





## THESIS APPROVAL SHEET

Title of Thesis: ENHANCED LOCAL ADDRESSABILITY OF A SPIN ARRAY WITH  
LOCAL EXCHANGE PULSES AND GLOBAL MICROWAVE  
DRIVING

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Master of Science 2023

Graduate Program: Physics

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NOTE: \*The Approval Sheet with the original signature must accompany the thesis or dissertation. No terminal punctuation is to be used.

# ANOOSHA FAYYAZ

## RESEARCH INTERESTS

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Quantum Information Theory, Quantum Communication, Quantum Algorithms, Quantum Control

## EDUCATION

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**Master of Science in Physics**, University of Maryland, Baltimore County, USA Jan 2021 - July 2023

- Thesis title: “Enhanced local addressability of a linear array of spins with local exchange pulses and global ESR driving”  
Advisor: *Jason Kestner*

**Bachelor of Science in Physics**, Lahore University of Management Sciences, Pakistan Sep 2016 - May 2020

- Final year project: “Dense Coding Capacity of a quantum channel”  
Advisor: *Nigum Arshed*
- Minor in Computer Science
- GRE general: 323 (Quantitative 168, Verbal 155).

## RESEARCH PROJECTS

---

**Enhanced local addressability of a linear array of spins with local exchange pulses and global ESR driving** Jan 2022 - present

- Developed a novel approach for addressing a single qubit. Employed a bin model based on resonant frequencies to organize the qubits into different bins, enabling targeted rotations.
- Utilized techniques to mitigate off-resonant rotation errors, such as crosstalk, through careful qubit tuning and precise control of driving strengths.
- Applied advanced mathematical and physics concepts, such as Lie algebra structure and group representations.
- Achieved high-fidelity operations with over 99% fidelity, demonstrating a deep understanding of quantum control principles and their application in addressing individual qubits.
- The manuscript with results and analysis is being prepared for submission which can be provided upon request.

**Dense Coding Capacity of a Quantum channel** Sep 2019 - May 2020

- Rederived the dense coding capacity for Pauli channels worked out by *R Laurenza*.
- Worked on possible ways to extend this to non-Pauli channels by solving the Lindblad master equation using entanglement-assisted channels in the presence of noise.

**Polymer Quantum Mechanics** Apr 2019 - Aug 2019

- Extensively studied deviations from usual quantization schemes making use of *Spectral Theory of Differential Operators*. Successfully quantized the Harmonic Oscillator using this scheme.
- Produced a report which was submitted to Office of Sponsored Programmes and Research.
- Gave a public seminar open to all students and faculty in the area on my research.
- Project type: Summer Research funded by the **Office of Sponsored Programmes and Research**, under Dr. Moez Hasan, *Lahore University of Management Sciences*.

**Measurement Theory** Jun 2019 - Aug 2019

- Reviewed an article on continuous dynamical coupling of spin chains and worked on possible extensions to use static and oscillating fields to generate entanglement in spin chains by removing spin-chain-environment interaction.
- Project type: Summer research under the supervision of Dr. Adam Zaman, *Lahore University of Management Sciences* where the article being reviewed was one of his published works.

## COURSE PROJECTS

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**Black hole binaries** Fall 2019

Course: Cosmology and Black holes

- Studied the Black hole binaries, analysed data from LIGO-Virgo and post-Newtonian treatment of the binaries under the supervision of Dr. Moez Hasan, *Lahore University of Management Sciences*.
- Produced a MAPLE code to analyse the data.

**NV Centers** Spring 2019

Course: Condensed Matter Physics

- Studied the mechanism of trapping of electrons in the centers of the N-V system

**Thermal Oscillations of a metal: Probing aspects of Fourier Analysis** Fall 2018

Course: Experimental Physics Lab II

- Reproduced the research work based on a published article of an existing faculty member Dr. Sabieh Anwer, *Lahore University of Management Sciences*.
- Produced a report with the results and my own analysis.

## ACADEMIC APPOINTMENTS

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**Research Assistant: *Kestner Group*** Spring 2022

**Teaching Assistant: *Introductory Physics II (UMBC)*** Spr 2021, Fall 2021, Fall 2022,

- Led three discussions every week, held office hours, and graded exams. (20 hours/week)

**Teaching Assistant: *Classical Mechanics (LUMS)*** Fall 2019

- Junior-year level, 10 hours/week.
- Responsible for grading homeworks, weekly office hours and giving tutorials.

**Teaching Assistant: *Mathematical Methods for Physicists I (LUMS)*** Spring 2018

- Sophomore year level, 10 hours/week.
- Responsible for grading quizzes, homeworks and giving tutorials.

## SKILLS

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<b>Programming Languages</b>	Python, Mathematica, Qiskit, C++, Maple, MATLAB
<b>Lab-Based Software</b>	LabView
<b>Technical</b>	Monte Carlo Simulations, Regression analysis, Empirical Orthogonal Function Analysis, HPC experience, Data Analysis

## AWARDS

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UMBC College of Natural and Mathematical Sciences Outstanding Teaching Assistant Award Fall 2021

LUMS Summer Researcher Award Summer 2019

## LEADERSHIP

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**Co-chair: Graduate Assistant Advisory Committee (GAAC)** 2022 - 2023

I spearheaded Meet and Confer meetings with the Graduate School (in the absence of unions) and successfully negotiated equal benefits for all Graduate Assistants. I also led a movement for Unionization rights for graduate students together with the College Park campus where we informed the students of Meet and Confer, Unions, and how to support Collective Bargaining rights bill in the Senate. This was the first time UMBC Graduate Student Association submitted a testimony in favor of the bill.

**Executive Council: Graduate Student Association** 2022 - 2023

**Committee Member: Graduate Assistant Advisory Committee** 2021 - 2022

## **PROFESSIONAL MEMBERSHIPS**

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American Physical Society 2019 - present

Physicists Coalition for Nuclear Threat Reduction Sep 2021 - present

## **EXTRA-CURRICULAR ACTIVITIES**

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Senior Representative Sports at LUMS 2018 - 2019

Player LUMS Basketball team 2016 - 2019

## ABSTRACT

Title of dissertation:      **ENHANCED LOCAL ADDRESSABILITY OF A  
SPIN ARRAY WITH LOCAL EXCHANGE  
PULSES AND GLOBAL MICROWAVE  
DRIVING**

Anoosha Fayyaz, Master of Science, 2023

Dissertation directed by:    **Professor Jason Kestner**  
University of Maryland, Baltimore County

Spin qubits have been an excellent candidate for scaled up quantum processors since a few decades not only due to the vast nanoelectronic device fabrication industry at their disposal but also the ability to address single spins by frequency-selective control using electron spin resonance (ESR). All the qubits having a distinguishable frequency is a prerequisite to this strategy which imposes an upper bound on the number of qubits that can be differentiated without crosstalk coming into play. There have been techniques in the literature to address individual spins for small scale devices. Here we theoretically propose a strategy to address an individual spin in a large array of spin qubits with a random distribution of g-factors by employing a combination of single-qubit and SWAP gates facilitated by a global microwave field and local exchange pulses. Consequently, only the target qubit undergoes the desired operation and all other qubits return to their original states, even qubits that share the same Larmor frequency as the target. Gate fidelities above 99% can thus be maintained for arrays containing tens of qubits.

ENHANCED LOCAL ADDRESSABILITY OF A SPIN ARRAY WITH  
LOCAL EXCHANGE PULSES AND GLOBAL MICROWAVE  
DRIVING

by

Anoosha Fayyaz

Dissertation submitted to the Faculty of the Graduate School of the  
University of Maryland, College Park in partial fulfillment  
of the requirements for the degree of  
Master of Science  
2023

Advisory Committee:  
Professor Jason Kestner, Chair/Advisor  
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## Preface

I would like to note that Chapter 3 of thesis reflects the content of my research paper, which is currently being prepared for submission.

## Dedication

I dedicate this thesis to my incredible parents, whose endless love and sacrifices have made all things possible and to my beloved husband, whose unwavering support, boundless love and willingness to make sacrifices have empowered me to reach new heights. Thank you for being my rock and joining me on this journey across the world.

## Acknowledgments

Firstly, I thank Allah for everything that I am and everything that He has bestowed upon me.

I would like to express my heartfelt gratitude to my advisor, Dr. Jason Kestner, for his support and encouragement throughout my journey. His exceptional patience and support have played a pivotal role in my academic growth. I greatly appreciate how he has mastered the art of striking the perfect balance between understanding his students' needs, motivating them to excel, and holding them accountable without instilling unnecessary fear. I am truly privileged to have learnt from and worked with such an exceptional mentor.

I would also like to thank Dr. Theodosia Gougousi for her invaluable assistance and warm hospitality. She played a crucial role in helping me transition smoothly and made me feel incredibly welcomed.

Finally, I would like to thank my esteemed professors and my fellow cohort mates, whose guidance and collaboration has been instrumental in shaping my academic journey.

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## Chapter 1: Introduction of Quantum Computing and Spin Systems

### 1.1 Organization of the thesis

This thesis is organized as follows. Chapter 1 serves as an introduction to the field of quantum computing. Chapter 2 discusses the key concepts related to silicon quantum dots. Chapter 3 presents the core research of the thesis focusing on how an individual spin qubit can be addressed in a spin array.

### 1.2 Introduction

The development of quantum computing can be traced back to the birth of quantum theory in the early 20th century. In 1936, Alan Turing laid the foundations of theoretical computer science with his revolutionary idea of a universal Turing machine [1]. This concept paved the way for future quests for powerful computational systems. However, the concept of harnessing quantum phenomena for computational purposes gained significant attention in the 1980s after Richard Feynman, in 1982, proposed the concept of using quantum systems to simulate quantum physics, arguing that classical computers struggle to accurately model quantum mechanical systems [2]. Feynman's idea marked a notable turning point in the field of quantum computation.

The pioneering work of Paul Benioff in 1980 introduced the idea of a quantum Turing machine, built upon Turing's original concept by employing quantum states and quan-

tum gates for computation processes [3]. Building on this foundation, in 1985, David Deutsch formulated the theory of quantum computation, highlighting the immense potential of quantum computers to solve problems that are intractable for classical computers [4]. Deutsch's work sparked interest in the computational power unleashed by the principles of quantum mechanics, setting the stage for groundbreaking advancements.

Furthermore, Peter Shor's remarkable discovery in 1994 was a major breakthrough for the field of quantum computation which showcased quantum superiority. Shor presented an efficient algorithm for factoring large numbers using a quantum computer, which posed a significant threat to classical cryptographic systems [5]. This breakthrough showcased that quantum computers have the potential to break widely-used cryptographic protocols, thereby revolutionizing the field of cryptography and necessitating the development of post-quantum cryptographic techniques.

These crucial works and subsequent research has sparked a revolution within the domain of quantum computing. With the ability to harness the unique characteristics of quantum bits (or qubits) such as superposition and entanglement, quantum computing holds the promise of achieving exponentially faster computation. This capability opens doors to solving complex problems that were once considered insurmountable. The quest to fully unlock the potential of quantum computing continues, with ongoing advancements in technologies like semiconductor quantum dots, spin systems, and g-factor tuning in silicon-based qubits. These advancements lay the foundation for a variety of applications across various fields, fueling the journey toward transformative breakthroughs.

### 1.3 Foundations of Quantum Computing

Quantum computing is rooted in the principles of quantum mechanics with concepts like superposition and entanglement at its core. Superposition allows quantum systems,

represented by qubits, to exist in a simultaneous combination of multiple states. This unique characteristic grants quantum computers the potential to explore a large solution space more efficiently than classical computers [6]. Entanglement, another fundamental aspect, establishes non-local correlations among qubits and facilitates the transmission of information and operations across different parts of a quantum system.

A qubit, short for quantum bit, is the most fundamental unit of information in quantum computation. In the most general form, a qubit can be represented as a linear combination of the two basis state, conventionally denoted as  $|0\rangle$  and  $|1\rangle$ . These two basis states serve as quantum counterparts of classical bits 0 and 1, respectively. This inherent superposition property of qubits enables quantum computations to explore multiple possibilities simultaneously. By performing quantum operations on these qubits, we can manipulate their states and perform computational tasks.

Encoding and manipulation of information in quantum computers, commonly known as quantum operations, are carried out using quantum gates and circuits. Quantum gates are analogous to the logic gates in classical computing, but they operate on the superposition of quantum states. These gates facilitate computations by enabling the manipulation of qubits. Quantum circuits are composed of a series of quantum gates, where each quantum gate is applied to specific qubits at specific time steps. Each unique configuration represents the sequence of operations performed on different qubits in the system. These circuits form the basis for designing quantum algorithms and are specifically designed to carry out the desired computational tasks [6].

## 1.4 Quantum Computing Architectures

Quantum computing encompasses a range of physical platforms, each with its own unique characteristics and challenges. Some prominent quantum computing architectures



are superconducting qubits, trapped ions, semiconductor qubits, photonic qubits and topological qubits. Superconducting qubits utilize circuits made of superconducting materials that can exhibit quantum behavior at extremely low temperatures. Trapped ions, on the other hand, involve the precise control of individual ions suspended in electromagnetic traps. Semiconductor qubits, which have emerged as a promising platform for quantum information processing, utilize quantum properties of electrons trapped in semiconductor materials. Similarly, the basis of photonic qubits are photons where quantum information is typically encoded in the quantum properties such as polarization. In contrast, topological qubits rely on the quantum states of topological materials to encode and manipulate quantum information. A detailed comparison of these different platforms can be found in [7].

Semiconductor quantum dots are nanoscale structures that confine electrons in a small region, enabling the precise control and manipulation of individual qubits. These qubits are typically implemented using the spin of an electron or the occupancy of a quantum dot level. The advantages of semiconductor qubits make them an excellent choice for quantum computing. One advantage is scalability, as quantum dots can be fabricated in large arrays with exceptional precision, allowing for the potential integration of a substantial number of qubits on a single chip. This scalability is vital for realizing large-scale quantum processors.

Another advantage of semiconductor qubits is their long coherence times, which refers to the duration that quantum information can be stored and manipulated without decoherence. For example, spin qubits benefit from the ability to isolate qubits from the environment, leading to longer coherence times compared to most other platforms. This extended coherence allows for more robust quantum operations and the implementation of complex quantum algorithms.

Moreover, one notable advantage of semiconductor qubits is their compatibility with well-established semiconductor technology, which has been optimized and perfected over

several decades. This compatibility allows seamless integration with the already developed industry, allowing for the realization of hybrid quantum-classical computing architectures by harnessing the strengths of both classical and quantum computing paradigms.

## 1.5 Spin qubits in semi-conductor dots

Spin qubits utilize the spin of an electron or a hole in a semiconductor quantum dot for information storage and processing. Semiconductor quantum dots are nanoscale pockets in semiconducting materials that are controllable due to their compatibility with the existing semiconductor technology. In this approach, a single or few electrons are confined within the quantum dot to create discrete energy levels that can be manipulated for quantum computation [8, 9].

The implementation of spin qubits in semiconductor quantum dots involves several key steps. First, the quantum dot is formed by confining the electrons or holes within a region of the semiconductor material using electrostatic confinement techniques [10]. The spin states of the confined charges can then be initialized by applying a magnetic field, which splits the energy levels according to their spin orientation. This initialization process prepares the qubit in a well-defined initial state, typically the spin-up or spin-down state.

To manipulate the spin qubits, electrical and magnetic control techniques are employed. Electrical control involves applying voltage pulses to the gates surrounding the quantum dot, which modulate the exchange potential and allow for the coherent manipulation of the electron spin states [11]. This modulation arises from the electrostatic coupling between the voltage on the gate and the charge distribution in the quantum dot. By adjusting the gate voltages, the effective magnetic field experienced by the spin qubit can be modified, leading to changes in its spin state. Magnetic control, on the other hand, relies on the application of external magnetic fields to induce spin rotations or to couple the spin qubits

with nearby nuclear spins [12]. These control techniques facilitate the implementation of various quantum gates, such as single-qubit rotations and two-qubit entangling operations, which are essential for quantum computation.

Readout of the spin qubits is achieved through a combination of electrical and magnetic measurements. Electrical measurements involve monitoring the current flowing through the quantum dot while applying voltage pulses, which allows for the detection of spin-dependent tunneling events [13]. Magnetic measurements, such as electron spin resonance spectroscopy, can also be employed to directly probe the spin states of the qubits [10]. The measured signal provides information about the state of the spin qubit, allowing for the determination of the computational outcome.

One example of a silicon-based spin qubit system is the double quantum dot formed in a silicon nanowire using electrostatic gating [14]. By applying appropriate gate voltages, the energy levels of the quantum dots can be controlled, creating a tunable two-level system based on the spin states of an electron in each dot. The exchange interaction between the spins enables the implementation of two-qubit gates, such as the controlled-not (CNOT) gate, essential for quantum computation [15].

In conclusion, spin qubits in semiconductor quantum dots possess unique properties that make them attractive for quantum information processing. Notably, they offer the potential for long coherence times, which is essential for preserving the fragile quantum states and enabling error correction. Additionally, the scalability of semiconductor technology allows for the integration of large numbers of quantum dots on a chip further increasing the feasibility of realizing multi-qubit systems. The compatibility of semiconductor qubits with existing semiconductor fabrication techniques also enables their incorporation into hybrid devices, such as those combining spin qubits with superconducting circuits.

## 1.6 Summary

In this chapter, we explored the field of quantum computing and discussed its historical background tracing it back to Richard Feynmann and Paul Benioff. The chapter then proceeded to discuss different quantum computing architectures, highlighting their unique characteristics and potential advantages, specifically spin qubits in semiconductor quantum dots.

## Chapter 2: Background on Silicon Quantum Dots

Spin qubits, based on electron or hole spins in quantum dots or impurities, offer promising potential for quantum information processing. Various control methods have been developed to manipulate and address these qubits, including Electron Spin Resonance (ESR) Control, Electric-Dipole Spin Resonance (EDSR), global control techniques such as electric control and strain engineering, and other methods based on spin-orbit interactions and hybrid systems. On the other hand, by manipulating the  $g$ -factor researchers can engineer the spin qubit's properties to optimize its performance and address specific challenges in quantum information processing. Several studies have highlighted the significance of  $g$ -factor tuning in spin qubits [11, 16, 17].

### 2.1 Electron Spin Resonance

Electron Spin Resonance (ESR) control is a powerful technique utilized in spin qubits to manipulate and readout the quantum states of individual electron spins in solid-state systems. ESR relies on the interaction between the spin of an electron and an externally applied magnetic field, leading to energy-level transitions that can be controlled and probed [7, 8, 11]. In this section, we will delve into the details of ESR control, its applications in spin qubits, and the effectiveness demonstrated in the literature.

ESR allows us to investigate the energy levels and dynamics of the electron spins by applying a resonant microwave field and observing the absorption or emission of microwave

radiation. The spin of an electron can be described by its spin angular momentum, given by the operator  $\mathbf{S}$ . In the presence of a magnetic field  $\mathbf{B}$ , the energy of the electron spin is given by the Zeeman interaction term, which can be written as

$$H_{Zeeman} = -g\mu_B\mathbf{B} \cdot \mathbf{S} \quad (2.1)$$

where  $g$  is the g-factor, which represents the gyromagnetic ratio and accounts for the coupling strength between the electron spin and the magnetic field,  $\mu_B$  is the Bohr magneton,  $\mathbf{B}$  is applied magnetic field vector, and  $\mathbf{S}$  is the spin operator.

To understand the effect of the magnetic field on the spin qubit, we consider the energy eigenstates of the system. In the absence of any magnetic field, the spin qubit is typically described by two energy eigenstates: the spin-up state  $|\uparrow\rangle$  and the spin-down state  $|\downarrow\rangle$ . These states have different energies, and the energy difference is proportional to the applied magnetic field. When a resonant microwave field is applied to the system, its frequency matches the energy difference between the spin states. This resonance condition allows for the absorption or emission of energy, leading to transitions between the spin-up and spin-down states. The resonance condition is given by

$$\Delta E = g\mu_B B_0 = \hbar\omega \implies \omega_0 = g\mu_B B_0/\hbar \quad (2.2)$$

where  $B_0$  is the magnitude of the applied magnetic field,  $\hbar$  is the Planck's constant, and  $\omega_0$  is the frequency of the microwave radiation. The resonance condition allows us to manipulate the spin qubit by selectively addressing specific transitions between its spin states. By tuning the frequency of the microwave radiation, we can control the spin dynamics of the qubit, induce rotations, and perform quantum operations [7, 8, 11].

In practical implementations of ESR, the experimental setup typically includes a res-

onant cavity or microwave resonator [18] that generates the microwave radiation. The resonant microwave field induces Rabi oscillations, causing coherent rotations between the spin states. It is important to consider the relaxation times  $T_1$  and  $T_2$  in ESR experiments.  $T_1$  represents the characteristic time scale for the decay of the spin's polarization, while  $T_2$  represents the characteristic time scale for the decay of the spin coherence. These relaxation processes can limit the fidelity and coherence of quantum operations performed using ESR.

The effectiveness of ESR control in spin qubits has been demonstrated through numerous experimental achievements. High-fidelity single-qubit gate operations with fidelities exceeding the fault-tolerance threshold have been reported in various systems [19]. Two-qubit gates have been achieved with high fidelity and minimal crosstalk, paving the way for scalable quantum architectures [15, 20]. Moreover, the long coherence times observed in ESR-controlled spin qubits have been crucial for preserving quantum information and enabling fault-tolerant quantum computation [11].

ESR in an array of spin qubits presents challenges for reliable and scalable operation. Decoherence of individual qubits and coupling between neighboring qubits are significant concerns. Environmental noise and interactions with surrounding nuclear spins can degrade qubit coherence and limit quantum processing capabilities [? ]. Managing and mitigating decoherence effects are crucial for long coherence times and high-fidelity operations. Secondly, selective addressing and manipulation of individual qubits in the array require precise control over local magnetic fields to tune resonance frequencies without affecting neighboring qubits. Achieving high spatial resolution and minimizing crosstalk are essential for large-scale arrays. Lastly, scaling up the array to a large number of qubits and integrating them coherently is a challenge that requires advanced fabrication techniques and device architectures [9, 21]. Scalability is crucial for realizing complex quantum algorithms and fault-tolerant computations [22, 23].

## 2.2 Global control

Global control refers to the technique of manipulating multiple spin qubits simultaneously in a spin qubit array. This approach enables the collective manipulation and coherent control of the entire array of qubits, offering significant advantages for scalable quantum information processing. Global control techniques have been developed and employed in various solid-state platforms, including semiconductor quantum dots, superconducting circuits, and trapped ion systems.

To understand global control, let's consider a quantum system described by a time-dependent Hamiltonian  $H(t)$ . The time evolution of the quantum state is governed by the Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle \quad (2.3)$$

where  $|\psi(t)\rangle$  is the quantum state at time  $t$ . In the context of global control, the goal is to find appropriate control strategies to manipulate the system's Hamiltonian  $H(t)$  in order to steer the quantum state towards desired states or achieve specific quantum operations. This can involve shaping the temporal profiles of control fields or applying feedback control techniques to adjust the system's dynamics.

One common approach in global control is to use external fields, such as electromagnetic fields, to interact with the quantum system. These external fields can be used to induce transitions between energy levels, manipulate quantum coherence, and perform quantum gates. For example, in the case of spin qubits, external magnetic fields can be used to control the spin states and perform operations such as rotations or entanglement.

The process of designing control strategies in global control often revolves around optimizing a control functional. This control functional serves as a measure of the desired performance or the objective of the control strategy and guides the selection of appropri-



ate control techniques. This could include maximizing the fidelity of a desired quantum operation, minimizing decoherence effects, or optimizing the energy efficiency of control fields. Various optimization techniques, such as optimal control theory or gradient-based algorithms, can be employed to find the optimal control fields that achieve the desired objectives.

In semiconductor quantum dot arrays, global control can be achieved by utilizing shared control lines, such as global gate electrodes, that are connected to all qubits in the array. These shared control lines enable the simultaneous application of control pulses, such as radiofrequency (RF) or microwave signals, to all the qubits, allowing for synchronized operations. This approach has been demonstrated in several experimental studies. For example, in Ref. [15], global microwave control is implemented in a two-qubit array of phosphorus donor qubits in silicon, showcasing the ability to perform collective single-qubit rotations and two-qubit entangling gates.

Another approach to global control in spin qubit arrays involves the use of spin resonance techniques, such as electron spin resonance (ESR) or nuclear magnetic resonance (NMR). By applying a global magnetic field gradient, it is possible to selectively address individual qubits within the array based on their resonant frequencies. This technique has been utilized in various systems, including quantum dots and NV centers in diamond. For instance, in Ref. [24], a scalable architecture for a room temperature solid-state quantum information processor based on electron spins in quantum dots is proposed. The paper discusses the utilization of a global magnetic field gradient for individual addressing of qubits [24].

The advantages of global control in spin qubit arrays are manifold. Firstly, it enables efficient and simultaneous manipulation of multiple qubits, reducing the time required for quantum operations and enhancing the overall processing speed. Secondly, global control techniques allow for the implementation of multi-qubit gates and entanglement operations,

facilitating the realization of complex quantum algorithms. Moreover, global control can improve the coherence and stability of qubits by minimizing the effects of local variations and noise sources.

However, one of the main challenges associated with global control is the necessity to address individual qubits for universal quantum computing. While global control allows for collective operations on qubit arrays, it lacks the ability to perform precise individual qubit manipulations, such as single-qubit rotations. This challenge is addressed by incorporating local exchange control, which facilitates the transfer of quantum information and the implementation of two-qubit gates. Achieving the required level of control and synchronization across the array poses additional challenges, including precise calibration of control pulses and the mitigation of crosstalk between qubits and control lines. Efforts to address these challenges involve advanced pulse shaping techniques, improved qubit designs, and the application of error correction strategies.

Experimental advancements in global control have demonstrated promising results. For example, in recent studies, researchers have demonstrated the implementation of two-qubit gates and entanglement operations in spin qubit arrays using global control techniques [17, 19, 25, 26]. These developments highlight the potential of global control for realizing large-scale quantum information processing and building more complex quantum circuits.

In summary, global control in quantum control refers to the application of control techniques that act globally on a quantum system to manipulate its dynamics and achieve desired quantum operations. This involves designing control strategies, optimizing control fields, and employing feedback control techniques. Global control has demonstrated significant success in various experimental platforms and is a key tool in the pursuit of quantum information processing.

## 2.3 g-factor tuning

Spin qubits rely on the manipulation of electron spins to store and process quantum information. The g-factor, also known as the Landé g-factor, plays a crucial role in spin qubits as it determines the coupling strength between the spin and external magnetic fields. It is a dimensionless quantity that quantifies the gyromagnetic ratio of the electron, representing the ratio of the magnetic moment to the angular momentum. The ability to control and tune the g-factor is of paramount importance in spin qubits as it allows for precise control of the spin states and enables various quantum operations.

One prominent approach for g-factor tuning is the utilization of electric fields to modulate the spin-orbit interaction, which influences the g-factor. The charge distribution and the electrical potential in a quantum dot can be controlled by applying gate voltages. Gate voltages, which are applied to electrodes located near the quantum dots, allow manipulation of the electric fields within the system. By adjusting these gate voltages, one can influence the distribution of charges within the quantum dots and modify the electric potential landscape that has a direct impact on the behavior of the qubit system, including the g-factor. The electric field-induced changes in the g-factor have been demonstrated in semiconductor quantum dots [15, 17, 27–29].

In addition to the utilization of gate voltages, g-factor can be tuned by strain engineering. Mechanical strain is applied to the material hosting the qubit, which makes the lattice structure undergo deformations. This changes the electron wavefunction, and consequently the g-factor. Strain can be introduced through different techniques such as piezoelectric actuators, nanomechanical devices, or substrate engineering. Changes in the g-factor due to strain primarily arise from modifications in the strength of spin-orbit coupling or modifications in the material's band structure. The influence of strain on the g-factor has been investigated in various quantum dots hosts such as GaAs/InGaAs [30–32].

## 2.4 Summary

In this chapter, we provided a detailed examination of ESR, global control, and g-factor tuning in spin qubits. We discussed their implementation, advantages, challenges, and experimental evidence of their effectiveness.

## Chapter 3: Enhanced Local Addressability of an electron spin in an array of spins

### 3.1 Theoretical model

We introduce here a method to address a single spin in a linear chain with nearest neighbor exchange coupling. For our model, we consider an arbitrary number of spin qubits, each with a different resonant frequency. The system can be described by the Hamiltonian

$$H = \sum_{i=1}^{N-1} J_i S_i \cdot S_{i+1} + \sum_{i=1}^N g_i \mu_B S_i \cdot B_i, \quad (3.1)$$

where  $J_i$  is the locally tunable exchange interaction between neighboring spins  $S_i$  and  $S_{i+1}$ , and  $B_i = (B_1 \cos \omega t, 0, B_0)$  is the magnetic field at the position of spin  $S_i$  consisting of a homogeneous in-plane dc field  $B_0$  and a perpendicular global microwave field of amplitude  $B_1$ . The resonant frequency of each qubit depends on its g-factor.

The g-factor is weakly dependent on the spin-orbit coupling [33] which in turn is affected by the random surface roughness and can be deliberately tuned by applying electric and/or magnetic fields [27]. The Larmor frequency of a given qubit can thus be shifted by tuning one of the top gates that define that dot such that its g-factor is Stark shifted [27, 34], which allows for a variety of straightforward methods to address a single qubit with global control by shifting it in and out of resonance [17, 21, 35].

Electrically controlled shifts on the order of  $\pm 5\text{MHz}$  at  $B = 1.4\text{T}$  have been demon-

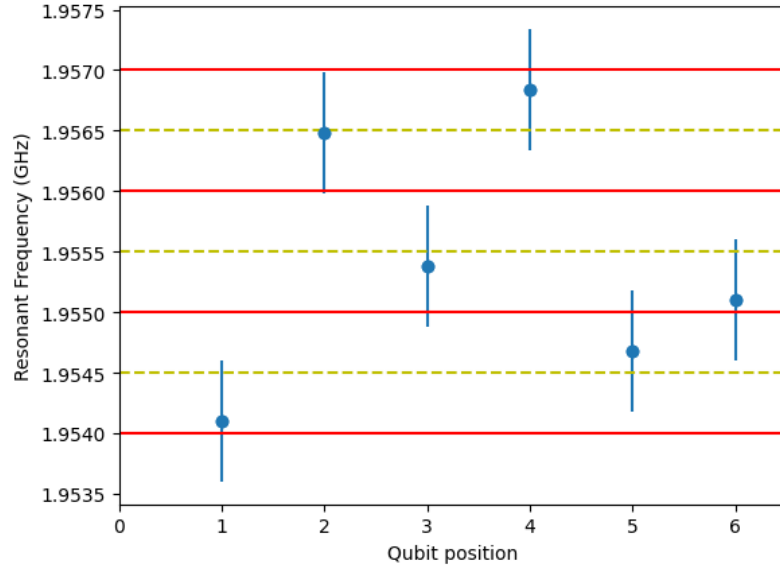


Figure 3.1: Example of resonant frequencies of spin qubits in an array. The red lines demarcate different frequency bins and the dashed lines indicate the central frequency of that particular frequency bin. Each qubit's frequency can be tuned up to 5 MHz in either direction (as shown by the vertical bars), so all qubits can be tuned to a central frequency.

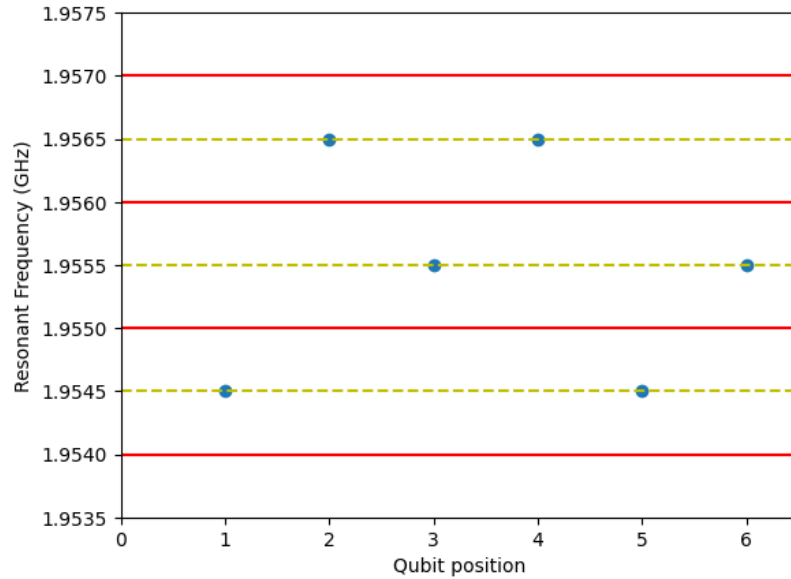


Figure 3.2: Example of resonant frequencies of spin qubits in an array after tuning to the central frequencies of the bins. Uniquely addressing a given target qubit now involves distinguishing it from the other qubits in the same bin using SWAPs as well as managing a discrete set of off-resonance rotations.

strated [15, 17, 20, 36], but this is not enough to neglect off-resonant effects when driving with Rabi frequencies  $\sim 1\text{MHz}$ . Additionally, schemes that park all idle qubits at resonance to take advantage of the enhanced robustness of the resulting dressed qubits [35] require enough tunability to encompass the entire spread of the qubits' frequency distribution, and the standard deviation of this distribution can be on the order of 60 MHz [29, 37].

Fig. 3.1 depicts an example of the resonant frequencies of spins in an array of six qubits. We group spins according to their resonant frequencies in frequency bins. The bin size is chosen to match the range of electrical tunability of the Larmor frequencies so that all qubits can be tuned to be at the center of a frequency bin. In this way the problem of addressability breaks into two separate parts: i) distinguishing the target qubit from other qubits in the same bin, which now all have identical frequencies, and ii) suppressing crosstalk between qubits in different bins, which now have a discrete set of frequency differences. We will consider these two tasks in the following subsections.

### 3.1.1 Pulse sequence to address one spin in a bin

Here, we propose a series of steps involving single qubit rotations and SWAP operations to address a single target spin. The steps are enumerated below, starting with all exchange couplings turned off and no microwave field.

1. Pulse the microwave to resonantly rotate the target qubit spin state around the  $x$ -axis. All the qubits in the same frequency bin as the target qubit also undergo a resonant rotation.
2. SWAP the target qubit with a qubit in a different bin which experiences negligible crosstalk with the target bin, by pulsing the exchange coupling links (usually it will suffice to pulse a single exchange link to SWAP with a nearest neighbor, but if the nearest neighbors are all in the same bin as the target, one must sequentially SWAP

the state further along the array). The spin state of the target qubit is thus transferred to a separately addressable bin.

3. Pulse the microwave at the new resonant frequency of the target state so as to rotate it about the y-axis. All the qubits in this frequency bin also undergo a resonant rotation.
4. SWAP the qubits again to return the spin states to their original locations.
5. Pulse the microwave at resonance with the target qubit as in step 1 but so as to perform the inverse rotation.
6. SWAP the target qubit again with the same neighbor in step 2.
7. Pulse as in step 3 but so as to perform the inverse rotation.
8. SWAP the qubits back again as in step 4.

At the end of this sequence only the target qubit acquires a net rotation, provided off-resonant rotations of qubits in one bin due to driving at the central frequency of another bin are negligible. In other words, by using the local exchange control we can effectively dynamically toggle the resonant frequency of the target qubit between values far beyond the range of the available in-situ tunability.

To make the logic more clear, the eight steps and the corresponding unitary operations are tabulated in Table 3.1 for the specific example of the g-factors taken in Figs. 3.1-3.2 and the target qubit being at position 1. The subscripts indicate the *initial location* of each spin state for this example. Note that we have swapped the target qubit with a qubit residing in bin 4 as an illustrative example, but the target qubit could be swapped with any qubit which does not share a frequency bin with the target qubit.

The ultimate rotation of the target qubit is

$$U = Y_{-\phi} X_{-\theta} Y_{\phi} X_{\theta} \quad (3.2)$$



Table 3.1: Evolution of the spins of Figs. 3.1-3.2 under the eight-step sequence.

1	Bin 1 rotation ( $X_\theta$ )	$X_{\theta,1}I_2I_3I_4X_{\theta,5}I_6$
2	SWAP qubits 1 and 4	$I_4I_2I_3X_{\theta,1}X_{\theta,5}I_6$
3	Bin 3 rotation ( $Y_\phi$ )	$I_4Y_{\phi,2}I_3(Y_\phi X_\theta)_1X_{\theta,5}I_6$
4	SWAP qubits 1 and 4	$(Y_\phi X_\theta)_1Y_{\phi,2}I_3I_4X_{\theta,5}I_6$
5	Bin 1 rotation ( $X_{-\theta}$ )	$(X_{-\theta}Y_\phi X_\theta)_1Y_{\phi,2}I_3I_4I_5I_6$
6	SWAP qubits 1 and 4	$I_4Y_{\phi,2}I_3(X_{-\theta}Y_\phi X_\theta)_1I_5I_6$
7	Bin 3 rotation ( $Y_{-\phi}$ )	$I_4I_2I_3(Y_{-\phi}X_{-\theta}Y_\phi X_\theta)_1I_5I_6$
8	SWAP qubits 1 and 4	$(Y_{-\phi}X_{-\theta}Y_\phi X_\theta)_1I_2I_3I_4I_5I_6$

where  $X_\theta \equiv \exp(-i\theta X/2)$  and  $X$  is a Pauli operator. One can verify that setting  $\phi = \theta$  produces the rotation

$$U = Z_\lambda X_\beta Z_\nu, \quad (3.3)$$

$$\lambda = -\frac{\pi}{4} - \arctan\left(\frac{\sin^2 \theta}{\sin^2 \theta + 2 \cos \theta}\right), \quad (3.4)$$

$$\beta = 2 \arcsin\left(\sqrt{2} \sin^2 \frac{\theta}{2} \sin \theta\right), \quad (3.5)$$

$$\nu = \frac{\pi}{4} - \arctan\left(\frac{\sin^2 \theta}{\sin^2 \theta + 2 \cos \theta}\right). \quad (3.6)$$

On the other hand, an arbitrary rotation can be formed by the Euler angle decomposition,

$$U(\alpha, \beta, \gamma) = Z_\alpha X_\beta Z_\gamma, \quad (3.7)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are the Euler angles. Thus, our sequence, supplemented by additional  $Z$  rotations which can be executed virtually (i.e., by instantaneous changes in the phase of the rotating reference frame) [38], is capable of producing any rotation such that  $|\beta| < 2 \arcsin\left(\frac{3\sqrt{3}}{4\sqrt{2}}\right) \approx 0.74\pi$ . Unitaries requiring larger  $\beta$  can be formed from two  $X_{\pi/2}$  rotations interleaved with virtual  $Z$  rotations. Thus this sequence is universal for single-qubit gates.

We now briefly note how the SWAP operations are performed. In the absence of driving,

the Hamiltonian in Eq. (3.1) for a given pair of neighboring qubits can be written as

$$H = \frac{J}{4}(XX + YY + ZZ) + \frac{\Delta E_z}{2}(IZ - ZI) + \overline{E_z}(IZ + ZI), \quad (3.8)$$

where  $J = J_i$ ,  $\Delta E_z = \mu_B B_0(g_{i+1} - g_i)$ , and  $\overline{E_z} = \mu_B B_0(g_{i+1} + g_i)/2$ . Note that the Hamiltonian can be split into two mutually commuting terms and belongs to an embedding  $\mathfrak{su}(2) \oplus \mathfrak{u}(1) \subset \mathfrak{su}(4)$ . The  $\mathfrak{u}(1)$  part of the Hamiltonian is

$$H_{\mathfrak{u}(1)} = \frac{J}{4}ZZ + \overline{E_z}(IZ + ZI) \quad (3.9)$$

and the  $\mathfrak{su}(2)$  part of the Hamiltonian can be written as

$$H_{\mathfrak{su}(2)} = \frac{J}{2}\tilde{Z} + \Delta E_z\tilde{X}, \quad (3.10)$$

where  $\tilde{Z} = (XX + YY)/2$  and  $\tilde{X} = (IZ - ZI)/2$ .

A gate equivalent to SWAP (up to local  $z$  rotations) is obtained by performing a  $\pm\pi/2$  rotation in the effective  $SU(2)$  about  $\tilde{Z}$ . If one can access high enough exchange coupling that the effect of the nonzero  $\Delta E_z$  is negligible, then the SWAP can be done in time  $T_{\text{SWAP}} = \pi/2J$ . Generally, though, one may not be able to access high enough exchange for this direct implementation, but one can compose the rotation with an exchange pulse sequence [39]

$$R(\hat{z}, \alpha) = R(\sin \gamma \hat{x} + \cos \gamma \hat{z}, \chi) R(\hat{x}, \varphi) R(\sin \gamma \hat{x} + \cos \gamma \hat{z}, \chi) \quad (3.11)$$

where  $R(\hat{n}, \chi) = \exp\{(-i\frac{\chi}{2}\hat{n} \cdot \vec{\sigma})\}$  is a rotation about the axis  $\hat{n}$  by an angle  $\chi$ . The solutions for  $\chi$  and  $\varphi$  in terms of  $\gamma$  and  $\alpha$  are (rearranging expressions from Ref. [40] and noting that

$J \geq 0$  implies  $-\pi/2 \leq \gamma \leq \pi/2$ )

$$\varphi = -2 \arcsin \left[ \tan \gamma \sin \left( \frac{\alpha}{2} \right) \right], \quad (3.12)$$

$$\chi = \operatorname{sgn} \alpha \arccos \left[ 1 - \frac{1 - \sqrt{\cos^2 \left( \frac{\alpha}{2} \right) - \frac{1}{4} \sin^2 \alpha \tan^2 \gamma}}{\cos^2 \left( \frac{\alpha}{2} \right) + \sin^2 \left( \frac{\alpha}{2} \right) \cos^2 \gamma} \right]. \quad (3.13)$$

In our application, the largest accessible value of exchange determines the available  $\gamma = \arctan(2\Delta E_z/J)$  and the middle segment is performed by turning the exchange off for a time. Note that Eqs. (3.12)-(3.13) are real only if  $|2\Delta E_z/J| \leq |\csc(\alpha/2)|$ , so for any neighboring pair of qubits where that is not the case for  $\alpha = \pm\pi/2$ , one can instead use  $\alpha = \pi/2n$  with the integer  $n$  chosen to ensure Eq. (3.12) remains real, and the composite rotation of Eq. (3.11) can be repeated  $n$  times to produce the SWAP. The total duration of the SWAP gate is thus

$$T_{\text{SWAP}} = n \frac{2\chi \cos \gamma + \varphi \cot \gamma}{J}, \quad n = \left\lceil \frac{\pi}{4|\arcsin(J/2\Delta E_z)|} \right\rceil \quad (3.14)$$

which ranges from  $(3.5\pi + \sqrt{2})/J$  when  $\Delta E_z \ll J$  down to  $\pi^2/2J$  when  $\Delta E_z \gg J$ .

### 3.1.2 Suppressing interbin crosstalk

Consider a constant amplitude global pulse intended to perform a resonant  $X_\theta$  rotation with exchange coupling turned off, as in step 1 of the sequence above. “Idle,” off-resonant qubits have a Hamiltonian of the form

$$H = g\mu_B S \cdot B = \hbar \begin{pmatrix} \omega_0/2 & \Omega \cos \omega t \\ \Omega \cos \omega t & -\omega_0/2 \end{pmatrix}, \quad (3.15)$$

where  $\omega_0$  is the resonant frequency,  $\Omega$  is the driving strength, and  $\omega$  is the driving frequency. In the rotating frame  $H_r = R^\dagger H R - i R^\dagger \dot{R}$ , with  $R = \exp(-i\omega t/2Z)$ , and rotating wave approximation (RWA) gives

$$H_r = \frac{\hbar\Omega}{2}X + \frac{\hbar m\delta}{2}Z \quad (3.16)$$

and an evolution operator in the rotating frame after time  $T$

$$U = e^{-iT(\Omega X + m\delta Z)/2}, \quad (3.17)$$

where  $m\delta = \omega_0 - \omega$  is the detuning from resonance with  $m$  as the integer difference between the index of the resonant bin and the off-resonant bin and  $\delta$  is the bin width.

While  $z$ -rotations are not worrisome since they can be absorbed into the local rotating frame (i.e., compensated by virtual  $z$ -rotations [38]), we wish to avoid rotation along any other axis resulting from the off-resonant driving. There is clearly no  $y$ -component to the off-resonant rotation of Eq. (3.17), but there is also no  $x$ -component if one chooses

$$\sqrt{\Omega^2 + (m\delta)^2}T = 2n\pi, \quad n \in \mathcal{Z}. \quad (3.18)$$

Substituting in that one must also have

$$\Omega T = \theta \quad (3.19)$$

for the driving to produce the intended resonant rotation, one obtains a condition similar to the synchronization condition found in [41],

$$\Omega = \pm \frac{m\delta\theta}{\sqrt{(2n\pi)^2 - \theta^2}}. \quad (3.20)$$

The same condition also holds for resonant y-rotations to have negligible off-resonant effects. Note that to ensure real values of  $\Omega$  the choice of integer  $n$  must satisfy

$$n > \frac{\theta}{2\pi}. \quad (3.21)$$

Thus, by careful choice of the driving amplitude (and time), one can completely suppress off-resonant errors at a given detuning. However, instead of aiming for complete error suppression in a given bin, we would like to minimize the overall infidelity arising from a large set of bins, most of which will be many bins away from the target bin. Thus we consider the limit of large  $m$ . Note that by choosing the free integer to be  $n = \ell m$ , the optimal driving strength of Eq. (3.20) becomes independent of bin index for large  $m$ ,

$$\Omega = \frac{\delta\theta}{2\ell\pi}, \quad \ell \in \mathcal{Z}. \quad (3.22)$$

In fact, although this is obtained for the large  $m$  limit, it is nearly optimal even for the nearest bin,  $m = 1$ , since the denominator of Eq. (3.20),  $\sqrt{(2\ell\pi)^2 - \theta^2} \approx 2\ell\pi$  for  $\theta \ll 2\ell\pi$ . So, by driving with one of these particular strengths we suppress off-resonant effects across all bins. Larger values of  $\ell$  produce a better approximation at the cost of longer pulse times.

### 3.2 Sequence performance

Now that we have established the pulse sequences to address the target qubit and the optimal parameters for suppressing off-resonant rotations, we analyze the effects of residual off-resonant rotations on the entire sequence. We will not include the effect of local pulse miscalibration errors in our analysis because their effect on the multi-qubit fidelity is negligible compared to the addressability errors that we are focusing on here. Furthermore, those errors can in principle be eliminated beforehand by local tuning procedures. Neither

do we consider the effect of charge noise, since again this is a separate issue whose effect on the multi-qubit fidelity is small in comparison when the total sequence time is much less than the dephasing time  $T_2^*$ . In principle, even this restriction could be relaxed by using dynamically correction via broadband pulses that can simultaneously invert all the spins or by more sophisticated shaped pulses. For the present though, since a typical dephasing time is  $T_2^* \sim 100\mu\text{s}$ , it is sufficient to restrict ourselves to sequences of length  $\sim 10\mu\text{s}$ .

The total time for the sequence is

$$T_{total} = 4 \times T + 4 \times T_{\text{SWAP}}, \quad (3.23)$$

where the local rotation step time is given by Eqs. (3.19) and (3.22) as

$$T = 2\ell\pi/\delta. \quad (3.24)$$

The multi-qubit fidelity for one local rotation in the sequence (steps 1, 3, 5, or 7) at the frequency of target bin  $t$  is given by

$$\mathcal{F}_{loc}(t) = \prod_{j \neq 0} \left| \frac{1}{2} \text{Tr}(U_j)^{N_{t+j}} \right|^2, \quad (3.25)$$

where  $U_j = e^{-iT(\Omega X + j\delta Z)/2}$ , and  $N_{t+j}$  is the number of qubits in the  $(t+j)$ -th bin.

The entire sequence for a particular configuration  $\vec{N} = (N_1, N_2, \dots)$  has a fidelity whose lower bound is the product of the fidelities of each step of the sequence. Assuming the fidelity of the nearest neighbor SWAP gates,  $\mathcal{F}_{\text{SWAP}}$ , are independent of the qubit frequencies, averaging over all possible target qubits in the configuration gives a sequence fidelity of

$$\mathcal{F}_{seq}(\vec{N}) \geq \mathcal{F}_{\text{SWAP}}^4 \sum_t \sum_{k \neq t} \frac{N_t}{N} \frac{N_k}{N - N_t} \mathcal{F}_{loc}(t)^2 \mathcal{F}_{loc}(k)^2, \quad (3.26)$$

since when randomly selecting a target qubit, the probability of it being in bin  $t$  is  $N_t/N$  and the probability that the nearest neighbor qubit (excluding qubits in the target bin) is in bin  $k$  is  $N_k/(N - N_t)$ . We set  $\mathcal{F}_{\text{SWAP}} = 1$  in the numerics below in order to focus on addressability errors.

To calculate the average total fidelity of the  $N$ -qubit system we randomly sample a large number qubit configurations. Each configuration is generated by randomly drawing  $N$  times from a normal distribution of mean  $\omega_0$  and standard deviation  $\sigma$ . We calculate the weighted average of Eq. (3.26) over these configurations,

$$\mathcal{F}_{\text{avg}} = \left( \sum_{\vec{N}} p(\vec{N}) \right)^{-1} \sum_{\vec{N}} p(\vec{N}) \mathcal{F}_{\text{seq}}(\vec{N}). \quad (3.27)$$

Here the probability of a given configuration is

$$p(\vec{N}) = \prod_{j=-\infty}^{\infty} p_j^{N_j} \binom{N - \sum_{k=-\infty}^{j-1} N_k}{N_j} \quad (3.28)$$

where

$$p_j = \int_{\omega_0 + (j-1/2)\delta}^{\omega_0 + (j+1/2)\delta} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\omega - \omega_0)^2 / 2\sigma^2} d\omega, \quad (3.29)$$

is the probability of a given qubit being in bin  $j$ .

For specificity, we numerically consider the case of where one wishes to apply a local  $X_{\pi/2}$  rotation, although it is easy to generalize to an arbitrary rotation. For this case,  $\phi = \theta = \pi/2$  in Eq. (3.2). We take an experimentally plausible bin width  $\delta = 10\text{MHz}$  and a distribution of width  $\sigma = 60\text{ MHz}$  for a magnetic field oriented along the  $[110]$  lattice direction [37]. The rms average of  $\Delta E_z$  is  $\sqrt{2}\sigma \approx 85\text{MHz}$  and taking a maximum exchange coupling of  $J \sim 50\text{MHz}$ , the average  $T_{\text{SWAP}} \sim 0.1\mu\text{s}$  from Eq. (3.14). Then the optimal driving strength for all local rotation steps is given by Eq. (3.22), choosing  $\ell = 4$  to keep the total time to  $\sim 10\mu\text{s}$ , as  $\Omega = 0.625\text{ MHz}$ . In that case there is suppressed off-resonant

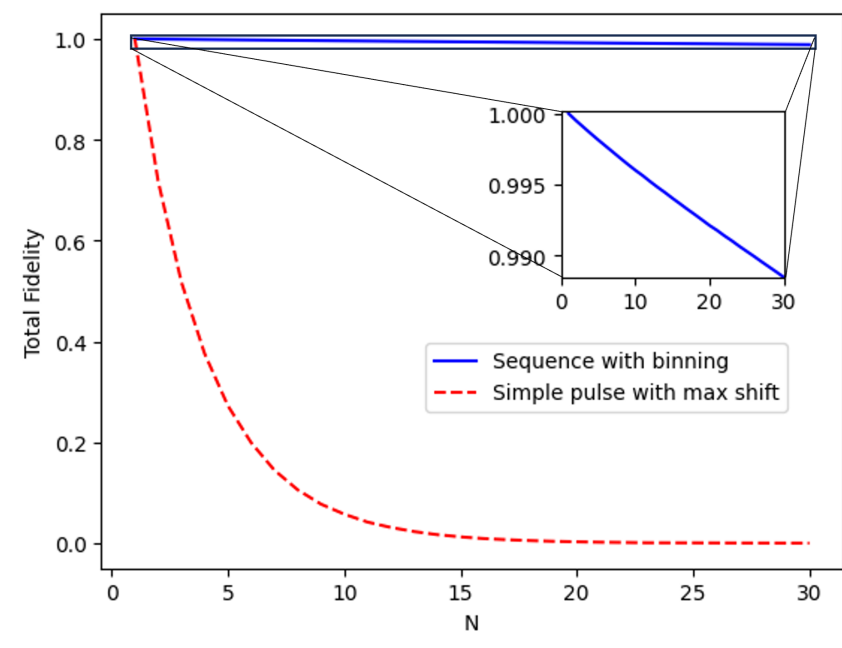


Figure 3.3: Average N-qubit fidelity vs the number of qubits for a single-qubit  $X_{\pi/2}$  rotation in a qubit array whose Larmor frequencies are normally distributed with standard deviation  $\sigma = 60$  MHz and individually tunable by  $\pm 5$  MHz. Both gates are of duration  $\sim 10\mu\text{s}$ .

rotation of qubits in neighboring bins such that they each undergo an identity operation with fidelities of

$$\left|\frac{1}{2}\text{Tr}(U_{\pm 1})\right|^2 = 0.9994, \quad \left|\frac{1}{2}\text{Tr}(U_{\pm 2})\right|^2 = 0.99985, \quad (3.30)$$

and so on, with higher fidelity in all further bins. As we increased the number of configurations included in our numerical computation of Eq. (3.27) from  $10^3$  to  $10^5$ , the resulting values of  $\mathcal{F}_{avg}$  only changed by  $\sim 10^{-5}$ , suggesting our numerics are well converged.

The results are plotted in Fig. 3.3, comparing the total fidelity of the proposed pulse sequence on the qubits after they are tuned to the central frequency of each frequency bin to the total fidelity in the naive case of a single, simple pulse at the resonant frequency of the target qubit after each idle qubit frequency is shifted as far away as possible from the target given the limited tunability of  $\pm\delta/2$ . Since slower pulses are more frequency



selective, we allowed the simple pulse to use the same time as the full sequence to make a fair comparison, so the driving strength of the simple pulse was reduced to  $\Omega_{simple} = \pi/2T_{total} \approx 0.157$  MHz. The significant advantage of our method is evident from Fig. 3.3 in that the decay of multi-qubit fidelity with the number of qubits is exponential for the simple pulse and only linear for the sequence.

## Chapter 4: Conclusion

We have demonstrated an effective method for addressing a specific qubit within a large array of qubits using a combination of pulse sequences, single qubit gates, and SWAP gates. We have found that the fidelity of operations performed on qubits located outside the bin the target qubit is in is limited by the ratio of the Rabi frequency to the detuning. By carefully adjusting the driving strength, we have consistently achieved a fidelity of over 99% for the whole pulse sequence for all other qubits in a system of as large as 28 qubits. In our analysis, we have considered a substantially large number of randomly generated configurations of qubits. However, to further enhance the relevance and applicability of our findings, it is crucial to delve into the specific arrangements employed in currently utilized quantum devices. By focusing on these popular quantum devices, we can gain valuable insights into their unique characteristics, constraints, and performance metrics. This targeted analysis will enable us to refine our understanding of the intricacies involved in manipulating and controlling qubits in real-world scenarios.

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