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# NIMROD resistive magnetohydrodynamic simulations of spheromak physics

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The physics of spheromak plasmas is addressed by time-dependent, three-dimensional, resistive magnetohydrodynamic simulations with the NIMROD code [C. R. Sovinec *et al.*, J. Comput. Phys. **195**, 355 (2004)]. Included in some detail are the formation of a spheromak driven electrostatically by a coaxial plasma gun with a flux-conserving geometry and power systems that accurately model the sustained spheromak physics experiment [R. D. Wood *et al.*, Nucl. Fusion **45**, 1582 (2005)]. The controlled decay of the spheromak plasma over several milliseconds is also modeled as the programmable current and voltage relax, resulting in simulations of entire experimental pulses. Reconnection phenomena and the effects of current profile evolution on the growth of symmetry-breaking toroidal modes are diagnosed; these in turn affect the quality of magnetic surfaces and the energy confinement. The sensitivity of the simulation results addresses variations in both physical and numerical parameters, including spatial resolution. There are significant points of agreement between the simulations and the observed experimental behavior, e.g., in the evolution of the magnetics and the sensitivity of the energy confinement to the presence of symmetry-breaking magnetic fluctuations. © 2008 American Institute of Physics. [DOI: 10.1063/1.2890772]

## I. INTRODUCTION

Spheromak plasmas are an interesting alternative confinement concept as compared to tokamaks. Spheromaks are typically more compact, have no center conductor, and operate at higher ratios of plasma pressure to magnetic pressure; and the investment in the confining magnetic structure and coils is more modest. These features in aggregate potentially make a spheromak power reactor more attractive than a tokamak reactor if the spheromak plasma can exhibit as much stability and favorable energy confinement. Electron temperatures  $T_e$  near 400 eV were observed transiently in the Los Alamos compact torus experiment (CTX) spheromak experiment,<sup>1</sup> and higher temperatures have been observed to persist in the SSPX spheromak<sup>2</sup> as long as proper discharge conditions are maintained.<sup>3,4</sup> Understanding the formation and decay of spheromaks and the energy confinement therein is a challenging problem. Results from numerical simulations with the NIMROD nonlinear resistive magnetohydrodynamics (MHD) code (at zero or finite plasma pressure) have shown that closed flux surfaces with net current can arise only after electrostatic drive is reduced.<sup>5,6</sup> Other calculations with NIMROD have directly investigated the importance of inductive effects on energy confinement including the evolution of the temperature and number density using thermal transport coefficients, electrical resistivity, and Ohmic heating. These simulations have elucidated the role of the current-profile evolution in influencing the growth of

symmetry-breaking modes, which in turn affect the quality of the magnetic surfaces and the energy confinement.<sup>6–8</sup>

NIMROD simulations have also demonstrated the intimate relationship between inductive effects, magnetic reconnection, and buildup of the magnetic field.<sup>9</sup> The MHD model combined with collisional transport is a reasonable approximation because the collisional mean free path is less than the axisymmetric flux-conserving radius  $R=0.5$  m for nominal SSPX plasma parameters on open magnetic field lines with  $n \sim 5 \times 10^{19} \text{ m}^{-3}$ ,  $T \leq 35$  eV, and singly charged ions.

After applying the integrated MHD/collisional transport simulations to address general questions of spheromak physics, development of a new 32-module capacitor bank for SSPX presented an opportunity to apply the model predictively. Modifications were made to include coupling between the simulated plasma load and each module of the new bank. A simulation was completed months before the new bank was tested on SSPX, using a reasonable guess for the programming parameters of each circuit. The simulation results predicted that the new bank would increase magnetic field strength and the plasma temperature, as intended. However, the improved performance also extended the range of temporal and spatial scales relative to earlier simulation. Detailed comparison between the new simulation and laboratory results finds substantial quantitative agreement during the formation phase of the experiment and in both global magnetic evolution and magnetic fluctuation levels during the controlled decay. However, the simulations produce temperatures that consistently fall below experimental results in well-conditioned discharges. This motivated a validation effort for the MHD/collisional transport model, as applied to

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high-performance spheromaks, and our findings are presented in this paper.

The NIMROD code<sup>10</sup> solves nonlinear time-dependent equations for particle number density ( $n_i=n_e=n$  with quasineutrality), plasma flow velocity ( $\mathbf{V}$ ), temperature (assuming  $T_i=T_e=T$ ), and magnetic field ( $\mathbf{B}$ ). In this, as in our earlier studies, it is used to provide whole-device, integrated simulation of SSPX including dynamo, transport, and inductive effects without resorting to empirically tuned models. In MKS units, the evolution equations considered here are

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = \nabla \cdot D \nabla n, \quad (1)$$

$$\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho \nu \nabla \mathbf{V}, \quad (2)$$

$$\begin{aligned} \frac{nk_B}{\gamma-1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = & -\frac{p}{2} \nabla \cdot \mathbf{V} + \nabla \\ & \cdot nk_B [\chi_{\parallel} \hat{\mathbf{b}}\hat{\mathbf{b}} + \chi_{\perp} (\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})] \\ & \cdot \nabla T + \frac{\eta \mathbf{J}^2}{2}, \end{aligned} \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \mathbf{J}), \quad (4)$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad (5)$$

where  $p \equiv 2nk_B T$  is the sum of electron and ion pressures, and  $\hat{\mathbf{b}} \equiv \mathbf{B}/|\mathbf{B}|$  is the evolving magnetic direction vector field. The simulations consider  $n$ ,  $T$ ,  $\mathbf{V}$ ,  $\mathbf{B}$ , and  $\mathbf{J}$  to be functions of all three spatial dimensions and time. Thus, induction of the spheromak equilibrium from magnetic fluctuations and energy transport from heat-flow fluctuations are modeled explicitly. The boundary conditions are  $\mathbf{E} \times \hat{\mathbf{n}} = \delta \mathbf{B} \cdot \hat{\mathbf{n}} = \mathbf{v} = \nabla n \cdot \hat{\mathbf{n}} = T = 0$  on the conducting surfaces. The vacuum magnetic fields from coils are assumed to have soaked through the bounding surfaces.

The computational grid has been constructed to conform to the conducting surfaces bounding the domain of the plasma gun and flux conserver. The simulations presented use bicubic and biquartic finite elements to represent the poloidal variations (here we typically use 24 elements in the direction normal to the electrodes and 32–48 elements parallel to the electrodes)<sup>11</sup> and various choices for the number of toroidal Fourier modes. Perturbations with smaller length scales in the azimuthal direction are encountered as the toroidal resolution is increased; however, this alone does not necessitate finer poloidal resolution when the elements already reproduce the smaller radial length scales. The time step in the NIMROD simulations is adaptive and governed by constraint conditions that we set conservatively. For a given flow velocity, a smaller time step automatically results when using a larger range of Fourier components and when the order of the finite-element basis functions is increased.

The parallel thermal diffusivity is  $\chi_{\parallel} = 387T^{5/2} \text{ m}^2/\text{s}$ .<sup>12</sup> We usually set  $\chi_{\perp}$  to a constant value motivated by experi-

mental measurements and calculations with the CORSICA transport and equilibrium code.<sup>13</sup> We use the core thermal conductivity found by fitting the data in this axisymmetric approximation; in our simulations, a large effective  $\chi_{\perp}$  arises near the edge from the large parallel thermal conductivity in the presence of stochastic field lines, behaving qualitatively like the effective perpendicular value in the axisymmetric fits. The magnetic diffusivity is computed as  $\eta/\mu_0 = 411(1 \text{ eV}/T)^{3/2} \text{ m}^2/\text{s}$ , using the toroidally averaged temperature. (Numerical tests show no significant deviation from results with a 3D computation of resistivity in the conditions of interest, as one would expect from the high parallel thermal conductivity in the presence of a toroidal magnetic field.)

We have typically employed an isotropic viscosity ( $\nu$ ) of 100–1000  $\text{m}^2/\text{s}$ , which is used to provide nonlinear numerical stability during the full-power stage of the current drive. However, in this study we will also examine the sensitivities to the value of the scalar viscosity and to using a simple, anisotropic model for the viscosity. With temperatures of approximately 30 eV in the edge plasma or during the spheromak formation stage, and a peak of  $\sim 200$ –350 eV in simulations of decay, the Lundquist number (computed as  $S = \mu_0 R v_A / \eta$ , where  $R$  is the radius of SSPX and  $v_A$  is the Alfvén speed), is of order  $10^5$ – $5 \times 10^6$  at the peak  $T_e$ . This is much larger than values considered in our earlier spheromak simulations, but the magnetic Prandtl number ( $\text{Pm} \equiv \nu \mu_0 / \eta$ ) is also larger for nonlinear numerical stability purposes;<sup>5,6</sup> thus,  $\text{Pm} \sim 10^2$ – $10^3$  at peak  $T_e$  and  $\sim 10$ – $10^2$  on and near the open-field line edge. The diffusion term in Eq. (1) keeps the density relatively smooth in the absence of particle transport and atomic fueling effects that are poorly understood and not present in the MHD model. The value of the artificial diffusivity ( $D$ ) is selected to help keep the computed minimum of the number density field above zero during strong drive when the MHD activity is violent. For the same reason, the diffusivity is increased locally in computational cells where  $n$  falls to 3% of its volume-averaged value. Use of artificial density diffusivity alters the physics of the simulation and affects energy conservation.<sup>14</sup> We have varied the value of  $D$  to determine its influence on the simulation results, however, and find its effects to be small. Because the radiation power is small compared to Ohmic power in most SSPX shots after proper wall conditioning, radiation is not modeled in the NIMROD simulations.

In conditions with sustained coaxial electrostatic drive, the cold edge plasma impedes parallel thermal conduction to the wall, despite the chaotic magnetic topology, allowing the plasma core temperature to reach tens of eV. Magnetic reconnection occurs rapidly in the cold outer plasma. When the drive is temporarily removed, relatively symmetric closed flux surfaces form following the resistive decay of symmetry-breaking modes, and core temperatures increase toward 100 eV or more. Applying a second, long current pulse (sustainment pulse) at currents below a formation threshold,<sup>15</sup> as in many SSPX discharges,<sup>3,16,17</sup> improves performance by delaying the onset of MHD modes that are resonant in the closed-flux region,<sup>7</sup> and larger volumes of closed flux can be achieved [observed in our simulations and sug-

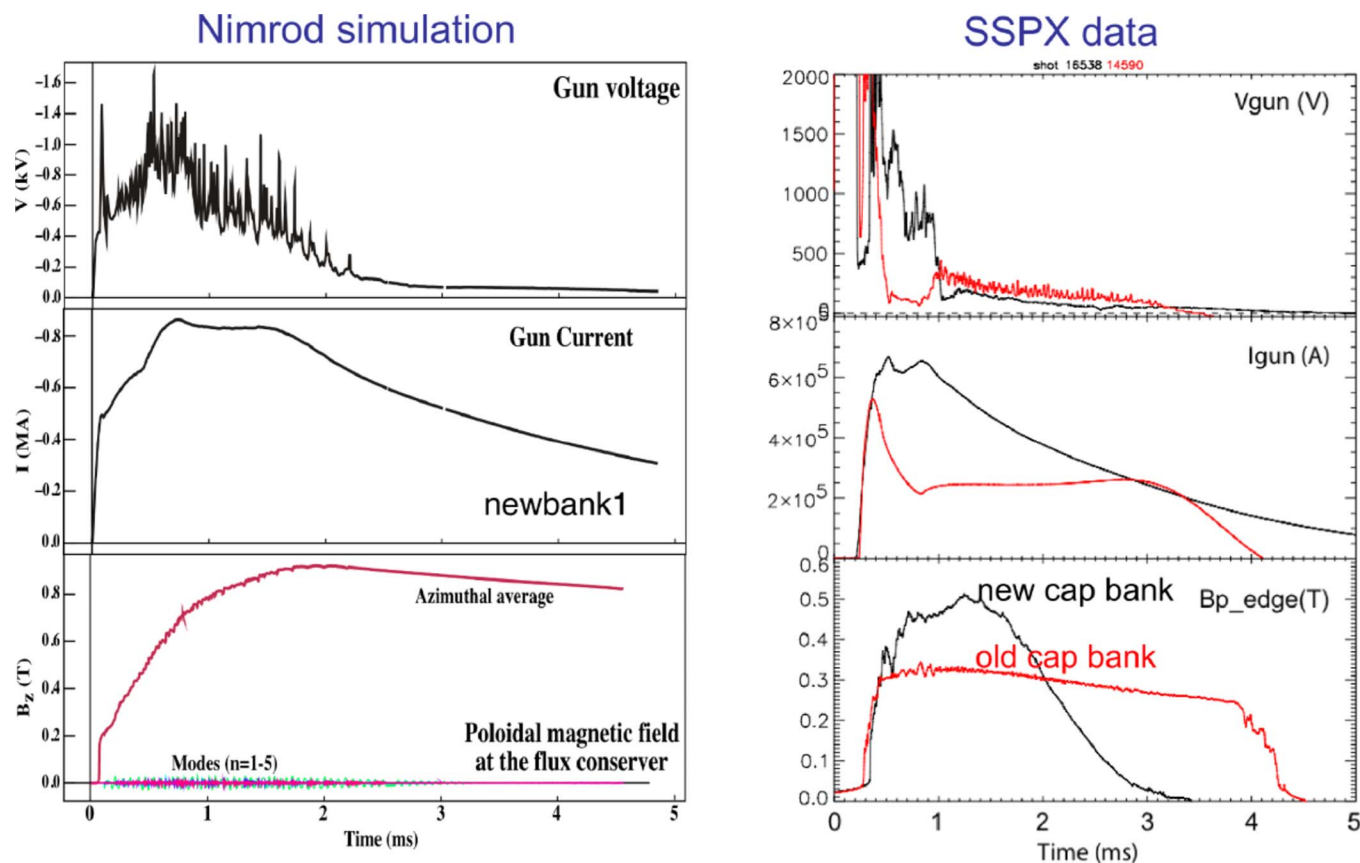


FIG. 1. (Color online) NIMROD simulation of the modular capacitor bank and representative SSPX data for discharges using the original and new capacitor banks: gun voltage, gun current, and poloidal magnetic field at the edge of the midplane in the flux conserver vs time.

gested by the spatial profiles of high- $T_e$  measurements in SSPX (Ref. 13)]. The simulations reveal the sensitivity of the magnetic surfaces and the energy confinement with respect to symmetry-breaking magnetic fluctuations and the close coupling of the magnetics and energy transport. We have presented detailed comparisons of nonlinear simulations with laboratory measurements from SSPX<sup>2,3</sup> and assessed confinement properties of the magnetic configuration in Refs. 7–9. In this paper, we present additional comparisons motivated by our validation efforts, and we assess sensitivities through simulations in which physical and numerical parameters have been varied. The simulation results here and in Refs. 7–9 show that magnetic fields and fluctuation amplitudes agree relatively well with SSPX observations, and temperature evolution data agree at least qualitatively with experimental behavior.

Figure 1 shows results from a NIMROD simulation that are typical of its use in modeling entire discharges in the SSPX experiment. Here we see a simulation in which the recently installed, 32-module capacitor bank has been used to drive higher currents in the spheromak resulting in higher magnetic fields. NIMROD was modified to incorporate the external LRC circuit equations for the modular capacitor bank. In complete analogy with the SSPX experiment, the inductance, resistance, capacitance, and time delay parameters for each module in the capacitor bank are selected as input so that various input current waveforms can be pro-

grammed. This bank is connected across the spheromak in parallel with the original “sustainment bank.” The latter is a pulse-forming network that is modeled as a known, time-dependent current pulse. The NIMROD simulation can thus legislate the same power-supply impedance as a given experimental discharge. The current from the power supply and the gun current through the plasma are equated by setting the instantaneous value of  $RB_\phi$  across the insulator gap between the cathode and the flux conserver. The plasma coupling to the power supply thus models that in the experiment and produces the time-varying gun and plasma currents, gun voltage, and spheromak parameters such as the magnetic field. The particular NIMROD simulation displayed was undertaken several months before the first exercise of the new capacitor bank. We note that the SSPX data show that with higher injected gun currents from the new bank, higher edge magnetic fields are achieved. The SSPX programmed current in discharge no. 16538, one of the first discharges following the upgrade, had a shorter pulse duration and was approximately 70% of the peak current programmed in the NIMROD simulation, and the magnetic field was correspondingly lower. The voltage spikes during formation are similar, however. Later experimental discharges, produced with conditions closer to those assumed in this simulation, had better temporal agreement.

The paper is organized as follows. Section II contains a description of simulations directly modeling spheromak for-



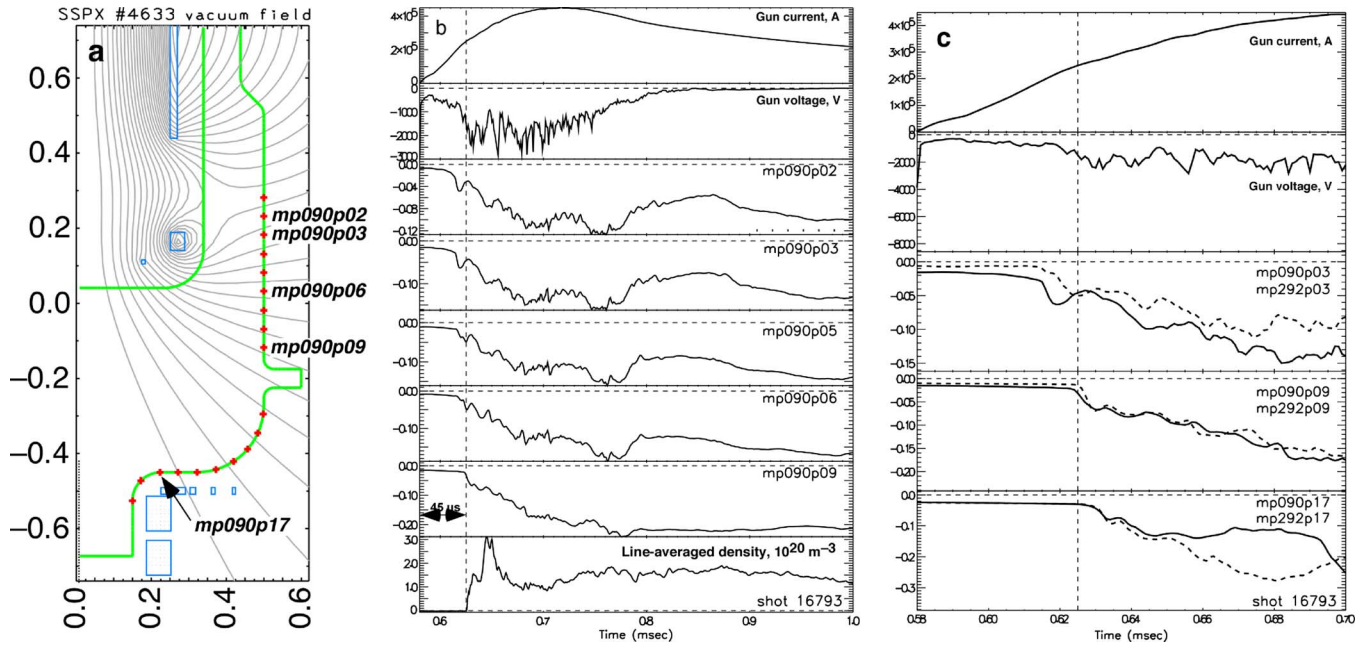


FIG. 2. (Color online) (a) SSPX geometry. The vacuum (bias) poloidal magnetic field is shown. Magnetic probes (measuring the field in T) installed in the flux-conserver wall are indicated by plus signs; the probes used in Figs. 2 and 3 are labeled. (b) Discharge in SSPX showing the movement of plasma down the gun (note probes 2–6). (c) Azimuthally located probes showing asymmetry during plasma ejection from the gun. Probe *mp292p03* (dashed line) is  $202^\circ$  azimuthally from *mp090p03* (solid line), etc.

mation. Section III describes simulations of controlled spheromak decay and how physical and numerical parameters affect the simulation results. Comparisons are made to experimental observations in the SSPX spheromak: the simulation results agree well with many features in the SSPX data and qualitatively or semiquantitatively with others and elucidate features that are highly sensitive to the simulation parameters. We conclude with a brief summary in Sec. IV that includes a table that summarizes the broad range of parameters explored in these simulations.

## II. SPHEROMAK FORMATION

The formation of a spheromak in SSPX is initiated by the injection of gas into the coaxial gun in the presence of a bias (poloidal) magnetic field, with an electrical breakdown following the application of high voltage across the coaxial gap. The increasing discharge current generates a toroidal magnetic field with a magnetic pressure somewhat in excess of the tension of the poloidal field, resulting in an ejection of the field into the flux-conserver volume. The resulting plasma pinches about the geometric axis, resulting in an unstable  $n=1$  (toroidal) mode that grows until the broken azimuthal symmetry results in magnetic reconnection which converts some of the injected toroidal magnetic flux into poloidal flux. The result in the presence of flux-conserving walls yields the spheromak magnetic configuration.

Resistive MHD simulation of formation in a pillbox geometry yielded a clear demonstration of the pinching and subsequent spheromak formation,<sup>5</sup> and a simulation of SSPX<sup>9</sup> yielded good agreement with the formation phase of the experiment including cathode voltage spikes associated with the reconnection events and the generation of poloidal

magnetic field. However, these simulations did not examine in depth the sensitivity of the results to the viscosity, the maximum toroidal mode number,  $n_{\max}$ , the plasma density, and other parameters. Several results are presented here with these quantities varied, with the goal of determining the corresponding sensitivities in the spheromak formation. The results also help to clarify the physics of the formation phase.

The electrical breakdown process cannot be handled in the model; instead, an initial background plasma is assumed. As will be seen below, the comparison of simulation and experiment is good despite this assumption and the large density diffusion coefficient ( $10^4 \text{ m}^2/\text{s}$  in these formation calculations).

### A. Experimental observations

Before examining formation simulations, we present results from two discharges in SSPX. Figure 2 shows the time history of a formation pulse, with magnetic probes on the gun and flux-conserver walls showing the propagation of the magnetic field from the coaxial region into the main flux conserver. The perturbed field appears at a probe near the midplane  $45 \mu\text{s}$  after breakdown. The formation in this shot is not fully symmetric, as seen in Fig. 2(c); two magnetic probes separated by  $202^\circ$  on the outer wall in the coaxial gun show a clear difference in their magnetic signals before plasma ejection. In contrast, the shot shown in Fig. 3 shows almost identical fields at the two probes prior to ejection. The difference is reflected in the cathode voltage pulse immediately following ejection.

As will be shown in the simulations discussed later, the rapid change in geometry as the plasma is ejected from the gun is accompanied by a corresponding change in gun induc-

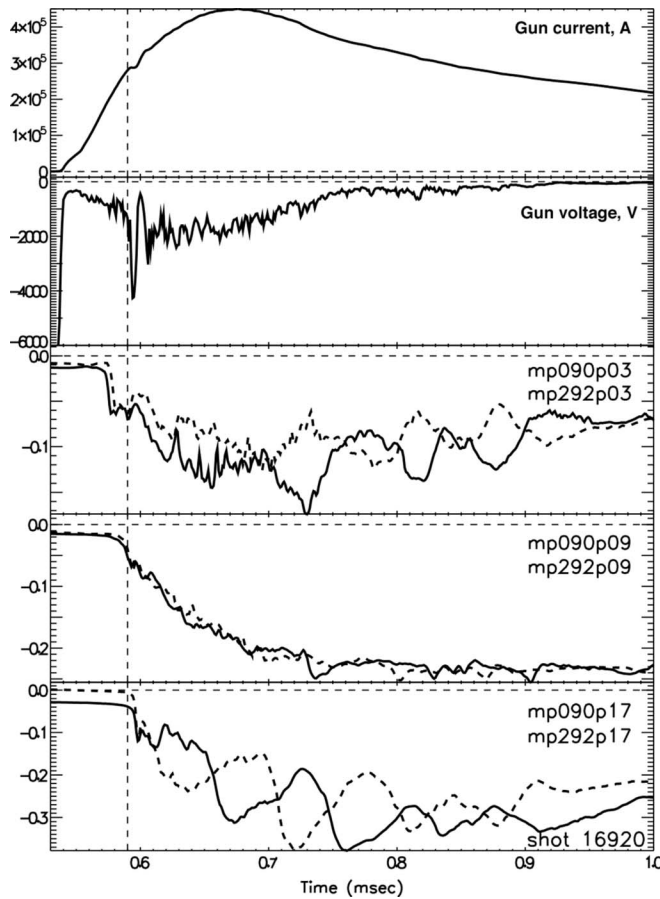


FIG. 3. A formation shot with a high degree of symmetry. Probes as in Fig. 2(c). Compare the voltage immediately following the vertical dashed line with Fig. 4.

tance which generates a corresponding gun voltage, as seen in Fig. 3. When the current is asymmetric, the current is localized and the inductance is likely already large before ejection from the gun, thereby reducing or eliminating the voltage pulse. It needs to be emphasized, however, that in the experiment breakdown and other processes during this initial formation stage are complex and poorly constrained. As a result, the detailed voltage time history varies more shot-to-shot than this simple description implies. These details are not modeled in the simulations.

The  $n=1$  mode can be seen clearly on the probes in the gun and on *mp090p17* at the bottom of the flux conserver, but is quite weak at the midplane probe, *mp090p09*. This midplane magnetic field is also “measured” in the simulations and plays a critical role in understanding spheromak formation.

### B. Ejection from a coaxial gun

In the experiment, the breakdown processes often generate initial asymmetries. However, the initial amplitudes of the nonaxisymmetric modes in the NIMROD simulations are usually small. As a result, their amplitude is negligible during the ejection of plasma from the gun and a voltage pulse is seen as in the experimental shot shown in Fig. 3. An example is shown in Fig. 4, which shows a clear correlation

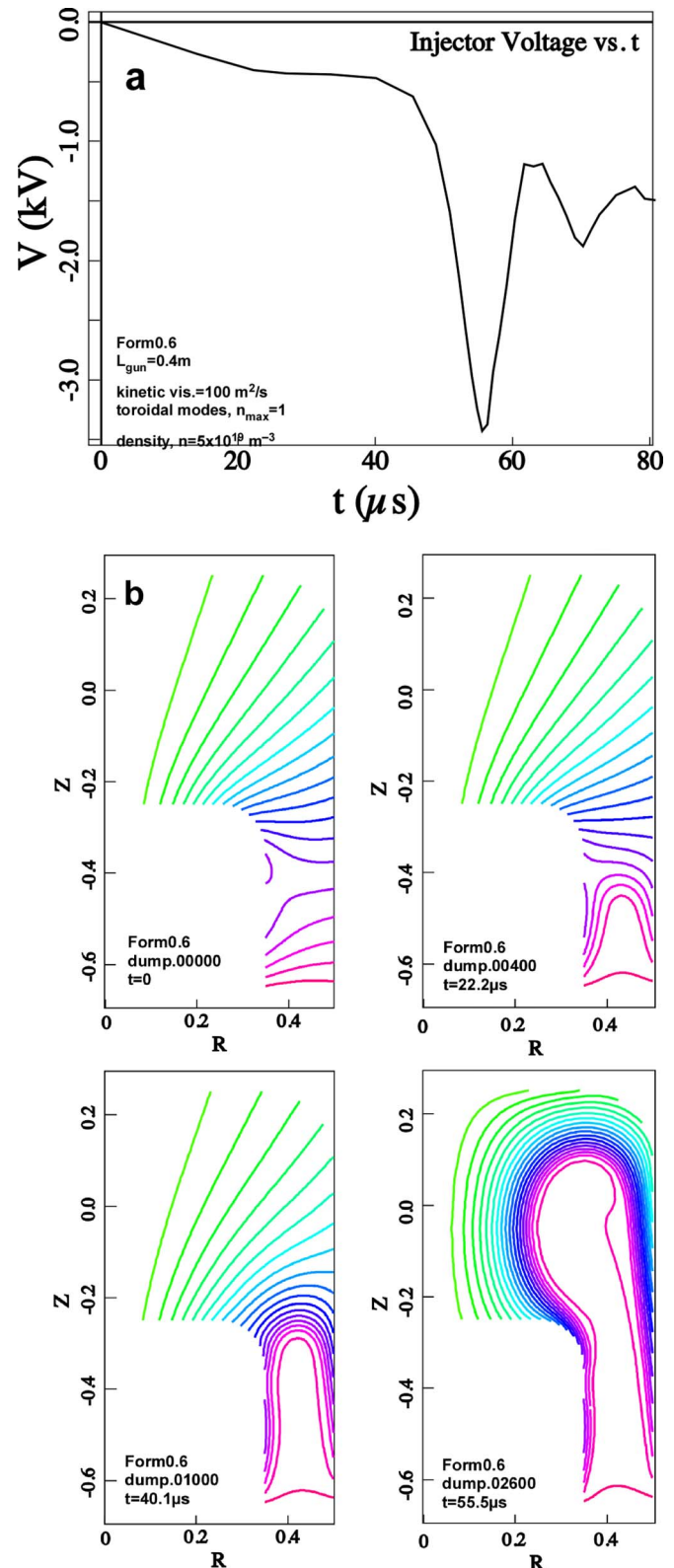


FIG. 4. (Color online) (a) Simulation showing voltage pulse as plasma is ejected from the gun. (b) Development of the poloidal flux ( $n=0$ ) during the initial stage of formation, showing the abrupt change in geometry as the plasma exits the gun.

between the time of ejection of plasma and the voltage pulse in a simulation. (A simulation with larger initial amplitudes had no initial voltage pulse,<sup>9</sup> consistent with the discussion of experimental data.) The time to the voltage peak is thus a measure of the time at which the plasma “bursts” from the gun. From the poloidal flux contours in Fig. 4, it is clear that until well after ejection, the gun current flows in a narrow layer between the injected toroidal magnetic field and the compressed poloidal field ahead of this advancing front. This characteristic has also been observed using magnetic probes during SSPX formation.<sup>18</sup> The time for an Alfvén wave to propagate the length of the gun is 1 to a few  $\mu\text{s}$ , and the plasma pressure is low, so the magnetic forces on the current are in near-balance. The rapid change in geometry as the plasma is ejected from the gun is apparent in Fig. 4(b); the voltage pulse in Fig. 4(a) corresponds in time to the ejection as discussed in the experimental data section.

A series of simulations showed that the time to ejection is only weakly dependent on the viscosity, density, gun length, and maximum toroidal mode number kept in the simulation. The ejection times in the experimental shots are shorter than the simulated time by 5–20  $\mu\text{s}$ . At least two effects may contribute to this. First, breakdown in the experiment may occur closer to the gun throat than is modeled in the simulation. The cusp magnetic field seen in the vacuum flux, Fig. 2(a), was added to the original bias poloidal field to make the breakdown more reliable by providing a long path for electrons in the gun which operates at low gas pressure. Breakdown likely occurs near this cusp, whereas in the simulation current starts to flow from the upstream end of the coaxial gun. The current front in the simulation takes about 20  $\mu\text{s}$  to reach the cusp, consistent with this speculation; cf. Fig. 4. Second, the initial asymmetry in most experimental shots results in larger local magnetic forces where the current flows; these are likely to equilibrate higher in the gun than in the nearly axisymmetric simulation. In any event, the difference in timing is small and appears to have no significant consequences on the later development of the spheromak. Consequently, it has not been explored further.

### C. Effects of viscosity and maximum toroidal mode number on poloidal field generation

Following the ejection of flux and plasma from the coaxial gun, the current pinches around the geometric axis, and the  $n=1$  and other column modes grow until symmetry breaking in the axisymmetric flux-conserving boundary becomes large.<sup>9</sup> The reconnection events cause a relaxation of the field into the spheromak geometry. Although the resulting spheromak is robust, the details of these processes can be expected to be sensitive to the parameters of the simulated plasma.

To examine the effect of viscosity, simulations were run with  $n_{\text{max}}=1$  and  $n=5 \times 10^{19} \text{ m}^{-3}$  but kinematic viscosities of 100  $\text{m}^2/\text{s}$  and 1000  $\text{m}^2/\text{s}$ . As seen in Fig. 5, the lower viscosity simulation shows more structure on the voltage. The magnetic field at the midplane flux-conserver wall shows that the  $n=1$  mode grows more rapidly and has more structure at lower than at higher viscosity, presumably be-

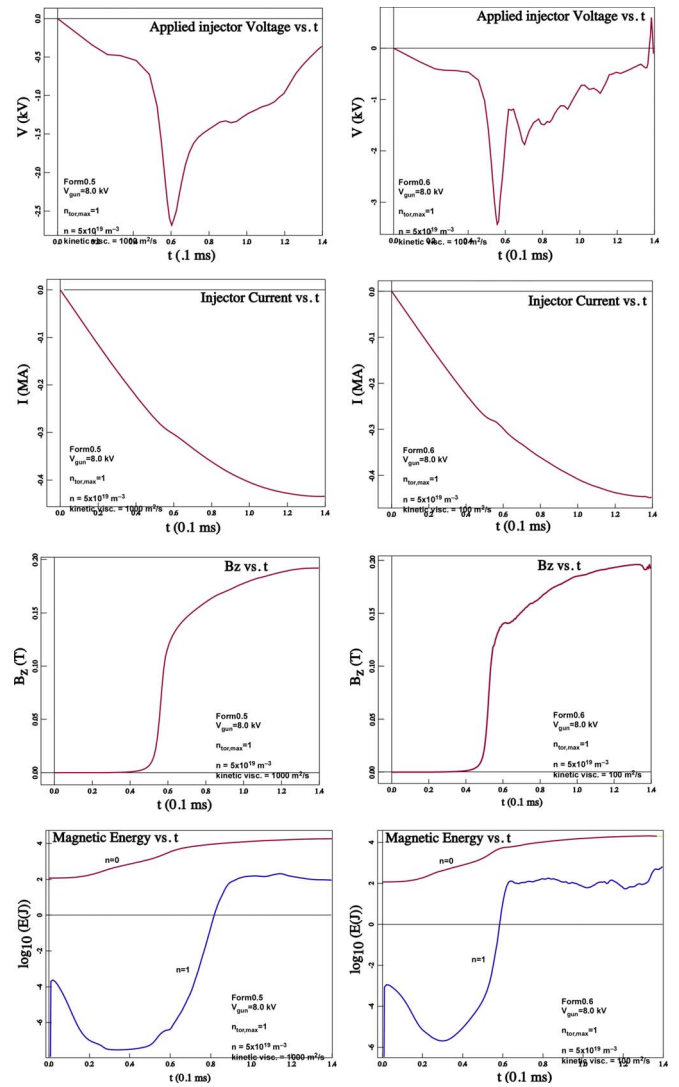


FIG. 5. (Color online) Effect of viscosity. Left,  $\nu=10^3 \text{ m}^2/\text{s}$ ; right,  $\nu=10^2 \text{ m}^2/\text{s}$ .

cause the fluid velocity in the simulations is sensitive to viscosity. We interpret the voltage spikes as arising from rapid changes in the spheromak inductance; the higher viscosity slows this change and reduces the spike amplitudes. The development of the axisymmetric ( $n=0$ ) magnetic field, however, is not sensitive to the viscosity, as seen in Fig. 6.

Viscosity becomes important when the velocity varies over sufficiently small distances as can be seen from the resistive MHD force equation, Eq. (2). For open field-line plasmas with  $T \sim 25\text{--}40 \text{ eV}$ , the speed of sound,  $(\gamma p/\rho)^{1/2} \sim 1 \times 10^5 \text{ m/s}$ ; we note from the simulations that flow velocities are the same order, as one would anticipate. The plasma is low  $\beta$ , so the magnetic field is nearly force-free and the  $\mathbf{J} \times \mathbf{B}$  term correspondingly small. Thus, we expect the viscosity to become important on scale lengths  $\sim \nu/V \sim 10^{-2} \text{ m}$  at  $\nu=100 \text{ m}^2/\text{s}$ . This is the scale at which current layers are seen for this viscosity<sup>9</sup> and allows for inductance changes on the microsecond time scale as observed in the corresponding simulations.

Formation calculations varying  $n_{\text{max}}$  from 1 to 10 are



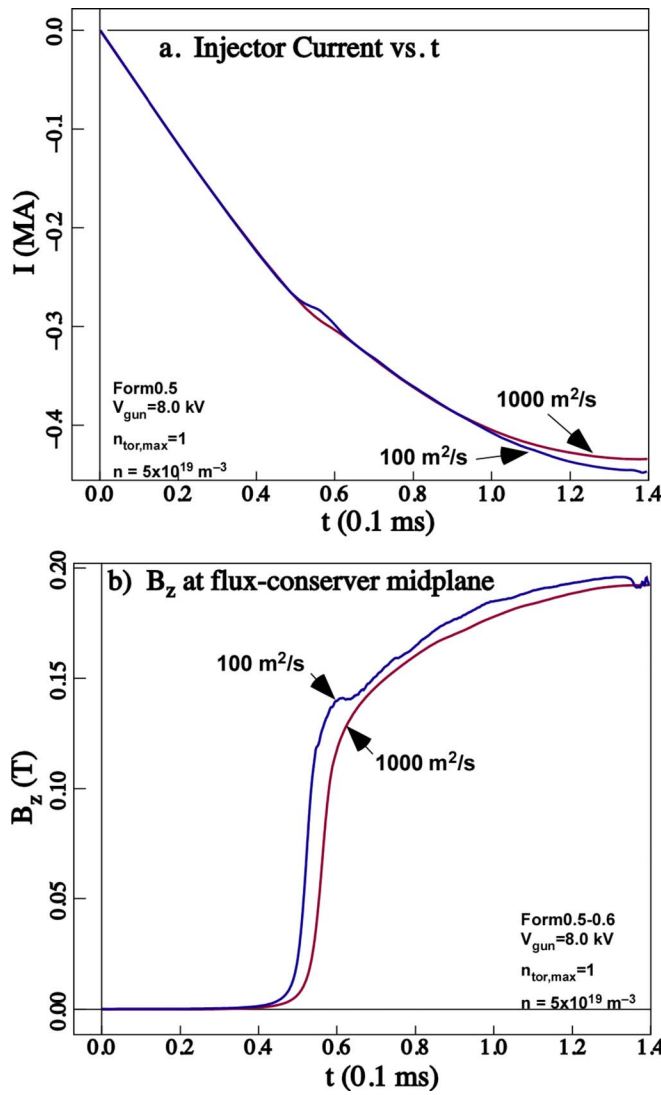


FIG. 6. (Color online) The mean-magnetic field is almost independent of viscosity. Shown are the currents and fields from the cases in Fig. 5.

compared with experiment in Fig. 7;  $n_{\text{max}} = 5$  and 10 behave qualitatively differently from  $n_{\text{max}} = 1$ . Following the initial voltage transient resulting from the ejection of flux from the coaxial gun, the voltage associated with the  $n_{\text{max}} = 1$  simulation drops significantly relative to that associated with higher toroidal mode-number simulations. At about  $140 \mu\text{s}$ , the  $n_{\text{max}} = 1$  voltage shows a strong variation in time; examination of the poloidal flux (not shown) shows that the x-point for the mean-field (azimuthally averaged) spheromak jumps to the vicinity of the cusp in the vacuum bias flux. This abrupt jump does not occur in the higher mode-number studies.

#### D. Electron temperature

For all the formation cases examined, the electron temperature is in the range 25–35 eV, in good agreement with the experimental range. The magnetic field lines are open to the cathode and flux conserver, so the temperature is determined by thermal parallel conductivity. The lack of sensitivity of the temperature to heating values and detailed geom-

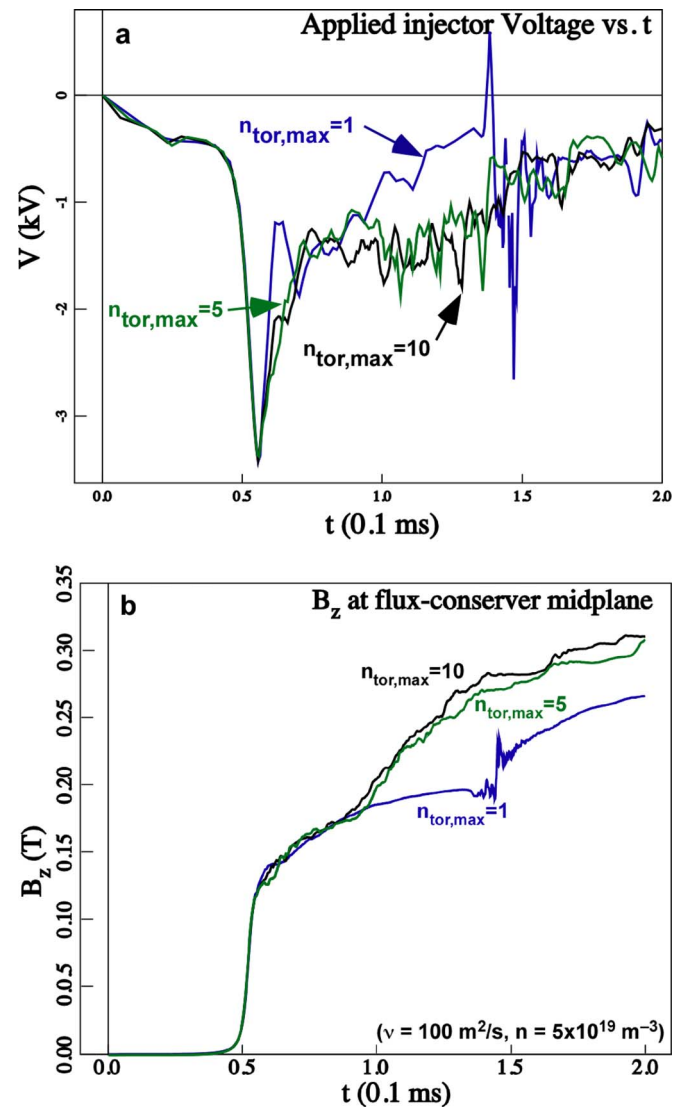


FIG. 7. (Color online) Effect of maximum mode number on detailed time history of voltage and magnetic field. Top, gun voltage; bottom, magnetic field.

etry results because the thermal conductivity is proportional to  $T_e^{5/2}$  and Ohmic heating is proportional to  $T_e^{-3/2}$ , so that small changes in  $T_e$  are sufficient to balance any effects of viscosity, mode numbers, etc. in the simulation, and of field-line length, impurities, and other parameters in the experiment.

#### E. Summary: Sensitivities during spheromak formation

Perhaps the best demonstration of agreement with experiment was found in simulating the buildup of plasma for long formation pulses,<sup>15,19</sup> The simulated results lay well within the scatter of experimental data (about  $\pm 20\%$ ) at fixed  $\lambda_{\text{gun}} = \mu_0 I_{\text{gun}} / \Psi_{\text{gun}}$  (with  $I_{\text{gun}}$  and  $\Psi_{\text{gun}}$  the gun current and bias flux, respectively) between a threshold of  $\lambda_{\text{gun}} = 10 \text{ m}^{-1}$  and about  $14 \text{ m}^{-1}$ . Above this range, the experiment lies below simulation for reasons that are not

understood.<sup>4</sup> The agreement below  $10 \text{ m}^{-1}$  is consistent with the present simulations as summarized below.

**Mode number.** The initial phase of plasma formation in the coaxial gun and its injection into the flux conserver are insensitive to the maximum mode number assumed in the simulations. The amplitudes of modes with  $n \geq 1$  remain low during this time.

The gun voltage during ejection of plasma from the coaxial region is found to vary with the symmetry of the breakdown and other parameters. Although the resulting detailed time history is affected, the final spheromak is not significantly changed by the initial symmetry: For example, the magnetic field in experiment and simulation typically agree within 10–20% for similar bias flux, gun current, and duration of the formation pulse.

If the maximum toroidal mode number is set to 1, the bias poloidal flux is amplified and a mean-field spheromak is formed. However, once reconnection becomes significant, the time history of the formation magnetic field and gun voltage differ considerably from experiment and from those when  $n_{\text{max}}=5$  or 10 are assumed in the simulation. The histories are almost identical for these two simulation cases.

**Viscosity.** The primary effect of viscosity in the range of 10 to  $1000 \text{ m}^2/\text{s}$  is on the detailed time histories, for example the magnetic field and gun voltage histories. The amplitude of the voltage spikes is sensitive to the viscosity. Although these details are important for comparing with and interpreting experimental results, they are found to have little impact on the mean-field spheromak parameters, e.g., the magnetic field strengths which, as for variations in the maximum mode number, agree with experiment within 10–20%.

**Density.** The density in the simulation has only minor effects on spheromak formation. Although it affects Alfvén times, these are an order of magnitude less than the evolution time of the mean field; and plasma resistivity is not playing a role in the dynamics, so changes in the Alfvén time are not important at the global level. Plasma inertial effects are small.

### III. CONTROLLED DECAY

We have undertaken a suite of simulations in which we have examined in a limited way the influence of a few of the physical and numerical parameters on the evolution of the magnetics and the electron temperature. We have varied the effective ionization state,  $Z_{\text{eff}}$ , which influences the resistivity and the parallel thermal conduction, the background electron number density, the scalar kinematic viscosity, the artificial density diffusivity, the toroidal mode resolution, and the poloidal resolution. We examine the influence of these parameter variations on the comparisons of the simulation magnetic and electron temperature evolution with the data observed in the SSPX experiment. We have also examined the sensitivity of the NIMROD simulation results with respect to using a tensor model of the plasma viscosity.

#### A. Effects of ionization state, scalar viscosity, plasma density, and density diffusivity

In the NIMROD fluid model, there are a number of important parameters to be set that affect coefficients in the system of equations. The choices of these parameters are guided by experimental data and by inferences drawn from modeling the experimental data. Here we consider limited variations in  $Z_{\text{eff}}$ , the background electron number density, the scalar kinematic viscosity, and the artificial density diffusivity. Before showing the results of the parameter studies, we first discuss some of the physics associated with these parameters.

The effective ionization state of the plasma directly affects the electron-ion collision rate and, hence, the plasma resistivity and electron parallel thermal conductivity.<sup>12,20</sup> Increasing  $Z_{\text{eff}}$  increases the electron-ion collision rate, increases the electrical resistivity, and decreases the electron parallel thermal conduction, e.g., increasing  $Z_{\text{eff}}=1$  to  $Z_{\text{eff}}=2.3$  increases the resistivity by  $\times 2.3$  and decreases the parallel thermal conductivity by  $\times 0.71$ . Considering Eq. (3) for the electron temperature evolution, for fixed current density and magnetic field structure, an increase in the resistivity  $\eta$  and decrease in thermal conductivity  $\chi_{\parallel}$  might be expected to increase the electron temperature. However, the evolution of the magnetics, the fluid motion, and the electron temperature are strongly coupled. Moreover, the electron temperature evolution in our spheromak simulations is very sensitive to the quality of the magnetic surfaces,<sup>6–9</sup> which in turn is very sensitive to the amplitudes of symmetry-breaking magnetic perturbations at levels  $|\delta B/B_0|$  less than or equal to a few percent.<sup>8,9,21</sup>

If the spheromak is limited by a power flow limit or a plasma stability limit that depends on  $\beta=2\mu_0 n_e k_B T_e/B_0^2$ , then if the average electron number density  $n_e$  is reduced,  $T_e$  might be expected to increase if the magnetic structure is unchanged. However, if the plasma number density is reduced, then the inertia term in the momentum equation, Eq. (2), is reduced, which in turn increases ideal magnetohydrodynamic growth rates. This can be deleterious to maintaining good magnetic surfaces because magnetic islands and stochastic field lines degrade the electron temperature owing to the enormous anisotropy,  $\chi_{\parallel}/\chi_{\perp} \gg 1$ .

We have used values of the scalar kinematic viscosity  $\nu=500$  or  $1000 \text{ m}^2/\text{s}$  in many of our NIMROD simulations. The nondissipative algorithm in NIMROD requires physical viscosity to limit the development of small-scale structures that cannot be resolved; simulations fail to converge in the absence of a sufficient viscosity. Evaluation of the Braginskii<sup>12</sup> tensor model for the viscosity yields parallel viscosity values  $\eta_0/\rho \sim 10^6 \text{ m}^2/\text{s}$  and perpendicular viscosity values  $\eta_1/\rho \sim 10^{-1} \text{ m}^2/\text{s}$  for  $T_i=150 \text{ eV}$ ,  $n=3.5 \times 10^{13} \text{ cm}^{-3}$ , and  $B=0.5 \text{ T}$ . The scalar viscosity used in the simulations, therefore, falls somewhere between the extremes. However, viscosity has a stabilizing influence for MHD modes, and the scalar-viscosity modeling may reduce symmetry-breaking magnetic perturbations that govern energy transport and the resulting temperature profile.

The artificial density diffusivity  $D$  in Eq. (1) affects the

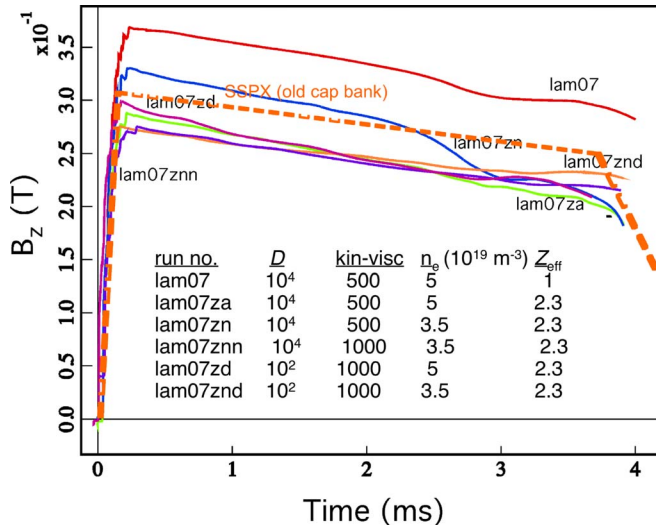


FIG. 8. (Color online) The poloidal magnetic field at the outboard midplane of the flux conserver vs time in the *lam07* NIMROD series of spheromak simulations. A composite SSPX  $B_z$  vs time trace (dashed line) is plotted for similar gun current input and typical operation with the older capacitor bank.

MHD physics in several ways as described in Ref. 14. The role of  $D$  is to maintain a relatively smooth plasma density, particularly during the violent formation stage of the spheromak. In the absence of good models for plasma sources (from bounding surfaces or ionization of neutral gas in the volume), we initialize a plasma density throughout the domain and use the density diffusivity to moderate the evolution of the plasma density. A direct consequence of the artificial diffusivity is that the evolution of the internal energy density acquires a fictitious term  $3TD\nabla^2 n$  that is a sink where the density peaks at the magnetic axis of the spheromak:

$$\begin{aligned} \frac{\partial 3nT}{\partial t} + \nabla \cdot (3nT\mathbf{V}) + 2nT\nabla \cdot \mathbf{V} \\ = \frac{1}{2} \eta J^2 + \nabla \cdot n\tilde{\chi}\nabla T + 3TD\nabla^2 n. \end{aligned} \quad (6)$$

This aspect of the modeling may artificially reduce the internal energy density near the magnetic axis. Note that Eq. (6) is implied by Eqs. (1)–(3) in NIMROD, but is not shown directly therein. Equation (3) for the electron temperature has no density diffusivity in it. The density diffusivity, when large, flattens the density, which is observed in the simulations. The result is that in the last term on the right side of Eq. (6), the product of  $D\nabla^2 n$  involves the large value of  $D$  (e.g.,  $10^2$ – $10^4 \text{ m}^2/\text{s}$ ) and the very small value of the second spatial derivative of the density. This term has to be comparable to or larger than the thermal conduction term to influence the internal energy density in the closed flux region, a term that scales as the product of  $\chi_{\perp} \sim 1$ – $20 \text{ m}^2/\text{s}$  and the second spatial derivative of  $T_e$  (whose scale length is comparable to the radius at the magnetic axis). If NIMROD is obtaining physical solutions for the internal energy and electron temperature, then the thermal conduction term in Eq. (6)

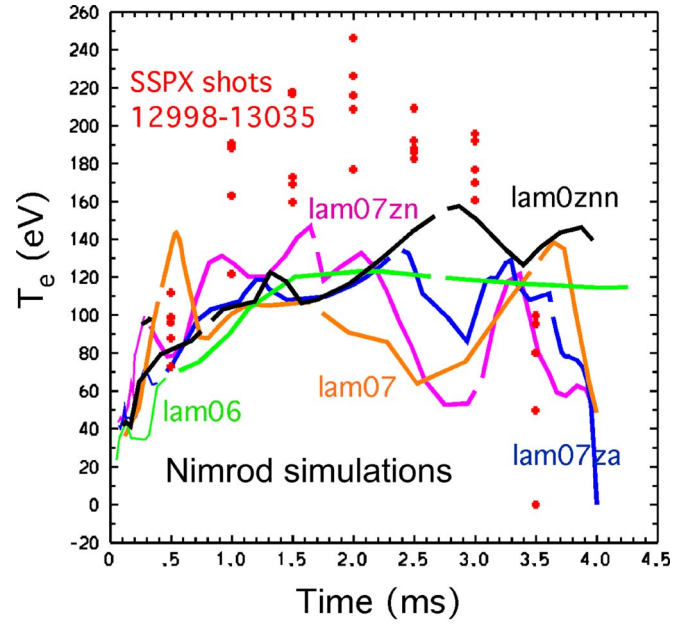


FIG. 9. (Color online) Peak electron temperature vs time in *lam07* NIMROD series of spheromak simulations and from a series of well-conditioned SSPX discharges prior to installation of the 32-module capacitor bank.

must dominate the density diffusivity term in competing with the Ohmic heating source term. We have explored the influence of the artificial diffusivity term on the magnetics and the temperature evolution, and report the results later in this section.

A suite of NIMROD simulations (*lam07* variants on the *lam06* simulation in Ref. 8) investigates sensitivity to the values of scalar viscosity, artificial particle diffusivity,  $Z_{\text{eff}}$ , and number density. Figures 8 and 9 show the evolution of the poloidal magnetic field at the outboard midplane of the flux conserver and the peak electron temperature from the simulations, each of which requires several weeks of run time on a local Beowulf cluster. The highest poloidal magnetic field occurred in the simulation with the lowest  $Z_{\text{eff}} = 1$  and therefore smallest resistivity (*lam07* in Fig. 8). (Viscosity has little effect on the buildup of the azimuthally averaged field, as seen in Fig. 6.) The other magnetic field traces in Fig. 8 do not show strong dependences on the parameter variations and are in general agreement with typical SSPX data for discharges with similar gun current. In contrast, the peak temperature histories for this series of simulations show considerably greater variation in Fig. 9, and there is also considerable spread in the SSPX temperature data. As has been established in earlier experimental and simulation work,<sup>5-9,13,16,17,21</sup> the plasma energy confinement and temperature are quite sensitive to the quality of the magnetic surfaces, which are profoundly influenced by the am-



plitudes of the symmetry-breaking magnetic perturbations at small amplitudes (a few percent or less). The time histories of the finite- $n$  magnetic fluctuations in the simulations corresponding to those in Fig. 9 differ significantly during controlled decay accounting for the observed differences in the temperature histories. The stochasticity of some of the magnetic field lines is an important factor influencing the transport, the electron temperature, the resistivity, and the magnetohydrodynamics in the strongly coupled system.

The plasma temperatures in the simulations get as high as 150 eV in this series, while the SSPX temperatures in similar discharges exceed 200 eV. The experiments exhibiting the highest electron temperatures tend to confine energy somewhat better than is suggested by NIMROD in this series of simulations. However, there are many SSPX discharges with electron temperatures in the range 100 – 150 eV for similar injected gun current histories. The NIMROD electron temperatures are typically  $\sim 40\%$  lower than the ensemble-averaged temperatures in the best SSPX shots. The simulated electron temperatures fall closer to the middle of the scatter plots when all SSPX shots are plotted, but the model does not include many effects that contribute to the spread of laboratory data, such as impurities and gas generation due to plasma-wall interactions. We believe that our results imply that the resistive MHD model is perhaps too simple to explain the temperatures in the best SSPX shots and extensions to the MHD model should be explored.

Given the extreme sensitivities of simulating the plasma temperature evolution in NIMROD and the limitations due to both finite computing resources (which restricts our attempts to examine numerical convergence as rigorously as we would like) and the simplifications of our resistive MHD physics model, our expectations on how close the simulations should agree with experiment on the temperature evolution should be tempered. Certainly the agreement shown here and in our earlier work<sup>7-9</sup> on the magnetics and voltages tends to be good; the agreement on temperature is better than just qualitative; and the gross behavior with respect to the importance of the  $n=1$  mode in formation, the relationship of the current-profile evolution to the emergence of symmetry-breaking modes which then influences the magnetic confinement,<sup>3,13,16-18</sup> and the ultimate crash of the discharge when the current profile sags at the edge leading to  $n=2$  mode activity seen in the experiment<sup>3,13,16-18</sup> are confirmed by the simulations.<sup>6-9</sup>

Figure 10 shows the results of restarting simulation *lam07znn* at  $t \sim 0.5$  ms after formation and reducing  $D$  from 10 000 to 100  $\text{m}^2/\text{s}$  in *lam07znd*:  $T_e$  rises faster to  $\sim 160$  eV at  $t \sim 2$  ms and grows to  $>180$  eV after 3.5 ms, before small-amplitude magnetic fluctuations ( $n=2$ ) degrade the magnetic surfaces and the discharge crashes. In these simulations, the density is allowed to evolve after the formation phase, and we observe a rather weak peaking of the density near the magnetic axis. The reduction of the value of  $D$  reduces the energy sink near the magnetic axis, which in turn does allow a somewhat higher plasma temperature to be achieved. However, the magnetic fluctuation histories do not remain unaffected and strongly influence the energy confinement. In Fig. 11, only higher values of  $D \sim 10^4 \text{ m}^2/\text{s}$  are

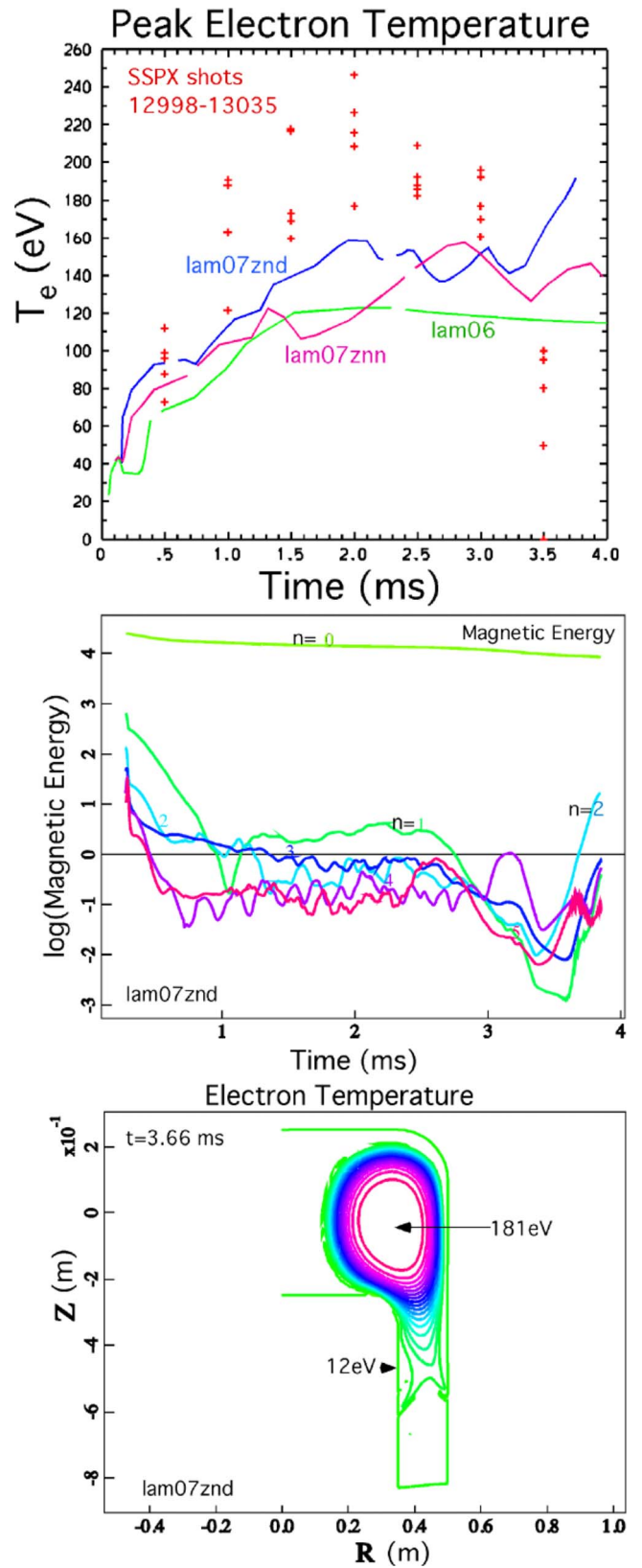


FIG. 10. (Color online) (a) Peak electron temperature vs time. (b) Magnetic energy for toroidal modes integrated over volume vs time. (c) Electron temperature contours at  $t=3.66$  ms showing the effects of decreasing the density diffusivity.



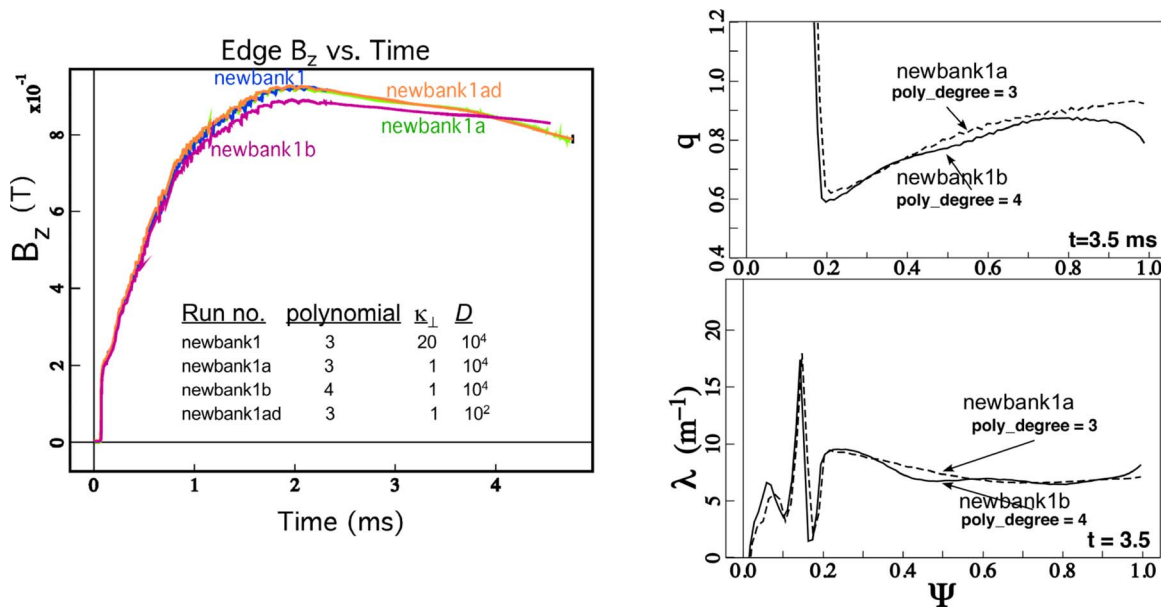


FIG. 11. (Color online) Toroidally averaged edge poloidal magnetic field vs time for several NIMROD simulations. The  $q$  and  $\lambda$  profiles as functions of magnetic flux  $\Psi$  at  $t = 3.5$  ms in *newbank1a* and *newbank1b*; note that the magnetic axis is at  $\Psi = 1$ .

shown, and the NIMROD simulations predicted lower temperatures than those observed in the SSPX data.

## B. Effects of poloidal resolution

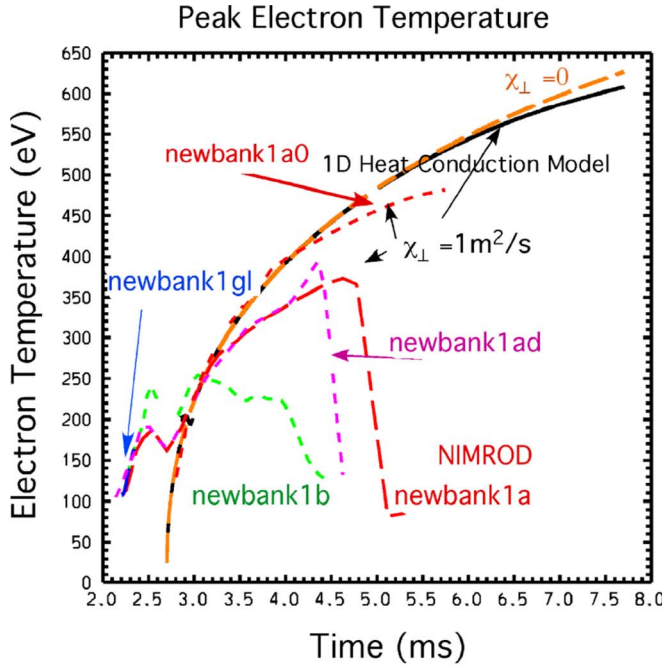
Numerical accuracy in SSPX simulations depends on the ability to resolve the MHD modes, their effect on magnetic topology, and the anisotropic thermal conduction along evolving and sometimes chaotic field lines. Sensitivity to error is heightened by the feedback of transport-inducing fluctuations on the evolving profiles that influence the MHD activity. To check the level of numerical error during the controlled decay phase of simulations for the new SSPX capacitor bank, we have undertaken a series of calculations, changing finite-element basis functions and isolating transport physics. We generally retained toroidal modes  $0 \leq n \leq 5$  and a poloidal resolution of  $24 \times 48$  elements in a mesh whose boundaries conform smoothly to the conducting surfaces bounding the spheromak. In some of the simulations, however, we restricted the toroidal resolution to just  $n = 0$  after formation to ensure perfect magnetic surfaces so as to obtain an upper limit on the energy confinement. We also investigated changing the value of the perpendicular thermal diffusivity ( $\chi_{\perp}$ ) from  $20 \text{ m}^2/\text{s}$  to  $1 \text{ m}^2/\text{s}$  to  $0 \text{ m}^2/\text{s}$  to understand the influence of these changes on the simulated temperature evolution.

When changing the finite-element bases from bicubic to biquartic polynomials in the full simulations, we find quantitatively small changes in magnetic profiles but significant discrepancies with respect to energy confinement. The *newbank1a* simulation completed with bicubic elements is refined to use biquartic elements in simulation *newbank1b* for the sensitive quiescent phase starting at 2 ms. After approximately 1500 Alfvén times, traces of the edge magnetic

field are not distinguishable, and the safety factor and parallel current profiles agree to within 10% over most of the profile (Fig. 11). Nonetheless, the biquartic computation does not maintain a state of high confinement as long as predicted in the bicubic computation, as shown in Fig. 12. At 3 ms, the value of the peak temperature obtained in *newbank1b* drops slightly to 230 eV, and after 4 ms it drops to approximately 100 eV. In contrast, the peak in *newbank1a* continues climbing to 370 eV before it drops at 5 ms. We note that the *newbank* series of SSPX simulations depicted in Figs. 11 and 12 modeled the new capacitor bank, which enabled higher injected gun currents in the formation and controlled decay phases (see Fig. 1 for an example), which resulted in higher plasma currents, increased Ohmic heating, and higher electron temperatures than in the *lam06* and *lam07* series.

An important consideration when modeling extremely anisotropic transport—and the ratio of thermal conductivities ( $\chi_{\parallel}/\chi_{\perp}$ ) exceeds  $10^6$  in SSPX plasmas with  $T_e > 100$  eV—is whether perpendicular transport is numerically enhanced by truncation error. The performance of high-order finite elements in resolving extremely anisotropic thermal conduction has been investigated in Ref. 10 in a two-dimensional calculation with a fixed magnetic structure. Favorable numerical behavior is reported for bicubic and biquartic finite elements for grid resolutions and  $\chi_{\parallel}/\chi_{\perp}$  values comparable to those in our SSPX simulation. However, here we include the additional complexity of the self-consistent evolution of the magnetics.

To investigate the accuracy of energy transport in simulations for the new capacitor bank, we varied  $\chi_{\parallel}/\chi_{\perp}$  and the artificial density diffusivity in bicubic computations, evolved temperature alone in bicubic and biquartic computations, and compared fluctuation-free results with a 1D model. The 1D calculation solves



Run no.	polynomial	$\kappa_{\perp}$	$D$
newbank1a0	3 (unif), $n=0$	1	$10^4$
newbank1a	3 (unif)	1	$10^4$
newbank1ad	3 (unif)	1	$10^2$
newbank1b	4 (unif)	1	$10^4$
newbank1gl	4 (Gauss-Leg)	1	$10^4$

FIG. 12. (Color online) Peak plasma temperature vs time for several NIMROD simulations and for a one-dimensional simplified model assuming perfect surfaces. The brief *newbank1gl* computation is very similar to *newbank1b* and confirms that the biquartic results do not change when the node locations of the polynomial basis functions are moved.

$$\frac{3}{2}nk_B\frac{\partial}{\partial t}T = \nabla \cdot nk_B\chi_{\perp} \nabla T + \eta(T)J(t)^2/2 \quad (7)$$

assuming perfect, closed magnetic surfaces over a region ( $0.1 \text{ m} \leq R \leq 0.4 \text{ m}$  in the examples considered) and using a time dependence for the plasma current that mimics the time dependence of the current decay in the NIMROD simulation for  $t > 2 \text{ ms}$ . The coefficients have  $\chi_{\perp} = 0, 1$  or  $20 \text{ m}^2/\text{s}$ , and  $\eta$  is the classical Spitzer–Braginskii temperature-dependent resistivity. (Modeling of the energy confinement in SSPX with the CORSICA code supports values of  $\chi_{\perp} = 1 \text{ m}^2/\text{s}$  during controlled decay in SSPX when the highest plasma temperatures are observed.<sup>13</sup>) When only the  $n=0$  component of the magnetic field is retained for  $t > 2 \text{ ms}$  in a bicubic NIMROD simulation (*newbank1a0*), the resulting temperature trace tracks the one-dimensional model for an even longer time (Fig. 12) and establishes a prediction for an upper limit on the plasma temperature that might be achieved in SSPX up to a given time in the discharge if the magnetic surfaces remain good. When Eq. (7) is evaluated for  $\chi_{\perp} = 20 \text{ m}^2/\text{s}$  and other parameters unchanged, the peak electron temperature

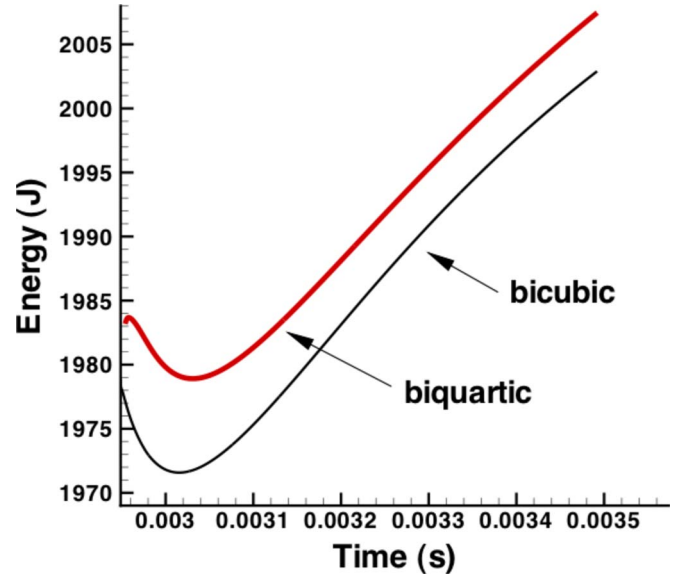


FIG. 13. (Color online) Comparison of internal energy evolution from bicubic and biquartic computations that solve Eq. (3) with  $n$ ,  $\mathbf{B}$ , and  $\mathbf{V}$  fixed at the state achieved at 3 ms in the *newbank1a* simulation.

obtained is  $\sim 270 \text{ eV}$ , while in NIMROD 190 eV is observed ( $\sim 70\%$  of the result from the one-dimensional model). In comparison,  $\sim 80\%$  of the peak temperature in the one-dimensional model was obtained in the NIMROD simulation with  $\chi_{\perp} = 1 \text{ m}^2/\text{s}$  for the same closed flux volume and model of the plasma current used in Eq. (7). Clearly the effects of decreasing the value of  $\chi_{\perp}$  on increasing the temperature are strongly moderated by the decrease in  $\eta$  at the higher temperature. The consistency with respect to changing the value of  $\chi_{\perp}$  of the 1D model in setting an upper bound for the electron temperature compared to the NIMROD results and the associated sensitivity of the electron temperatures argue against there being significant pollution of the temperature equation from the very large anisotropy in the parallel and perpendicular thermal conduction. If there were pollution of the thermal conduction, the more likely result would be a complete insensitivity of the electron temperatures with respect to changes in  $\chi_{\perp}$  for  $\chi_{\perp} \ll \chi_{\parallel}$  and lower and flatter temperatures.

Regarding the artificial particle density diffusivity, a bicubic computation with  $\chi_{\perp} = 1 \text{ m}^2/\text{s}$  and  $D$  reduced by two orders of magnitude (*newbank1ad* in Figs. 11 and 12) nearly matches the *newbank1a* result until an earlier thermal collapse occurs at 4.5 ms. Another pair of computations compares bicubic and biquartic resolution when evolving temperature and its effect on thermal conductivities and Ohmic heating in the presence of fixed profiles of  $n$ ,  $\mathbf{B}$ , and  $\mathbf{V}$ . They are started from 3 ms into the *newbank1a* bicubic computation, and the resulting internal energies only differ by a small and relatively constant offset, as shown in Fig. 13. Moreover, peak temperatures of 246.0 and 247.4 at the end of the thermal computations agree to within 1%.

Returning to analysis of the full simulations, we note that while the symmetry-breaking magnetic fluctuation amplitudes are small and the magnetic confinement is good, the

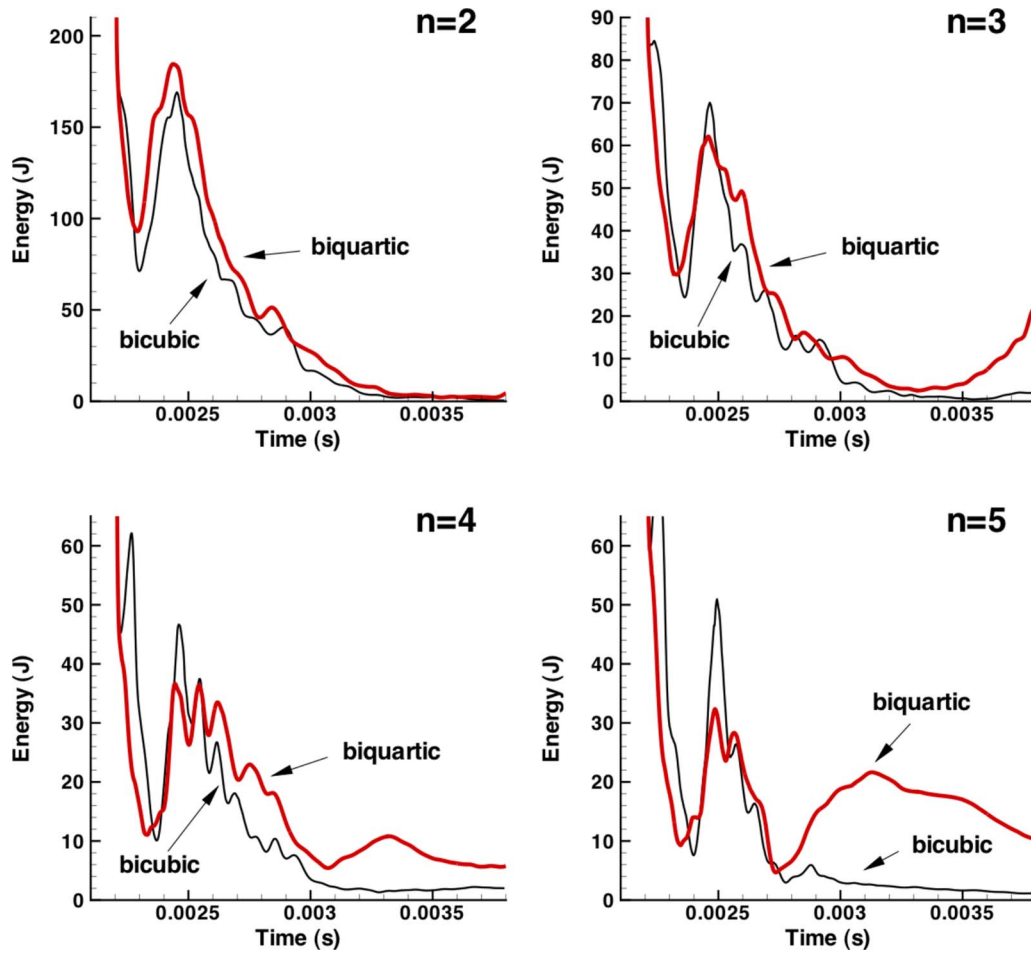


FIG. 14. (Color online) Comparison of the evolution of magnetic fluctuation energy by toroidal Fourier index ( $n$ ) from the bicubic *newbank1a* (black) and the biquartic *newbank1b* (red).

temperatures in the bicubic *newbank1a* track values computed from a one-dimensional model. These results also provide confidence in the poloidal resolution of anisotropic thermal conduction with the mesh of bicubic elements.

Having shown that the bicubic elements represent anisotropic transport with reasonable accuracy, we infer that differences in the MHD activity may indirectly account for the energy confinement discrepancy between *newbank1a* and *newbank1b*. Evidence is found in the magnetic fluctuation energy histories, shown in Fig. 14. We first note that the  $n > 1$  fluctuation energies are at least an order of magnitude smaller than the internal energy, and the discrepancy between the two simulations is mostly a matter of detail until just before 3 ms for  $n > 2$  and throughout the plotted time for  $n = 2$ . Neither the  $n = 2$  fluctuation nor the  $n = 1$  fluctuation (not shown) are resonant within the core plasma ( $\Psi > 0.2$  in Fig. 11) during this period of time. The  $n = 1$  fluctuation energy is an order of magnitude larger, but like the  $n = 2$  evolution, there is reasonably good agreement between the bicubic and biquartic results. Unlike the  $n = 1$  column mode and its  $n = 2$  harmonic, the  $n = 5$  mode shows distinct behavior in the biquartic computation starting at 2.9 ms, followed by  $n = 4$  excitation at 3.1 ms. The  $q$ -profile evolution has  $q > 4/5$  at 2.2 ms, dips below  $4/5$  at  $\Psi \approx 0.25$  by 3 ms, and

decreases below  $3/4$  by 3.5 ms, which is shown in Fig. 11. The biquartic computation thus finds instability when the respective modes become resonant, unlike the bicubic computation, and as evident in Fig. 15, this leads to a larger region of magnetic stochasticity inside the location of minimum  $q$  value. This is also the period of time when the peak temperature of *newbank1b* falls below that of *newbank1a* in Fig. 12. Figure 14 shows that the  $n = 3$  mode starts growing at 3.5 ms in the biquartic computation. Its saturation leads to the thermal collapse at 4 ms. The  $q$ -profile evolution in *newbank1a* trails that in *newbank1b* slightly, and the parallel current profile has less of a gradient in the outer region of the core plasma, possibly due the absence of  $n = 4$ – $5$  activity. The delayed excitation of the  $n = 3$  in the bicubic computation may be related to these profile differences, but less poloidal resolution may also contribute. On a more qualitative level, excitation of the  $n = 3$  eventually leads to the thermal collapse in both computations.

While the discrepancies between the bicubic and biquartic computations indicate a need for greater numerical resolution, the biquartic simulation is already very intensive. More importantly, there is still something to be learned from the results we have obtained. First, sensitivity of energy confinement to excitation of resonant MHD modes is in general



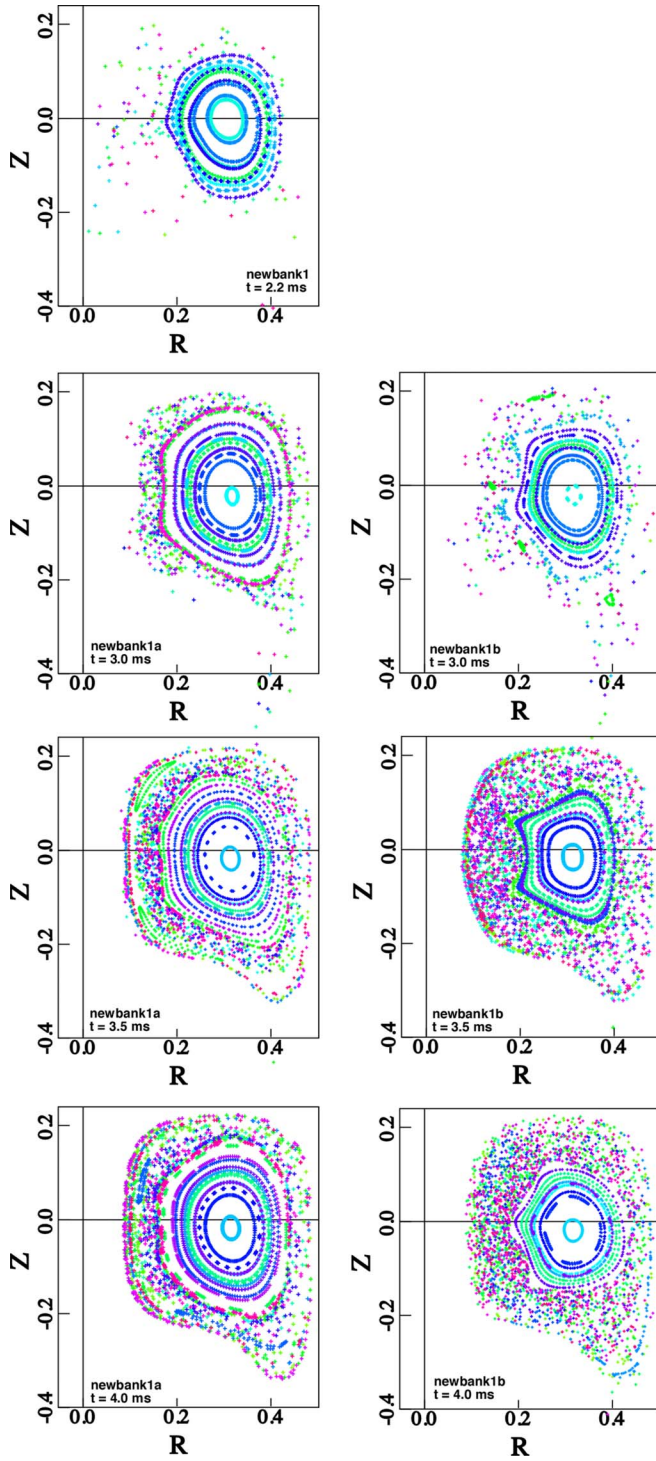


FIG. 15. (Color online) Poincaré surface-of-section plot of the magnetic field lines from NIMROD simulations *newbank1*, *newbank1a* (third-order polynomial), and *newbank1b* (fourth-order polynomial) at various times.

agreement with experimental results<sup>13</sup> and likely contributes to the spread in laboratory data shown in Fig. 9, for example. Second, the NIMROD simulation without magnetic fluctuations suggests that  $T_e > 450$  eV can be achieved with good control over the magnetic fluctuations, and SSPX has obtained electron temperatures approaching 500 eV with the new capacitor bank.<sup>4</sup> Finally, while the overall evolution of energy confinement agrees at least qualitatively with the ex-

periment, the trend with increasing resolution is toward lower peak temperatures than what is achieved in experiment. These observations also point to an inadequacy of the MHD model with collisional transport for the conditions achieved in recent SSPX discharges.

### C. Effects of tensor viscosity and variations in kinematic viscosity

Kinematic viscosity,  $\nu$ , is used in NIMROD, Eq. (2), both to smooth fine-grained structures and to provide a boundary layer of finite volume on the surfaces of the spheromak electrodes.<sup>5</sup> In this boundary layer, the magnitude of kinematic viscosity is increased in SSPX calculations by a factor  $\approx 30$  above its value in the volume.

Actual viscosity in the plasma is extremely anisotropic and differs significantly from the functional form of the kinematic viscosity described above. The version of the code used in these calculations allows parallel viscous stress that is in the Braginskii form albeit with a coefficient that is independent of the plasma parameters. The form of the viscous force available for the simulations is thus

$$\mathbf{F}_{\text{visc}} = \nabla \cdot \nu \rho \nabla \mathbf{V} - \nabla \cdot \mathbf{P}_{\parallel}, \quad (8)$$

where

$$\mathbf{P}_{\parallel} = -\eta_0 \left( \hat{\mathbf{b}} \cdot \nabla \mathbf{V} \cdot \hat{\mathbf{b}} - \frac{1}{3} \nabla \cdot \mathbf{V} \right) (3\hat{\mathbf{b}}\hat{\mathbf{b}} - \mathbf{I}). \quad (9)$$

The Braginskii coefficient is a function of the plasma parameters,

$$\eta_0 = 0.96 n k_B T_i \tau_i \quad (10)$$

with the temperature in eV and

$$\tau_i = 2.09 \times 10^{13} T_i^{3/2} \mu^{1/2} / n \ln \Lambda. \quad (11)$$

The density,  $n$ , is in MKS, and  $\mu = m_i/m_p$ . Evaluating  $\eta_0$  at the representative parameters,  $T_i = 150$  eV,  $n = 3.5 \times 10^{19} \text{ m}^{-3}$ , and  $\mu = 1$ , yields  $\eta_0/nm_i = 1.1 \times 10^6 \text{ m}^2/\text{s}$ .

To evaluate effects of an anisotropic viscous stress, a series of simulations were made with and without the parallel viscosity and kinematic (essentially perpendicular) coefficients of  $\nu = 10, 100$ , and  $1000 \text{ m}^2/\text{s}$ . Other varied parameters were the number of toroidal modes and the particle diffusion coefficient. The effect of adding the large parallel viscosity is shown in Fig. 16, which compares the  $q$  profiles at  $\nu = 1000 \text{ m}^2/\text{s}$  with and without the parallel viscosity. The added viscosity shifts and broadens the minimum of  $q$  after about 2 ms. Reducing the kinematic viscosity to  $10 \text{ m}^2/\text{s}$  has a much larger effect, as also seen in the figure. The quality of the magnetic surfaces is sensitive to these changes, and a reflection of this effect is seen in Fig. 17; however, results of the preceding section lead to resolution concerns for simulations with reduced values of perpendicular viscosity.

In these runs, the relatively small particle diffusion coefficient of  $100 \text{ m}^2/\text{s}$  reduced the density in the bottom of the coaxial gun by about a factor of 20 relative to calculations with a much larger diffusion coefficient (e.g.,  $10\,000 \text{ m}^2/\text{s}$ ).



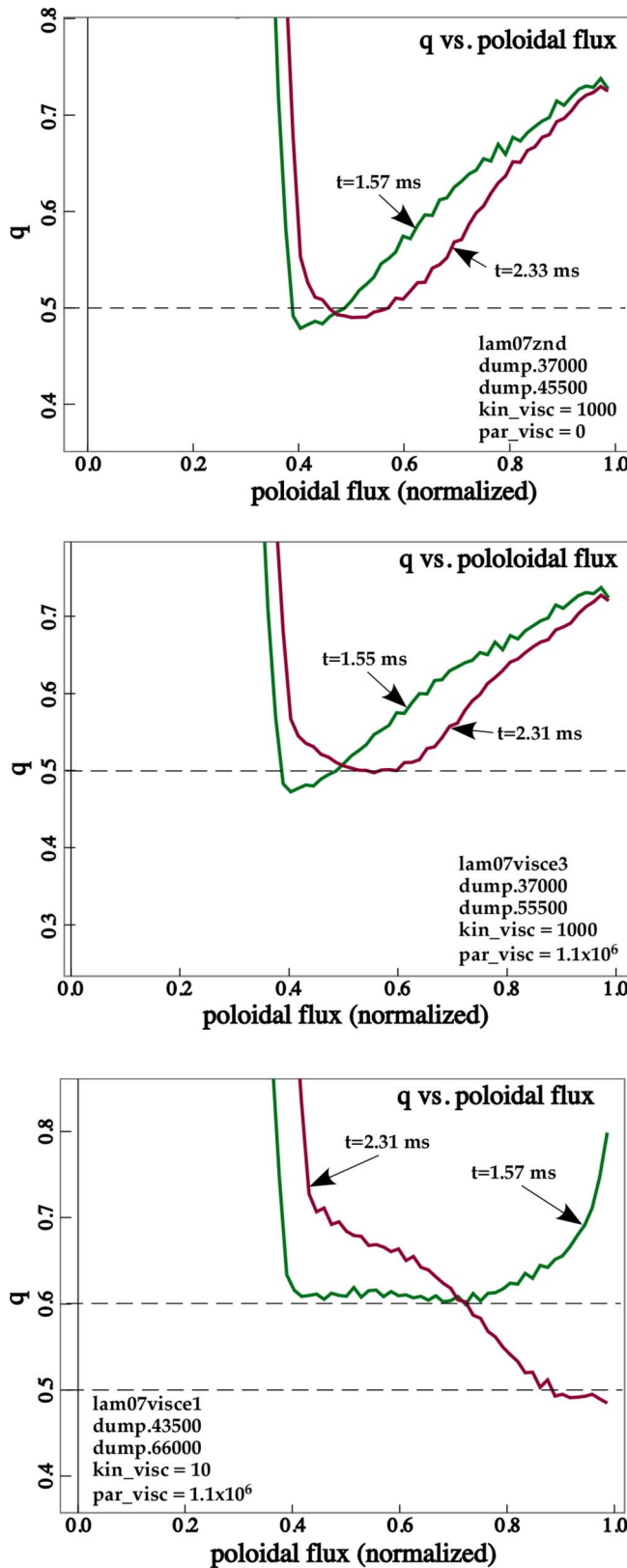


FIG. 16. (Color online) The  $q$  profile is affected by parallel viscosity and by reducing the kinematic viscosity. (a)  $\nu=1000 \text{ m}^2/\text{s}$ . (b)  $\nu=100 \text{ m}^2/\text{s}$ . (c)  $\nu=10 \text{ m}^2/\text{s}$ . The volume of “good” surfaces decreases for changes from  $\nu=1000 \text{ m}^2/\text{s}$ .

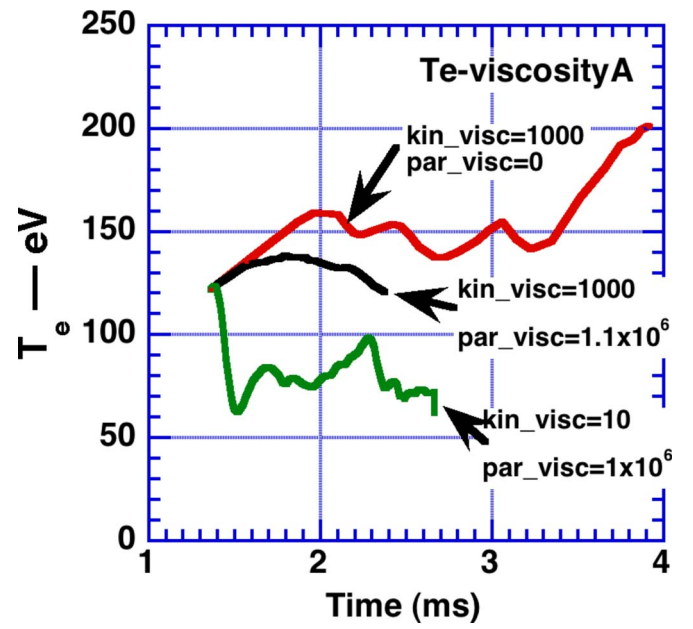


FIG. 17. (Color online) Electron temperature evolution for the three cases shown in Fig. 16.

The effect of parallel viscosity on the density was examined by setting  $n_{\text{max}}=0$ . The primary effect was to spread a density peak that exists near the top of the gun close to the magnetic axis, with no significant consequences.

A rotation boundary condition is applied to all these simulations to give a mode frequency of about  $10^4 \text{ Hz}$ ; evidently the dominant plasma rotation mechanism in the laboratory plasma is not MHD in nature. The resulting rotation is nearly rigid rotation at  $1000 \text{ m}^2/\text{s}$ , whereas at  $10 \text{ m}^2/\text{s}$  the effect of the magnetized plasma is to yield a drop of about half the rotation near the flux-conserver wall.

Finally, a scaling of mode magnetic energies ( $n_{\text{max}}=20$ ) was made for kinematic viscosities from  $10$  to  $1000 \text{ m}^2/\text{s}$  (Table I). As seen in Fig. 18, in each case there is a rapid drop in mode energy above  $n=5$  to  $7$ . Only small changes in the  $q$  profile are seen for the three cases.

This reduction of energy with mode numbers indicates that simulations with  $n_{\text{max}}=5$  capture the dominant spheromak physics, especially for  $\nu=100\text{--}1000 \text{ m}^2/\text{s}$ . This result is consistent with experiment, in which mode numbers above

TABLE I. Parameter variations in NIMROD runs. For these simulations,  $n=3.5 \times 10^{19} \text{ m}^2/\text{s}$ , parallel viscosity  $=1.1 \times 10^6 \text{ m}^2/\text{s}$ , perpendicular thermal conductivity  $=1 \text{ m}^2/\text{s}$ , and bicubic finite elements were used.

Run name	$n_{\text{max}}$	$\nu$	D
K_v=1000_n=0	0	1000	100
K_v=10_n=0	0	10	100
Kin_visc=10	5	10	100
Kin_visc=1000	5	1000	100
Kv=e1_n=21	21	10	10 000
Kv=e2_n=21	21	100	10 000
Kv=e3_n=11	10	1000	10 000
Kv=e3+n=21	21	1000	10 000

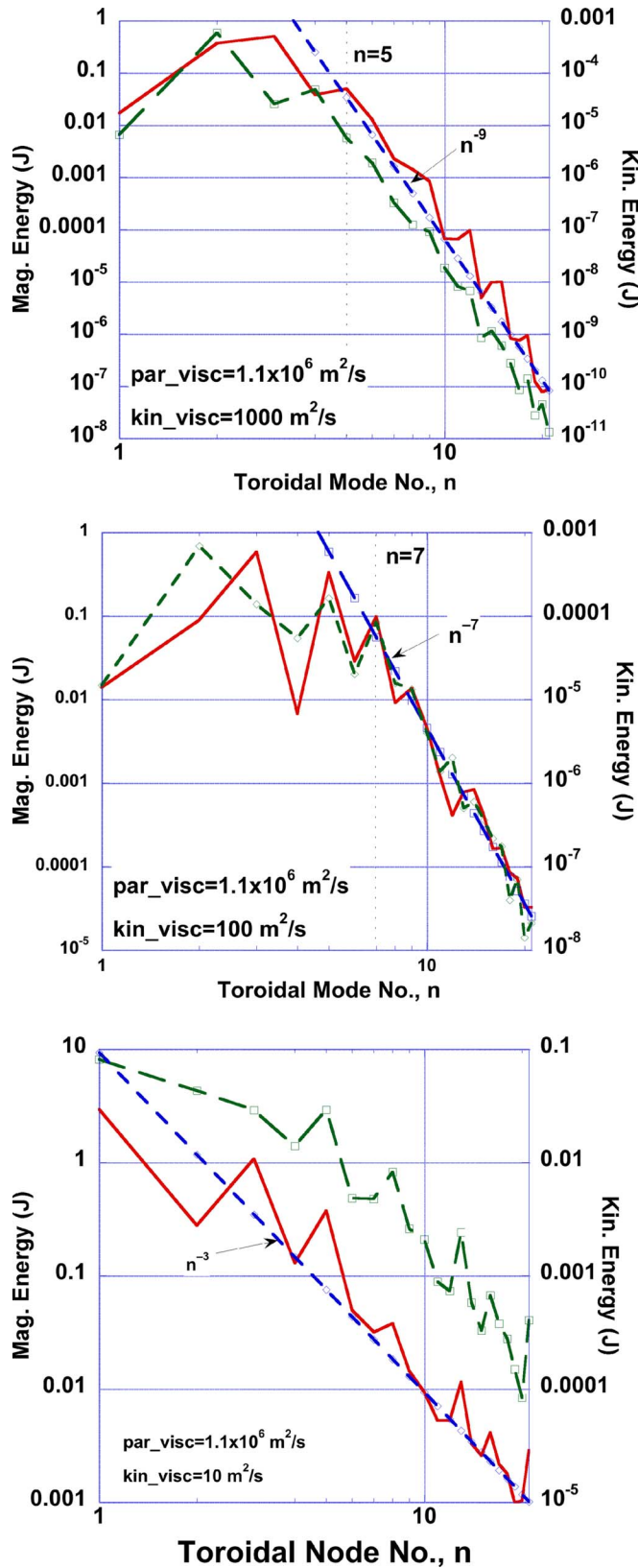


FIG. 18. (Color online) Mode amplitude scaling with toroidal mode number for the simulations shown in Fig. 16.

4–5 usually have small amplitudes. Parallel viscosity has quantitative effects, especially on the quality of mode surfaces and the electron temperature, but otherwise has little effect on the mean-field spheromak.

TABLE II. Parameter variations.

Parameter	Spheromak formation		Slowly decaying spheromak	
	Range	Section	Range	Section
Kinematic viscosity, $\nu - \text{m}^2/\text{s}$	$10^2 - 10^3$	II C	$10 - 10^3$	III A and III C
Diffusion, $D - \text{m}^2/\text{s}$	$10^4$	—	$10^2 - 10^4$	III A and III C
Perp. thermal cond., $\chi_{\perp} - \text{m}^2/\text{s}$	20	—	0–20	III B
Density, $n - \text{m}^{-3}$	0.15– $1.5 \times 10^{20}$	II E <sup>a</sup>	0.35– $1.0 \times 10^{20}$	III A
Eff. ionic charge, $Z_{\text{eff}}$	1	—	1–2.3	III A
Max. tor. mode no. $n_{\text{max}}$	0–10	II C	0–21	III C
Finite-element order, polynomial	3	—	3–4	III B

<sup>a</sup>Not discussed in detail.

#### IV. SUMMARY

In this paper, we have presented selected parameter studies of spheromak plasma formation and controlled decay using NIMROD simulations and have compared selected results to SSPX observations. The simulations illuminate the sensitivities of the physics with respect to variations in physical and numerical parameters. The parameter ranges examined for both the spheromak formation phase and the slowly decaying phase are listed in Table II, together with the section in which they are discussed. These sections also contain quantitative comparisons with experimental results.

The simulation results presented here and in previous publications show that magnetic fields and fluctuation amplitudes agree relatively well with SSPX observations, and temperature evolution data agree at least qualitatively with experimental behavior. There are significant differences between the formation and slowly decaying phases. During formation, the field lines are stochastic and reach the walls, at least most of the time. Because of the very strong dependence of the parallel thermal conductivity on electron temperature, its value is highly insensitive to the discharge parameters.<sup>22</sup> The values of the azimuthally averaged magnetic field, time-averaged gun voltage, etc. are also insensitive to the kinematic viscosity and maximum toroidal mode number (if  $\geq 5$ ). Detailed time histories are fairly sensitive to these parameters, however, as one might expect for a discharge in which repeated reconnection events are fundamental to generating the magnetic topology.

In the slowly decaying phase, on the other hand, the comparisons of the simulation temperature evolution are at best semiquantitative. That NIMROD predicts peak electron temperatures that are systematically smaller than the peak temperatures observed in the best SSPX shots suggests that the resistive MHD/collisional transport model in NIMROD is probably too simple and that extended MHD models should be explored, e.g., a model that includes kinetic closure modifications wherein limits on parallel heat conduction

can allow higher temperatures to be achieved, or a model that evolves both the ion and electron temperatures.

We are unable to demonstrate numerical convergence of the detailed evolution of high- $n$  symmetry-breaking fluctuations and their effect on plasma temperature during the quiescent phase of the spheromak evolution. The presence of some stochastic field lines and the close coupling of the system of governing equations contribute to the difficulty of achieving strict numerical convergence of all of the detailed time histories in the simulations. Similar effects may play a role in the observed shot-to-shot variations seen in the SSPX experiment. However, the information described in Sec. III B indicates that the MHD/collisional transport model predicts more fluctuation activity and less energy confinement than what is achieved in the experiment. The importance of flux-limiting effects beyond the scope of collisional transport is presently being investigated through integral closures.<sup>23,24</sup> The possibility of drift effects in the experiment leading to greater stability of the symmetry-breaking modes is a topic for future modeling efforts, and numerical development for Hall, gyroviscous, and magnetization heat-flow effects is underway. Relaxing the assumption of rapid electron-ion thermal equilibration is also worth investigation.

While discrepancies with respect to fluctuation histories and peak electron temperatures preclude full validation of the existing model, the simulations described here capture the most important features of the gross behavior of spheromak formation and controlled decay. In particular, the buildup of the azimuthally averaged field is fairly robust with respect to variations in plasma density, viscosity, and maximum toroidal mode number despite significant variations in detailed time histories of, e.g., reconnection events. These results suggest that spheromak field buildup is insensitive to the details of the physics in the reconnection layers, although more detailed physics studies with two-fluid and other models will be required to substantiate this.

Finally, it is important to emphasize that the NIMROD simulations and the SSPX experimental data are consistent on a fundamental level. When driving the spheromak with electrostatic helicity injection, by relaxing and tailoring the drive during partial decay, good magnetic surfaces form and energy confinement improves until such time as symmetry-breaking magnetic fluctuations degrade the surfaces and the confinement. The simulations and experimental results elucidate the interrelation of the current-profile evolution, the emergence of magnetic fluctuations that are associated with specific resonance surfaces that appear in the plasma, and the effects of the fluctuations on the quality of the magnetic surfaces and the energy confinement in what is a closely coupled system.

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