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The Impedance of a Rocket-Borne Capacitive Ionospheric Probe

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In a recent paper *Herman* [1964] obtained an expression for the expected impedance of a capacitive ionospheric probe as a function of electron density N , collision frequency ν , temperature T , magnetic field H , and rocket attitude γ with respect to the magnetic field. It was assumed, however, that the magnetic field was along the rocket's axis of symmetry. This restriction is easily removed.

The ratio of the probe capacitance C' to its free space value C_0 is given by

$$\frac{C'}{C_0} = \frac{\int (E_i \epsilon_{ij} E_j / |E|) dS}{\epsilon_0 \int |E| dS} \quad (1)$$

where

E_i = i th component of the electric field.

ϵ_{ij} = the complex dielectric tensor, $D_i = \epsilon_{ij} E_j$.

ϵ_0 = dielectric constant of free space.

From the Appleton-Hartree equations, the dielectric tensor can be written

$$\epsilon = \epsilon_0 + \frac{X}{U(Y^2 - U^2)} \begin{bmatrix} U^2 - Y_x^2 & -iUY_z - Y_xY_y & iUY_y - Y_xY_z \\ iUY_z - Y_yY_x & U^2 - Y_y^2 & -iUY_z - Y_yY_z \\ -iUY_y - Y_zY_x & iUY_z - Y_zY_y & U^2 - Y_z^2 \end{bmatrix} \quad (2)$$

where

$X = Ne^2/m\omega^2$

$Y = \mu_0 eH/m\omega$

$Z = \nu/\omega$

$U = 1 - iZ$

(all the symbols have their standard meaning). Then

$$(C'/C_0) = (1/2\epsilon_0)$$

$$\cdot \cos^2 \delta [\epsilon_{11} + \epsilon_{22} - 2\epsilon_{33}] + (\epsilon_{33}/\epsilon_0)$$

where cylindrical symmetry has been used, and δ is the half-angle of the nose cone.

Define γ as the angle between the rocket's symmetry axis and the magnetic field H . When H is taken in the y - z plane

$$\epsilon_{11} + \epsilon_{22} - 2\epsilon_{33} = \frac{XY^2}{U(Y^2 - U^2)} [3 \cos^2 \gamma - 1] \quad (3)$$

$$\epsilon_{33} = \epsilon_0 + \frac{X(U^2 - Y^2 \cos^2 \gamma)}{U(Y^2 - U^2)} \quad (4)$$

When we use these quantities, the real and imaginary parts of C'/C_0 are

$$\begin{aligned} \text{Re } \frac{C'}{C_0} &= 1 + \frac{A}{D} XY^2 \frac{\cos^2 \delta}{2\epsilon_0} (3 \cos^2 \gamma - 1) \\ &\quad - \frac{A}{D} \frac{X}{\epsilon_0} Y^2 \cos^2 \gamma \\ &\quad + \frac{A}{D} \frac{X}{\epsilon_0} (1 - Z^2) - \frac{B}{D} \frac{2ZX}{\epsilon_0} \end{aligned} \quad (5)$$

$$\begin{aligned} \text{Im } \frac{C'}{C_0} &= -\frac{B}{D} XY^2 \frac{\cos^2 \delta}{2\epsilon_0} (3 \cos^2 \gamma - 1) \\ &\quad + \frac{B}{D} \frac{X}{\epsilon_0} Y^2 \cos^2 \gamma \\ &\quad - \frac{B}{D} (1 - Z^2) X/\epsilon_0 - 2 \frac{A}{D} \frac{ZX}{\epsilon_0} \end{aligned}$$

where $A \equiv Y^2 - 1 + 3Z^2$, $B = (3 - Y^2 - Z^2)Z$, $D = A^2 + B^2$.

These can be associated with the capacitance

C and conductance G as measured by a bridge contained within the rocket; that is,

$$\frac{C'}{C_0} = \frac{C}{C_0} - i \frac{G}{\omega C_0} \quad (6)$$

Precisely the same result can be obtained if the calculation proceeds from

$$\epsilon_{ii} = \epsilon_{ii}' + \frac{1}{\omega} \sigma_{ii}' \quad (7)$$

where

$$D_i = \epsilon_{ii}' E_i$$

$$J_i = \sigma_{ii}' E_i$$

$$J_i = \text{current density}$$

Then

$$\frac{C}{C_0} = \frac{\int (E_i \epsilon_{ii}' E_i / |E|) dS}{\epsilon_0 \int |E| dS} \quad (8)$$

$$\frac{G}{C_0} = + \frac{\int (E_i \sigma_{ii}' E_i / |E|) dS}{\omega \epsilon_0 \int |E| dS} \quad (9)$$

Thus an expression has been obtained that permits the analysis of capacitive rocket probes for any attitude. The expression, of course, does not contain any information about possible ion

sheath effects. These must be computed separately and combined with the above values through an appropriate equivalent circuit of lumped parameters.

Clearly, from the form of (8) and (9) the analysis is not restricted to any particular dielectric tensor ϵ_{ii} , such as that derived from Appleton-Hartree theory. Using a more general dielectric tensor obtained from a paper by Sen and Wyller [1960], P. Crouse (private communication) has made numerical computations that show only small differences when compared with the Appleton-Hartree dielectric tensor. The major differences appeared around 50-km altitude (a region of high collision frequency) for the ionospheric models considered.

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