ELECTION INVERSIONS BY THE U.S. ELECTORAL COLLEGE

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February 7, 2011
Revised April 28, 2011

Abstract

It is well known that the U.S. Electoral College, like other two-tier electoral systems, is subject to *election inversions* such as occurred in the 2000 Presidential election, in which the candidate or party that wins the most votes from an electorate fails to win the most electoral votes or parliamentary seats and therefore loses the election. In so far as this phenomenon may be 'paradoxical,' it is of a different character from most other voting paradoxes in that it may arise even if there are only two candidates or parties and it is straightforward in nature and its occurrence is readily apparent. However, the likelihood of election inversions and the factors that produce them are less apparent, and there has been considerable confusion about the circumstances under which election inversions occur. This paper identifies the sources of election inversions by the Electoral College, establishes logical bounds on the phenomenon, and estimates the frequency and magnitude of inversions on the basis of historical state-by-state Presidential election data.

To appear in Dan S. Felsenthal and Moshé Machover, eds., *Electoral Systems: Paradoxes, Assumptions, and Procedures*, Berlin: Springer, 2012 (forthcoming).

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An *election inversion* occurs when the candidate (or party) that wins the most votes from an electorate fails to win the most electoral votes (or parliamentary seats) and therefore loses the election. Public commentary commonly uses terms such as 'reversal of winners,' 'wrong winner,' 'divided verdict,' and 'misfire'' to describe this phenomenon; the academic social choice literature adds such terms as 'representative inconsistency,' 'compound majority paradox,' 'referendum paradox,' and 'majority deficit.' Election inversions can occur under any two-tier electoral system, including the U.S. Electoral College. As is well known, the Electoral College actually produced a 'wrong winner' in the 2000 Presidential election, and it has done so twice before.

In so far as this phenomenon may be 'paradoxical,' it is of a somewhat different character from most other paradoxes in the theory of voting and social choice, in that it may arise even if there are only two candidates, it is straightforward in nature, and its occurrence is readily apparent. However, the likelihood of inversions and the factors that produce them are less apparent, and there has been considerable confusion about the circumstances under which election inversions occur. For example, the susceptibility of the Electoral College to inversions is sometimes blamed on the small-state bias in the apportionment of electoral votes and/or the 'non-proportional' or 'winner-take-take-all' manner of casting state electoral votes, but inversions can occur in the absence of both factors.

With specific respect to the U.S. Electoral College, I first note the three historical manifestations of election inversions and identify and discuss one massive but 'latent' inversion in more detail. I then use 'uniform swing analysis' based on historical election data in order to estimate the frequency, magnitude, and direction of potential election inversions. Along the way, I identify three sources of election inversions — 'rounding effects,' 'apportionment effects,' and 'distribution effects' — and examine their separate impacts on the likelihood of election inversions by the Electoral College.

1. The Problem of Election Inversions

The President of the United States is elected, not by a direct national popular vote, but by an indirect Electoral College system in which (in almost universal practice since the 1830s) separate state popular votes are aggregated by adding up state electoral votes awarded, on a winner-take-all basis, to the plurality winner in each state.[1] Therefore the U.S. Electoral College is a two-tier electoral system: individual voters cast votes in the first tier to choose between rival slates of 'Presidential electors' pledged to one or other Presidential candidate, and the winning elector slates then cast blocs of electoral votes for the candidate to whom they are pledged in the second tier. Each

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[1] At present Maine (since 1972) and Nebraska (since 1992) use the 'modified district system,' under which electoral votes may be split. The 2008 election was the first actually to produce a split (in Nebraska).

state has electoral votes equal in number to its total representation in Congress and since 1964 the District of Columbia has three electoral votes. At the present time, there are 538 electoral votes, so 270 are required for election and a 269-269 electoral vote tie is possible.

To the best of my knowledge, the first theoretical work on election inversions was May (1948), who attempted to calculate the *a priori* probability of inversions based on a particular probability model of election outcomes. Several years earlier Schattschneider (1942) had noted in passing the '25%–75% rule' pertaining to election inversions that will be discussed later. Sterling (1981) provided an insightful geometric analysis of 'Electoral College misrepresentation,' a modified version of which will be fruitfully employed here. More recently, Nurmi (1999, 2001, and 2002), Laffond and Laine (2000) and Feix et al. (2004) have addressed the general phenomenon of election inversions in social choice terms, and Chambers (2008) has demonstrated (in effect) that no neutral (between candidates or parties) two-tier electoral rule can satisfy 'representative consistency,' i.e., preclude election inversions. Merrill (1978) and Ball and Leuthold (1991) have provided empirically based estimates of the expected frequency of Electoral College election inversions, and Lahrach and Merlin (2010) have done related work with respect to French local government elections. The fact that Electoral College can produce inversions is regularly cited by its critics (e.g., Peirce and Longley, 1981; Abbott and Levine, 1991; Longley and Peirce, 1996; Edwards, 2004), so its defenders (e.g., Best, 1971; Diamond, 1992; Ross, 2004) must also address the question.

'Westminster' single-member-district parliamentary systems in the U.K., Canada, Australia, India, and New Zealand (prior to 1993) are likewise two-tier voting systems and have produced election inversions about as frequently as the U.S. Electoral College. Some examples are listed in Table 1. It can be seen that most of these election inversions were very close with respect to both votes and seats, but the case of Canada in 1979 shows that this is not always the case.

These parliamentary systems differ in two important respects from the U.S. Electoral College. First, 'Westminster' systems have *uniform districts* — that is, the districts have equal weight (namely a single parliamentary seat), reflecting (approximately) equal populations and/or numbers of voters. In contrast, Electoral College 'districts' (i.e., states) are unequal in both population and voters and likewise have unequal electoral votes.

Second, the popular vote percentages shown in Table 1, which can be seen to add up to substantially less than 100%, indicate that many of these parliamentary inversions occurred in elections in which third and perhaps additional minor parties received a substantial percent of the vote (and some seats). The presence of third parties can distort the relationship between votes and seats for the two leading parties. In contrast, Electoral College inversions, like most U.S. elections, have occurred in what were for all practical purposes two-candidate contests. Indeed, the following analysis deals entirely with two-party popular vote percentages and, with the exception of special consideration of the 1860 election, excludes Presidential elections in which third candidates carried one or more states and thereby won some electoral votes.

The U.S. Electoral College has produced the three manifest election inversions listed in Table 2.[2] All were very close with respect to popular votes, and two were very close with respect to electoral votes as well.

In addition to these three historical instances, the Electoral College produced one massive but "latent" election inversion, which has been recognized as such by Sterling (1981) but by few others. Abraham Lincoln won an electoral majority 1860 on the basis of a plurality of less than 40% of the popular vote. The Democratic Party had split into Northern and Southern wings, each with its own Presidential candidate (Stephen Douglas and John Breckinridge, respectively) and a fourth candidate, John Bell, had been nominated by the remnants of the Southern Whig Party under the label of the Constitutional Union Party. The popular and electoral vote totals, as shown in Table 3, entail two manifest but inconsequential inversions — namely, Douglas won more popular votes but fewer electoral votes than either Breckinridge or Bell. Under a system of direct popular vote, the two Democratic candidates would have been 'spoilers' against each other if we can suppose that, in the event of the withdrawal of one, the other would have inherited most of his support and would therefore have defeated Lincoln. However, under the Electoral College system, Douglas and Breckinridge were *not* spoilers against each other. Indeed, we can make the following strong counterfactual suppositions and still preserve a Lincoln electoral vote victory: (1) the Democrats successfully hold their Northern and Southern wings together and thereby win *all* the votes captured by each wing separately, (2) the election is a typical 'straight fight' and the Democrats also inherit *all* the votes of the Constitutional Union party; and, for good measure, (3) the Democrats win *all* of New Jersey's electoral votes (which, for peculiar reasons, were split between Lincoln and Douglas). Even so, Lincoln still would have won the 1860 election on the basis of electoral votes. The final column of Table 3 shows the results of this counterfactual 1860 election. The consequence of all these suppositions is that only 11 electoral votes (in California, Oregon, and New Jersey) would switch from the Republican to Democratic column. We will examine this counterfactual two-party variant of the 1860 election in more detail later.

In sum, a first cut at estimating the expected frequency of election inversions under the Electoral College — based on the historical record since 1828 (the first election in which almost every state selected presidential electors by popular vote) — is either 3/46 = .06 or (counting the

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[2] The 1876 election was decided (just before inauguration day) by an Electoral Commission that, by a bare majority and straight party-line vote, awarded all of 20 disputed electoral votes to Hayes. The 1824 election is sometimes counted as an inversion, in that John Quincy Adams was elected President even though Andrew Jackson had received more popular votes (in the 18 out of 24 states in which presidential electors were popularly elected) than Adams. However, Jackson also won more electoral votes than Adams but not the required majority, so the election was decided by the House of Representatives, which elected Adams. In 1960, peculiarities with respect to Presidential ballot in Alabama make it unclear exactly how to determine the 'popular vote' for President in that state, and thus also nationwide. One (somewhat implausible) reckoning of the Alabama popular vote makes Nixon the national popular vote winner, thereby making 1960 an election inversion. In any event, the 1960 election is excluded from this analysis because a third candidate won electoral votes from 'unpledged electors' (see the Appendix).

counterfactual1860) 4/46 = .087. However, with the exception of the counterfactual 1860 election, all inversions occurred in close elections and, considering only elections in which the popular vote winner's margin was no greater than about three percentage points, the expected frequency of inversions is considerably higher, namely 3/12 = .25. Clearly an important determinant of the probability of an election inversion is the probability of a close division of the popular vote.

2. Popular Votes and Electoral Votes

We now turn to a more informative empirical analysis of election inversions that uses historical state-by-state popular vote percentages to construct the 'Popular Vote-Electoral Vote' (PVEV) step-function for each.[3] The PVEV function is based on the kind of 'uniform swing analysis' pioneered by Butler (1951), which has also been called 'hypothetical (single-year) swing analysis' (Niemi and Fett, 1986) and the 'Bischoff method' (see Peirce and Longley, 1981), and which has been employed by Nelson (1974), Garand and Parent (1991), and others in the context of assessing 'partisan bias' in the Electoral College.

The PVEV function is a cumulative distribution function and is therefore (weakly) monotonic. It is a step-function because the 'dependent variable' (EV) is discrete, assuming only whole number values and jumping up in discrete steps as the 'independent variable' (PV) increases (essentially) continuously.

Let us consider the 1988 election as an example. We set up the template used in Figure 1, showing the Democratic popular vote percent on the horizontal axis and the Democratic electoral vote on the vertical axis.[4] The Democratic nominee Michael Dukakis received 46.10% of the two-party national popular vote and won 112 electoral votes (one of which was lost to a 'faithless elector'). This combination of Democratic popular and electoral votes is plotted in Figure 1a.

Figure 1b zooms in on the neighborhood of this plotted point. Of all the states that Dukakis carried, he won Washington (with 10 electoral votes) by the smallest margin of 50.81%, so if the Democratic national popular vote of 46.10% were to decline by 0.81 percentage points (to 45.29%) *uniformly across all states*, Washington would tip into the Republican column and thereby reduce the Democratic electoral vote to 102, as shown in Figure 1b. In like manner, of all the states that Dukakis failed to carry, he lost Illinois (with 24 electoral votes) by the smallest margin of 48.95%, so if the Democratic national popular vote of 46.10% were to increase by 1.05 percentage points (to 47.15%) uniformly across all states, Illinois would tip Democratic and thereby increase the Democratic electoral vote to 136, as also shown in Figure 1b.

More generally, we can 'swing' the Democratic vote downwards until the Democratic electoral vote falls to the logical minimum of zero and upwards until it increases to the logical

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3 See the Appendix for details concerning this data.

4 Remember that, here and elsewhere, popular vote percentages are put on a strictly two-party basis, excluding votes cast for third or other minor candidates, and (with the 1860 exception already noted) we consider only elections in which the two major candidates won all the electoral votes, thus putting everything on a strictly two-party basis. We therefore would reach exactly the same conclusions if we organized the figures in terms of Republican popular and electoral votes.

maximum of 538, as is shown in Figure 1c.[5] This chart displays the PVEV function for 1988, over which Democratic popular support rises or falls uniformly across the states and translates into corresponding Democratic electoral vote totals.

While the full PVEV function in Figure 1c appears to go through the *two-way tie point* corresponding to PV = 50% and EV = 269, a moment's thought suggests that almost certainly it does not go *precisely* through this point. This becomes evident when we zoom in on the center of the chart in Figure 1d. We see that (i) if Dukakis had won exactly 50% of the popular vote, he would have lost the election with only 252 electoral votes, and (ii) if he had won anything between 50.00% and 50.08% of the popular vote, he still would have lost the election with no more than 260 electoral votes. His popular vote percent required for an electoral vote majority is 50.08%. Thus there is an *inversion interval* that is 50.08% − 50% = 0.08 percentage points wide, within which Dukakis would have won the popular vote but lost the election on the basis of electoral votes. Given the Democratic orientation of our analysis, the fact that the width of the interval is positive (e.g., +0.08) means that the Democratic candidate must win that more than 50% of the popular vote in order to win a majority of the electoral vote and reflects an anti-Democratic bias in the PVEV function. Conversely, a negative inversion interval means that the Democratic candidate can win a majority of the electoral vote with less than 50% of the popular vote and reflects a pro-Democratic bias in the PVEV function. In either event, the absolute width of the inversion interval is more consequential than the electoral vote split at the 50% popular vote mark, since the likelihood of an election inversion depends on the absolute width of the inversion interval, while the specific number of electoral votes that the 'wrong winner' receives within this interval does not affect who is elected President. Moreover, while the Democratic electoral vote at the 50% popular vote mark determines whether the inversion interval lies below or above the 50% mark, the *magnitude* of the Democratic electoral vote deficit or surplus at the 50% mark is logically unrelated to the *width* of the inversion interval.

With an even number of electoral votes, there may also be a 'tie interval,' within which neither candidate has the required electoral vote majority. The historical PVEV functions for 1872, 1972, and 2008 exhibit tie intervals. If a 'tie interval' were to span the 50% popular vote mark, there would be no inversion interval at all, but no historical PVEV exhibits such a 'spanning' tie interval.

As is well known, the 2000 election produced an actual election inversion. We might think that this was because the PVEV function for 2000 was quite different from that in 1988 and, in particular, that it entailed a larger pro-Republican bias. Indeed, with exactly 50% of the popular vote, Gore would have won only 237 electoral votes, less than Dukakis's 252. But Gore would have won the election with 50.2664% of the popular vote, hardly more than that required for a Dukakis victory. Indeed, the two PVEV functions are very similar, as a comparison of Figure 2a with Figure 1d shows. The crucial difference between the two elections was the obvious fact that the actual 2000 election itself was *much* closer. The Democratic two-party popular vote percent was 50.2664%, putting it

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[5] Defining the uniform swing in terms of the absolute percent of the total popular vote means that highly lopsided state popular votes in conjunction with extreme swings can create hypothetical popular vote percentages that are less than 0% or greater than 100%. But this is of no practical concern because our focus is on hypothetical elections that are close to the 50% mark with respect to the national popular vote.

(just) within the 2000 inversion interval, as shown in Figure 2b. (Even so, Gore would have won if the inversion interval in 2000 had been as small as in 1988.)

In contrast to both 1988 and 2000, in the counterfactual 1860 election a unified Democratic ticket would have won only 134 electoral votes out of 303; more astoundingly, the Democrats would have needed to win 61.26% of the popular vote for an electoral vote majority, producing an inversion interval 11.26 percentage points wide.

For every two-candidate Presidential election since 1828, Figure 3 shows the Democratic percent of the electoral vote at the 50% popular vote mark. (Electoral vote percentages are given, since the number of electoral votes has changed over time.) Figure 4a shows the Democratic popular vote percent required for an electoral vote majority (or tie). Figure 4b is derived from 4a and explicitly shows the width and direction (i.e., negative or pro-Democratic vs. positive or pro-Republican) of the inversion interval in each election.

Several observations can be drawn from these charts. First, as previously observed, if a bar that falls short of 50% in Figure 3, the corresponding bar exceeds 50% in Figure 4a and vice versa. However, the magnitudes of the two deviations are by no means strongly associated. For example, in 1888 the Democrat (Cleveland) would have won only about 36% percent of the electoral vote at the 50% popular vote mark, but he still would have won a majority of electoral votes with about 51% of the popular vote. (In one of the three historical election inversions, he lost the election with about 50.4% of the popular vote.)

Second, Figure 3 shows that the Democratic (and therefore also Republican) percent of the electoral vote at the 50% popular vote mark often deviates strikingly from 50%. Moreover it is apparent that, over the whole time period, the Democratic electoral vote percent at the 50% mark has been far more likely to fall below 50% than to exceed it, indicating a historical anti-Democratic bias in the Electoral College system. At the same time, the resulting inversion intervals shown in Figure 4b are typically quite small, rarely exceeding two percentage points, and the mean of the absolute intervals (ignoring whether they reflect pro-Republican or pro-Democratic bias) is only 0.076 percentage points. Like the data in Figure 3, they have exhibited an anti-Democratic bias more often than not. However, considering only elections from the mid-twentieth century on, the intervals have been smaller, rarely exceeding one percentage point and averaging about 0.5 percentage points, and exhibit no particular party bias.[6] Furthermore, the 1988 election turns out to have the smallest inversion interval on record.

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[6] Garand and Parent (1991) employ a similar uniform swing analysis for Presidential elections from 1872 through 1984 but, instead of using each PVEV function directly, they use it to estimate the best fitting (with two parameters, 'representational form' and 'partisan bias') logistic S-curve to predict the electoral vote for the Republican candidate at the 50% popular vote mark. Using such a curve produces quite different and usually much smaller Republican electoral vote percentages at the 50% popular vote mark. (Garand and Parent do not report inversion intervals, but their S-curves imply substantially wider intervals as well.) This is because the logistic curve estimated on the basis of the PVEV function is by assumption a symmetric S-shape and partisan bias shifts merely shifts the S-curve up or down the popular vote line. This mean that asymmetry anywhere in the PVEV data can shift the curve in the vicinity of the 50% popular, even if the PVEV function passes close to the perfect tie point. (Also see footnote 7.)

Based as they are on state-by-state data for all two-candidate Presidential elections, these results provide a more refined basis for estimating the likelihood election inversion by the Electoral College. Over all these elections, the Democratic two-party popular vote percent is approximately normally distributed with a standard deviation of about 6.2%. If we set its mean value at 50% (it is actually 49.17%), the mean absolute inversion interval of 0.76% implies a probability of an election inversion of approximately .048. If we consider only 1952 onwards, the popular vote SD increases to about 7.0% while the mean inversion interval falls to 0.47%, which implies a probability of an election inversion of only about .027. Considering only the six most recent elections, the average inversion interval falls further to 0.43%, but these elections have all been relatively close with a SD of 3.7%, which raises the probability of an inversion to about 0.046.

How the probability of inversion depends on the closeness of elections is more comprehensively displayed in Figures 5a and 5b. The first figure separately stacks negative (pro-Democratic) and positive (pro-Republican) tie or inversion intervals on top of each other in order of their widths to give a sense of the how the frequency of inversions varies with closeness of the popular vote; this chart makes the historical Republican advantage very evident. Figure 5b stacks *absolute* (tie or) inversion intervals in the same manner; this chart can reasonably be interpreted as indicating the approximate probability of an election inversion as a function of the popular vote winner's margin above 50% of the two-party vote. If that margin is arbitrarily close to 50%, we can expect *a priori* that the probability of an inversion is about 0.5; if it is about 50.5%, Figure 4(b) shows that the probability is about .25; if it is about 51%, the probability is about 0.125, and if it exceeds 52%, the probability is almost zero (in the absence of extreme sectional conflict like 1860). It is worth noting that Merrill (1978) and Ball and Leuthold (1991) produced quite similar estimates based on rather different methods.

3. Rounding Effects

The PVEV function for 1988 is almost symmetric. Figure 6 shows that, if we construct the Republican PVEV and superimpose it on the Democratic one, the two step functions, while distinct in detail, come very close to coinciding, not only in the vicinity of the 50% popular vote mark but throughout. If the two functions were to coincide precisely at the 50% mark, no inversion interval could exist. The small inversion interval that does exist results from what we might characterize as 'rounding error' necessarily entailed by the fact that a PVEV function moves up or down in discrete steps as the popular vote swings up or down. For example, as the Democratic popular vote swings upwards, the pivotal state that gives the Democratic candidate 270 or more electoral votes almost certainly will not tip into the Democratic column *precisely* as the Democratic popular vote crosses the 50% mark but rather a little below or above the 50% mark, so an inversion interval of some magnitude essentially always exists. Clearly a specific PVEV function allows a 'wrong winner' of one party only, depending on whether the inversion interval lies above or below the 50% Democratic popular vote mark.

However, suppose we plot other PVEV functions produced by small random perturbations in the actual 1988 state-by-state popular vote data. These PVEV functions will likely fall almost entirely within the 'thickening' and 'smoothing' of the actual PVEV function, as suggested in Figure 7. The resulting 'fuzzy' PVEV function passes through the two-way tie point even though almost certainly no specific 'crisp' PVEV function does so. Figure 7 suggests that, if the 1988 election had been much closer and state-by-state votes had been slightly perturbed, Dukakis as well as Bush could have emerged as a 'wrong winner.'

In contrast, Figure 8 shows the PVEV functions for both parties in 1940, which are clearly distinct almost everywhere. Moreover, even the 'fuzzy' Democratic PVEV function clearly misses the two-way-tie point, as shown in Figure 9. Figure 10 presents the most extreme case, namely the counterfactual version of 1860; clearly the 'fuzzy' PVEV function would miss the two-way tie point by an even larger margin than in 1940. Given these PVEV functions in contrast to the 1988 one, inversions are much more likely to occur because they result, not from mere 'rounding effects,' but from a fundamental asymmetry in the general character of the PVEV function, particularly in the vicinity of the 50% mark. Moreover, even with fairly substantial perturbations of the state-by-state votes, 'wrong winners' would almost always be Republicans, not Democrats.

The 1940 PVEV exemplifies in typical form — and the 1860 PVEV in exaggerated form — the substantial pro-Republican bias in historical PVEV functions in the vicinity of the 50% mark that resulted largely from the electoral peculiarities of the old 'Solid South' throughout the first two-thirds of the twentieth century — in particular, its overwhelmingly Democratic popular vote percentages, combined with its strikingly low voting turnout. Though the *overall* bias might be deemed pro-Democratic, in the vicinity of the 50% popular vote mark the bias is pro-Republican. Consider the party PVEV functions for 1940 displayed in Figure 8: the Democrats win more electoral votes than the Republicans do for almost all levels of popular vote support, but the Republicans win more in a narrow range in the vicinity of the 50% mark, which of course is precisely the range that matters.[7] The counterfactual 1860 case provides an even more extreme example.

Bias in the PVEV function can result from either or both of two distinct phenomena: *apportionment effects* and *distribution effects*. The former refers to disproportionality between the popular votes cast within states and the electoral votes cast by states. As an example, the old 'Solid South' had very low voting turnout (mostly reflecting the effective disenfranchise of potential black voters), with the result that Southern electoral votes were based on a much lower total popular vote than those of other regions. The latter reflects geographical patterns in the popular vote for the two candidates or parties that makes one candidate's distribution of popular votes more 'efficient' in winning electoral votes than the other. As an example, the overwhelmingly Democratic popular votes in the 'Solid South' did not win the Democrats any more electoral votes than more modest popular

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[7] I believe that this consideration in considerable part determines the Garrand and Parent (1991) conclusion, based on smooth S-curves estimated on the basis of the entire PVEV, that the Electoral College has historically had a pro-Democratic bias.

vote majorities would have, whereas the Republicans won most non-Southern states by more modest margins. Both apportionment of distribution effects by can produce election inversions by themselves and, in combination, they can either reinforce or counterbalance each other.

4. Apportionment Effects

In order to assess the magnitude and direction of apportionment effects, we start with the theoretical benchmark of a *perfectly apportioned* two-tier electoral system, in which apportionment effects are eliminated because electoral votes are apportioned among the states in a way that is precisely proportional to the total popular vote cast within each state (which requires that states be apportioned fractional electoral votes). In a perfectly apportioned system, a candidate who wins X% of the electoral vote carries states that collectively cast X% of the total popular vote.[8] This concept is introduced as an analytical tool; as a practical matter, an electoral system can be perfectly apportioned only retroactively — that is, after the popular votes in each state are cast and counted.

Apportionment effects encompass whatever may cause deviations from perfect apportionment. The U.S. Electoral College system is imperfectly apportioned, for at least six reasons.

(1) House seats (and therefore electoral votes) must be apportioned in small *whole numbers*, and therefore cannot be *precisely* proportional to *anything*.

(2) There are many *different methods* of apportioning whole numbers of seats or electoral votes on the basis of population, none of which is uniquely best (Balinski and Young, 1982).

(3) House (and therefore electoral vote) apportionments are anywhere from *two to ten years out-of-date* at the time of a Presidential election.

(4) The apportionment of electoral votes is *skewed in favor of smaller states*, because all states are guaranteed a minimum of three electoral votes and (approximate) proportionality begins only after that.

(5) The size of the House is not fixed by the Constitution and can be changed by law (as it frequently was until the early twentieth century), so the magnitude of the small-state bias can be reduced (or enhanced) by law, by increasing (or reducing) the size of the House.[9]

(6) House seats (and therefore electoral votes) are apportioned to states on the basis of their *total population* and not on the basis of their (i) voting age population, or (ii) voting eligible

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[8] Note that this says nothing about the popular vote margin by which the candidate wins or loses states and, in particular, it does say or imply that the candidate wins X% of the national popular vote,

[9] See Neubauer and Zeitlin (2003) for an analysis of how changes in House size would have affected the 2000 Presidential election.

population (excluding non-citizens, etc.), or (iii) number of registered voters, or (iv) number of actual voters in a given election.[10]

Similar apportionment imperfections apply (in greater or lesser degree) in all two-tier electoral systems.

While imperfect apportionment *may* create bias in a PVEV function, it *need not* do so. Overall bias depends on the extent to which states' advantages or disadvantages with respect to apportionment effects are correlated with their support for one or other candidate or party. The logically maximum bias that can arise from imperfect apportionment can be determined by (i) ranking the states by their degree of advantage with respect to actual apportionment relative to perfect apportionment, (ii) cumulating both electoral votes and total popular vote shares over this ranking until an electoral vote majority is achieved, and (iii) noting the corresponding share of the national popular vote that has been accounted for. In recent elections this popular vote share has been about 45% but it was considerably smaller (about 32%) in the early twentieth century.

Let us first examine the impact of apportionment effects in the counterfactual 1860 election, which was based on especially imperfect apportionment.

(1) The southern states (in most cases for the last time[11]) benefitted from the three-fifth compromise giving them partial credit for their non-voting slave populations.

(2) Southern states had on average smaller populations than northern states and therefore benefitted disproportionately from the small-state advantage in apportionment.

(3) Even within the free population, suffrage was more typically restricted in the South than elsewhere (where close to universal adult male suffrage prevailed).

(4) Turnout among eligible voters was generally lower in the South than the North.

But *all* of these apportionment effects favored the South and therefore the Democrats. Perfect apportionment would have *increased* the popular vote required for a Democratic electoral vote majority. Thus the massive pro-Republican election inversion was entirely due to distribution effects, and the inversion interval would have been even wider in the absence of the counterbalancing apportionment effects.

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[10] In addition, until slavery was abolished by the Thirteen Amendment in 1865, House seats were apportioned on the basis of the total free population plus three fifths of 'all other persons' (who certainly could not vote). While the Nineteenth Amendment, requiring all states to give women the right to vote on an equal basis with men, took effect in 1920, there was a preceding period in which some states allowed women to vote while others did not. This produced major apportionment effects. For example, in 1916 considerably more popular votes were cast in Illinois (which allowed women to vote) than in New York (which did not), even though Illinois had only 29 electoral votes compared with New York's 45.

[11] The few slave states that did not secede from the union retained this apportionment advantage in the 1864 election. By the time of the 1868 election, there were no 'other persons.'

While perfect apportionment is presumably not feasible in practice, we can use it analytically. Figure 11 compares the 1988 Democratic PVEV functions and inversion intervals under actual and perfect apportionment. Clearly apportionment effects were very small in this election. Figures 12 and 13 make the same comparisons for 1940 and 1860. Here apportionment effects are very substantial at low Democratic popular vote percentages and remain quite substantial up to a bit over the 50% mark in 1940 and up to about the 60% mark in 1860. The 1940 chart reflects the typical 'Solid South' effect that was displayed in a monolithic fashion in the 1860 election.

Note that the percent of the popular vote required for the Democrats to win a majority of electoral votes in 1860 is 61.26% under the actual apportionment and 62.51% under perfect apportionment. The difference between 61.26% and 62.51% of −1.25% indicates the impact of imperfect apportionment on the Democratic vote required for an electoral vote majority and, being negative, it indicates that the actual apportionment benefitted the Democrats. At the same time, the difference between 62.51% and 50% of +12.51% indicates the huge impact of distribution effects on the Democratic vote required for an electoral vote majority and, being positive, it indicates that the distribution effects harmed the Democrats.

Figures 14 and 15 correspond to Figures 3 and 4b by showing summary data for all elections under perfect apportionment.[12] One might expect that 'perfecting' apportionment would typically reduce the width of overall inversion intervals and thereby reduce the frequency and of election inversions. Indeed, under perfect apportionment, Gore would have won the 2000 election with 274.92 electoral votes, and Tilden would have won the 1876 election with 182.174 electoral votes. However, under perfect apportionment Cleveland would have lost even more decisively to Harrison in 1888, winning only 135.76 electoral votes to Harrison's 265.24. Moreover, Wilson would have lost the 1916 election with only 238.57 electoral votes out 531, despite a modest majority of the popular vote. So with respect to actual election inversions, perfect apportionment would eliminate two but not all three instances and would create one new instance. Perhaps surprisingly, perfect apportionment actually increases the overall degree of Republican bias in the Electoral College system and, as a consequence this, considerably increases the average magnitude of absolute inversion intervals from 0.76% to 01.22%.

Finally, Figure 16 decomposes each inversion interval into the contributions made by apportionment effects and distribution effects. We will look more closely at this figure in the concluding section.

5. Distribution Effects

We can measure the impact of distribution effects on inversion intervals simply by calculating the difference between 50% and the percent of the vote received the Democrats at the 50% popular vote mark under perfect apportionment (as displayed in Figure 16), but we can also examine distribution effects more directly.

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[12] Since perfect apportionment requires fractional electoral votes, no electoral votes ties occur.

Distribution effects in two-tier electoral systems result from the winner-take-all feature at the state (or district) level. Distribution effects can be powerful even with small uniform districts and/or perfect apportionment. If one candidate's (or party's) popular vote is more 'efficiently' distributed over states (or districts) than the other's, an election inversion can occur even with perfect apportionment.

The simplest possible example of distribution effects producing an election inversion in a small, uniform, and perfectly apportioned district system is provided by nine voters in three districts. Suppose that the individual votes for candidates D and R in each district are as follows: (R,R,D) (R,R,D) (D,D,D). Thus the election outcome is as follows:

	Popular Votes	*Electoral Votes*
D	5	1
R	4	2

Since R's votes are more 'efficiently' distributed than D's (whose support is 'wastefully' concentrated in the third district), R wins a majority of districts with a minority of popular votes.

More generally, suppose there are k uniform districts each with n voters. To avoid the problem of ties, let us assume both k and n are odd numbers. A candidate can win by carrying a bare majority of $(k + 1) / 2$ districts each with a bare majority of $(n + 1) / 2$ votes. Thus a candidate can win with as few as $[(k + 1) / 2] \times [(n + 1) / 2] = (n\times k + n + k +1) / 4$ efficiently distributed total votes. With $n = 3$ and $k = 3$, the last expression is 4/9 = 44.4%, but as n and/or k become large, the last expression approaches a limit of $(n \times k) / 4$, i.e., 25% of the total popular vote.

Stated more intuitively, if the number of districts is fairly large and the number of voters is very large, the most extreme logically possible election inversion in a perfectly apportioned system results when one candidate or party wins just over 50% of the popular votes in just over 50% of the uniform districts or in non-uniform states that collectively have just over 50% of the electoral votes. These districts also have (just over) 50% of the popular vote (because apportionment is perfect). The winning candidate or party therefore wins just over 50% of the electoral votes with just over 25% of the popular vote. The other candidate, though winning almost 75% of the popular vote, loses the election, producing a massive election inversion. In the resulting PVEV, the inversion interval is just short of 25 percentage points wide. (If the candidate or party with the favorable vote distribution is also favored by imperfect apportionment, the inversion interval could be even greater.) This '25% - 75%' rule pertaining to distribution effects was noted in passing by Schattschneider (1942, p. 70) and more formally by May (1948), Laffond and Laine (2000), and perhaps by others as well.

In the counterfactual 1860 Lincoln vs. anti-Lincoln scenario, the popular vote distribution over the states approached the logically extreme 25%- 75% pattern more closely than in any actual Presidential election. In the counterfactual election, Lincoln carried all the northern (free) states except New Jersey, California, and Oregon, mostly by modest popular vote margins that rarely exceeded 60% and typically were closer to 50%. These states held somewhat more than half the electoral votes and a larger majority of the (free) population. The anti-Lincoln opposition carried all the slave states by essentially 100% margins. (No Lincoln-pledged electors ran in any of the state that would subsequently secede from the Union.) The opposition also carried California and Oregon by substantial margins and New Jersey by a narrow margin. All together these states held somewhat less than half of the electoral votes and substantially less than half of the (free) population.

Sterling (1981) has devised an insightful geometric construction to visualize 'Electoral College misrepresentation.' A 'Sterling diagram' is a histogram that displays the popular vote split between the two candidates in each state, where states are ranked in order from the strongest to weakest for the winning party, with the width of each state 'bar' proportional to its total popular vote.[13]

Figure 17a shows the Sterling diagram for the 1988 election. Selected 'bars' are explicitly drawn in and labeled by state. Running from the most Republican state of Utah to the least Republican 'state' of the District of Columbia, it is Michigan, beating out Colorado by about 0.03%, that tips the Republican electoral vote over the 270 mark. Once Michigan is in their column, the Republicans are carrying states with 49.43% of the national popular vote, as indicated by the vertical dashed line. The fact that this falls below the 50% mark reflects the (very small and previously noted) apportionment effect favoring the Republicans in 1988.[14] The area of the whole rectangle making up the Sterling diagram represents all 100% total national (two-party) popular vote. The shaded area below the tops of the bars represents the 53.9% of the popular vote won by Republican Bush and the unshaded area above the top of the bars represents the 46.1% of the popular votes won by Democrat Dukakis.

Figure 17b demarcates different portions of the total Republican popular vote in 1988. The dark shaded portion represents the portion of the total popular vote essential for 270 electoral votes, This is essentially the 25% given by the '25% – 75% rule' (except that, because apportionment effects work slightly in favor of the Republicans, it is actually slightly less than 25%). The lightly shaded portion represents 'surplus' (or 'wasted') Republican popular votes, which fall into three different quadrants of the diagram: (i) 'surplus' Republican votes (in excess of 50%) in the states

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[13] I follow Sterling by orienting these charts to the party that actually won the election, rather than to the Democratic party.

[14] Note that the interval between 49.43% and 50% is not directly related to the inversion interval. The inversion interval is the difference between 50% and *the smallest national popular vote percent* for a candidate that produces an electoral vote majority. This interval is the difference between 50% and *the share of the total popular vote cast by the smallest set of states* (ranked by party strength) *that produces an electoral vote majority*. In the absence of apportionment effects, this interval would be zero. If states were instead ranked from in order from strongest to weakest for the Democratic party, Michigan would again be the pivotal state and, once the Democrats win Michigan, they would be carrying states with 54.58% of the national popular vote; note that Michigan (which cast 4.01% of the national popular vote) counts in both totals. Such percentages (and the corresponding electoral vote splits) can deviate substantially from a 50-50 split, because pivotal states are typically big states, and tiny shifts the national popular vote split between the two candidates can shift a pivotal state one way or another and thus have a big impact on the percent of the national popular (and electoral) vote cast by states carried by one or other candidate.

essential for 270 electoral votes, (ii) all Republican votes up to 50% in ‘surplus’ states (in excess of 270 electoral votes), and (iii) all Republican votes that are ‘surplus’ in both respects.

We can get a direct measure of overall distribution effects in this or any other election by comparing the percent of all votes that are ‘surplus’ for each party and thereby determine which party has the most efficient distribution of votes. In Figure 17b, it appears that the Republicans ‘wasted’ slightly more more ‘surplus’ votes than the Democrats. This comes about in part, because the Republicans have a (very small) advantage due to apportionment effects and, much more important, because they won the election by a substantial margin (so the shaded portion of Figure 17b extends into the upper-right quadrant). We can modify the Sterling diagram by making two adjustments to remove these factors. First, we reallocate electoral votes among the states so that they are perfectly apportioned. Now the horizontal axis now shows both the cumulative percent of the popular vote cast in, and the cumulative percent of electoral votes cast by, the states, so we no longer must specify where an electoral vote majority is achieved. Second, we ‘swing’ the Republican vote uniformly downward (by simply shifting the tops of the state bars uniformly downward) until the election is a *perfect tie*, in the specific sense that the popular vote is tied in the pivotal state (Michigan) that produces 270 electoral votes, and the median (state) bar has a height of 50% and no Republican votes appear in the upper-right quadrant.[15]

Figure 18 shows the Sterling diagram for 1988 with these two adjustments. In the adjusted diagram, 50% of the total popular votes (precisely those in the upper-left and lower-right quadrants) are ‘surplus’ to one other party. The adjusted diagram shows how this fixed proportion of surplus votes is divided between the two parties. In the absence of distribution effects (and in a perfect tie election with no apportionment effects), surplus votes would be equally divided between the two parties (25% for each). We see that in 1988 surplus votes are almost equally divided at the perfect tie point: 25.24% for Republicans and 24.76% for Democrats. This fact that the Republicans ‘wasted’ slightly more votes than the Democrats demonstrates that there is no logical connection between the *overall* distribution effects displayed in Figure 18 and the impact of distribution effects on the width and direction of the inversion interval. If in 1988 the Republicans had won their strongest states by more modest margins, they would have had fewer ‘surplus’ votes than the Democrats, rather than slightly more, but this would have had no impact on the inversion interval.

The 1988 election provides an example of an election with very small distribution effects. Once again the counterfactual 1860 election provides by far the most spectacular example of huge distribution effects (that totally overwhelm somewhat more modestly pro-Democratic apportionment effects). Figure 19 shows the standard Sterling diagram for 1860. Due the Democratic apportionment advantage, the Republicans had to carry states casting 60% of the popular to win, and they actually carried states casting about 67% of the popular vote. Figure 20 shows the Sterling diagram adjusted to show perfect apportionment and a uniform swing against the Republicans just sufficient to bring about a perfect tie election. It thereby isolates distribution effects and shows the

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[15] In this sense, the 2000 election was only 537 votes away from a perfect tie. Note that a perfect tie in this sense is almost certainly not a ‘two-way tie,’ since almost certainly the national popular vote is not tied. While the popular vote is likely to be very close, the only logical constraint remains that given by the 25% – 75% rule.

massive Republican advantage in this respect: of the 50% of all votes that are by definition surplus, 38.9% were 'wasted' by the Democrats and only 11.1% by the Republicans.[16]

Figure 21 shows the Democratic advantage or deficit in each election with respect wasted votes as the difference between 25% and the actual percent of Democratic popular votes that are surplus, along with the net impact of distribution effects on the inversion interval previously shown in Figure 16. It can be seen that these two quantities track each other quite closely but, as we would expect given the considerations previously mention, they are less than perfectly related.

6. Conclusions

Let us more closely examine Figure 16, which shows the magnitude and direction of inversion intervals and their decomposition into apportionment and distribution effects for every election. A number of observations are in order.

First, it may seem surprising that the impact of apportionment effects on inversion intervals is precisely zero in as many as fourteen elections, given the highly imperfect apportionment underlying the Electoral College system. However, we must recall that even highly imperfect apportionment can have little impact on an inversion intervals if party support over states is not correlated with their apportionment advantages. Beyond this, zero impact on an inversion interval does not mean that the PVEV functions under perfect and actual apportionment are everywhere identical, only that they coincide at the 50% popular vote mark — that is, when we cumulate both actual and perfectly apportioned electoral votes, the same state is pivotal under both apportionments. The fact that pivotal states are likely to be large states with many electoral votes (under both apportionments) reinforces this observation.

Second, perhaps the most striking fact conveyed by Figure 16 is out of the 22 elections in which both apportionment and distribution effects have an impact on the inversion interval, these effects work in opposite rather than reinforcing directions in 19 of these cases, and they thereby tend to produce relatively small overall inversion intervals.

Third, the fact the apportionment and distributions typically work in opposite directions is largely an artifact of another overall pattern Figure 16, which is that in general apportionment effects have favored Democrats while distribution effects have favored Republicans. Since the latter effects have generally been somewhat stronger than the former, the Electoral College has historically exhibited a small but significant pro-Republican bias. (In this respect, the counterfactual 1860 election merely exaggerates the overall pattern.) The pro-Republican bias is further enhanced by the fact that, in all four elections in which apportionment and distribution effects reinforced one another, they did so in the pro-Republican direction.

Such patterns become more understandable when we take account of the chronological order of these cases. It then become evident the 'overall pattern' noted above really is the product of a

[16] This Republican advantage with respect to surplus vote could be reckoned as even greater, since the 11.1% takes no account of the fact that the swing required to create a tie election makes the Republicans popular vote negative in the Southern states in which they actually won (literally) zero votes.

particular historical era extending across the first two-thirds of the twentieth century. Until the early twentieth century, no consistent pattern is evident, perhaps reflecting relatively loose party ties in the early party system followed by the disruptive events leading to and during the Civil War and subsequent Reconstruction. During most of this period — though with the conspicuous exceptions of 1856 (excluded from this analysis) and 1860 (included in its counterfactual variant) — politics was largely non-sectional, with both parties typically carrying states and winning electoral votes in all regions of the country.

Electoral maps that show which party carried each state may suggest that the ‘solid South’ emerged in immediately in the first election following the end of Reconstruction in 1877, for such maps show that the Democrats won the electoral votes of every Southern state in 1880 and for decades thereafter. However, such maps do not show that until the early twentieth century, Republicans consistently won a substantial minority of votes in Southern states, based in large part on the support of not yet disenfranchised black voters. But beginning in 1890, Southern states began to establish “Jim Crow’ regimes that entailed (in addition to racial segregation) suppression of both the black and Republican vote (and a good deal of the white vote as well). ‘Jim Crow’ becomes fully evident in the 1908 election, which begins a string of ten elections (plus three excluded from this analysis) through 1956 in which apportionment effects consistently favored the Democrats (as a result of vote suppression and low turnout in the South) and distribution effects favored the Republicans (as a result of ‘wastefully’ large Democratic popular vote majorities in the South), with the latter outweighing the former in their impact on inversion intervals and producing an overall pro-Republican bias in the Electoral College system.[17]

This string of elections ends with collapse of the ‘Jim Crow’ system in the late 1950s and early 1960s under the pressure from the civil rights movement and federal intervention. Beginning in 1964, after passage of the Civil Rights Act sponsored by Democratic President Johnson and opposed by the Republican nominee Barry Goldwater, the old white Democratic South began to switch its party allegiance from the Democratic to the Republican side, so the partisan impact of apportionment and distribution effects was reversed. Thereafter, as the federal Voting Rights Act took full effect, turnout increased to normal levels in the South and heavy black support provided the basis for a substantial (but rarely winning) Democratic popular vote in Southern states. In the modern era, apportionment and distribution effects (and inversion intervals) are relatively small and, in so far as they exist, typically reverse of the earlier pattern by favoring Republicans (who typically win by large margins in the small states of the Great Plain and inter-mountain West that are favored by the apportionment of electoral votes) and Democrats respectively.

Further research along these lines can proceed in a number of directions, including the following.

The elections with more than two candidates that have been excluded from this analysis can be included in various ways. First, in the manner of the treatment of the 1860 election here, scenarios may be created by combining the votes for candidates in various ways to produce counterfactual two-party contests. Second, the popular vote for a third candidate can be frozen, while major party votes

[17] Key (1949) provides the definitive treatment of politics in the old ‘solid South.’

are allowed to 'uniformly swing' against each other. Typically this will show an Electoral College 'deadlock interval' (of which a tie interval is a special case) in addition to or, more commonly, instead of an inversion interval.[18] Finally, multi-candidate or multi-party elections (such as are increasingly found in 'Westminster' parliamentary systems) may be analyzed in their full complexity by considering hypothetical uniform swing between all pairs of parties. However, this will require complicated analytical methods.

A number of 'reforms' of the U.S. Electoral College (apart from its total abolition and replacement by a one-tier direct election system) have been proposed over the years, including several 'district' and 'proportional' plans. In addition, the method of apportioning electoral votes among the states might be revised in various ways — in particular, to reduce or remove the small-state apportionment advantage. It remains to be determined whether such reforms would make election inversions more or less likely.

The notion of perturbing of a 'crisp' PVEV function to create a 'fuzzy' one, which was treated informally here, can be treated more formally by simulating elections on the basis of given PVEV function with random fluctuations (i.e., *non-uniform* swings) at the state and/or regional level.

Finally, a theoretically productive approach would be to estimate the probability of election inversions in random or 'Bernoulli' elections, in which voters decide how to vote by independently flipping fair coins. Such elections can be easily simulated, and this is the same probability model that provides a practical interpretation of the *absolute Banzhaf voting power measure* — namely, that voting power is the probability of casting a decisive vote in such an election. Voting power analysis has well-known applications to two-tier electoral system such as the Electoral College (Felsenthal and Machover, 1998). Indeed, Feix et al. (2004) have already estimated, by means of simulations, the probability of election inversions in uniform and perfectly apportioned two-tier electoral systems. This probability quickly approaches a limit of about .205 as the number of districts increases. My own preliminary work along the same lines indicates that the probability of inversions is somewhat greater than this in Electoral College simulations, but the extent to which this is due to non-uniform districts or to imperfect apportionment is as yet unclear. One advantage of the random election approach is that systematic distribution effects are removed and estimates of inversion probabilities therefore reflect only of the properties of the electoral institutions themselves (i.e., with respect to the Electoral College, the manner of apportioning and casting electoral votes) and not more contingent features pertaining to the geographical basis of party support in any particular historical period.

In the meantime, we can conclude that the probability of an election inversion by the existing U.S. Electoral College is quite small and largely dependent on the closeness of the popular vote. This probability is now smaller than in some times past and, unlike those earlier times, is more or less equally likely to make the Democratic or Republican candidate the 'wrong winner.'

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[18] Elections with substantial third-candidate popular votes, such as 1980, 1992, and 1996) may also be fruitfully analyzed in this way, even if the third candidate won no electoral votes in the actual election.

Appendix: Presidential Election Data

The 1828-2004 Presidential election data used here comes from Congressional Quarterly's *Guide to U.S. Elections* (2005), which is based on the Interuniversity Consortium for Political and Social Research (ICPSR) Historical Election Returns file. See the p. xvi in the *Guide* for further details. The 2008 data comes from David Leip's *Atlas of U.S. Elections* at **http://uselectionatlas.org/**, which is based on information from state election agencies. For present analytic purposes, it was necessary or expedient to make the following adjustments in the data.

1. All state and national popular vote percentages are based on the major two-party vote only, excluding popular votes cast for third-party and other minor Presidential candidates.

2. Apart from 1860 (for which we consider the 'Republican vs. anti-Republican' counterfactual two-party variant), the following elections are set aside because third-party candidates won electoral votes by carrying at least one state:

   1832 Wirt (Anti-Masonic Party) won 8 electoral votes;

   1856 Fillmore (American Party) won 8 electoral votes;

   1860 Brekinridge (Southern Democrat) won 72 electoral votes and Bell (Constitutional Union Party) won 39 electoral votes;

   1892 Weaver (Populist Party) won 22 electoral votes;

   1912 T. Roosevelt (Progressive Party) won 88 electoral votes;

   1924 LaFollette (Progressive Party) won 13 electoral votes;

   1948 Thurmond (Southern Democrat) won 38 electoral votes;

   1960 Byrd (Southern Democrat) won 14 electoral votes (cast by 'unpledged' electors);

   1968 Wallace (American Independent Party) won 45 electoral votes.

3. Despite significant third-candidate popular votes in 1980 (Anderson), 1992 (Perot), and 1996 (Perot), these elections are not excluded because Anderson and Perot carried no states and therefore won no electoral votes. The popular votes for Anderson and Perot are excluded from popular vote totals (like popular votes for minor candidates in all elections).

4. Because of the general-ticket system for electing party-pledged electors, each state's electoral vote is normally undivided. However, divisions in state electoral votes occur in three circumstances:

   (a) when a 'faithless' elector violates his or her pledge and casts a 'protest' electoral vote for another candidate, which occurred in 1948, 1956, 1960, 1968, 1972, 1976, 1988, 2000, and 2004);

(b) when electors are elected from districts rather than statewide, and each major-party candidate carries at least one district, as happened in Michigan in 1892 and Nebraska in 2008; and

(c) when electors are elected at-large but individually rather than on a general ticket, as happened with some frequency in the 19th century, in several states in 1912, and in Alabama in 1960.

Consistent with the almost universal practice and present analytical purposes, all calculations assume that states cast undivided electoral votes for the state popular vote winner. (Thus, McCain in 2008 is credited with all five electoral votes from Nebraska and Gore in 2000 is credited with 267 electoral votes). When electors are elected at-large but not on a general ticket system, standard records of the Presidential vote by state (including those relied on here) credit each Presidential candidate with the popular vote for his party's leading elector.

5. The South Carolina legislature appointed presidential electors through 1860. These electors were always Democrats, but in 1832 and 1836 they cast their electoral votes for an 'Independent Democrat' rather than the national Democratic party nominee. The Delaware legislature appointed electors in 1828 (pledged to National Republican J. Q. Adams), the Florida legislature appointed electors in 1868 (pledged to Democrat Horatio Seymour), and the Colorado legislature appointed electors in 1876 (pledged to Republican Rutherford Hayes).

   In the calculations pertaining to the actual Electoral College, South Carolina is counted as voting 100% Democratic but casting 0% of the national popular vote, and Delaware, Florida and Colorado are treated in a parallel manner. For purposes of making perfect apportionment calculations for the Delaware, Florida, and Colorado, I use the total popular vote for governor in the same year (or in 1829 in the case of Delaware) to take the place of the (non-existent) popular vote for Presidential electors. This data came from Tables 7.8, 7.9, and 7.6, pp. 264-275, of Walter Dean Burnham's *Voting in American Elections* (2010). However, the South Carolina legislature appointed the governor as well as Presidential electors through 1860. Therefore I use the total vote for U.S. House candidates to take the place of the Presidential vote. This was calculated using Tables 2.2-2.4 (Potential Electorate Estimates), pp. 115-119, together with Table 8.3b (Estimated House Turnout), pp. 401-410, in Burnham's book.

6. In 1836, Whig presidential electors were pledged to different candidates in different states. The popular and electoral votes for the three Whig candidates are simply added up to get a national Whig popular vote percent and electoral vote total, so the calculations treat 1836 as a normal two-party election.

7. In 1860, Democratic 'fusion' (i.e., anti-Lincoln) elector slates that included both prospective electors pledged to Douglas and others to Breckinridge (and, in at least one state, several pledged to Bell) were run in a number of Northern states, sometimes in competition with "pure" Douglas slates (see Fite, 1911, p. 223). None of the 'fusion' slates won, but they

make apportioning popular vote support between Douglas and Breckinridge a somewhat arbitrary matter. But since we use only the counterfactual version of 1860 in which Lincoln runs against a unified opposition, we can sidestep these complexities.

8. In 1872, the Democratic (and 'Liberal Republican') candidate Horace Greeley died after the Presidential election but before the casting of electoral votes. Three Democratic electors in Georgia cast electoral votes for their deceased nominee, while the other Democratic electors scattered their votes among four living candidates. Congress refused to count the three Greeley electoral votes from Georgia, and it also refused to count electoral votes (cast for Republican Ulysses Grant) from Arkansas and Louisiana, due to disruptive conditions in those states. The scattered Democratic electoral votes (including the three rejected votes for Greeley) are counted toward the Democratic total and the Arkansas and Louisiana popular and rejected electoral votes are counted toward the Republican total, so the calculations treat 1872 as a normal national election (apart from the absence of Mississippi, Texas, and Virginia, which had not yet been readmitted to the union).

Table 1. Election Inversions in 'Westminster' Parliamentary Systems

Country	*Election*	*Leading Parties*	*Pop. Vote %*	*Seats*
Britain	1929	Conservative	38.06	260
		Labour	37.12	287
Britain	1951	Labour	48.78	297
		Conservative	47.97	302
Britain	1974 (Feb.)	Conservative	37.90	297
		Labour	37.18	301
New Zealand	1978	Labour	40.44	40
		National	39.80	51
New Zealand	1981	Labour	39.01	43
		National	38.77	47
Canada	1971	Liberal	40.11	114
		Conservative	35.89	136

Table 2. The Three Historical Election Inversions by the U.S. Electoral College

Election	*EC Winner* [EV]	*EC Loser* [EV]	*EC Loser's 2-P PV%**
1876	Hayes (R) [85]	Tilden (D) [184]	51.53%*
1888	Harrison (R) [233]	Cleveland (D) [168]	50.41%
2000	Bush (R) [271]	Gore (D) [267**]	50.27%

* Two-party popular vote percent.
** Gore lost one electoral vote to a 'faithless elector.'

Table 3. The 1860 Election: A Latent But Massive Inversion

Candidate	*Party*	*Pop. Vote*	*EV*	*Unified Dem.*	*Unified Opp.*	*EV*
Lincoln	Republican	39.82%	180	39.82%	39.82%	169
Douglas	Northern Democrat	29.46%	12	47.55%	60.16%	134
Breckinridge	Southern Democrat	18.09%	72			
Bell	Constitutional Union	12.61%	39	12.61%		

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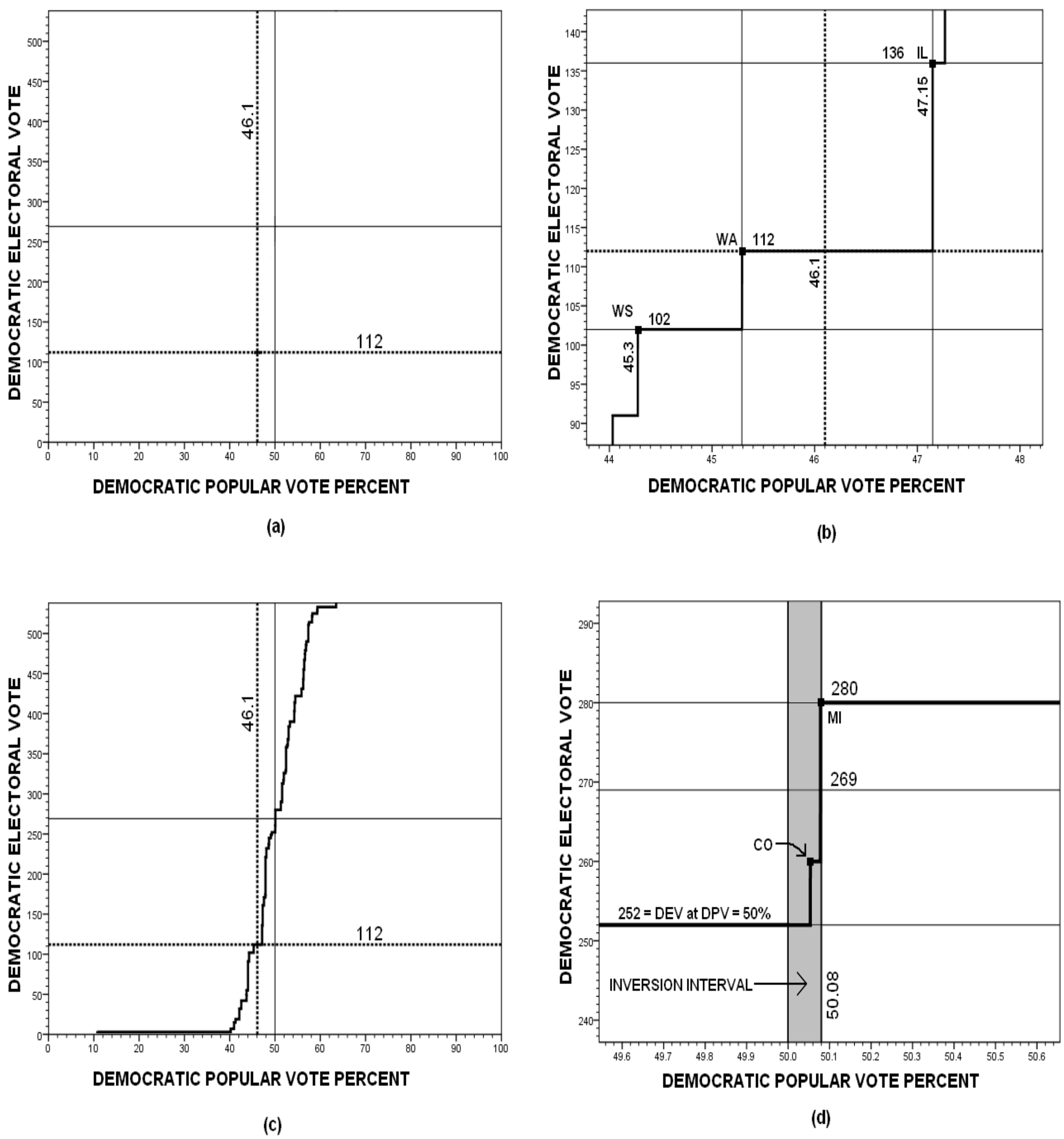
Figures


DEMOCRATIC ELECTORAL VOTE
DEMOCRATIC POPULAR VOTE PERCENT
46.1
112
(a)
136 IL
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WA 112
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45.3
(b)
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280
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50.08
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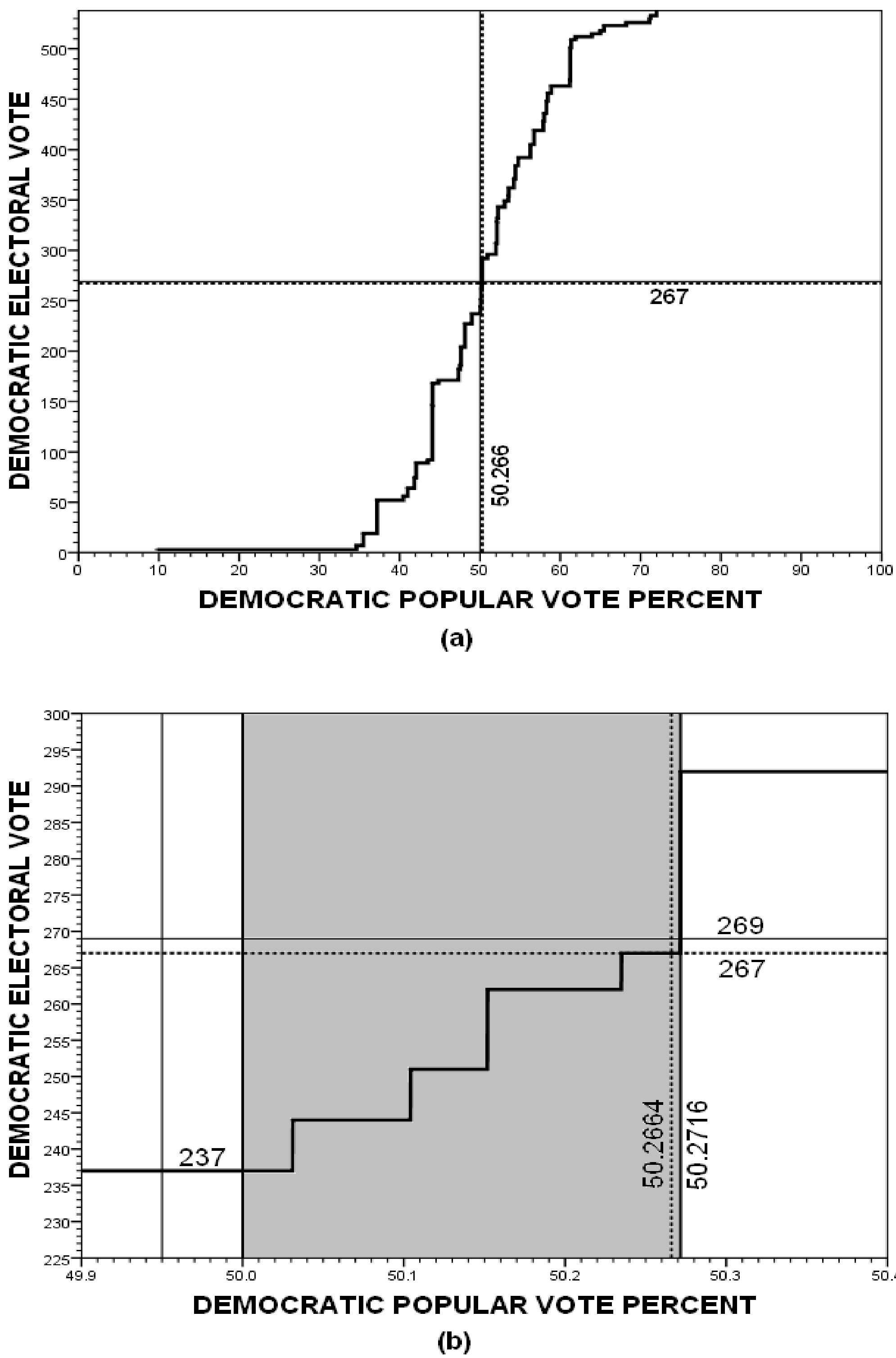
Figure 1


DEMOCRATIC ELECTORAL VOTE
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267
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(a)
DEMOCRATIC ELECTORAL VOTE
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269
267
237
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(b)

Figure 2

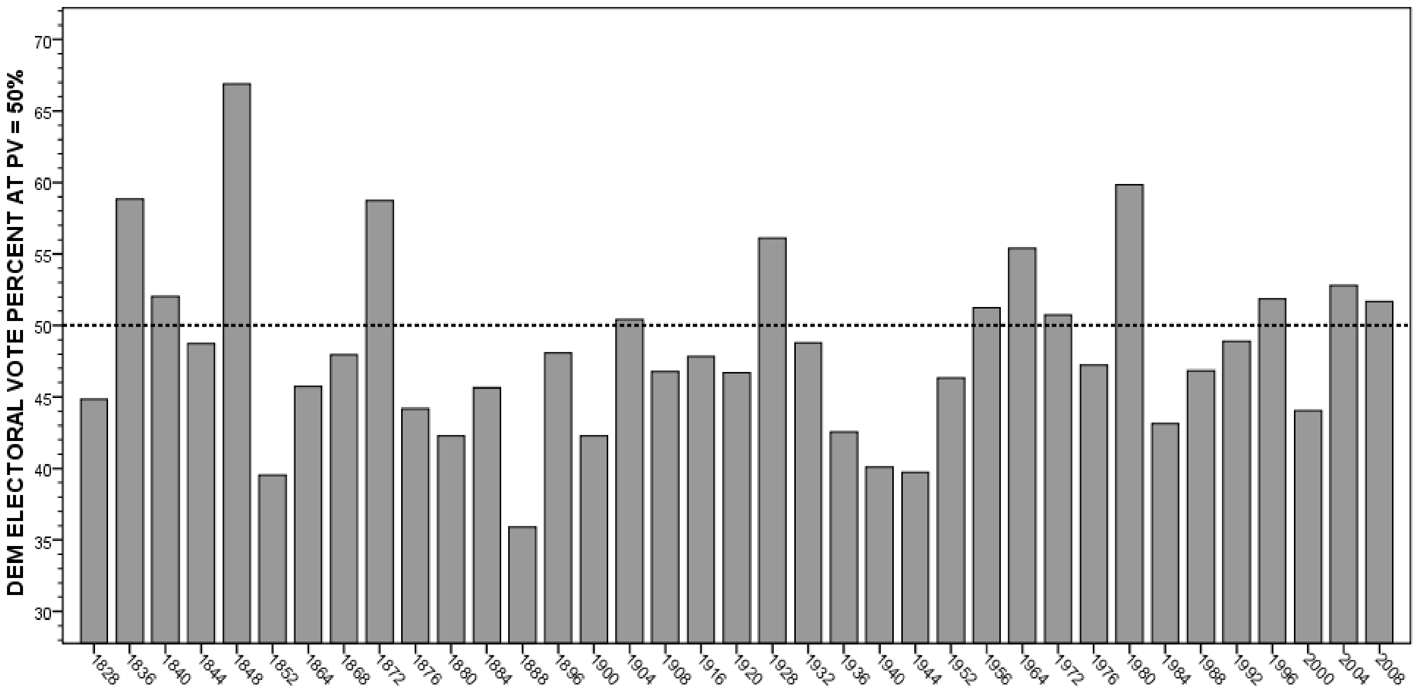

DEM ELECTORAL VOTE PERCENT AT PV = 50%
70
65
60
55
50
45
40
35
30
1828
1836
1840
1844
1848
1852
1864
1868
1872
1876
1880
1884
1888
1896
1900
1904
1908
1916
1920
1928
1932
1936
1940
1944
1952
1956
1964
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008

Figure 3

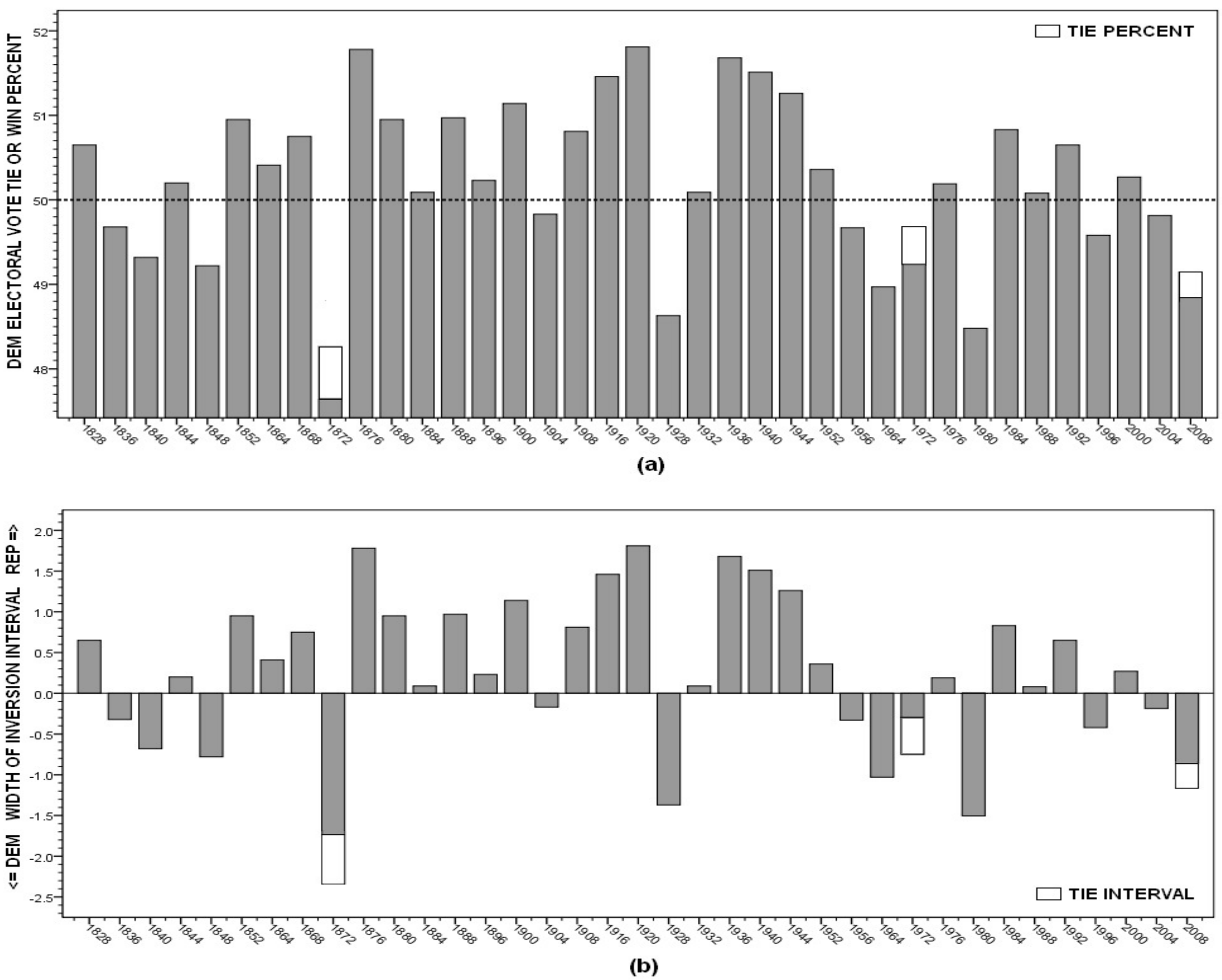
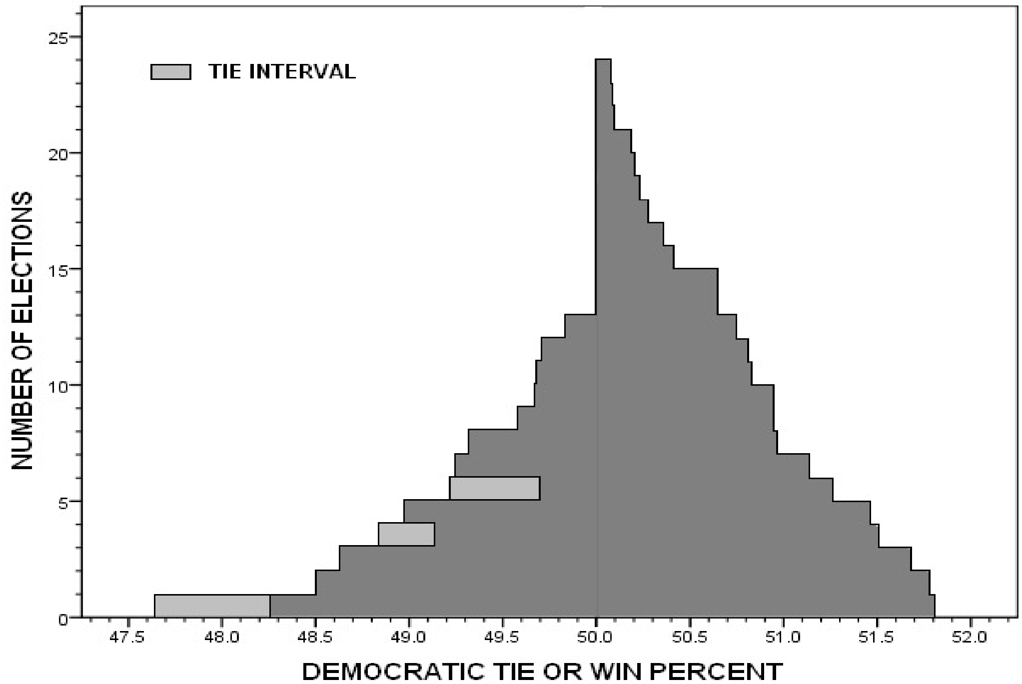
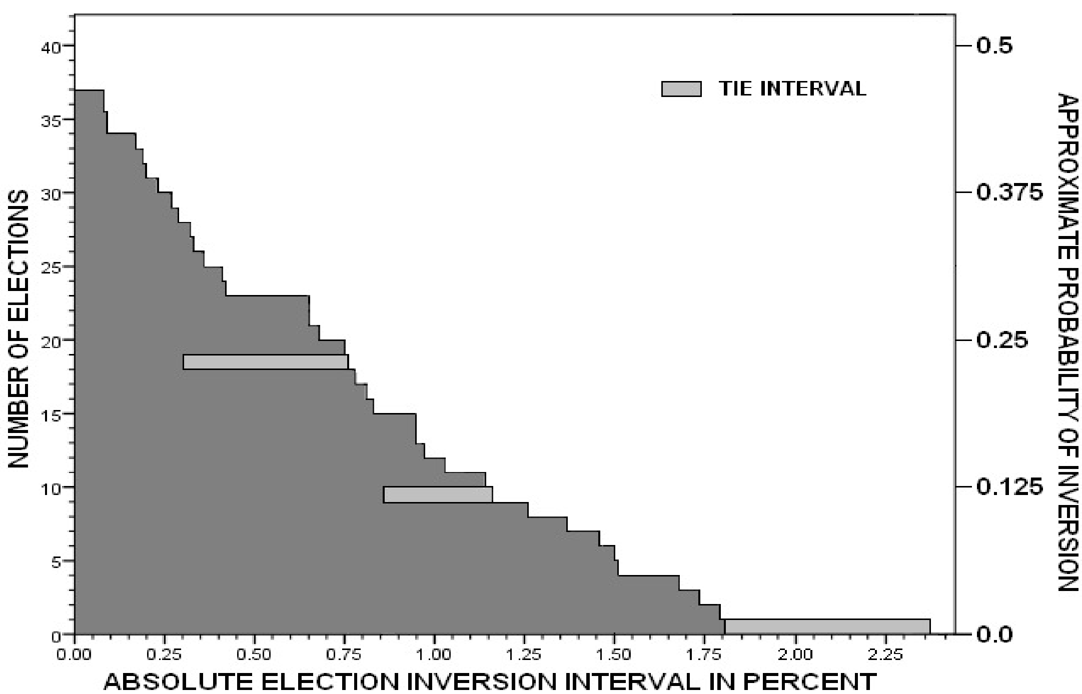

DEM ELECTORAL VOTE TIE OR WIN PERCENT
TIE PERCENT
52
51
50
49
48
1828 1836 1840 1844 1848 1852 1864 1868 1872 1876 1880 1884 1888 1896 1900 1904 1908 1916 1920 1928 1932 1936 1940 1944 1952 1956 1964 1972 1976 1980 1984 1988 1992 1996 2000 2004 2008
(a)
<= DEM WIDTH OF INVERSION INTERVAL REP =>
TIE INTERVAL
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
1828 1836 1840 1844 1848 1852 1864 1868 1872 1876 1880 1884 1888 1896 1900 1904 1908 1916 1920 1928 1932 1936 1940 1944 1952 1956 1964 1972 1976 1980 1984 1988 1992 1996 2000 2004 2008
(b)

Figure 4

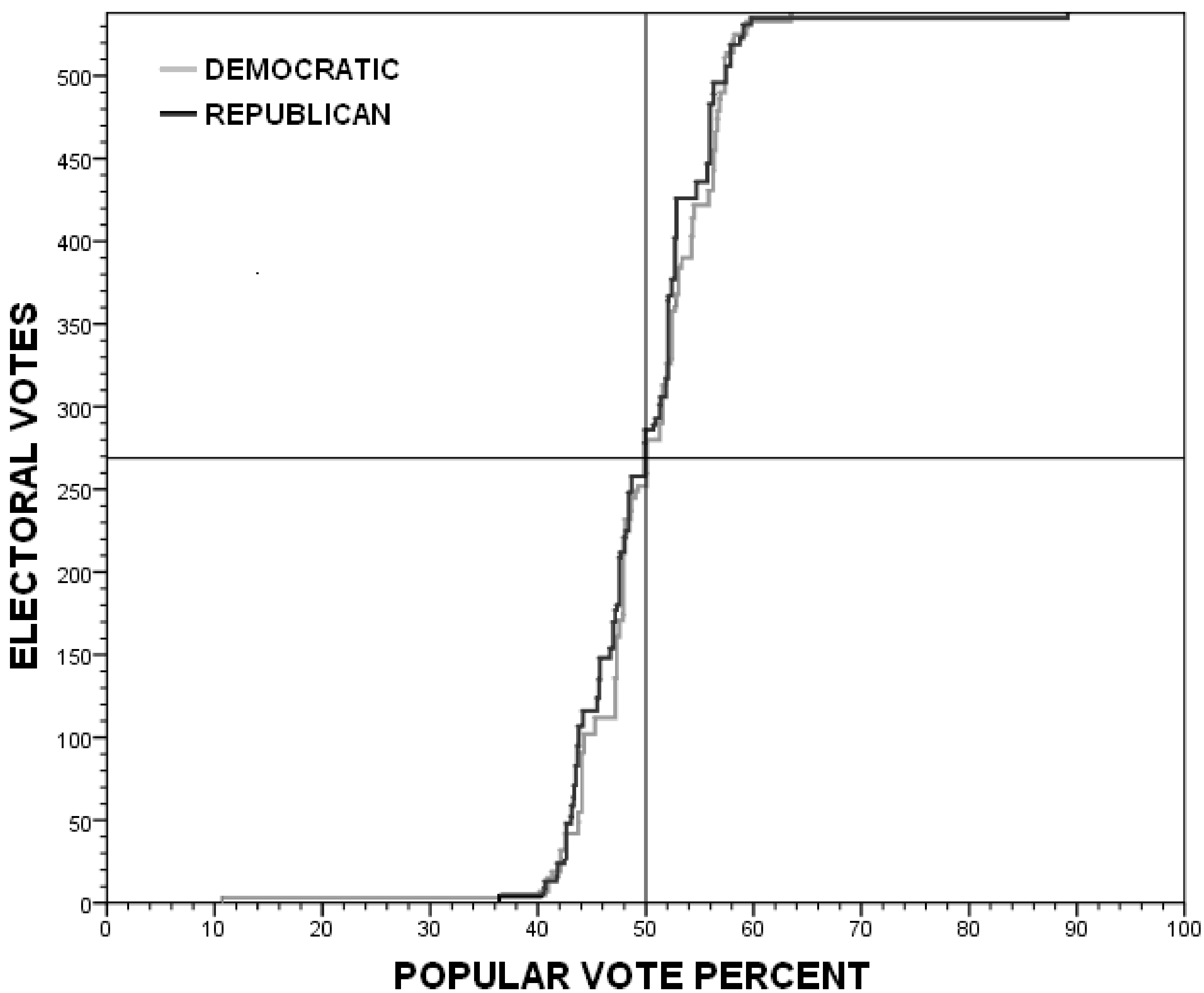

TIE INTERVAL
NUMBER OF ELECTIONS
0
5
10
15
20
25
47.5 48.0 48.5 49.0 49.5 50.0 50.5 51.0 51.5 52.0
DEMOCRATIC TIE OR WIN PERCENT

(a)


TIE INTERVAL
NUMBER OF ELECTIONS
0
5
10
15
20
25
30
35
40
APPROXIMATE PROBABILITY OF INVERSION
0.0
0.125
0.25
0.375
0.5
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00 2.25
ABSOLUTE ELECTION INVERSION INTERVAL IN PERCENT

(b)

Figure 5



DEMOCRATIC
REPUBLICAN
ELECTORAL VOTES
0
50
100
150
200
250
300
350
400
450
500
0
10
20
30
40
50
60
70
80
90
100
POPULAR VOTE PERCENT

Figure 6

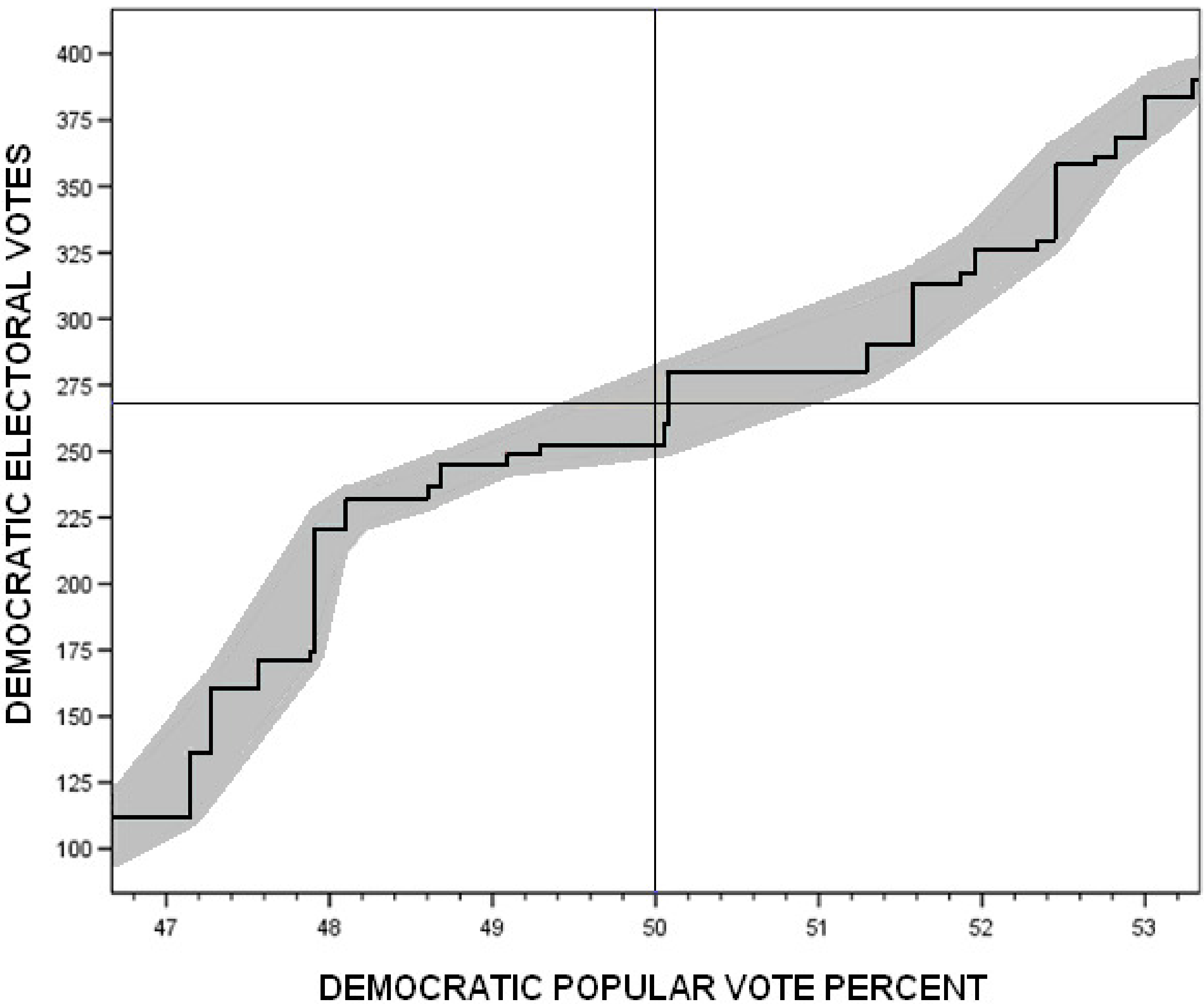

DEMOCRATIC ELECTORAL VOTES
400
375
350
325
300
275
250
225
200
175
150
125
100
47
48
49
50
51
52
53
DEMOCRATIC POPULAR VOTE PERCENT

Figure 7

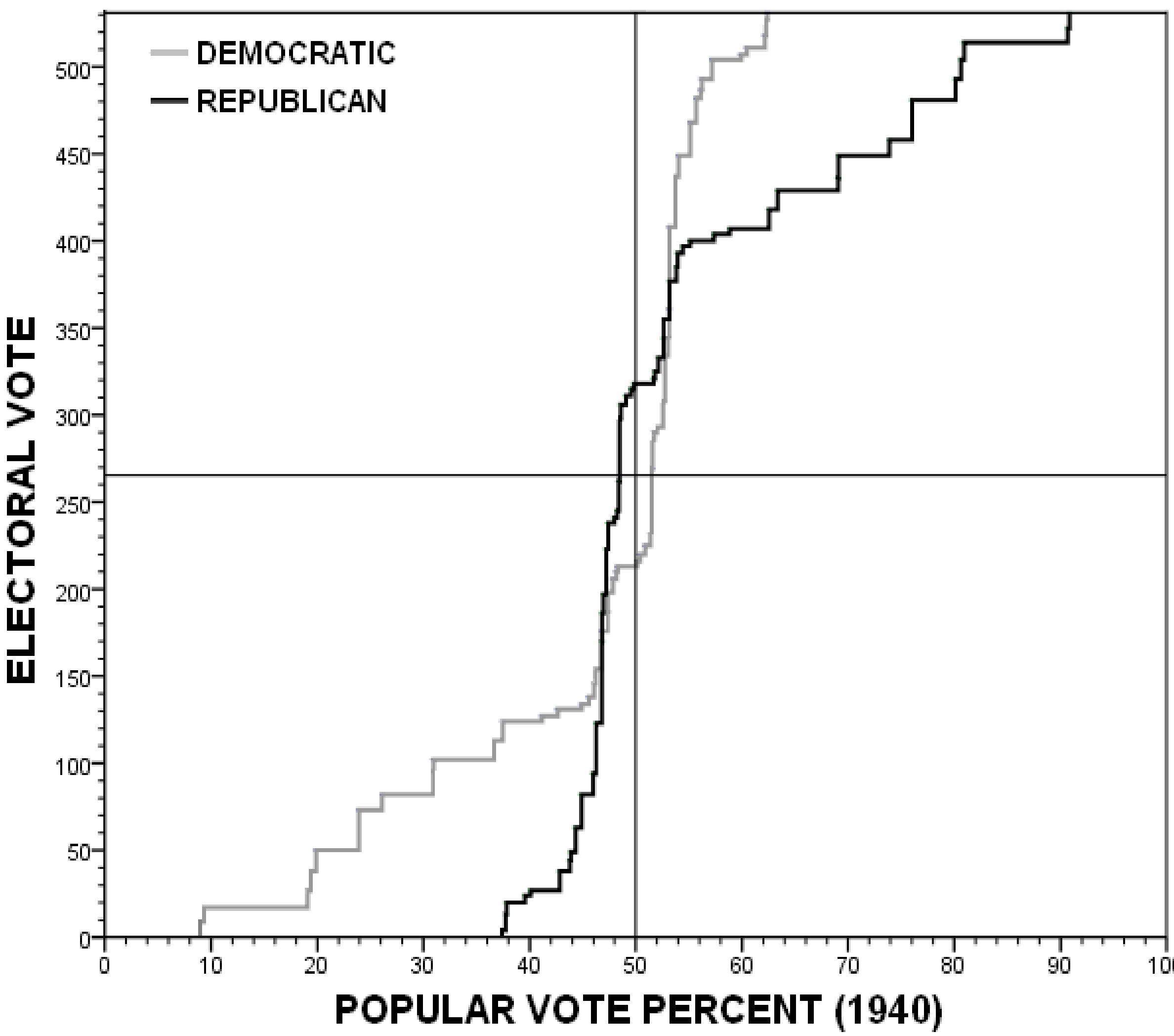

DEMOCRATIC
REPUBLICAN
ELECTORAL VOTE
0
50
100
150
200
250
300
350
400
450
500
0
10
20
30
40
50
60
70
80
90
100
POPULAR VOTE PERCENT (1940)

Figure 8

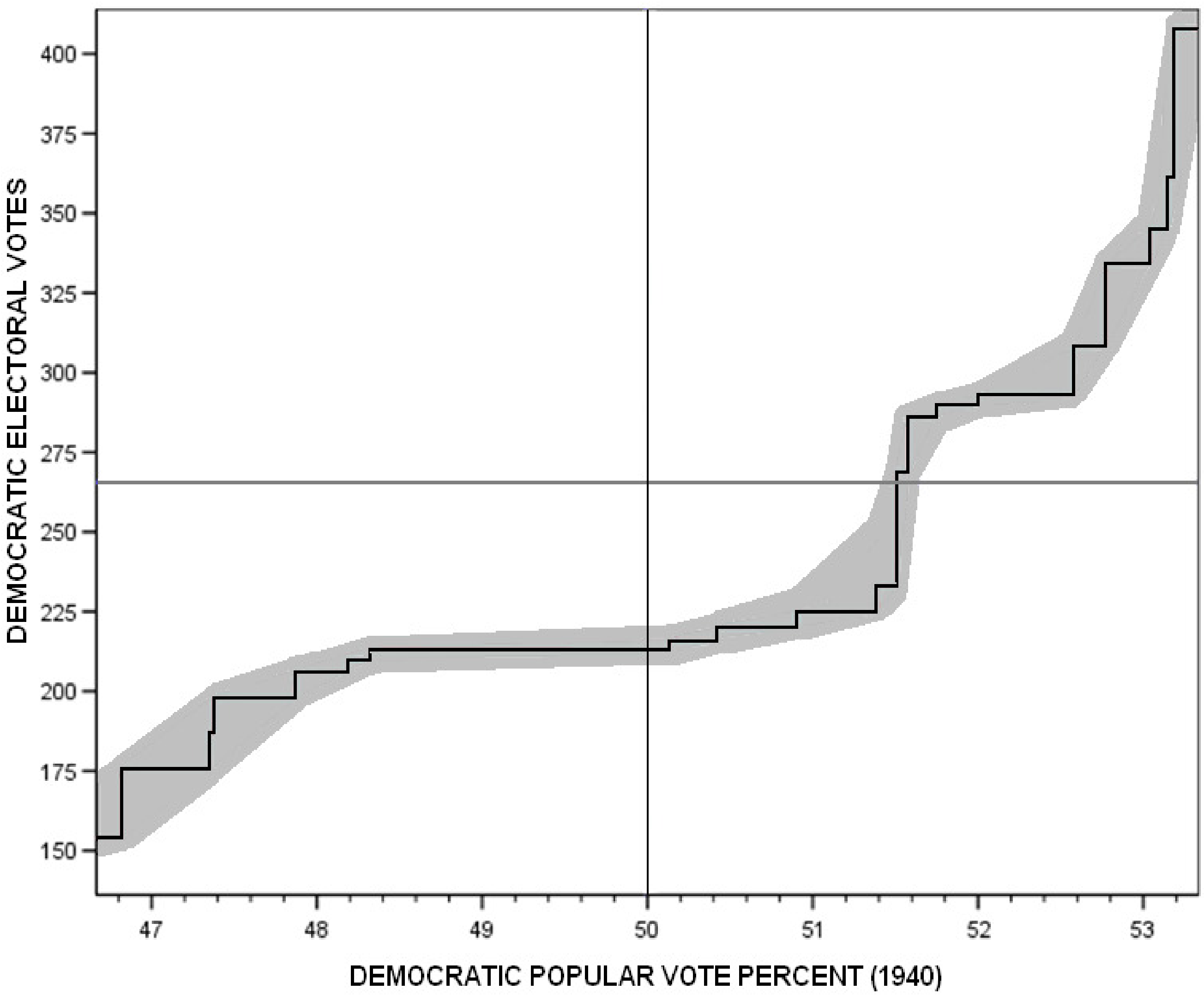
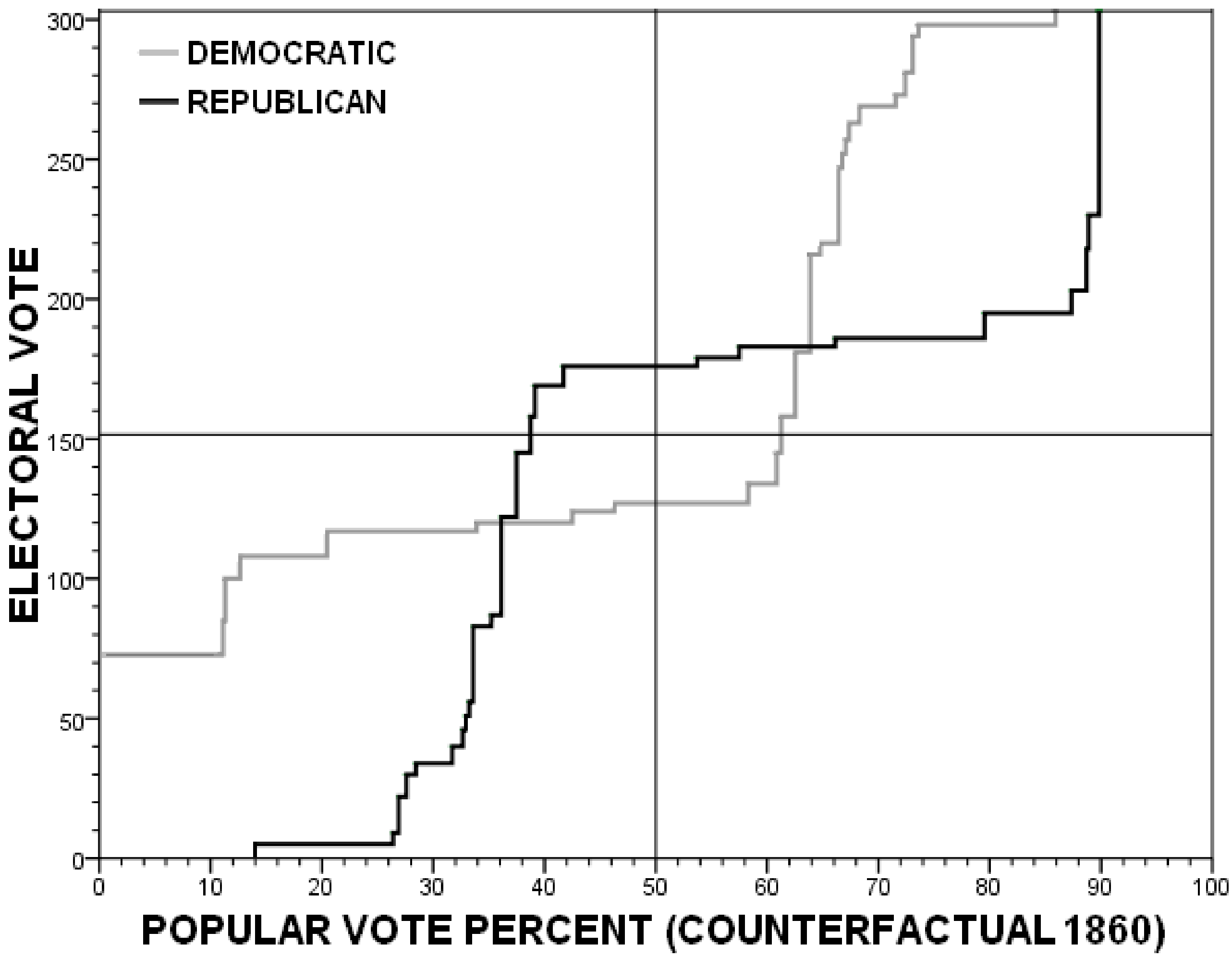

DEMOCRATIC ELECTORAL VOTES
400
375
350
325
300
275
250
225
200
175
150
47
48
49
50
51
52
53
DEMOCRATIC POPULAR VOTE PERCENT (1940)

Figure 9



DEMOCRATIC
REPUBLICAN
ELECTORAL VOTE
0
50
100
150
200
250
300
0
10
20
30
40
50
60
70
80
90
100
POPULAR VOTE PERCENT (COUNTERFACTUAL 1860)

Figure 10

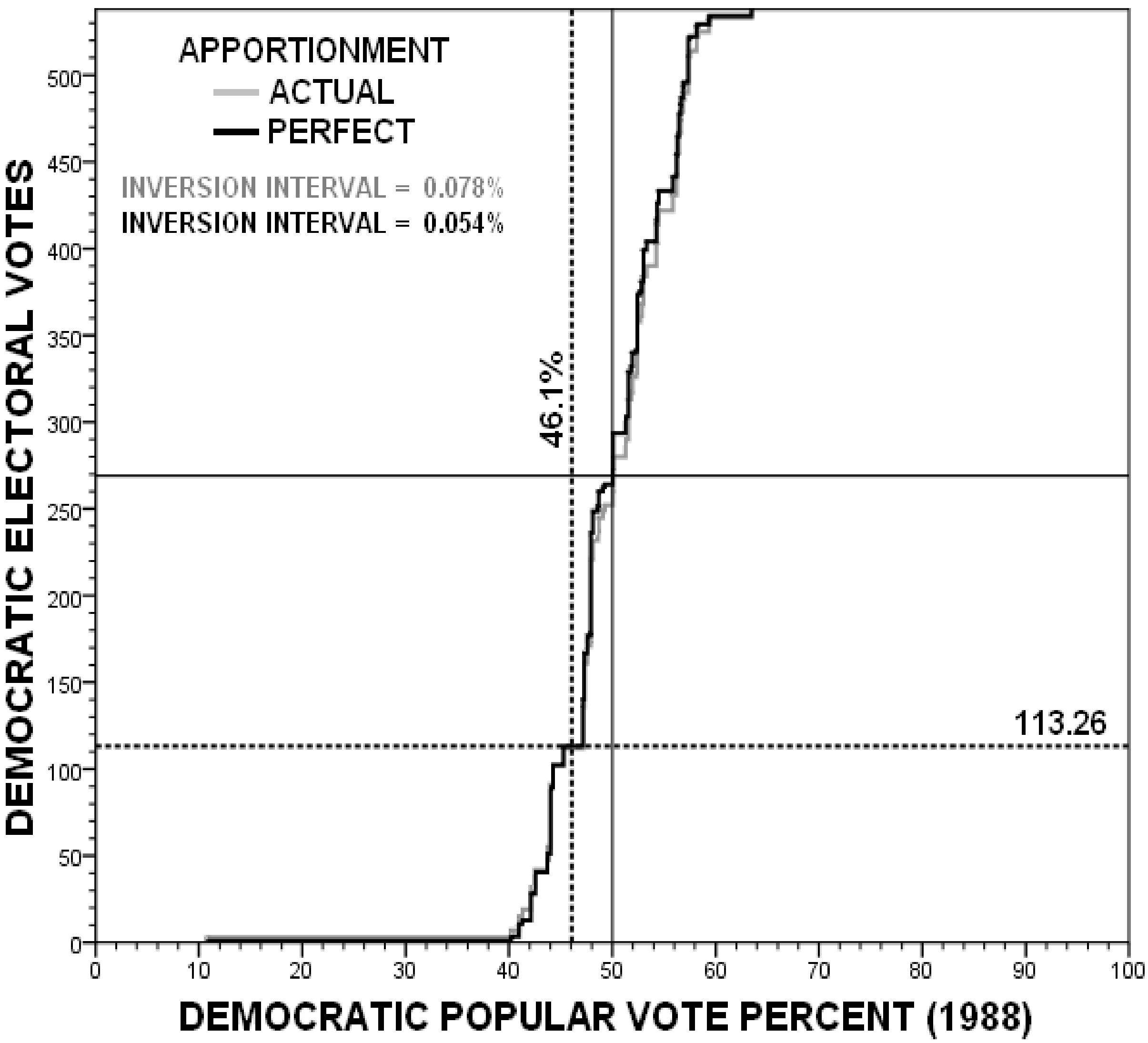

APPORTIONMENT
ACTUAL
PERFECT
INVERSION INTERVAL = 0.078%
INVERSION INTERVAL = 0.054%
46.1%
113.26
DEMOCRATIC ELECTORAL VOTES
DEMOCRATIC POPULAR VOTE PERCENT (1988)
0
50
100
150
200
250
300
350
400
450
500
0
10
20
30
40
50
60
70
80
90
100

Figure 11

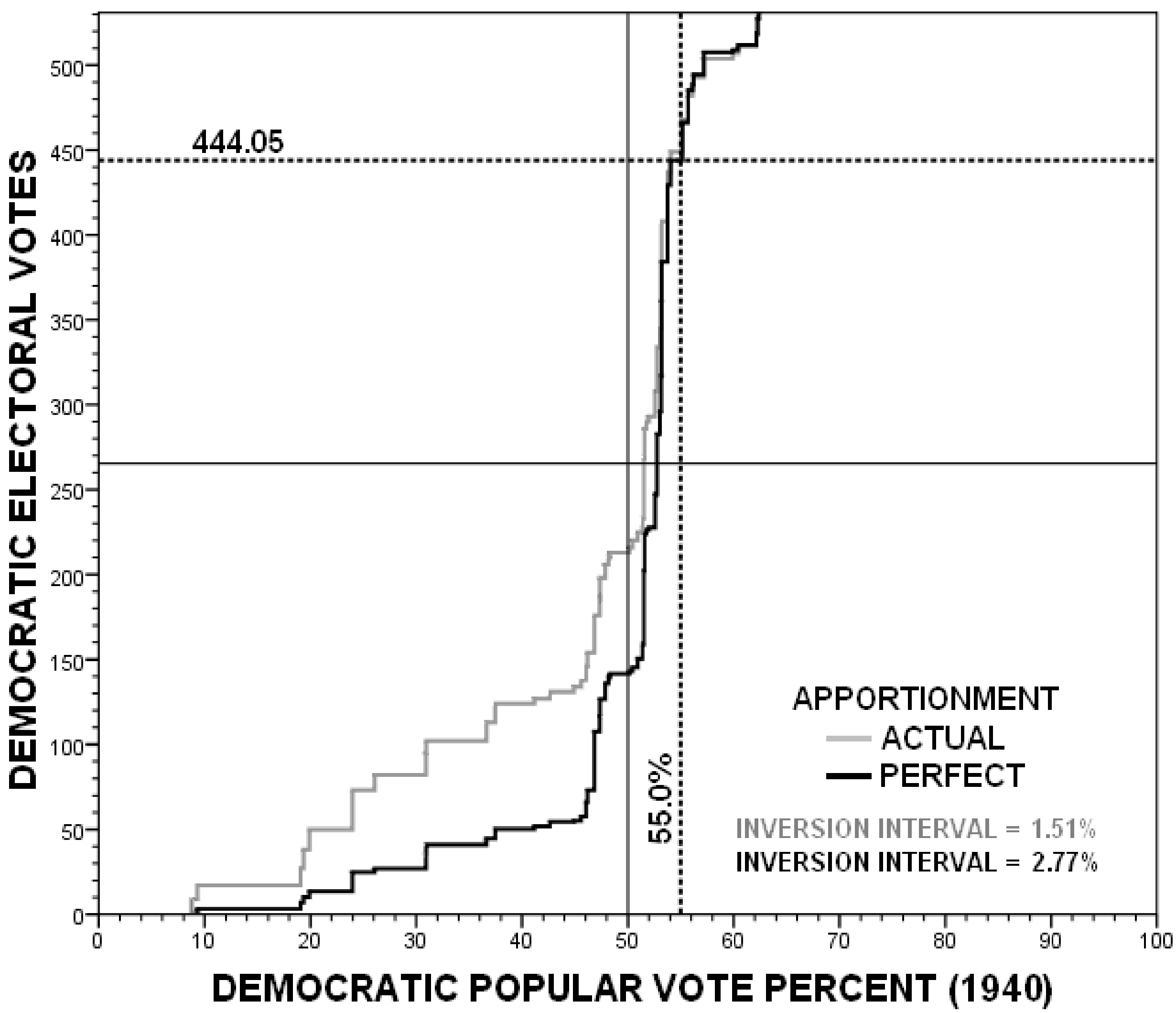

444.05
55.0%
APPORTIONMENT
ACTUAL
PERFECT
INVERSION INTERVAL = 1.51%
INVERSION INTERVAL = 2.77%
DEMOCRATIC ELECTORAL VOTES
DEMOCRATIC POPULAR VOTE PERCENT (1940)
0
50
100
150
200
250
300
350
400
450
500
0
10
20
30
40
50
60
70
80
90
100

Figure 12

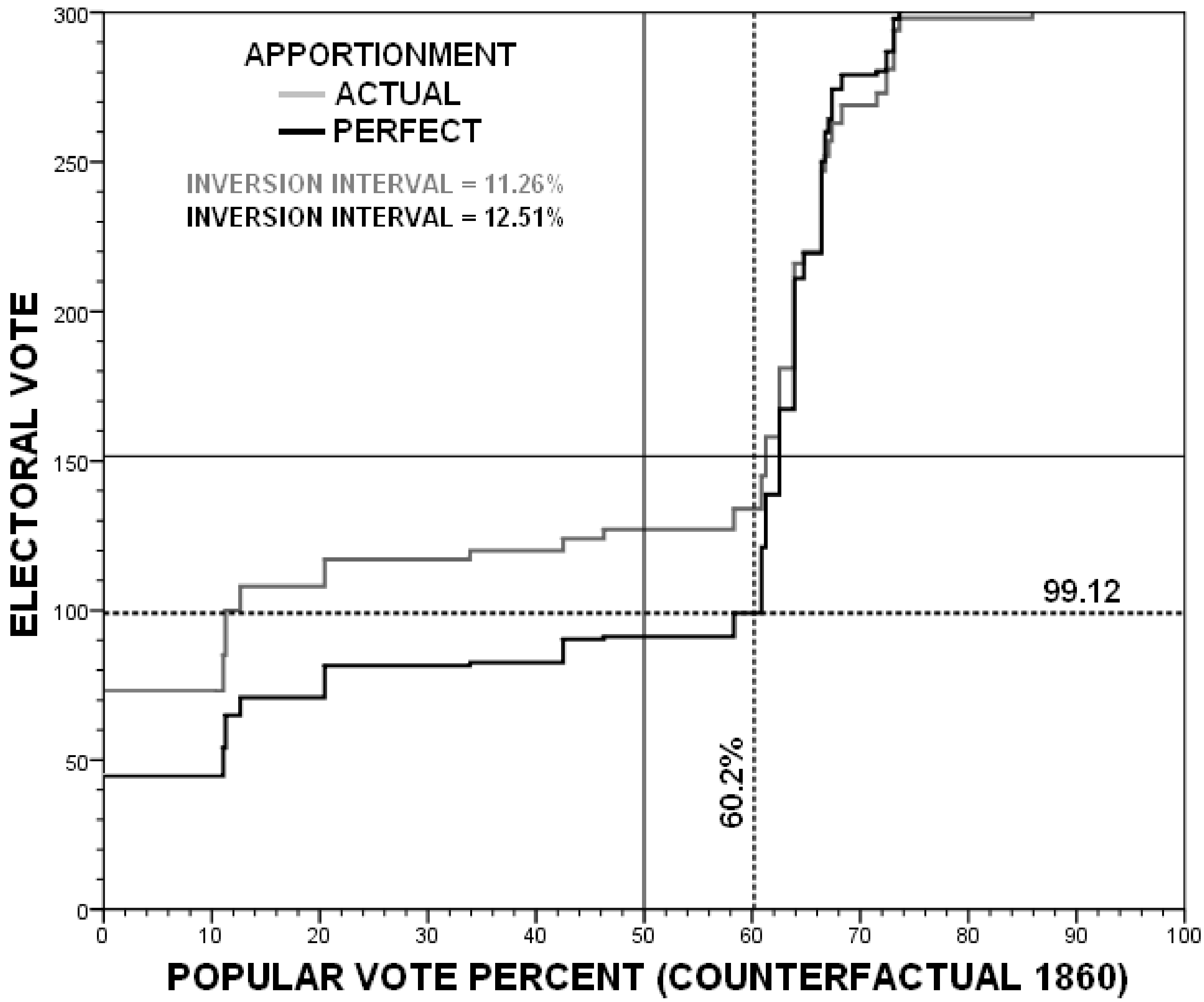

APPORTIONMENT
ACTUAL
PERFECT
INVERSION INTERVAL = 11.26%
INVERSION INTERVAL = 12.51%
ELECTORAL VOTE
300
250
200
150
100
50
0
99.12
60.2%
0
10
20
30
40
50
60
70
80
90
100
POPULAR VOTE PERCENT (COUNTERFACTUAL 1860)

Figure 13

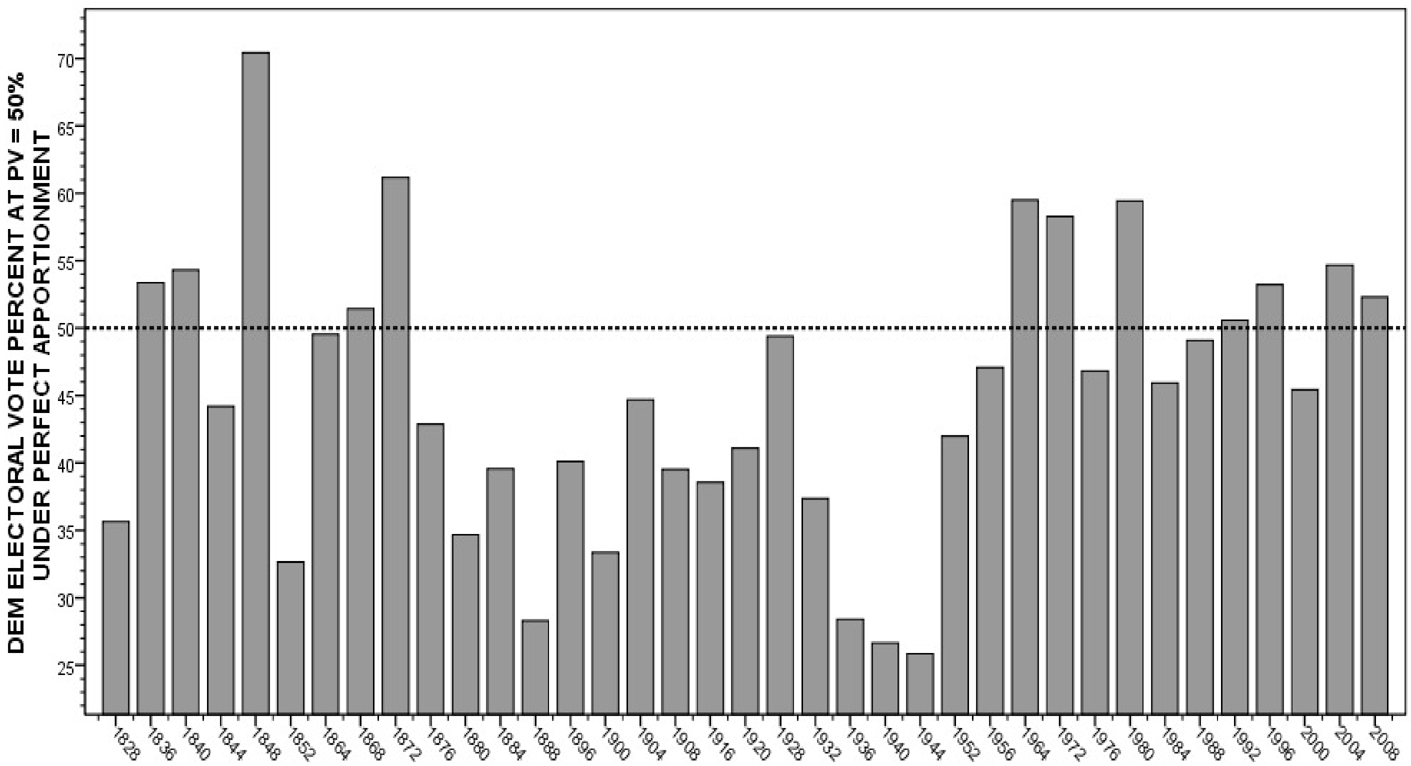

DEM ELECTORAL VOTE PERCENT AT PV = 50%
UNDER PERFECT APPORTIONMENT
70
65
60
55
50
45
40
35
30
25
1828
1836
1840
1844
1848
1852
1864
1868
1872
1876
1880
1884
1888
1896
1900
1904
1908
1916
1920
1928
1932
1936
1940
1944
1952
1956
1964
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008

Figure 14

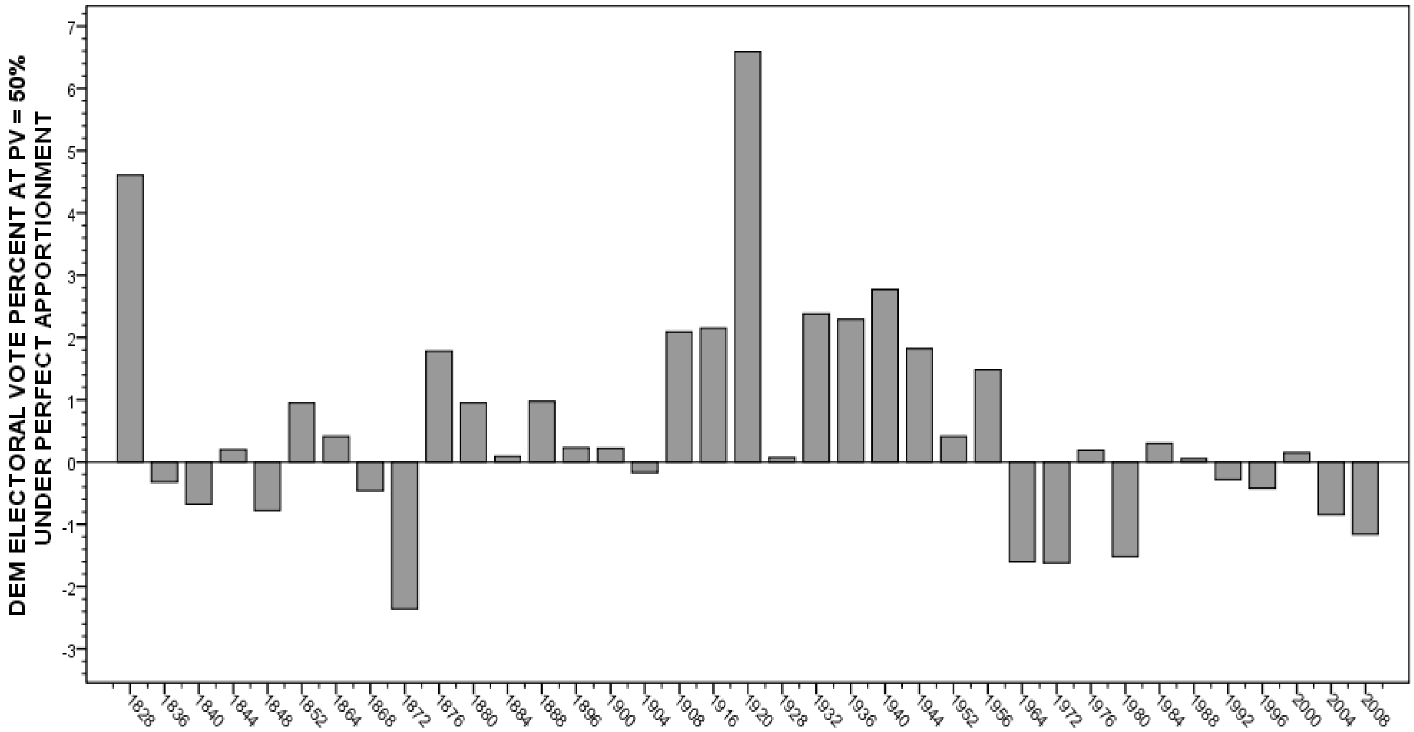

DEM ELECTORAL VOTE PERCENT AT PV = 50%
UNDER PERFECT APPORTIONMENT
7
6
5
4
3
2
1
0
-1
-2
-3
1828 1836 1840 1844 1848 1852 1864 1868 1872 1876 1880 1884 1888 1896 1900 1904 1908 1916 1920 1928 1932 1936 1940 1944 1952 1956 1964 1972 1976 1980 1984 1988 1992 1996 2000 2004 2008

Figure 15

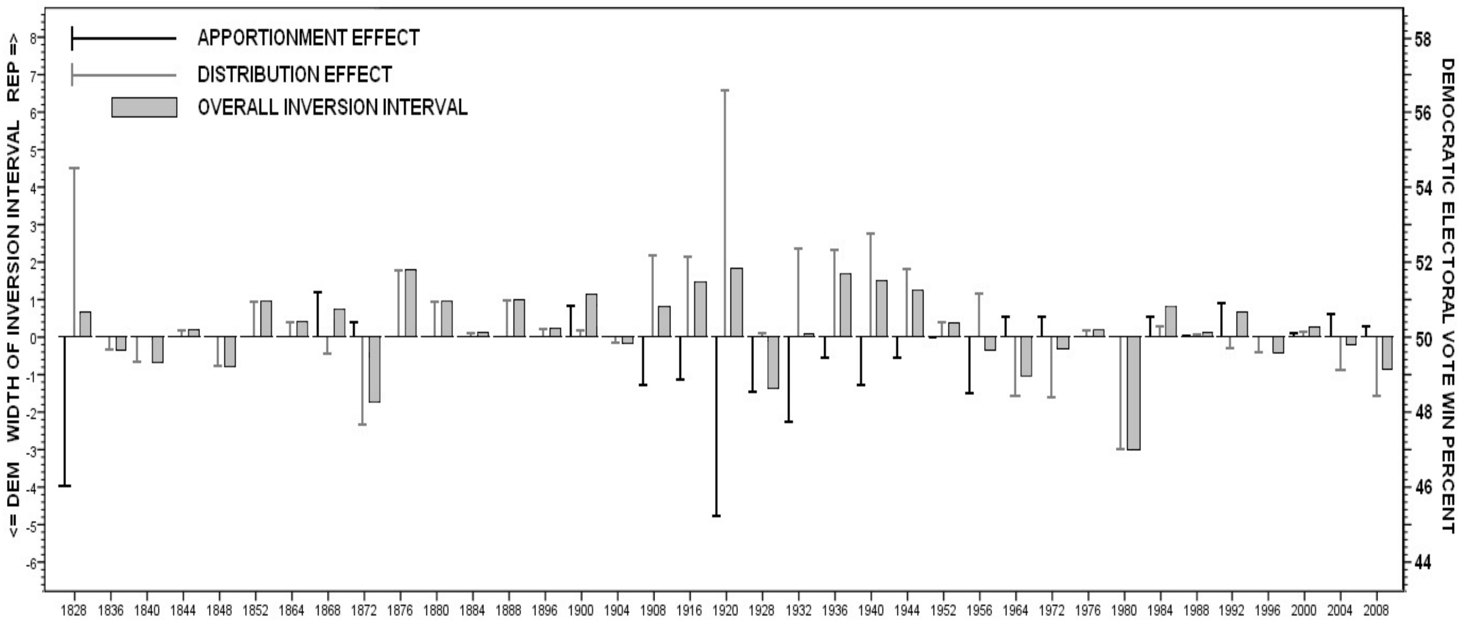

APPORTIONMENT EFFECT
DISTRIBUTION EFFECT
OVERALL INVERSION INTERVAL
<= DEM WIDTH OF INVERSION INTERVAL REP =>
DEMOCRATIC ELECTORAL VOTE WIN PERCENT
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
58
56
54
52
50
48
46
44
1828 1836 1840 1844 1848 1852 1864 1868 1872 1876 1880 1884 1888 1896 1900 1904 1908 1916 1920 1928 1932 1936 1940 1944 1952 1956 1964 1972 1976 1980 1984 1988 1992 1996 2000 2004 2008

Figure 16

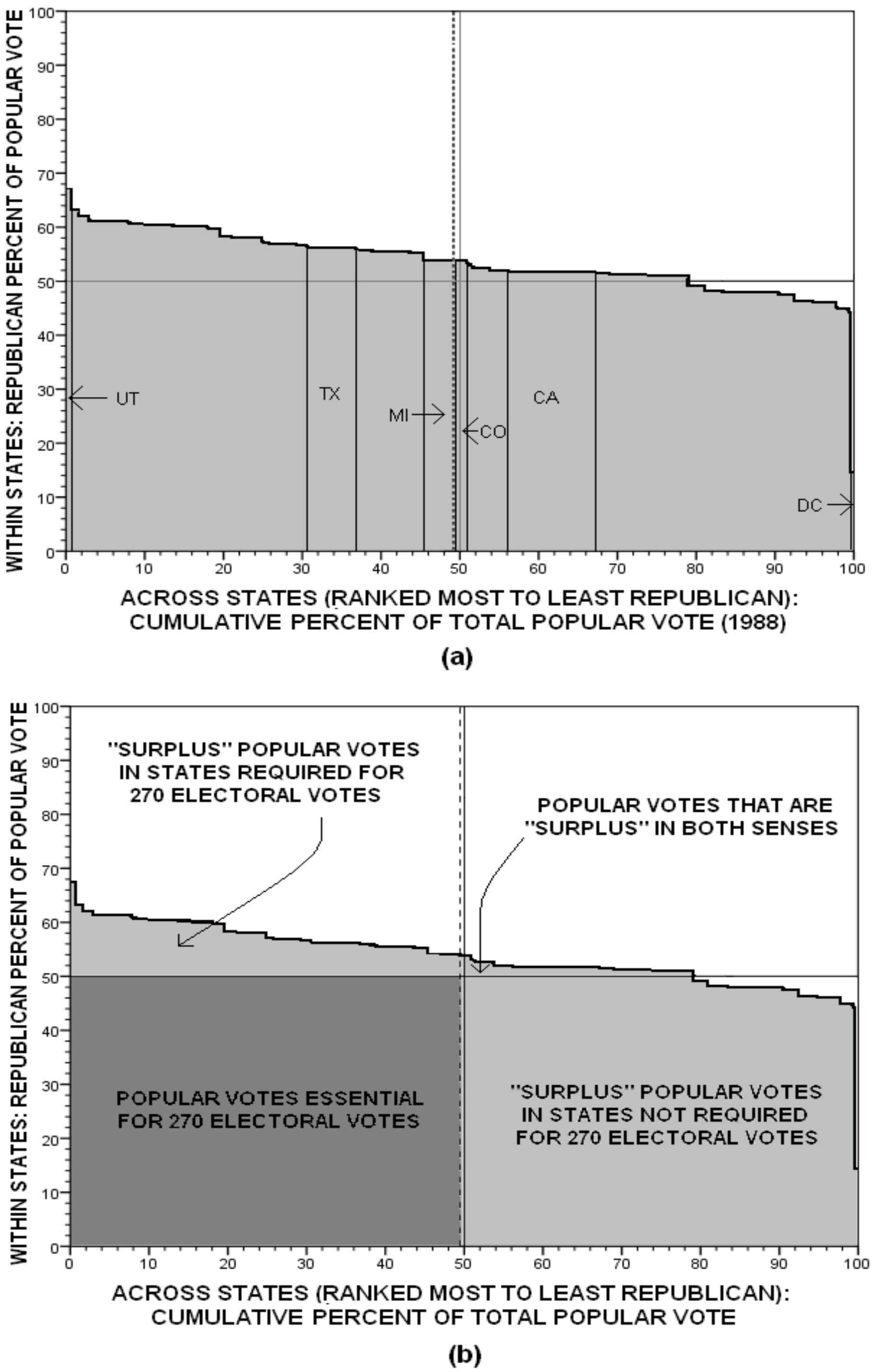

WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
UT
TX
MI
CO
CA
DC
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN):
CUMULATIVE PERCENT OF TOTAL POPULAR VOTE (1988)
(a)
WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
"SURPLUS" POPULAR VOTES
IN STATES REQUIRED FOR
270 ELECTORAL VOTES
POPULAR VOTES THAT ARE
"SURPLUS" IN BOTH SENSES
POPULAR VOTES ESSENTIAL
FOR 270 ELECTORAL VOTES
"SURPLUS" POPULAR VOTES
IN STATES NOT REQUIRED
FOR 270 ELECTORAL VOTES
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN):
CUMULATIVE PERCENT OF TOTAL POPULAR VOTE
(b)

Figure 17

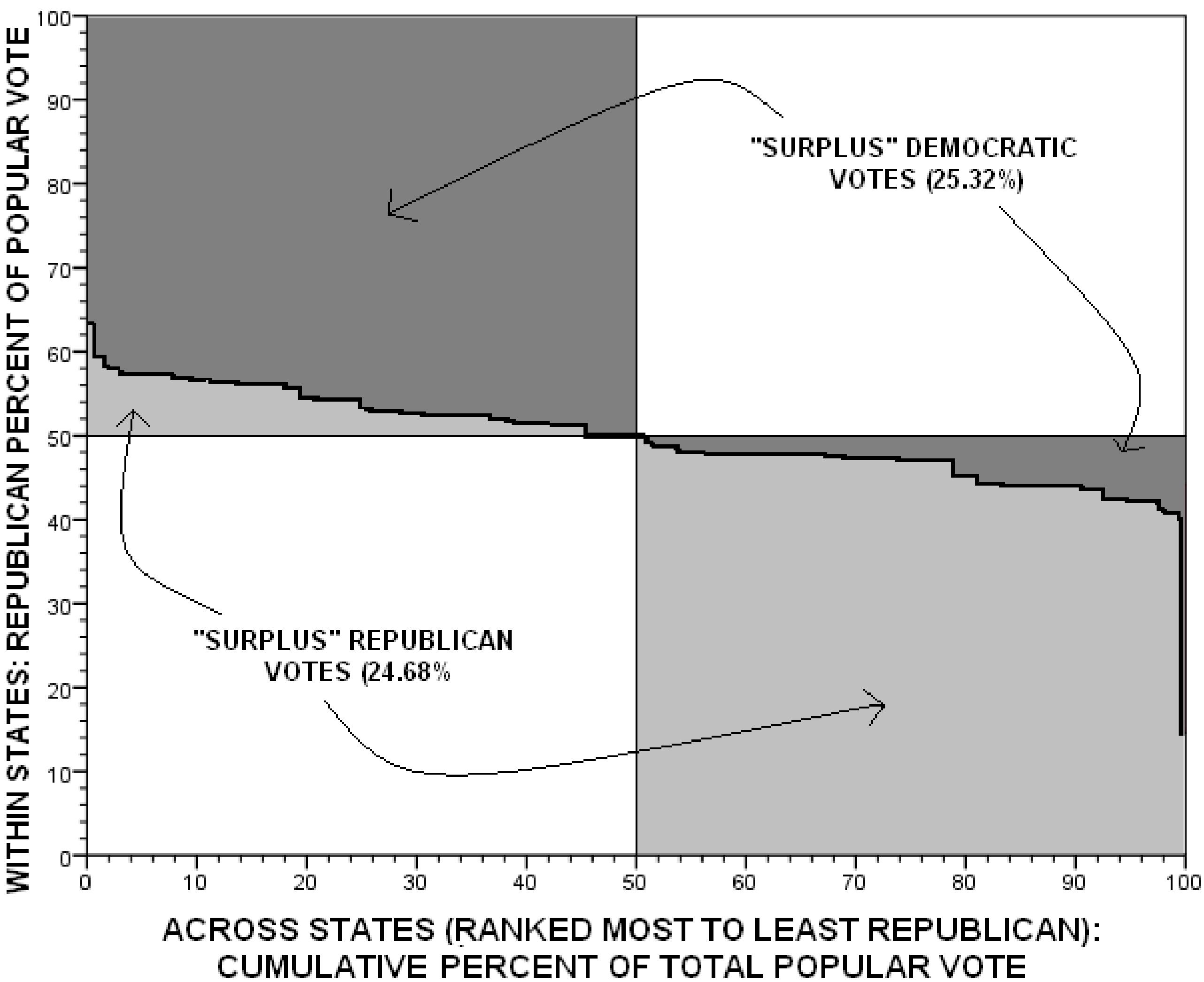

"SURPLUS" DEMOCRATIC VOTES (25.32%)
"SURPLUS" REPUBLICAN VOTES (24.68%
WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN): CUMULATIVE PERCENT OF TOTAL POPULAR VOTE
0
10
20
30
40
50
60
70
80
90
100

Figure 18

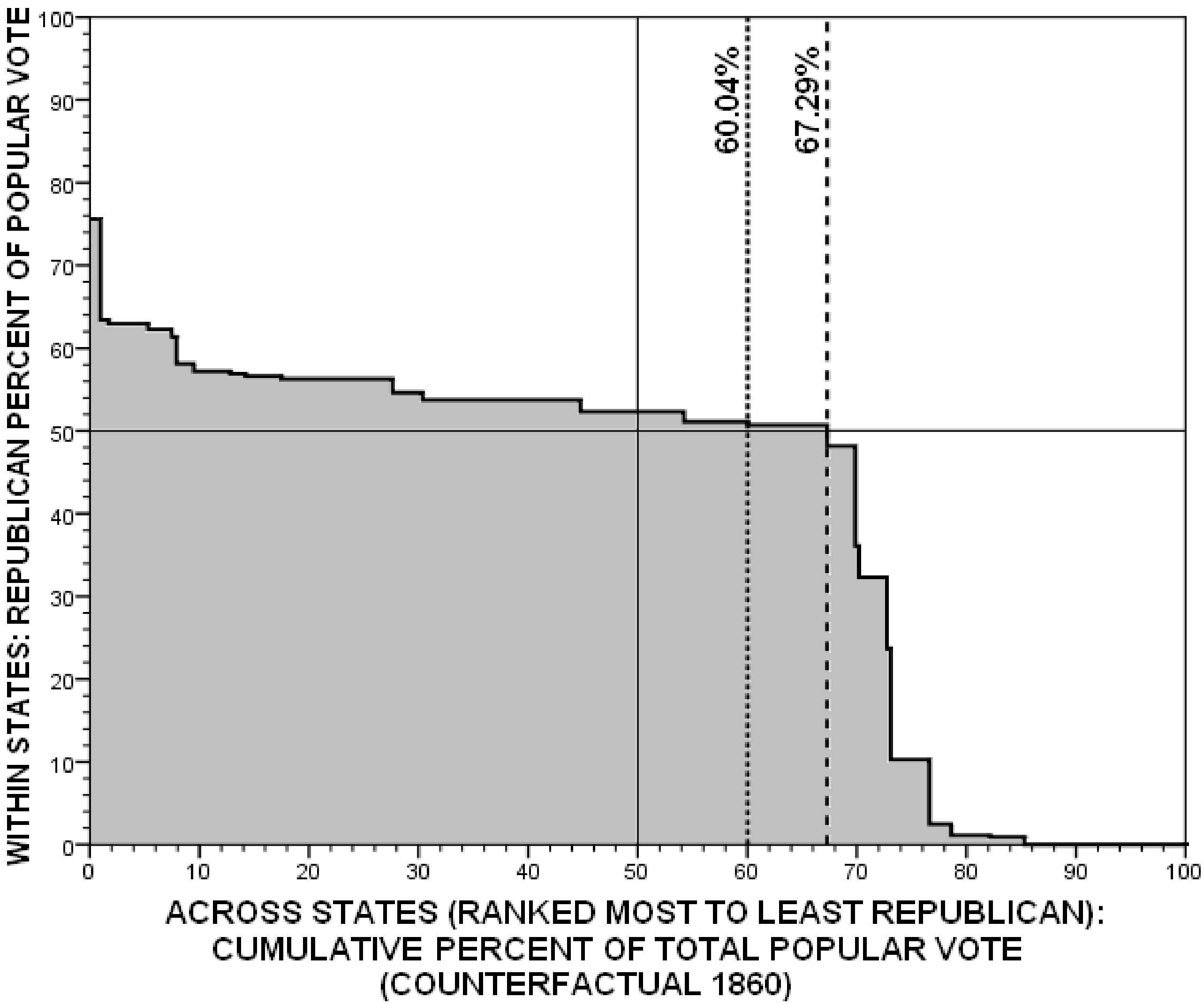

WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
100
90
80
70
60
50
40
30
20
10
0
60.04%
67.29%
0
10
20
30
40
50
60
70
80
90
100
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN):
CUMULATIVE PERCENT OF TOTAL POPULAR VOTE
(COUNTERFACTUAL 1860)

Figure 19

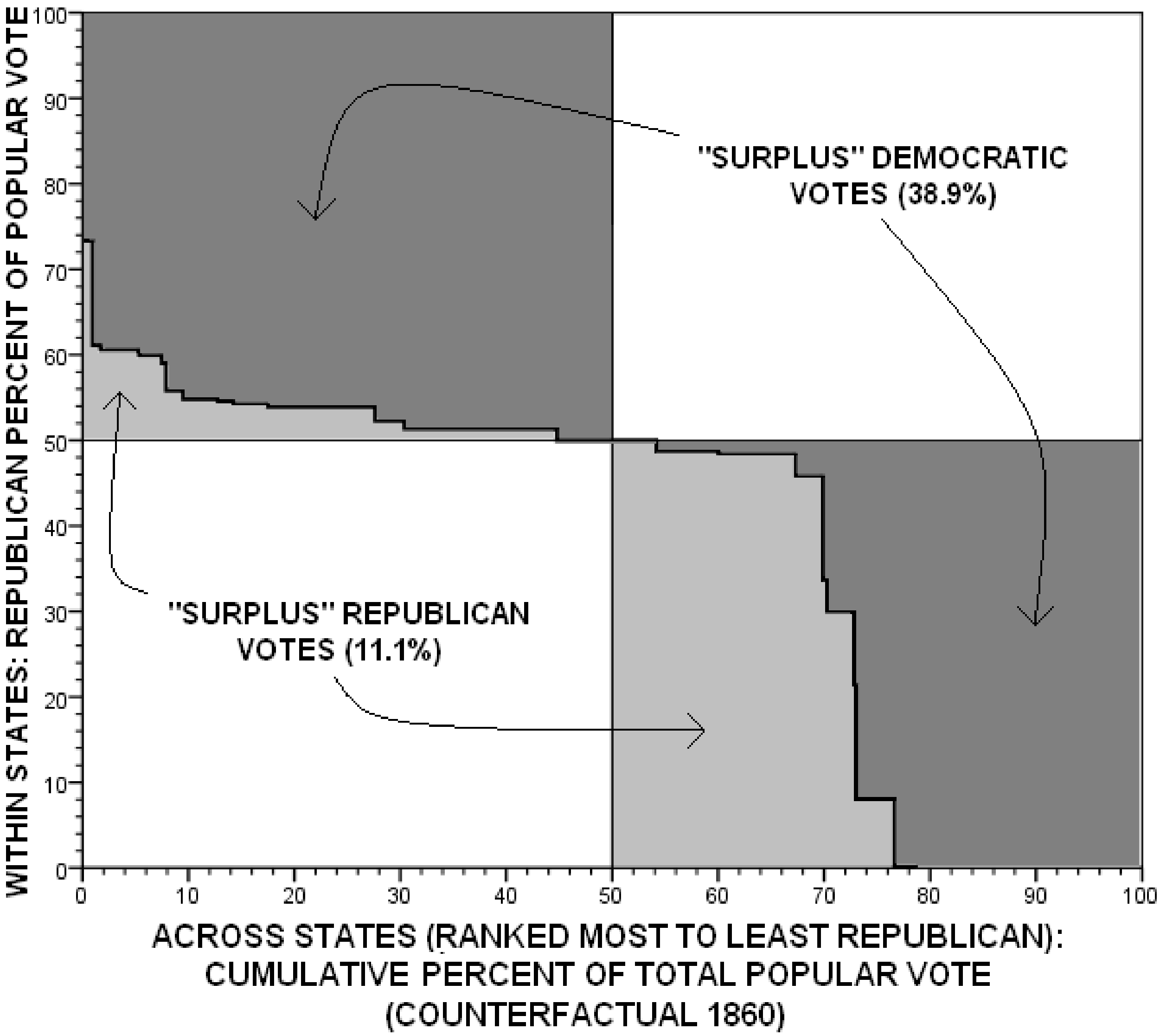

WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
100
90
80
70
60
50
40
30
20
10
0
"SURPLUS" DEMOCRATIC VOTES (38.9%)
"SURPLUS" REPUBLICAN VOTES (11.1%)
0
10
20
30
40
50
60
70
80
90
100
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN):
CUMULATIVE PERCENT OF TOTAL POPULAR VOTE
(COUNTERFACTUAL 1860)

Figure 20

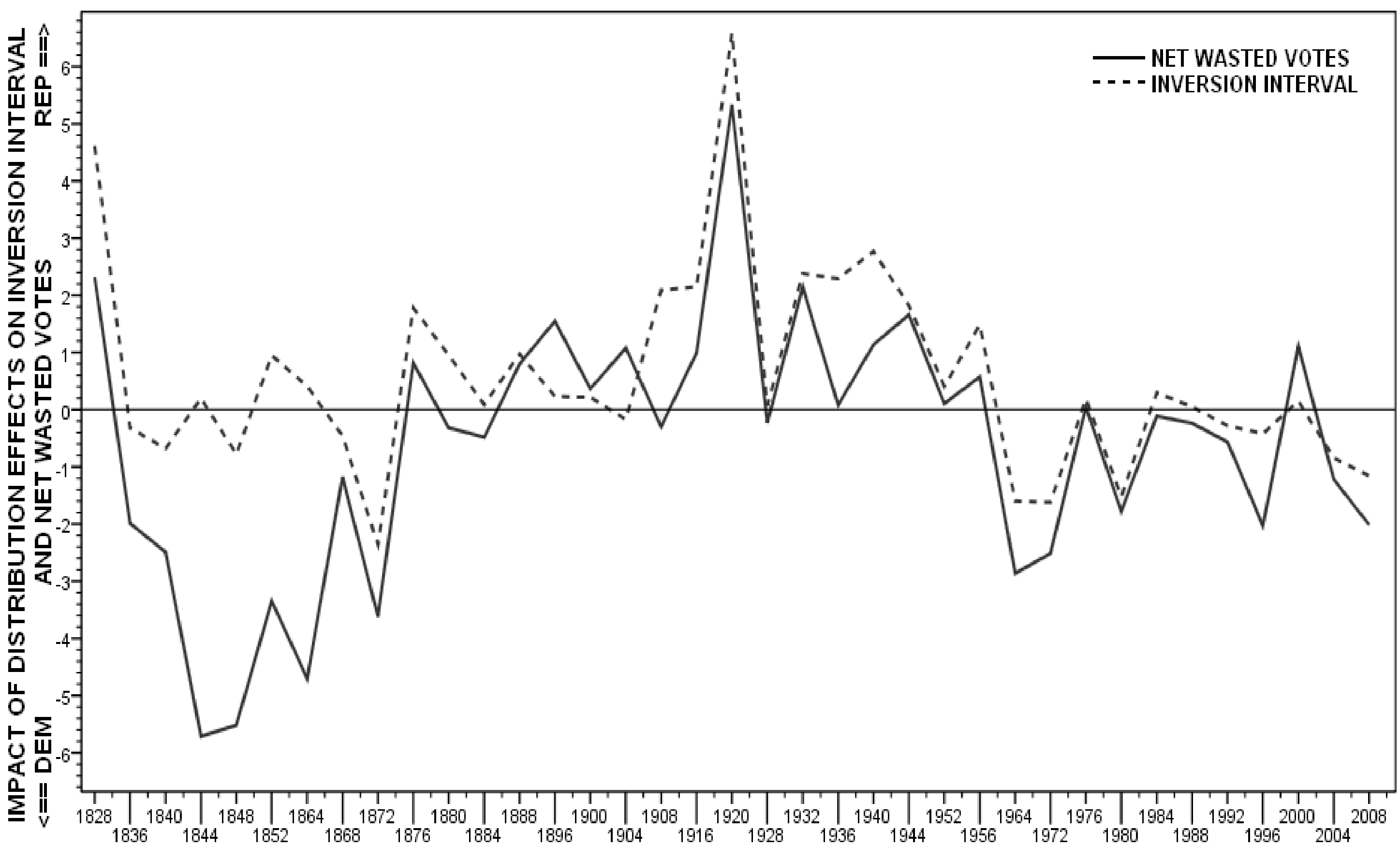

IMPACT OF DISTRIBUTION EFFECTS ON INVERSION INTERVAL
AND NET WASTED VOTES
<== DEM
REP ==>
NET WASTED VOTES
INVERSION INTERVAL
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
1828
1836
1840
1844
1848
1852
1864
1868
1872
1876
1880
1884
1888
1896
1900
1904
1908
1916
1920
1928
1932
1936
1940
1944
1952
1956
1964
1972
1976
1980
1984
1988
1992
1996
2000
2004
2008

Figure 21