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THE STRUCTURE OF HELICAL INTERPLANETARY MAGNETIC FIELDS

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Abstract. The interplanetary magnetic field is known to be highly helical. Although the detailed spatial structure of the fields has yet to be elucidated, the helicity spectrum has been conjectured to result from a random walk in direction of a constant magnitude magnetic field vector. We demonstrate that a model using three-dimensional fluctuations with variations in $|B|$ gives a good fit to the helicity spectrum as well as to other properties of the interplanetary magnetic field.

1. INTRODUCTION

In this letter we present results of an investigation into the nature of the spatial structure of interplanetary magnetic fluctuations. We offer a model that accounts for the observed helical structure of the fields as measured by the magnetic helicity. (Magnetic helicity represents the degree of twist in the field lines; for example, circularly polarized waves are maximally helical with a sign indicating the handedness of the twist [Moffatt, 1978].) Understanding the helical structure is important for studies of energetic charged particle propagation in the heliosphere, for although theoretical treatments of pitch-angle scattering have included the effects of helicity [see, for example, Hasselmann and Wibberenz, 1968, 1970; Goldstein *et al.*, 1981], it is difficult to ascertain the quantitative effects of helicity in the absence of an understanding of the actual spatial structure of the interplanetary fields. Knowledge of the helical properties of the solar wind fields at high time resolution, especially in the inner heliosphere, might also provide information about coronal and/or photospheric fields. In this Letter we develop a model that is consistent with the observed statistical properties of the magnetic helicity spectrum.

An objective of this investigation has been to examine the paradigm that the helicity spectrum arises from a random walk in angle of a constant magnitude magnetic fluctuation vector. After briefly describing the relevant characteristics of the interplanetary fluctuations in §2, we show in §3 that the constant magnitude paradigm yields incorrect helicity spectra. We argue that the difficulty with the model is the constraint of constant field magnitude and show that relaxing that constraint leads to model fields that have statistical properties virtually indistinguishable from the observed fields. We use two properties of the magnetic helicity spectrum in developing our model: the behavior of the normalized spectrum as one increases the number of degrees of freedom used in the spectral analysis; and the occurrence probability distribution of values of the normalized spectrum.

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Rugged invariants

The fluctuations in the interplanetary magnetic field and plasma with wavelengths between approximately 10^2 – 10^6 km are known to represent a dynamically evolving nonlinear system which exhibits many properties of a turbulent magnetofluid. Matthaeus *et al.* [1982] and Matthaeus and Goldstein [1982] showed that in a super-Alfvénic incompressible magnetofluid in which the “frozen-in-flux” approximation [Taylor, 1983] holds, single point measurements of the magnetic field are sufficient for determining the magnetic helicity and its reduced spectrum in the direction of the fluid flow. The magnetic helicity H_m is one of three quadratic, “rugged” invariants of incompressible, ideal magnetohydrodynamics (MHD) [Frisch *et al.*, 1975; Matthaeus and Goldstein, 1982; Woltjer, 1958a,b; Kraichnan and Montgomery, 1980]. H_m is defined by

$$H_m = \int d^3x \mathbf{A} \cdot \mathbf{B} \quad (1)$$

where \mathbf{A} is the vector potential and \mathbf{B} is the magnetic field. The second invariant is the cross helicity defined by

$$H_c = \frac{1}{2} \int d^3x \mathbf{v} \cdot \mathbf{b} \quad (2)$$

In eq. (2), \mathbf{v} and \mathbf{b} are the fluctuating velocity and magnetic fields, respectively, and \mathbf{b} is normalized so that $\mathbf{b} = \delta\mathbf{B}/\sqrt{4\pi\rho}$. Belcher and Davis [1971] provided extensive measurements of the cross helicity in the solar wind and showed that at certain times the cross helicity was large and indicated propagation of highly Alfvénic fluctuations away from the sun. The third invariant, the total energy per unit mass is

$$E = \frac{1}{2} \int d^3x (\mathbf{v}^2 + \mathbf{b}^2) \quad (3)$$

The properties and evolution of the cross helicity and total energy in the interplanetary medium have been studied extensively during the past decade (see the recent review by Roberts and Goldstein [1991]). In contrast, the magnetic helicity spectrum in the inertial range of the fluctuation spectrum has not received much attention since the initial work of Matthaeus and Goldstein [1982] and Matthaeus *et al.* [1982] and their subsequent investigation of the effects of the magnetic helicity on the scattering of charged particles [Goldstein and Matthaeus, 1981; Matthaeus and Goldstein, 1981]. At larger scales, Bieber *et al.* [1987a,b] have studied the helical properties of the global Parker spiral field of the solar wind.

2. CHARACTERISTICS OF THE INTERPLANETARY FIELDS

The general characteristics of the magnetic helicity spectrum were described by Matthaeus and Goldstein [1982]. In the wave number range 10^{-6} – 10^{-2} km $^{-1}$, the

reduced spectrum, $H_m(k)$, oscillates rapidly in sign such that the magnitudes of the positive and negative values follow a power law with slope $-8/3$. A convenient dimensionless representation of $H_m(k)$ which illustrates both the degree of twist in the field and its handedness is the normalized quantity $\sigma_m(k)$ defined by

$$\sigma_m(k) \equiv kH_m(k) / E_B(k) \quad (4)$$

where $E_B(k)$ is the reduced magnetic power spectrum. An intuitive view of $\sigma_m(k)$ is that it represents the correlation coefficient between the two transverse components of \mathbf{B} with one of the components shifted $\pi/2$ in phase.

An example of $\sigma_m(k)$, plotted against frequency is shown in Figure 1 (top panels). During this period, the speed of the solar wind was approximately 350 km/s so that the wave numbers are given by $k = 2\pi f/V_{SW}$. This example was constructed from magnetometer data acquired near 1 AU by Voyager 1 on day 251 of 1977. In this analysis we used 4096 magnetic field vectors averaged to a time spacing of 9.6 sec. The magnetic helicity spectra were constructed using the fast Fourier transform technique with $n = 2, 6$, and 26 degrees of freedom. When $n = 2$, $\sigma_m(k)$ oscillates fairly randomly between ± 1 . As more spectral smoothing is employed, the positive and negative excursions of $\sigma_m(k)$ decrease, but for $n > 6$ the bounds within which $\sigma_m(k)$ oscillates decrease only slowly with further increases in n . The values of $\sigma_m(k)$ are nearly equivalent to white noise in that the spectrum of $\sigma_m(k)$ is nearly flat and the standard deviation of the σ_m values scales as $1/\sqrt{n}$. Another important property of the interplanetary magnetic field is that, in the absence of transients such as shock waves or stream interaction regions, the magnitude

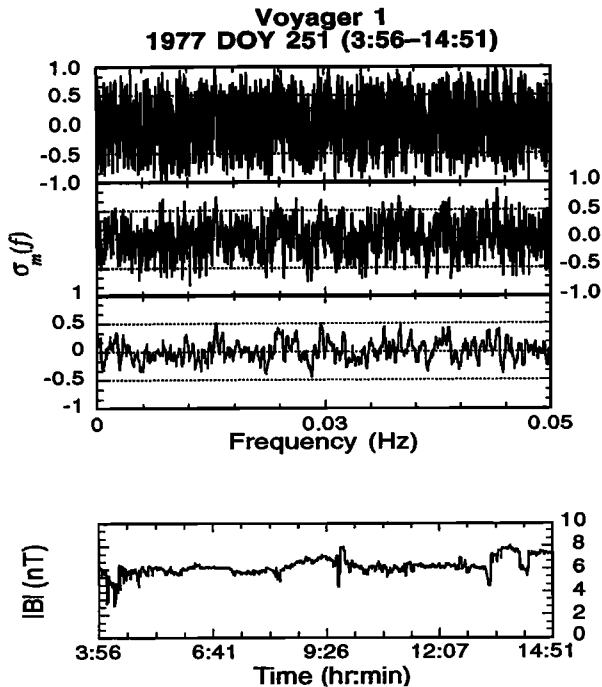


Fig. 1. (Top panels) The spectrum of $\sigma_m(k)$ determined from Voyager 1 9.6 sec magnetometer data near 1 AU with $n = 2, 6$, and 26 in the top three panels, respectively. (Bottom panel) The $|B|$ time series of the Voyager interval.

of the interplanetary magnetic field is approximately constant as can be seen from both the bottom panel of Figure 1, which shows $|B|$, and Figure 2, which shows the trace of the power spectral matrix and the power in the magnitude of the magnetic field for $n = 26$. Figure 2 illustrates the typical situation of ten times more power in the component fluctuations than in the magnitude fluctuations (solid and dashed lines, respectively). The additional fact that the plane of minimum variance of the fluctuations tends to align parallel to the mean magnetic field [e.g. Belcher and Davis, 1971] suggests that the fluctuations can be described as plane waves, although Burlaga [1979] and Denskat and Burlaga [1977] reported evidence for nonplanarity.

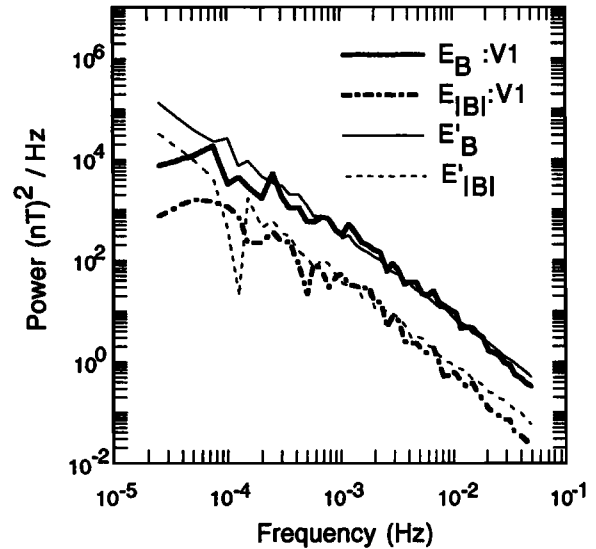


Fig. 2 The power spectral density of the Voyager 1 data shown in Figure 1 (thick lines) and the artificial three-dimensional data with varying field magnitude (thin lines). Solid and dashed lines are the trace of the power spectral matrix and the power in the field magnitude, respectively.

3. MODEL FIELDS

The behavior of the magnetic helicity spectrum implies that the magnetic fluctuations are significantly polarized, albeit in a random way. One model of the spatial structure of these helical magnetic fields is motivated by measurements of the cross helicity which show that in the solar wind \mathbf{b} and \mathbf{v} are highly correlated and have approximately equal magnitudes [Belcher and Davis, 1971; Matthaeus and Goldstein, 1982], characteristic of Alfvén waves. These observations suggest a simple model of plane-polarized Alfvén waves whose sense of polarization varies rapidly and randomly with frequency (or wave number). Barnes [1981] developed a model of the field in which a constant magnitude fluctuating magnetic field vector executes a random motion on a sphere. Noting that constant magnetic field strength still allows for Alfvén wave solutions of the MHD equations [Barnes, 1976; Goldstein et al., 1974], Barnes [1981] incorporated nonplanarity in his model, but retained the constraint of constant field magnitude. He showed that this construction results in a minimum variance direction aligned along $\langle \mathbf{B} \rangle$.

Construction of Artificial Data — Constant $|B|$

Using an algorithm first developed by J. R. Jokipii (private communication), we have constructed helical data sets with specific spectral slopes for comparison with Voyager observations. The model fields are constructed by defining a complex function of frequency with an amplitude that is a power law in frequency with slope $\alpha/2$ and a random phase. The desired power spectrum then has a spectral index α , and the inverse Fourier transform of the appropriately symmetrized complex amplitudes is the function ϕ used to construct the desired time series. Here we choose $\alpha = -5/3$ for a Kolmogoroff spectrum. We investigate two generic field models: one in which the fluctuation vector is confined to a circle and a more general model which allows the field vector to move on a sphere. To ensure that the steps of the “colored noise” walk are small, we require that $\phi(t) < 1$ for all t which constrains the walk to a relatively small segment of a circle. For the motion on a circle, the fluctuating components of the model field are defined by $\delta B(t) \equiv (B_x, B_y, B_z)$ with $B_x = 0$ and

$$\begin{aligned} B_y(t) &= a[\sin(\phi(t) + \pi/4)] \\ B_z(t) &= a[\cos(\phi(t) + \pi/4)] \end{aligned} \quad (5)$$

The phase shift $\pi/4$ is introduced to keep the field values away from their extrema of ± 1 and the value of a controls the overall power level of the series. The resulting values of $\sigma_m(f)$ are systematically too small and average to zero very rapidly as n increases (see Figure 3).

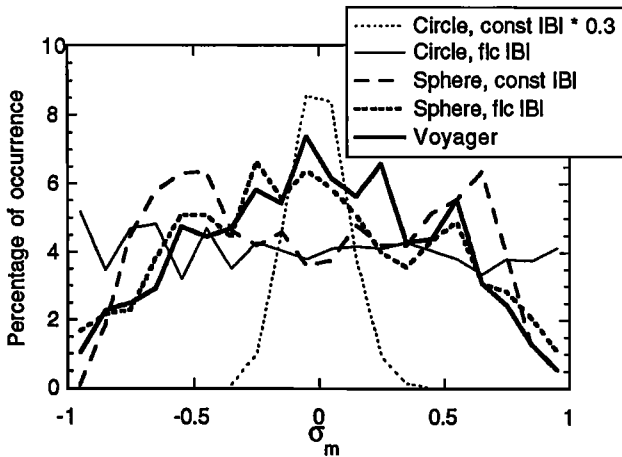


Fig. 3 The percentage of occurrence of values of $\sigma_m(f)$ from the data (Fig. 1) and several models.

New model — Variations in $|B|$

Although one can affect the magnitude of the magnetic helicity by varying the properties of the function ϕ , it does not appear possible to both increase the helicity and preserve the $-5/3$ power spectral slope. We conclude, therefore, that the limitations of the model lie with either the constraint that $|B|$ is constant, or the restriction to two-dimensionality. As we now demonstrate, it seems necessary to relax both constraints. Let us consider a three-dimensional fluctuating field of the form

$$\begin{aligned} B_y(t) &= a[\sin(\phi_1(t) + \pi/4)\cos(\phi_2) + \Phi_y(t)] \\ B_z(t) &= a[\cos(\phi_1(t) + \pi/4)\cos(\phi_2) + \Phi_z(t)] \\ B_x(t) &= a[\sin(\phi_2) + \Phi_x(t)] \end{aligned} \quad (6)$$

where $\phi_{1,2}$ are independent realizations of the “colored noise” function as are $\Phi_{x,y,z}$. The $\Phi_{x,y,z}$ are normalized to control the power in the magnitude fluctuations. Again, $\phi_{1,2} < 1$ and the walk is on a (small) part of a sphere.

With $\phi_2 = 0$, eq. (6) does, in fact, produce values of $\sigma_m(f)$ which approximate some aspects of the Voyager data, but further investigation indicates that one must also relax the two dimensionality of these fields. The problem is best illustrated by constructing the occurrence probability of values of $\sigma_m(f)$ as shown in Figure 3 where the thick solid line is the Voyager data, the thin and dotted lines are the two-dimensional results from eqs. (5) and (6). The constant field on a circle produces too few large values of $\sigma_m(f)$, while the varying field on a circle produces too many large values; this is unphysical because the definition of $\sigma_m(f)$ includes a denominator that is the trace of the power spectral matrix, and thus in three dimensions values of $\sigma_m(f)$ close to ± 1 are improbable.

The results using three dimensional fields generated from eq. 6 are shown in Figures 3 and 4. Using $|B| = \text{constant}$ as in Barnes [1981] model, yields occurrence probabilities with too many large values (long-dashed line). The best approximation to the Voyager data is found using a three-dimensional field with varying magnitude (short dashed line, Fig. 3). Figure 2 shows the power spectrum resulting from the three-dimensional field (thin lines). Note that for the parameters chosen, the power level for the fluctuations in $|B|$ is comparable to that in the Voyager data. The spectrum of $\sigma_m(f)$ for this field, shown in Figure 4, is statistically virtually indistinguishable from the Voyager 1 spectrum. The direction of minimum variance of this artificial field is aligned with $\langle B \rangle$, as expected from Barnes [1981].

The fields obtained from eq. (6) (with $\phi_2 \neq 0$) are similar to the picture developed by Burlaga and Turner [1976] (also see Hollweg [1975]). Basically, the small values of $\sigma_m(f)$ in the $|B| = \text{constant}$ model arise from the near linear

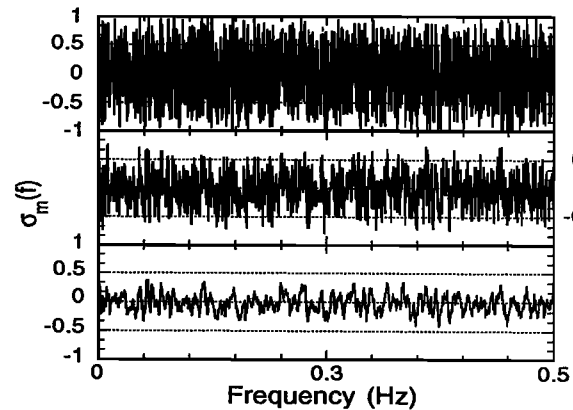


Fig. 4 The normalized magnetic helicity spectrum determined from a three-dimensional field of varying magnitude with $n = 2, 6$, and 26 in the top three panels, respectively.

polarization of small-scale fluctuations along a short arc of a circle, while the introduction of \mathbf{B} fluctuations allows the small scales to execute more circular motions, thus increasing $\sigma_m(f)$.

4. SUMMARY AND DISCUSSION

We have shown that the magnetic helicity spectrum of interplanetary magnetic fluctuations appears inconsistent with the paradigm that the fields are randomly circularly polarized at all scales and are magnitude preserving. When the magnitude is allowed to vary in a random way and the field vector moves near a sphere the resulting spectrum of magnetic helicity very closely resembles interplanetary spectra. These artificial data sets replicate many properties of the interplanetary field, particularly the statistical properties of $\sigma_m(f)$ and are consistent with the description of Alfvénic fluctuations developed by Turner and Burlaga [1976]. In a more detailed paper we will show that the interplanetary magnetic helicity spectrum is scale-invariant from approximately 10^{-5} km^{-1} to 0.1 km^{-1} and has virtually unchanging statistical properties from 0.5 AU–5 AU. We defer discussion of the implications for these results for the origin of interplanetary magnetic fields.

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