

*Geophysical Research Letter*

Supporting Information for

**Properties of electromagnetic fields generated by tsunami first arrivals:  
Classification based on the ocean depth**

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**Introduction**

[1] In the main text, we omitted the detail of the analytical solution in Eq. (2) for simplicity. This supporting material provides the derivation and detailed expression of Eq. (2).

23 **Text S1**

24 **EM fields generated by linear dispersive tsunamis**

25 [2] Here we derive an analytical solution of EM fields generated by linear dispersive  
26 waves (LDW), assuming a flat seafloor and a homogeneous half-space conductor  
27 beneath the seafloor. We assumed irrotational and incompressible tsunami flows and  
28 solve the Laplace equation of the stream potential,

$$\nabla^2 \Phi = 0, \quad (\text{S1})$$

29 with the following linear boundary conditions,

$$\frac{\partial \Phi}{\partial t} + g\eta = 0 \quad (z = 0), \quad (\text{S2})$$

$$\frac{\partial \eta}{\partial t} - \frac{\partial \Phi}{\partial z} = 0 \quad (z = 0), \quad (\text{S3})$$

$$\frac{\partial \Phi}{\partial z} = 0 \quad (z = -h). \quad (\text{S4})$$

30 Here,  $z = 0$  and  $z = -h$  correspond to the sea surface and the flat seafloor,  
31 respectively.  $\mathbf{u} = \nabla \Phi$  gives the oceanic velocity. Provided that  $\Phi \propto e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$ , and  
32 the initial phase of  $\eta$  is zero, the solution is written as:

$$\Phi = -iac \frac{\cosh(k(z + h))}{\sinh(kh)}, \quad (\text{S5})$$

$$\eta = a, \quad (\text{S6})$$

$$u_H = a\omega \frac{\cosh(k(z + h))}{\sinh(kh)}, \quad (\text{S7})$$

$$v_z = -ia\omega \frac{\sinh(k(z + h))}{\sinh(kh)}, \quad (\text{S8})$$

33 where  $e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$  is omitted for simplicity.  $c = \omega/k$  and  $a$  are the phase velocity and

34 the amplitude of the sea level change, respectively. Substituting Eq. (S7) into Eq. (1) in  
 35 the main text, the governing equation for  $b_z$  in the ocean layer is rewritten as:

$$\left(\frac{\partial^2}{\partial z^2} - \alpha^2\right)b_z = A \cosh(k(z+h)) \quad (-h \leq z \leq 0), \quad (\text{S9})$$

$$\text{where } A = \frac{F_z}{K} \frac{ia\omega k}{\sinh(kh)} \text{ and } \alpha = \sqrt{k^2 - i\omega\mu\sigma}.$$

36 Here  $\sigma$  is the homogeneous conductivity in the ocean layer. Adopting continuity of  $b_z$   
 37 and  $b_y$  at the ocean surface and bottom as boundary conditions, the vertical component  
 38 of the magnetic field generated by LDW are expressed as:

$$\begin{aligned} \frac{b_z}{F_z} = & \left\{ \frac{\sinh(\alpha z) - C_1 \cosh(\alpha z) - e^{kh}\{C_2 \cosh(\alpha(z+h)) + \sinh(\alpha(z+h))\}}{(1 + C_1 C_2) \sinh(\alpha h) + (C_1 + C_2) \cosh(\alpha h)} + \cosh(k(z+h)) \right\} \\ & \times \frac{k\eta}{\sinh(kh)} \quad (-h \leq z \leq 0), \end{aligned} \quad (\text{S10})$$

39 where  $C_1 = \alpha/k$ ,  $C_2 = \alpha/\alpha_e$ ,  $\alpha_e = \sqrt{k^2 - i\omega\mu\sigma_e}$ , and  $\sigma_e$  is the homogeneous  
 40 conductor beneath the ocean layer. At  $z = -h$ , Eq. (S10) is reduced to:

$$\frac{b_z}{F_z} = C_{LDW}(\omega, h, \sigma, \sigma_e) \times \frac{k\eta}{\sinh(kh)}, \quad (z = -h)$$

$$\text{where } C_{LDW}(\omega, h, \sigma, \sigma_e) = \frac{C_1 C_2 \sinh(\alpha h) + C_2 \cosh(\alpha h) - C_2 e^{kh}}{(1 + C_1 C_2) \sinh(\alpha h) + (C_1 + C_2) \cosh(\alpha h)}. \quad (\text{S11})$$