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RESEARCH ARTICLE | NOVEMBER 01 1999

Magnetohydrodynamic turbulence in the solar wind

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Phys. Plasmas 6, 4154–4160 (1999)

<https://doi.org/10.1063/1.873680>



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Magnetohydrodynamic turbulence in the solar wind

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(Received 4 June 1999; accepted 2 July 1999)

Low frequency fluctuations in the solar wind magnetic field and plasma velocity are often highly correlated, so much so that the fluctuations may be thought of as originating near the Sun as nearly perfect Alfvén waves. Power spectra of these fluctuations from 10^{-7} Hz to several Hz suggest that the medium is turbulent. Near 1 AU, fluctuations below 10^{-5} Hz have a relatively flat slope (~ -1) and contain most of the energy in the fluctuating fields. From 10^{-5} Hz to ~ 0.1 Hz, the spectra exhibit a power law inertial range similar to that seen in ordinary fluid turbulence. At the highest frequencies, the rapid fall-off of the power suggests that strong dissipation is occurring. From *in situ* measurements, it is clear that the fluctuations emanate from the solar corona. The turbulent cascade appears to evolve most rapidly in the vicinity of velocity shears and current sheets. Numerical solutions of both the compressible and incompressible equations of magnetohydrodynamics in both Cartesian and spherical geometry corroborate this interpretation. There are conflicting interpretations of observations suggesting that much of the power in magnetic field fluctuations resides in quasi-two-dimensional structures and simulations have helped to elucidate some of these issues. [S1070-664X(99)02811-6]

I. INTRODUCTION

The existence of the solar wind was first deduced by Birkeland¹⁻³ and predicted theoretically by Parker⁴ (also see Holzer *et al.*⁵ for a recent review). Soon after, Gringauz *et al.*⁶⁻⁸ confirmed the presence of a continuous flow of a hot, supersonic and super-Alfvénic plasma. Subsequent observations confirmed that the flow was continuous, filled the heliosphere, and existed primarily in two states: a fast wind with velocities ranging up to 800 km/s, and a slow wind with velocities in the range 250–350 km/s (see Parker⁹ for more details). This “wind” is, in fact, more tenuous than the best vacuum we can produce in terrestrial laboratories; its density varies from about 1–20 particles/cm³ at 1 Astronomical Unit (AU) from the Sun.

Much of the nearly continuous variability in the solar wind arises from low-frequency plasma waves. Of course, the flow is also punctuated by more intermittent and more dramatic variations, including collisionless shock waves, magnetic clouds, coronal mass ejections, and, at and beyond 1 AU, corotating interaction regions (see Burlaga¹⁰ for a comprehensive review). The fluctuations in the solar wind magnetic field and plasma velocity are often highly correlated, so that at times they can be thought of as nearly perfect Alfvén waves.¹¹ Alfvén waves are defined by $\mathbf{v} = \pm \mathbf{b}$, where \mathbf{v} and \mathbf{b} are the fluctuating components of the velocity and magnetic fields, respectively, and where the magnetic field has been normalized to cgs velocity units by dividing by $\sqrt{4\pi\rho}$ where ρ is the mass density. The sign in the definition of Alfvén waves indicates the propagation direction with respect to the local mean magnetic field: a “+” sign denoting propagation antiparallel to the mean magnetic field \mathbf{B}_0 and a “–” sign denoting propagation parallel to \mathbf{B}_0 . In the solar wind, especially in the inner heliosphere, the direction of propagation of Alfvén waves is predominantly outward from

the Sun. This is thought to be because most of the Alfvénic flux is generated in the photosphere or lower corona where the outward flow of the atmosphere is both subsonic and sub-Alfvénic. Thus, when the solar wind becomes super-Alfvénic, only outward propagating fluctuations will be convected out into the heliosphere. In addition, compressive waves, such as slow mode or, to a lesser extent, fast mode waves will be Landau damped,¹² leaving nearly all of the fluctuating wave energy in the Alfvén wave mode. Most of the data analysis and simulations described below use a somewhat different convention for the sign of the correlation between the velocity and magnetic fields, namely, the “+” sign will denote fluctuations propagating outward into the heliosphere regardless of the sign of \mathbf{B}_0 and the “–” sign will denote inward propagating fluctuations.

Early analyses by Coleman¹³⁻¹⁵ indicated that these fluctuations, whatever their origin, had properties reminiscent of turbulence. In particular, power spectra of the magnetic field or velocity fluctuations often contained an “inertial” range with a slope of approximately $-\frac{5}{3}$, which is the value predicted and observed for ideal (i.e., dissipationless) isotropic incompressible Navier-Stokes fluid turbulence.¹⁶⁻¹⁸ These early solar wind observations could not distinguish clearly between a $-\frac{5}{3}$ slope and the $-\frac{3}{2}$ slope predicted by Kraichnan¹⁹ for ideal isotropic incompressible magnetohydrodynamic (MHD) turbulence. More recent work²⁰ indicates that the spectral slope is more often $-\frac{5}{3}$, but in either case the observation is striking because the solar wind is neither isotropic,²¹ nor incompressible,^{22,23} nor dissipationless.

Coleman proposed that turbulence in the solar wind was dynamically evolving, driven by stream-shear instabilities. He argued further that turbulent dissipation at high wave numbers could account for the anomalously high proton temperatures observed in the wind at 1 AU. On the other hand,

the nearly pure Alfvénicity²⁴ cast doubt on Coleman’s hypothesis because pure Alfvén waves are exact solutions of the ideal incompressible MHD equations, so that to the extent that the solar wind was governed by these equations, all nonlinear interactions should have been suppressed strongly. Furthermore, it was thought that the mean magnetic field of the solar wind would suppress the shear-driven Kelvin–Helmholtz instability,²⁵ further inhibiting the generation of instabilities at stream shear interfaces. Bavassano *et al.*²⁶ noted that, even if excited, shear instabilities could not produce fluctuations on the large scales observed.

These two seemingly conflicting interpretations of solar wind observations have been reconciled in recent years. It is now clear that regions of strong velocity shear, especially in proximity to the heliospheric current sheet where the average spiral magnetic field reverses direction, are locations where the interplanetary medium is stirred and Alfvénic correlations are sharply reduced. In the absence of strong velocity shears, however, Alfvénic fluctuations are convected into the outer heliosphere with surprisingly little evolution although the *spectra* do evolve even in relatively “pure” streams.²⁷ For a more detailed discussion, the reader is referred to a review of magnetohydrodynamic turbulence theory²⁸ and to reviews of the application of MHD turbulence to solar wind observations in particular.^{29–31}

In the next section, we review briefly some of the more striking evidence that the solar wind is a dynamically evolving turbulent magnetofluid. In Sec. III we discuss some numerical solutions of the MHD equations which show how velocity shears and spherical expansion influence and control the formation of a Kolmogorov-type spectrum. We also discuss briefly the role of both spherical expansion and the background magnetic field in determining the symmetry of the fluctuations. In Sec. IV we summarize the present status of the field and indicate avenues for future research.

II. OBSERVATIONAL BACKGROUND

At 1 AU the solar wind consists of a mix of Alfvénic fluctuations, convected structures, streams of various amplitudes, and propagating compressive structures. All of these interact after leaving the solar corona. At periods shorter than the solar rotation period, the interaction between fast and slow solar wind streams drives nonlinear couplings, producing a flow of energy in wave number space from large to small scales that is ultimately dissipated by kinetic effects. Although even small amplitude waves will be distorted by velocity shear,^{32,33} or by density gradients,³⁴ including linear mode coupling,³⁵ both simulations and observations indicate that velocity shears in the solar wind drive nonlinear interactions that reduce the Alfvénicity of the turbulence. The interaction appears to be primarily one which generates inward propagating Alfvén fluctuations rather than strongly compressive non-Alfvénic fluctuations.³⁶

A. Power spectra

As illustrated in Fig. 1, at time scales of a month to a year, coherent motions are visible.³⁷ The spectrum at these large scales ($\approx 10^{-6}$ Hz) has a slope of approximately $-1/3$,

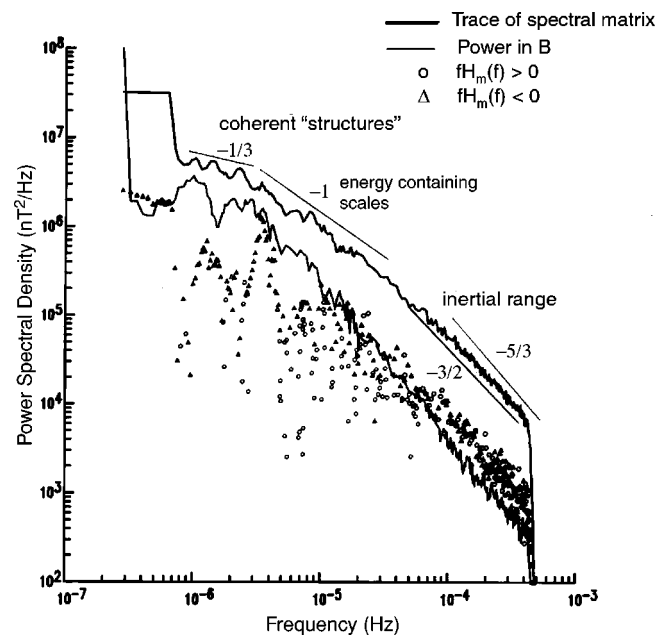


FIG. 1. A power spectrum of the solar wind magnetic field from a time series spanning more than a year. The upper curve is the trace of the power spectral matrix of the three components of \mathbf{B} , the lower solid curve is the power in $|\mathbf{B}|$, and the circles and triangles are the positive and negative values of the [reduced (Ref. 91)] magnetic helicity (Refs. 39 and 92) respectively.

although this is highly variable. Often solar rotation peaks occur at periods of 27 days and its harmonics. In the decade surrounding $\approx 10^{-5}$ Hz, the spectrum is steeper (approximately -1)—this is the “energy containing scale.” The remainder of the frequency spectrum in Fig. 1 shows part of an inertial range spectrum that extends out to the proton cyclotron frequency (see below).

Figure 1 also shows the power in the magnitude of the magnetic field B which, in the inertial range, is more than an order of magnitude below the power in \mathbf{B} . This lack of power in B corroborates the notion that solar wind fluctuations are primarily incompressible and has led to work that describes solar wind turbulence in the context of a nearly incompressible magnetofluid theory.³⁸ Also included in Fig. 1 is the spectrum of the magnetic helicity^{20,39} $H_m(f)$ showing that in the inertial range the handedness of the magnetic fluctuations is random. At larger scales structures do appear that may be preferentially left- or right-handed.⁴⁰

At 0.5 AU the inertial range ends near 1 Hz where the spectrum steepens (Fig. 2) and $H_m(f)$ tends toward a single sign, suggesting that the ion cyclotron component is being damped while the electron whistler branch is continuing to higher frequencies, albeit at a much reduced amplitude.^{41–43}

As energy cascades to smaller scales one might expect the energy containing portion of the spectrum to shrink, and this is observed. For example, in Fig. 3 we show the evolution of the spectra of the Elsässer variables with heliocentric distance. (The Elsässer variables^{44,45} are defined by $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b}$, where \mathbf{z}^+ and \mathbf{z}^- refer to waves propagating “outward” and “inward” with respect to the average background magnetic field \mathbf{B}_0 .) As is clear from this and other work,⁴⁶ the k^{-1} portion of the spectrum is eroded away as

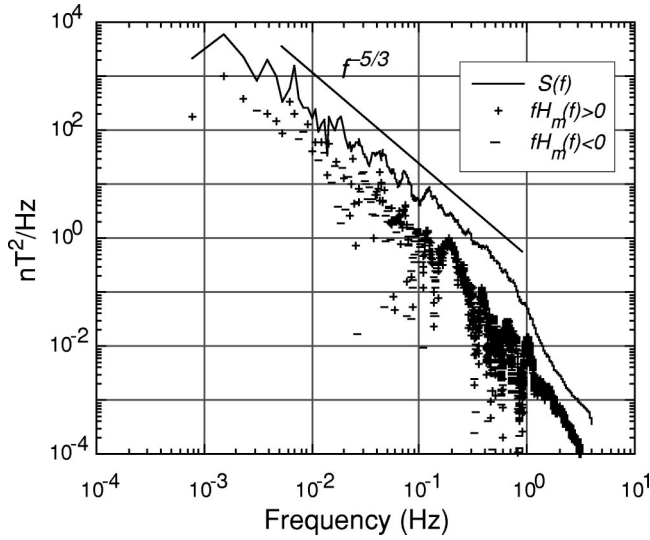


FIG. 2. A power spectrum of Mariner 10 data from 0.5 AU showing the dissipation range of magnetic fluctuations. Also shown are positive and negative values of $fH_m(f)$.

one moves outward in the heliosphere; a trend which continues to several AU.^{47–49}

B. Evolution of cross helicity

The other major piece of evidence that the solar wind is an actively evolving magnetofluid comes from studies using

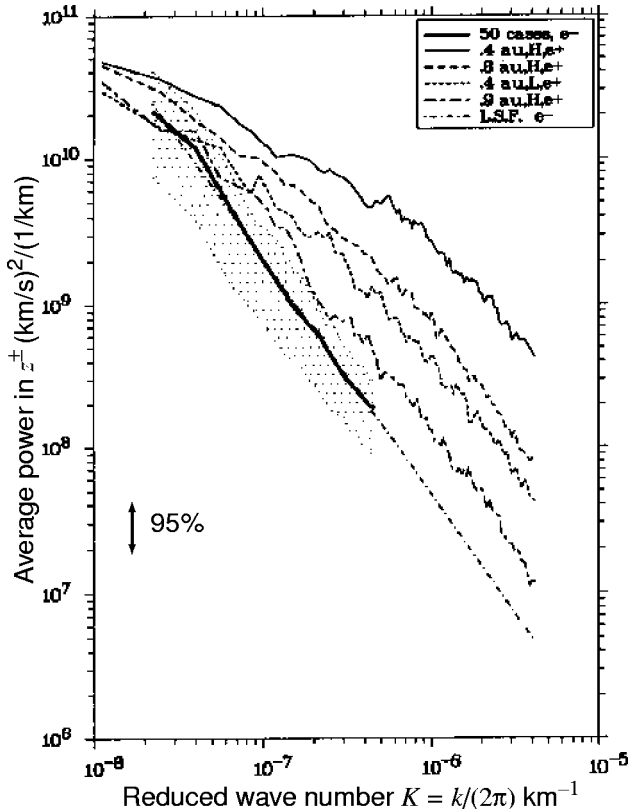


FIG. 3. Evolution spectra of the Elsässer variable z^+ from 0.4 to 1 AU. Also plotted is a least-squares fit to 50 z^- spectra (lower dashed-dotted line). The spectral range of the 50 spectra is indicated by the dappled band (adapted from Ref. 93). The power in $z^±$ is indicated in the legend as $e^±$.

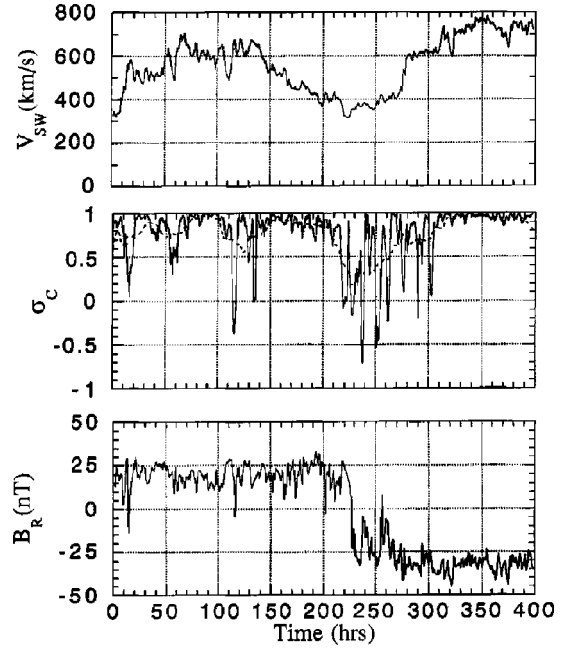


FIG. 4. Hour-averaged Helios 2 data obtained near 0.3 AU (days 93–110 of 1976). Plotted are V_{SW} , σ_c at the scale of about 3 hours (solid line) and a 25-hour running mean (dashed line), together with the radial component (B_R) of the magnetic field (from Ref. 36).

data from the Helios and Voyager spacecraft.^{27,50–53} These studies showed clearly that nearly all regions near the ecliptic are characterized by rapid decreases with heliocentric distance in the Alfvénicity of the fluctuations as measured by the reduced cross helicity σ_c . The cross helicity (global) invariant H_c (Refs. 54–56) is defined by $H_c = (1/2) \int dx^3 \mathbf{v} \cdot \mathbf{b}$. A more useful quantity is the normalized quantity $\sigma_c(k) \equiv 2H_c^r(k)/E^r(k)$. $E^r(k)$ is the reduced spectral energy. Evidence for the dynamical evolution of solar wind turbulence is found in the decrease in σ_c with increasing radial distance from values very close to unity at 0.3 AU. Figure 4 shows an example of this decrease near the boundary between fast and slow solar wind flows. The plot uses one-hour-averaged Helios data from near 0.3 AU and shows a strong decrease in σ_c in association with the center of the heliospheric current sheet and regions of strong velocity shear.

C. The Alfvén effect

Magnetofluid turbulence was predicted to have, on average, equal magnetic and kinetic energy (per unit wave number) in the inertial range of the turbulence spectrum.¹⁹ The physical reasoning behind this conjecture can be understood by recalling that Alfvénic fluctuations ($\mathbf{v} = \pm \mathbf{b}$) of arbitrary amplitude are solutions of the ideal incompressible MHD equations. A perturbation of $\mathbf{v} \neq \mathbf{0}$ (with $\mathbf{b} = \mathbf{0}$) can therefore be described as an initial condition for two Alfvén wave packets with magnetic fluctuations $+\mathbf{b}$ and $-\mathbf{b}$, respectively. These packets should then propagate away from each other with speeds $\pm B_0/(4\pi\rho)^{1/2}$, where $\mathbf{B}_0 = \langle \mathbf{B} \rangle$ and $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, indicating that approximate equipartition should result. A measure of this is the “Alfvén ratio” r_A

$=E_V(k)/E_B(k)$, where $E_V(k)$ and $E_B(k)$ are the reduced spectra of the kinetic energy and magnetic energy, respectively. Although some evidence for equipartition has been reported in two-dimensional simulations,⁵⁷ solar wind observations consistently show that r_A computed for fluctuations in the inertial range of the spectrum lies between 0.4–1.0.^{20,27,36,50,58–60} At the large, ~ 10 day scale, r_A is dominated by the large fluctuations in the radial component of the solar wind velocity, even within streams, while at the smaller scales at ~ 0.3 AU, $r_A \approx 1$, at least in one high speed stream, but by 1 AU it decreases to ~ 0.5 , or less.⁶¹ In the outer heliosphere, as seen by Voyager 1 at 8 AU, r_A still has a most probable value ~ 0.5 .

D. The symmetry of solar wind turbulence

We have very little idea of the symmetry properties or solar wind turbulence. We do know that the variances of solar wind fluctuations are not isotropic.²⁴ Determining the spectral symmetry properties of the fluctuations requires specifying the three-dimensional wave vector dependence of the power spectrum. From single spacecraft data this is difficult to do. Nonetheless, even early work suggested that significant anisotropies were present.⁶² A second approach to determining the symmetry of the fluctuations is to use long stationary data intervals to form an ensemble from which a two-dimensional correlation function can be constructed.²¹ The results indicate that the ensemble is dominated by two populations: fluctuations with large correlation lengths perpendicular to $\bar{\mathbf{B}}_0$ (Alfvénic) and fluctuations with large correlation lengths parallel to $\bar{\mathbf{B}}_0$ (quasi-two-dimensional).

In quasi-two-dimensional turbulence, wave vectors are nearly perpendicular to the large-scale magnetic field $\bar{\mathbf{B}}$, while the magnetic fluctuations are orthogonal to both \mathbf{k} and the local mean magnetic field $\bar{\mathbf{B}}_0$. Subsequent analysis suggested that nearly 80% of interplanetary turbulence might consist of fluctuations with $\mathbf{k} \perp \bar{\mathbf{B}}_0$.⁶³ In contrast, other work suggests that the non-Alfvénic component of the fluctuations involves “structures” with magnetic fluctuations parallel to \mathbf{B}_0 .^{64,65} Two studies^{33,66} analyzed the role of small velocity shears in generating a significant population of fluctuations with wave numbers *nearly* orthogonal to \mathbf{B}_0 and Ghosh *et al.*⁶⁷ have explored with numerical simulations how a two-component model of solar wind fluctuations might arise by time-averaging spectra in different regions.

III. SIMULATIONS

A. Previous results in rectangular geometry

Early attempts to model the development of interplanetary turbulence solved the incompressible (i.e., $\nabla \cdot \mathbf{v} = 0$) MHD equations in two periodic spatial dimensions.⁵⁸ As mentioned above, these studies showed that proximity to both the heliospheric current sheet and regions of strong velocity shear were important factors controlling the evolution of the cross helicity. Three-dimensional incompressible, but still periodic, simulations⁶⁸ confirmed the two-dimensional results.

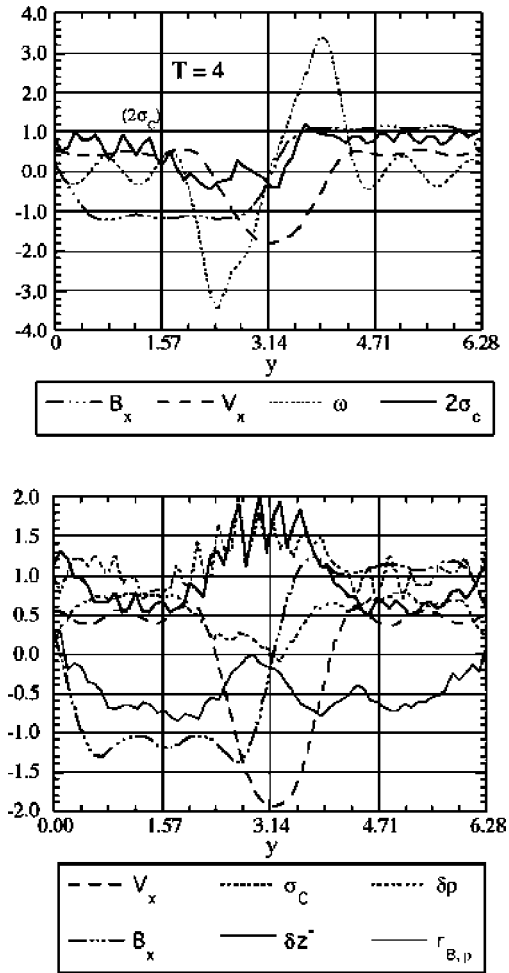


FIG. 5. (top): Results at $T=4$ eddy-turnover times from an incompressible MHD simulation (ω is the vorticity) (adapted from Ref. 58). (bottom): Results at $T=2$ eddy-turn-over times from a solution of the compressible equations. Similar to the top panel with the addition of density fluctuations, $\delta\rho$, δz^- , and the correlation between density and magnetic field magnitude, $r_{B,\rho}$ ($\delta\rho$ and δz^- are normalized to twice their maximum values of 0.042 and 0.075, respectively) (adapted from Ref. 58).

The next step was to use compressible MHD codes to simulate magnetofluid turbulence. These were used to test the incompressible results and to explore phenomena that could not be addressed in incompressible MHD, such as the magnitude and sign of the correlation between density and magnetic field magnitude. Observationally, compressive regions are not highly correlated with a decrease in Alfvénicity,^{27,60} however, density fluctuations may be significant in other ways.³⁰ For example, especially at solar minimum, regions with structured density and other plasma properties show both developed turbulence spectra and strong cross helicity depletions,^{53,61,69–71} but these phenomena are not necessarily due to effects of compressibility.

In many respects, the compressible simulations provided support for the nearly incompressible model (Fig. 5).⁵⁸ In one two-dimensional compressible MHD simulation with a current sheet, the results were nearly identical for the evolution of the “incompressible” quantities to those found using incompressible MHD codes (cf. Fig. 5). The compressive case also showed that density fluctuations arose naturally from the

incompressible evolution.^{23,72} The anticorrelation between $|B|$ and ρ (denoted by $r_{B,\rho}$ in Fig. 5), indicates that the density fluctuations are in approximate pressure balance with the magnetic field. The density fluctuations also correlate well with the Elsässer variable, δz^- .⁷³

The evolution of the Alfvén ratio r_A has not been studied in great detail, especially with three-dimensional compressible MHD codes (and not at all in the spherical geometry discussed in the next section). Analytic work for quasi-two-dimensional or very large-scale fluctuations suggests that waves r_A should decrease with distance approximately as r^{-2} , but this is not observed in the solar wind. The two-dimensional simulations generally find $r_A \approx 1$ where a strong mean magnetic field is present.^{36,58} For small mean magnetic fields, r_A can be less than unity, but no systematic behavior has been described. As yet, no satisfactory explanation has emerged to account for the fact that in the solar wind $r_A < 1$, and is most often ≈ 0.5 .

B. Role of expansion and velocity shear

The simulations discussed heretofore did not include the expansion of the wind into the heliosphere. Depending on the time scale for the generation of a turbulent cascade and the spatial volume of interest, the expansion of the wind might act to suppress the generation of turbulence. In fact, it was proposed that the observed fluctuations are essentially frozen-in because the expansion of the wind is too rapid to allow for development of a cascade.^{77–80} The argument was based on solutions of the MHD equations in a comoving coordinate system that expanded as time progressed. To further examine these and other questions, we developed a three-dimensional MHD code using inflow-outflow boundary conditions in spherical geometry^{81,82} (also see Ref. 83).

Because the flow in the simulations by Goldstein *et al.*^{81,82} was super-Alfvénic and supersonic, the frozen-in-flow approximation⁸⁴ could be used to construct reduced spectra from the time series of the magnetic field collected at specific points in the simulation volume. The simulation domain contained two velocity shear layers in the middle of the volume while slower, denser plasma occupied the middle third of the box to model the slow solar wind. For a detailed description of the three-dimensional spherical simulations discussed here, the reader is referred to Ref. 82.

In Fig. 6 we show that velocity shear in three-dimensional expanding flows affects σ_c as it does in two dimensions. As before, $|\sigma_c|$ is reduced significantly from the initial value of ~ 1 , especially near regions of strong gradients in velocity and in the vicinity of the current sheet. For this particular simulation, the radial expansion was modest.

To address the issues raised by Grappin *et al.*,⁸⁰ a series of numerical simulations was run to study whether or not a true spectral cascade could be generated in a spherically expanding flow. The simulation conditions varied from a rapid expansion to nearly Cartesian situations. Power spectra from two such runs are shown in Fig. 7. In the weakly expanding run, the power has cascaded to frequencies well beyond the highest frequency of the initial Alfvénic wavepacket ($f \approx 18$ in code units). In contrast, in the strong expansion run,

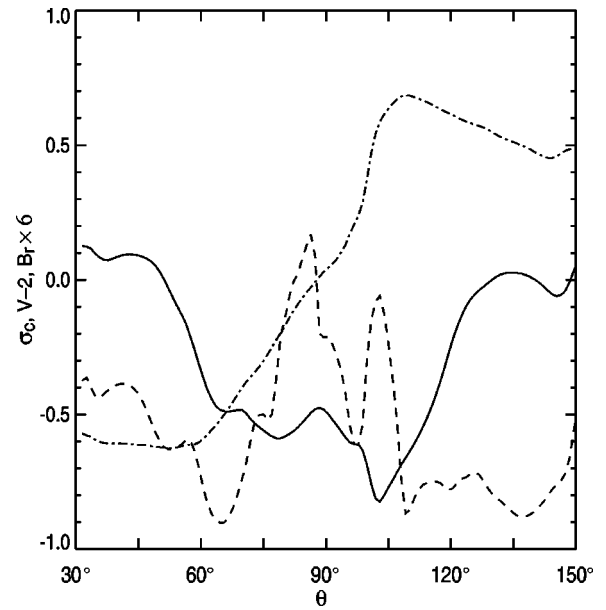


FIG. 6. Velocity shear and σ_c from the three-dimensional spherical expansion code. The current sheet is indicated by the change in sign of B_r (dotted-dashed line); the velocity shear by V_r (solid line), and σ_c (dashed line). The values have been averaged over the interval $r=3-4$ (from Ref. 82).

there is little evidence for a spectral cascade and the spectrum steepens sharply above mode 18. The input wave packet, which was initially propagating outward along the dc magnetic field, had a white noise spectrum and it is curious that even in the strong expansion run the input noise relaxes to an approximately $-\frac{5}{3}$ spectral slope.

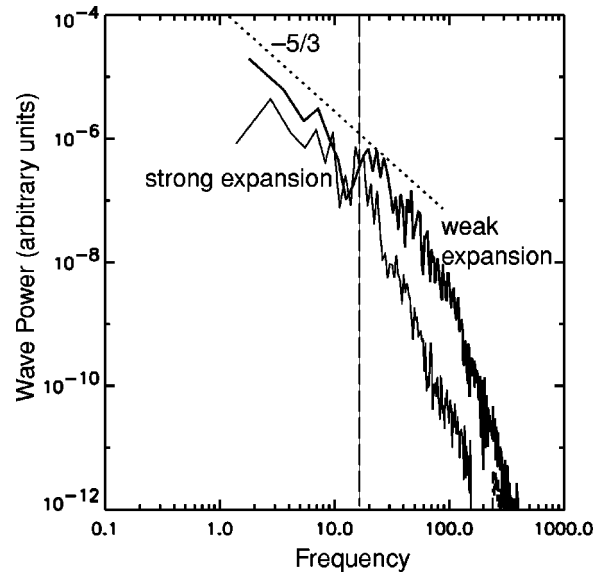


FIG. 7. A comparison of two three-dimensional solutions of the MHD equations in spherical geometry. Reduced one-dimensional power spectrum of the magnetic field for a nearly Cartesian situation ("weak expansion," upper curve) and for strong expansion (lower curve). The short-dashed line has a slope of $-\frac{5}{3}$ and the vertical dashed line indicates the highest frequency of the initial wave packet.

IV. SUMMARY AND CONCLUSIONS

We have reviewed some of the evidence that the solar wind is an actively evolving turbulent magnetofluid, stirred by solar rotation and shears between and within streams. The Alfvénic nature of the solar wind originates in the solar corona, but is modified and reduced in the heliosphere by velocity shears.

Many mysteries remain. We do not understand the symmetry of the fluctuations. How much of the observed population of fluctuations with wave vectors highly oblique to the mean magnetic field originates in the corona and how much is generated *in situ* by velocity shear (“phase mixing”⁸⁵)? This distinction is particularly important because quasi-two-dimensional turbulence originating in the solar corona will not pitch-angle scatter energetic particles, but the effect of radial velocity shear is to leave the power in the parallel fluctuations unchanged so that pitch-angle scattering is unaffected by the phase mixing or refraction of wave numbers to highly oblique directions. When the quasi-two-dimensional component of solar wind fluctuations was first described,²¹ it was assumed to have arisen due the effect of the background magnetic field. Both theoretical⁸⁶ and numerical^{87–89} work has shown that large anisotropies form in MHD fluids in the presence of a strong dc magnetic field. However, there are other mechanisms that can produce a large component of wave vectors peaked orthogonal to the background magnetic field. These include velocity shear,^{33,66} pressure-balanced structures,⁶⁷ and quasi-static conditions in the solar corona.^{65,90} Resolution of the nature and origin of the symmetries of solar wind fluctuations will have to await further observations, analysis, and simulation.

We have shown that spherical expansion influences the development of a turbulent cascade for parallel propagating fluctuations—the effect on nearly-two-dimensional fluctuations is similar.⁹⁰ Although a turbulent cascade can be sustained in a spherically expanding magnetofluid, there is no known theoretical reason for the slope of the resulting power spectra to approach the Kolmogorov value of $-\frac{5}{3}$ since the medium is not isotropic. Nor is it understood why the fluctuating power in the input wave packet should also relax to an approximate $-\frac{5}{3}$ slope.

Work continues on understanding the role of compressibility in determining the detailed properties of the turbulence. With the three-dimensional simulations now available, it will also be possible to elucidate more fully how turbulence is influenced by corotating interaction regions and interplanetary shock waves.

ACKNOWLEDGMENTS

This research was supported, in part, by NASA’s Sun Earth Connections Theory Program in “The Role of Turbulence in Heliospheric Plasmas” at the Goddard Space Flight Center.

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