H(0 \leq z)$ can

be obtained using (4) and have the same form as in (13) but divided by i .

[12] The great simplification is that the term in brackets in (13) depends only on environmental parameters and hence is independent of ω and κ . At the sea surface the exponential term is unity and therefore (13) shows that $b_z(z=0)$ and η are related through a parameter which depends on the generic behavior of long surface gravity waves but does not depend on the specific wave parameters. The specific wave number κ is needed, however, for calculating the magnetic fields at altitude but this effect can also be easily understood and included by simple spatial low-pass filtering of data.

[13] Hence, it is sufficient to examine only the behavior of the term in brackets in (13) which gives the conversion formula for b_z/η at sea level. We note that this term is complex because c_s is complex.

[14] In Figure 1 (top) we show the amplitude which describes the proper scaling. The phase angle (Figure 1 (bottom)) describes the phase lag between b_z and η . Physically, it describes whether the process falls in the diffusive regime (phase $\approx 90^\circ$) or frozen-flux regime (phase $\approx 0^\circ$). Indeed, in the frozen-flux regime, $c \gg c_d$ and (13) at the surface simply states that $b_z/F_z = \eta/h$ (i.e. fractional changes in depth due to flow convergence are associated with fractional changes in the vertical magnetic field.) Perhaps this can be made more clear by writing (13) as

$$\frac{b_z}{F_z} = \frac{c}{c_s} \frac{\eta}{h} e^{-\kappa z}. \quad (14)$$

[15] A peculiarity of the situation for the Earth's ocean is that c and c_d are comparable in amplitude over most of the deep ocean. Typical values are $c \approx 200$ m/s and $c_d \approx 80$ m/s. In the deepest waters the first ratio on the right of

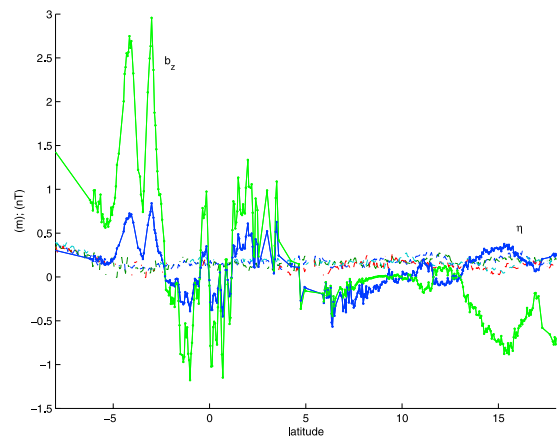


Figure 2. The upward vertical magnetic field b_z (nT) at sea level (green) associated with the observed sea surface height anomalies (m) along a Jason 1 track (extending from about 82 E, -10 S to 93 E, 20 N) two hours after the start of the December, 2004 tsunami (blue). Only the real (in-phase) part of b_z is shown. Jason observations for three pre-tsunami tracks are shown for reference. Estimates for b_z are the same at the seafloor, while at satellite altitude the high-wave number features (wavelengths \ll satellite altitude) will be strongly attenuated.

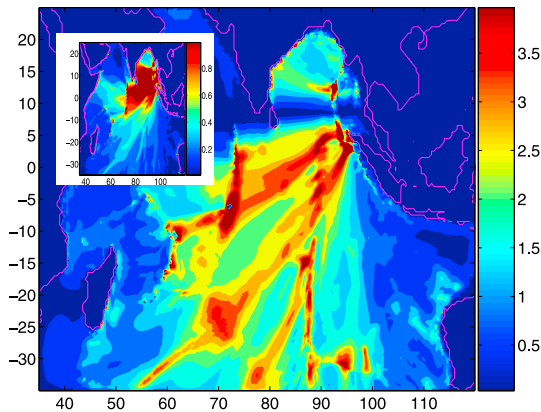


Figure 3. Maximum sea surface elevation (m) from simulation of 26 December, 2004 tsunami by V. Titov (shown in inset) is used to produce the maximum amplitudes (nT) for the upward magnetic field component generated by the tsunami flow. The amplitudes of about 3 nT from this simulation are in agreement with those calculated from sea surface observations in Figure 2. As expected, amplitudes near the magnetic equator (occurring in the region at $\approx 8\text{--}9$ degrees N. latitude) are small.

(14) is close to unity and therefore (14) describes a simple situation where fractional changes in vertical magnetic field b_z/F_z vary directly with fractional changes in the height of the water column η/h .

[16] Contrarily, in shallower water $c \ll c_d$ and $c/c_s \approx c/(ic_d)$. This factor is notably: 1) less than unity; and 2) imaginary. In this case the amplitude of the magnetic fields per unit η is always smaller than in the frozen-flux regime and b_z is out of phase with η (although \mathbf{b}_H and η are then in phase). But recall that the approximations used break down at distances approaching a wavelength from the coastline. Hence, while these estimates remain valid on the shallow shelves for swell, for example, they are not valid for tsunamis having long wavelengths comparable to or greater than the shelf width.

[17] The results in Figure 1 indicate that for a tsunami of 0.5 m wave height, the peak to peak magnetic anomalies can reach about 20 nT but only in deep-water regions near the magnetic poles. The recent tsunami in Sumatra, for example, occurred near the magnetic equator and the expected amplitudes for the magnetic field are expected to be located not near the epicenter but rather in the southern Indian Ocean. Recently, several research groups (e.g., Institute of Ocean Sciences, NOAA/PMEL, and NASA/JPL) have shown websites (<http://www-sci.pac.dfo-mpo.gc.ca/osap/projects/tsunami>, <http://www.pmel.noaa.gov/tsunami/>, <http://www.jpl.nasa.gov/news/news.cfm?release=2005-013>) with satellite observed sea surface elevation anomalies associated with this tsunami. As an illustration, we show in Figure 2 sea surface elevations from the Jason 1 altimetry satellite on its 109th repeat along track 254. Alternatively, estimates of the expected magnetic fields can be obtained from simulated tsunami flow. In Figure 3 we show results using data from a tsunami simulation in

viewing the results, it should be kept in mind that, as discussed above, the estimates are not valid in two regions shown: 1) near the magnetic equator, but as discussed the magnetic fields are anyway small here; 2) close (≈ 300 km) to coastlines. Also, note that the sea surface data used are for the maximum amplitudes rather than a snapshot of η at one time. For a snapshot, the dominant variations would be due to the sign changes in b_z between the crests and troughs of the wave (as in Figure 2).

[18] In summary, ocean flow associated with the propagation of tsunamis belong to a class (non-dispersive long gravity waves) for which the magnetohydrodynamically generated magnetic fields are related to the sea surface perturbations in a very simple, if approximate, manner. (This can be contrasted with the usual case where magnetic fields must be obtained from large-scale integrals over flow sources.) This simple relationship (described by either (13) or (14)) can be used for inspecting the ground and satellite magnetic records for the presence of past tsunamis. But a primary and timely use will be in determining the benefit of adding magnetic sensors to tsunami warning systems. This will require finding methods for identifying (rapidly presumably) the tsunami contributions in magnetic measurements. In this regard we note that the simple relationship above is of the type needed, and that it predicts elliptically polarized magnetic fields in the atmosphere which will rotate in planes containing the vertical axis and decay exponentially upward. Given the facts that the expected tsunami signals are episodic and weak, and that the frequency/wave numbers overlap with that of other magnetic sources, it is expected that the described vector behavior and structure of these fields must also be used in any method aimed at identifying tsunami magnetic signals.

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