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Turbulent heating and temperature evolution in the solar wind plasma

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Abstract. We calculate the turbulent dissipation rate of incompressive hydromagnetic fluctuations and the resulting radial evolution of temperature in the solar wind using generalizations of Kolmogorov and Kraichnan MHD turbulence phenomenologies that include the suppression of dissipation by high cross helicity. The results for the temperature evolution are compared to a variety of data sets to test the phenomenologies over a wide parameter range. Motivated by the observations, we use different power laws in radius for the amplitudes of Alfvénic and nonAlfvénic fluctuations to determine the cascade rates. To explain the observations using Kolmogorov-like models, we found it necessary to suppress the dissipation rates for the high cross helicity streams even further than predicted by simple models; this may be due to the nonequilibrium nature of the spectrum or to other causes as yet unknown. The Kolmogorov-like model gives rise to a significant amount of turbulent heating, implying that turbulent heating, while likely dominant only in the inner heliosphere, may be competitive with heating by shocks and the assimilation of interstellar pickup ions in the outer heliosphere. In contrast, the generalized Kraichnan phenomenology yields less turbulent heating than the Kolmogorov-like model and seems inadequate to explain the observations. We conclude that while no existing turbulence model adequately explains the observed radial dependence of temperature in the solar wind, there appears to be sufficient energy available for turbulent heating to contribute significantly, even in the outer heliosphere.

1. Introduction

The proton temperature T of the solar wind, regarded here as a simple magnetofluid parameter, is observed to decrease systematically with increasing heliocentric distance r beyond the solar corona. The details of the form of this radial decrease depend on the type of flow, but generally the decrease is slower than that expected for an adiabatic gas, implying that some heating must be occurring. The adiabatic cooling, however, always dominates the heating. For a density variation of r^{-2} and a 5/3 polytropic index, the adiabatic temperature variation of $T(r) \propto r^{-4/3}$ (see section 2) represents a rather strong radial dependence, especially in the inner heliosphere.

Schwenn [1983], *Freeman and Lopez* [1985], and *Lopez and Freeman* [1986] studied the temperature evolution in the inner heliosphere using Helios data. They cal-

culated power law fits to $T(r) \propto r^{-\gamma}$ for solar wind in various velocity ranges and found γ to be between 0.78 and 1.33. They also found that during their transit, slow streams tended to cool adiabatically, while fast and intermediate streams were heated. However, a more recent analysis by *Freeman et al.* [1992] that corrected the handling of velocity gradients and included a better treatment of the nonuniform spatial distribution of data along the spacecraft orbit found γ to be approximately in the range of 0.80 to 1.00 for both fast and slow speed flows, indicating heating of the slow wind in contrast to the earlier results. In these analyses the index γ was obtained by averaging over streams in various velocity ranges. *Marsch and Richter* [1987] found that the specific entropy of both fast and slow streams grows with increasing distance, consistent with *Freeman et al.*'s [1992] conclusion that all the streams in the inner heliosphere are heated.

Fast streams in the inner heliosphere at solar minimum are often characterized by a very high cross helicity (a measure of the correlation between the fluctuating magnetic field and velocity field vectors and thus of "Alfvénicity"; see, for example, *Belcher and Davis*

[1971], *Matthaeus and Goldstein* [1982], *Roberts et al.* [1987a, b], and the review by *Marsch* [1990]). The sign of the correlation indicates that the fluctuations mainly propagate outward from the Sun. Slow streams at solar minimum, while they sometimes also contain high cross helicity fluctuations propagating outward, more typically contain fluctuations with smaller cross helicity and both inward and outward propagation. Away from solar minimum conditions, other factors, such as proximity to the heliospheric current sheet, are better correlated with Alfvénicity (see *Goldstein et al.* [1995] for a review), but the spectral characteristics of the Alfvénic flows are still similar to the fast flows when the Sun is relatively inactive (D. A. Roberts, manuscript in preparation, 1995). We will refer to the high cross helicity flows as being “Alfvénic” and the lower cross helicity flows as “non-Alfvénic,” although these are simple idealizations. Because both Alfvénic and non-Alfvénic streams exist in the inner heliosphere, we can conclude from the recent *Freeman et al.* [1992] observations that both Alfvénic and non-Alfvénic streams are heated.

Gazis [1984] has conducted a study of the temperature evolution in the outer heliosphere within 10 AU using Voyager 1 data and reported that streams having a variety of velocities all had approximately similar temperature gradients, although the faster ones were hotter. He concluded that γ was close to 0.7 for all streams. This results may hold much farther out as well [*Gazis et al.*, 1994], although the uncertainties in the observations become large. Because non-Alfvénic streams dominate over Alfvénic streams in the outer heliosphere, the *Gazis* results primarily apply to non-Alfvénic streams. In contrast, *Tu* [1987] concluded from his study of the temperature evolution of an Alfvénic stream in the outer heliosphere that there was no significant heating.

Various mechanisms have been invoked for heating the solar wind, but none appear to account satisfactorily for the observed temperature of the wind. Because corotating interaction regions do not generally form shocks inside of 1 – 2 AU, shocks will not contribute significant heating (in the sense of entropy generation) in the inner heliosphere, although they may well make important contributions beyond a few AU. In the outer heliosphere, *Goldstein and Jokipii* [1977] and *Whang et al.* [1989, 1990] have studied the importance of shock heating. *Whang et al.* [1990] calculated the cumulative effects of the corotating shocks and found that $T(r) \propto r^{-0.53}$ in the outer heliosphere. However, *Burlaga* [1994] concluded that because few shocks were observed between 18.9 – 30.2 AU during 1986 – 1989, heating from shocks could not explain the slower than adiabatic fall-off in proton temperature in the far outer heliosphere.

The pickup of interstellar neutrals by the solar wind is another potentially important heat source as concluded by *Isenberg et al.* [1985]. Although compressions that are precursors to shocks will increase the plasma temperature locally, this is a reversible, adiabatic increase,

offset by cooling in rarefactions; true heating by compressions does not occur until shocks form. The electron temperature varies only very slowly with r due to a strong heat conduction from the Sun, but the coupling between electrons and protons outside the corona is probably too weak to provide significant proton heating [*Marsch et al.*, 1983]. Broadband Landau and transit-time damping [*Barnes*, 1979] associated with compressional coupling due to the tendency for wave vectors to remain radial while the mean field spirals in the outer heliosphere are too small to contribute significant heating [*Hollweg and Lilliequist*, 1978]. The “saturation” of the fluctuations as b_{rms}/B increases is neither observed in spacecraft data [*Roberts*, 1989] nor in simulations [e.g., *Roberts and Wiltberger*, 1995], which implies that the “saturation” mechanism also cannot provide enough energy to heat the solar wind plasma.

These considerations leave turbulent heating due to a nonlinear cascade of energy to small wavenumbers, where cyclotron damping occurs, as a major candidate for the proton heating mechanism in the inner heliosphere and perhaps also in the outer heliosphere. The main point of this paper is to investigate the consequences of turbulence models for proton heating. These calculations are in the same spirit as those of *Coleman* [1968], *Tu et al.* [1984], and *Tu* [1988] but will generalize the work of these authors to lay the groundwork for more detailed models.

Our philosophy is to try to estimate the extent to which turbulent heating can account for the solar wind observations. Because of the quantitative uncertainties present in all of the heating scenarios, we will be unable to determine just what the primary mechanism is, but we shall show that turbulent heating is likely to be competitive with other processes. Our approach is to calculate the turbulent dissipation in the solar wind using a Kolmogorov-like model [*Matthaeus and Zhou*, 1989; *Zhou and Matthaeus*, 1990b; *Marsch*, 1990] and Dobrowolny et al.’s generalized Kraichnan model [*Kraichnan*, 1965; *Dobrowolny et al.*, 1980; *Grappin et al.*, 1983]. We will emphasize the generalized Kolmogorov phenomenology because its predictions seem most consistent with the observed spectral index of $-5/3$ for both “inward” and “outward” propagating Alfvén wave energy spectra in the solar wind (see *Matthaeus and Goldstein* [1982] and *Marsch and Tu* [1990], but see discussion below), and because recent simulations show that these models yield more correct cascade rates [*Pontius et al.*, 1993; *Verma et al.*, 1993]. The approach we are taking is a generalization of that of *Tu* [1988], which we apply to a variety of data sets to test the ideas over a broad parameter range.

The organization of this paper is as follows: In section 2, an expression for the radial evolution of temperature for the solar wind is derived for arbitrary heating sources. In section 3 we describe briefly the dissipation rates in the existing MHD turbulence phenomenologies. In section 4 we describe the method and assumptions of

our analysis. We consider the heating of non-Alfvénic and Alfvénic wind separately because of their different evolution. In section 5 we use Kolmogorov-like MHD turbulence phenomenology to calculate turbulent heating and temperature evolution in the solar wind, and in section 6 we use Dobrowolny et al.'s generalized Kraichnan model to calculate the evolution. Section 5 also compares our results with those of *Tu* [1988]. Section 7 contains a discussion and summary.

2. Variation of Temperature and Entropy With Distance

In this section we derive an expression for the temperature evolution of the wind taking into account dissipation and the radial expansion. If N and M are the number of moles and the mass of the wind in volume V as shown in Figure 1, then from the first law of thermodynamics

$$NC_V dT = M\epsilon dt - p_{th} dV \quad (1)$$

where ϵ is the dissipation rate per unit mass, C_V is the specific heat at constant volume, T is the temperature, and p_{th} is the thermal pressure of the wind. In time dt the solar wind will travel a radial distance $dr = Udt$. For the volume element V , dV/dr is given by

$$\frac{1}{V} \frac{dV}{dr} = \frac{2}{r} \quad (2)$$

so that (1) becomes

$$\frac{dT}{dr} + \frac{2R}{C_V} \frac{T}{r} = \frac{\Sigma}{UC_V} \epsilon(r, U) \quad (3)$$

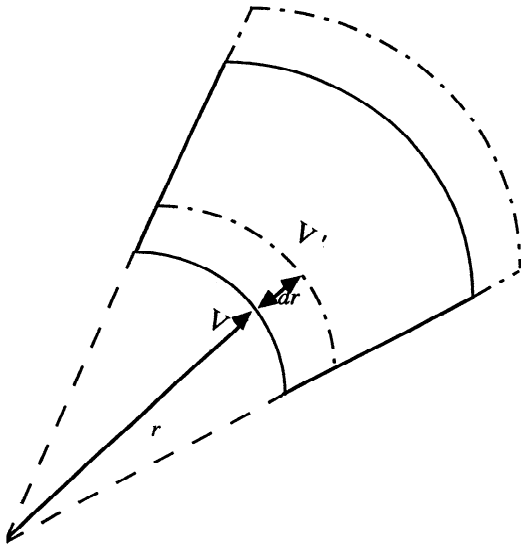


Figure 1. The volume V at a distance r expands to V' . The element travels a distance $dr = Udt$ distance in time dt .

where Σ is the mass per unit mole (≈ 1 gm for solar wind plasma). Equation (3) has the immediate solution

$$T(r, U) = \left(\frac{r}{r_0} \right)^{-4/3} \times \left[T_0(U) + \frac{\Sigma}{UC_V} \int_{r_0}^r \left(\frac{s}{r_0} \right)^{4/3} \epsilon(s, U) ds \right] \quad (4)$$

where $T_0(U)$ is the temperature at r_0 for a stream with velocity U . From (4) we can determine the temperature evolution of the solar wind given the radial dependence of ϵ , and $T_0(U)$ and $\epsilon(U)$ on a boundary. We have assumed that the speed of the stream is constant, which implies the $\rho \propto r^{-2}$. We make this assumption to simplify the analysis even though the density is observed to vary somewhat differently. However, this simplification breaks down for the Alfvénic stream analyzed in the outer heliosphere. There $\rho \propto r^{-2.79}$ [Bavassano and Smith, 1986; Tu, 1987]; for that case we modify our analysis accordingly.

For the empirical boundary conditions of non-Alfvénic streams, we selected streams with uniform speed and density to ensure that we are studying the evolution of a single stream and not a mixture of many streams. We selected four streams at 1 AU during low solar activity from the OMNItape spacecraft data set available from the National Space Science Data Center. These streams had speeds of 372, 439, 578, and 650 km/s. For Alfvénic conditions, we followed particular streams using Helios data [Bavassano et al., 1982] for the inner heliosphere and the results of Bavassano and Smith [1986] for the outer heliosphere. (We would have used Helios data for recurring non-Alfvénic streams, but the temperature fluctuations in these regions were too high to form meaningful radial evolution profiles for particular streams.)

Finally, we derive the expression for entropy increase due to heating. From thermodynamics

$$T ds^* = \epsilon dt \quad (5)$$

where $s^* = c_V^* \ln(p_{th}/\rho^\zeta)$, c_V^* is the specific heat per unit mass, and $\zeta = C_P/C_V$ [Priest, 1982; Siscoe, 1983]. Using $dr = Udt$, we obtain

$$T \frac{ds^*}{dr} = \frac{\epsilon}{U} \quad (6)$$

which implies that dissipation produces an increase of entropy, as expected.

3. Cascade and Dissipation in MHD Turbulence Phenomenologies

We can obtain the temperature evolution due to turbulent heating upon substituting the turbulent dissipation rate and initial temperature $T_0(r_0)$ into (4) under the assumption that turbulent dissipation is the sole

source of heat. The turbulent dissipation can then be calculated once ϵ is estimated from the existing incompressible MHD turbulence models described briefly below. (For additional details see *Matthaeus and Zhou* [1989], *Zhou and Matthaeus* [1990b], *Marsch* [1990], *Kraichnan* [1965], and *Dobrowolny et al.* [1980].) In the following, we use the Elsässer variables, $\mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b}$, where \mathbf{v} is the bulk velocity, $\mathbf{b} = \mathbf{B}/(4\pi\rho)^{1/2}$ is the magnetic field in Alfvén speed units, and ρ is the mass density. We define $E^\pm = (\mathbf{z}^\pm)^2/2$ as the (ideally conserved) “pseudo-energies” of the Elsässer fields, which gives a total kinetic plus magnetic energy $E = (E^+ + E^-)/2$. Note that pure Alfvén waves have \mathbf{z}^+ or $\mathbf{z}^- = 0$, depending on the propagation direction of the wave with respect to the mean field. We shall choose “plus” to always indicate the species propagating outward from the Sun.

All of the turbulence phenomenologies make the following assumptions:

1. The physics in the inertial range depends neither on the large (energy containing) scales, nor on the small (dissipation) scales, that is, the physics is scale-invariant.

2. The energy distribution is isotropic at the intermediate scales. This assumption is suspect due to the presence of the magnetic field. Simulations and solar wind observations indicate that the fluctuations are, in fact, not isotropic [*Belcher and Davis*, 1971; *Matthaeus et al.*, 1986, 1990].

3. The interaction is local in wavenumber space. (This assumption has recently been supported in the hydrodynamic case by *Zhou* [1993a, b].)

4. The energy fluxes or cascades Π^\pm of “outward” (\mathbf{z}^+) and “inward” (\mathbf{z}^-) propagating Alfvén waves flow through the inertial range from the energy-containing scales until they reach the dissipation range where they are damped. Furthermore, the fluxes are independent of wavenumber in the inertial range and therefore equal the dissipation rates ϵ^\pm . The energy spectra E^\pm of \mathbf{z}^\pm in the inertial range depend only on the local quantities k and the energy fluxes.

Kolmogorov-like MHD Turbulence Phenomenology

The generalized Kolmogorov phenomenology additionally assumes [*Matthaeus and Zhou*, 1989; *Zhou and Matthaeus*, 1990b; *Marsch*, 1990] is that the mean magnetic field or the magnetic field of the largest eddies in velocity units (C_A) is much less than the amplitudes of the Alfvénic fluctuations in the inertial range. When these assumptions are satisfied, the dissipation rates ϵ^\pm of the “outward” and “inward” propagating Alfvénic fluctuations are

$$\epsilon^\pm = C^{-3/2} [E^\pm(k)] [E^\mp(k)]^{1/2} k^{5/2} \quad (7)$$

where k is wavenumber and C is analogous to Kolmogorov’s constant C_K of fluid turbulence.

From (7) we find

$$\frac{E^-(k)}{E^+(k)} = \left(\frac{\epsilon^-}{\epsilon^+} \right)^2 \quad (8)$$

which implies that the majority species in the inertial range has a higher dissipation rate than does the minority species. For convenience, we write the dissipation rates in terms of a parameter α_1 , defined as the ratio of $E^-(k)$ and $E^+(k)$. Then (7) can be rewritten as

$$\epsilon^+ = C' \sqrt{\alpha_1} [E^+(k)]^{3/2} k^{5/2} \quad (9)$$

$$\epsilon^- = C' \alpha_1 [E^+(k)]^{3/2} k^{5/2} \quad (10)$$

where $C' = C^{-3/2}$. The quantity α_1 is related to the normalized cross helicity or Alfvénicity ($\sigma_c = (E^+ - E^-)/(E^+ + E^-)$) by

$$\alpha_1 = \frac{1 - \sigma_c}{1 + \sigma_c} \quad (11)$$

Note that pure Alfvén waves ($\alpha_1 = 0$) do not dissipate because there are no nonlinear interactions.

The total dissipation rate ϵ is $(1/2)(\epsilon^+ + \epsilon^-)$. Using (9) and (10) we find that, in terms of $E^+(k)$, ϵ is

$$\epsilon = C' \frac{\alpha_1 + \sqrt{\alpha_1}}{2} [E^+(k)]^{3/2} k^{5/2}. \quad (12)$$

In frequency space, using the frozen-in hypothesis [*Taylor*, 1938] $f = (U/2\pi)k$, this becomes

$$\epsilon = C^{-3/2} \frac{\alpha_1 + \sqrt{\alpha_1}}{2} [E^+(f)]^{3/2} f^{5/2} \left(\frac{2\pi}{U} \right) \quad (13)$$

In section 5 we use this expression to obtain heating rates of solar wind streams for the generalized Kolmogoroff phenomenology using the observed values of $E^+(f)$ and α_1 . It follows from (13) that in the Kolmogorov-like phenomenology if the energy levels of the majority species of Alfvénic and non-Alfvénic streams are equal, then the Alfvénic stream will have the lower dissipation rate.

The energy spectra $E^\pm(k)$ from the above development both have $-5/3$ power law exponents for constant dissipation rates. These spectral slopes are observed often in the solar wind (especially in non-Alfvénic cases) [*Matthaeus and Goldstein*, 1982; *Marsch and Tu*, 1990], consistent with the Kolmogorov model, which is somewhat surprising because the ratio of the amplitudes of the largest fluctuations (10 to 20 km/s) to C_A (40 to 50 km/s) is only 0.2 to 0.5. This apparent contradiction is still unresolved. One possible resolution is that the intermittency of the fluctuations increases the observed exponent compared to an “underlying” exponent of $-3/2$ expected from Kraichnan’s MHD model [e.g., *Carbone*, 1993; *Ruzmaikin et al.*, 1995], although the analyses of *Burlaga* [1993, and references therein] and *Marsch and Liu* [1993] show that the situation is at least

more complex than implied by the cases analyzed by *Ruzmaikin et al.* [1995]. Numerical simulations [*Verma et al.*, 1993] show that for small σ_c , the numerical results for the flux of energy in the inertial range are consistent with the predictions of the Kolmogorov-like model for the ratio z_{max}^\pm/C_A in the (observed) range of 0.2 to 0.5. Moreover, for high cross helicity, while no model worked very well, the Kraichnan-like model discussed below gave very inaccurate results. Similar conclusions were reached by *Pontius et al.* [1993] for the decay of the energy containing eddies. Thus we tend to believe that the above Kolmogoroff-like phenomenology is closer to the truth than the Kraichnan-like one developed below, but we analyze both here for further insight on this issue.

Kraichnan's Phenomenology and Dobrowolny et al.'s Generalization

Kraichnan [1965] assumed that the mean magnetic field or the magnetic field of the largest eddies C_A is greater than the amplitudes of the fluctuations. With this assumption,

$$\epsilon = A^{-2} C_A^{-1} [E_b(k)]^2 k^3 \quad (14)$$

where E_b is the magnetic energy and A is a constant. *Dobrowolny et al.* [1980] generalized this model by accounting for the dissipation of the z^+ and z^- wave fluxes independently. The dissipation rates can then be written as

$$\epsilon^+ = \epsilon^- = A^{-2} \frac{1}{C_A} E^+(k) E^-(k) k^3 \quad (15)$$

In section 6, we use this expression to calculate the temperature evolution in the solar wind. This will be termed the generalized Kraichnan or Kraichnan-like case.

Matthaeus and Zhou's Model

Matthaeus and Zhou [1989] and *Zhou and Matthaeus* [1990b] developed a generalization of both Kolmogorov-like and Kraichnan-like models. In their model the dissipation is given by

$$\epsilon^\pm = \frac{A^2 E^+(k) E^-(k) k^3}{C_A + \sqrt{k E^\pm(k)}} \quad (16)$$

When $E^+ \approx E^-$, *Matthaeus and Zhou* [1989] showed that the small wavenumbers ($k E^\pm(k) \gg C_A$) have a 5/3 energy spectrum, whereas large wavenumbers ($k E^\pm(k) \ll C_A$) have a 3/2 energy spectrum. The break point k_b between 5/3 and 3/2 spectrum can be estimated using $k_b E(k_b) \approx C_A$, which for the solar wind at 1 AU is approximately 10^{-6} km^{-1} . However, in this region of the energy spectrum the spectral index is more often -1 , and at higher wavenumbers the observations tend to show $-5/3$ rather than $-3/2$ spectra. Hence it does not appear that the Zhou and Matthaeus model

works in the solar wind near (and inside of) 1 AU. For this reason we have not used it in this study. The models we do consider are limiting cases of the Zhou and Matthaeus model, but we apply them outside the limits of their expected range of validity to gain insight into what form of dissipation may be relevant to the observations independent of the simple scaling arguments that motivated their derivation.

4. Modeling Temperature Evolution

The radial dependence of the dissipation $\epsilon(r, U)$, derived from the amplitude evolution of solar wind fluctuations, is one of the inputs to (4). We separate our study into Alfvénic and non-Alfvénic streams, because the amplitude and spectral evolution of streams appears to be more dependent on their Alfvénicity, than it is on stream speed. *Roberts et al.* [1987b] noted that fast streams are not necessarily Alfvénic, nor are slow streams necessarily non-Alfvénic. This observation and the prediction that the dissipation rate depends critically on the Alfvénicity of the stream motivates our categorization. In contrast, earlier temperature evolution studies [*Freeman et al.*, 1992; *Gazis*, 1984] grouped solar wind streams according to whether they were fast or slow.

The non-Alfvénic streams appear to be in a quasi-steady state beyond 0.3 AU. Consequently, the energy entering and leaving modes in the inertial range will be approximately equal while both decline due to expansion. In this sense the inertial range fluctuations of non-Alfvénic streams are “adiabatic” which probably accounts for the observed fact that their amplitude evolution is WKB-like [*Roberts et al.*, 1990; *Zhou and Matthaeus*, 1989; *Zhou and Matthaeus*, 1990a; *Verma and Roberts*, 1993], that is $E_b \approx \langle b^2 \rangle / \rho \propto r^{-1}$.

In contrast, the Alfvénic streams, especially inside of 1 AU, are still evolving and have yet to reach a steady state. The radial evolution of their fluctuation amplitudes at inertial range wavenumbers was found for the particular cases to be studied here to be $E^+ \propto r^{-2.2}$ and $r^{-1.5}$ in the inner and outer heliosphere, respectively [*Bavassano et al.*, 1982; *Bavassano and Smith*, 1986; *Tu et al.*, 1984; *Tu*, 1988].

In the following sections we use the radial variation of the amplitudes of the Alfvénic and non-Alfvénic streams to estimate the dissipation rates from (13) or (15) for use in (4). Unknown in this procedure are the value of the constants C and A . These are generally taken to be ~ 1 [*Matthaeus and Zhou*, 1989; *Zhou and Matthaeus*, 1990b], but there is no strong restriction placed on them from scaling arguments. However, in a numerical simulation using $\sigma_c = 0.25$ we found $C \approx 3.6$ [*Verma et al.*, 1993]. One of the purposes of the present study is to test whether or not the temperature evolution predicted by the models for this range of C ($\approx 1 - 4$) agrees with observations.

5. Temperature Evolution Using Kolmogorov-like Phenomenology

Temperature Variation of Non-Alfvénic Streams

In this section we derive the temperature variation of non-Alfvénic solar wind streams. As noted above, the amplitudes of non-Alfvénic inertial range fluctuations vary with heliocentric distance as

$$E(k, r) = E_0(k) \left(\frac{r_0}{r} \right), \quad (17)$$

with $E^+ \approx E^- \approx E$. Therefore, from (13), the radial dependence of $\epsilon(r)$ is

$$\epsilon(r) = \epsilon_0 \left(\frac{r_0}{r} \right)^{3/2} \quad (18)$$

When (18) is substituted into (4), we find that the radial evolution of temperature is

$$T(r, U) = \left(\frac{r}{r_0} \right)^{-4/3} \left[T_0(U) + \frac{\epsilon_0 r_0}{UC_V} \frac{6}{5} \left\{ \left(\frac{r_0}{r} \right)^{5/6} - 1 \right\} \right] \quad (19)$$

We estimated ϵ_0 and T_0 using the hourly averaged plasma and magnetic field data of 1975 and 1976 obtained from the OMNI tape. We identified streams in the data, then found T_0 by averaging the temperature data over the stream interval. Computation of ϵ_0 requires knowledge of α_1 , $E^+(k)$, and C_K . To find $\alpha_1 = E^-(k)/E^+(k)$ we use fast Fourier transforms to compute $E^+(k)$ and $E^-(k)$. The non-Alfvénic streams, by definition, have α_1 close to unity. The specific heat at constant volume, C_V , is 12.5 Joule K⁻¹ mole⁻¹. The total dissipation rates for the streams were found using (13) and (19) to obtain $T(r)$ in the range of 0.3 AU to 100 AU. The best match to the observations of *Freeman et al.* [1992] and *Gazis* [1984] were found when $C = 1.0$. Figure 2 shows the predicted temperature evolution of these streams for $C = 1.0$ and Table 1 lists the temperature gradients for both $C = 1.0$ and $C = 1.5$.

The temperature gradients of the streams with $U_0 = 372$, 439, and 578 km/s (for which we assumed that

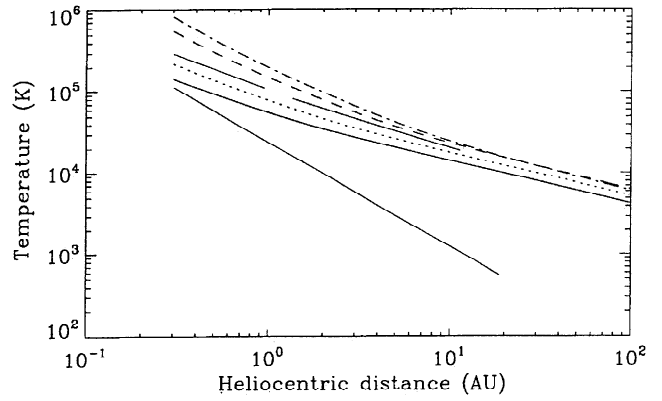


Figure 2. $T(r)$ versus r as predicted by the Kolmogorov-like phenomenology for the non-Alfvénic streams with speed of 372 (solid), 439 (dotted), 578 (dashed), and 650 (chained) km/s ($C = 1.0$). The straight line with slope $-4/3$ represents adiabatic cooling. The two line segments have the observed slopes of 0.9 and 0.7 in the distance intervals 0.3–1.0 and 1–10 AU, respectively.

$\sigma_c = 0$) are all consistent with the observed values of 0.8 to 1.0 for the inner heliosphere [*Freeman et al.*, 1992] and about 0.7 for 1–10 AU [*Gazis*, 1984]. The predicted evolution may also be consistent with the evolution out to 40 AU or so, given the uncertainties involved [*Gazis et al.*, 1994]. For these three streams, our model predicts that on the average $T(r) \propto r^{-0.91}$ from 0.3–1 AU, $T(r) \propto r^{-0.67}$ from 1–10 AU, and $T(r) \propto r^{-0.53}$ for 10–100 AU. For the fourth stream, for which $U = 650$ km/s, we used $\sigma_c = 0.33$ on the assumption that fast streams tend to be more Alfvénic. This stream has somewhat higher temperature gradients in the three regions, $\gamma = 1.16, 0.90$, and 0.61, respectively.

These results indicate that as one moves outward from the Sun the temperature gradient becomes less steep. For $r \gg r_0$, (19) yields $\gamma \rightarrow 0.5$, which is the minimum allowable value. As expected, the computed values of the temperature gradients depend on the value of C because it controls the dissipation rate. For $C = 1.5$, the value of Kolmogorov's constant in fluid turbulence,

Table 1. Predicted Temperature Gradients γ by the Kolmogorov-like Phenomenology for non-Alfvénic Streams

C	U , km/s	$10^{-5}T_0$, K	α_1	$10^3\epsilon$, km ² /s ³	γ		
					0.3 – 1 AU	1 – 10 AU	10 – 100 AU
1.0	372	0.56	1.0	1.1	0.78	0.59	0.51
	439	0.78	1.0	1.6	0.87	0.64	0.52
	578	1.50	1.0	2.5	1.08	0.80	0.57
	650	2.00	0.5	2.6	1.16	0.90	0.61
1.5	372	0.56	1.0	0.6	1.07	0.79	0.56
	439	0.78	1.0	0.9	1.11	0.83	0.58
	578	1.50	1.0	1.3	1.20	0.97	0.64
	650	2.00	0.5	1.4	1.24	1.05	0.70

we find $T(r) \propto r^{-1.13}$ for $0.3 - 1$ AU, $T(r) \propto r^{-0.86}$ for $1 - 10$ AU, and $T(r) \propto r^{-0.59}$ for $10 - 100$ AU (see Table 1). In this case turbulent heating is less than the observed heating. For $C = 3.6$, the value derived from the simulations of Verma *et al.* [1993], there is significantly less turbulent heating. However, because $C = 3.6$ was estimated using just one low resolution 3-D MHD simulation, we do not know how representative it is. Further high-resolution simulations are required to make definite predictions. It is possible that C may be greater than 1.0 and that the additional dissipation required might be provided, at least in the outer heliosphere, by shocks, and interstellar pickup ions. If the inner heliospheric flows are nearly adiabatic, as some early studies implied, then the dissipation associated with $C = 3.6$ would be adequate, but we believe that the more recent observational studies are probably more accurate.

Temperature Variation of Alfvénic Streams

In this section we study the temperature evolution of two Alfvénic streams, one in the inner and the other in the outer heliosphere. The two examples we study have been discussed previously by Bavassano *et al.* [1982] and Bavassano and Smith [1986]. Both of these flows were located within the trailing edges of high speed streams.

Inner Heliosphere Bavassano *et al.* [1982] studied the spectral and amplitude evolution of an Alfvénic stream in the inner heliosphere using Helios data. They found that the magnetic fluctuations did not evolve as expected from WKB theory; instead the inertial range wave energy varied as $r^{-2.2}$. The steeper amplitude evolution compared to non-Alfvénic streams is due to the dissipation and nonequilibrium spectral evolution of the stream towards a steady state which, presumably, would result in a $k^{-5/3}$ energy spectrum [Tu *et al.*, 1984; Tu, 1988]. Tu [1988], using a model also motivated by turbulence theory, was able to account for the observed spectral evolution and also calculated the temperature variation of this stream. The differences between Tu's model and our own will be described in the next subsection. We first calculate the temperature variation expected from our model.

For an Alfvénic stream $E^+(k) \approx 4E_b(k)$ and $E^-(k) \approx 0$. Therefore $E(k) \approx 2E_b(k)$. Since $E_b(k) \propto r^{-2.2}$ [Bavassano *et al.*, 1982], we obtain

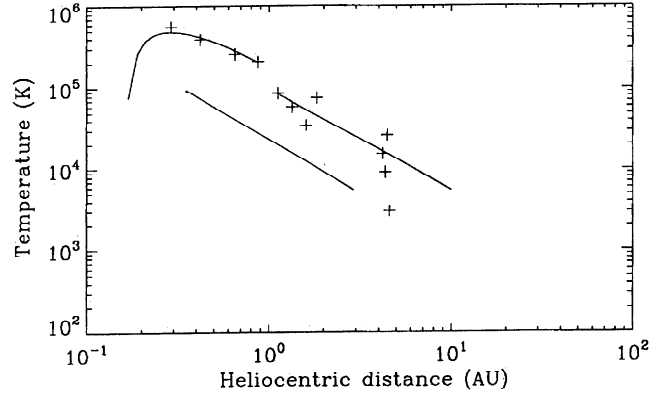


Figure 3. $T(r)$ versus r as predicted by the Kolmogorov-like phenomenology for the Alfvénic stream with speed of 640 km/s (inner heliosphere) and 500 km/s (outer heliosphere) ($C = 8.0$). The observed temperature of these streams at various distances are shown by “plus”. The straight line with slope $-4/3$ represents adiabatic cooling.

$$E(k, r) = E_0(k) \left(\frac{r_0}{r} \right)^{2.2} \quad (20)$$

The radial dependence of ϵ follows from (13) and is given by

$$\epsilon(r) = \epsilon_0 \left(\frac{r_0}{r} \right)^{3.3} \quad (21)$$

From (4) we then find the temperature variation in the inner heliosphere to be

$$T(r, U) = \left(\frac{r}{r_0} \right)^{-4/3} \left[T_0(U) + \frac{\epsilon_0 r_0}{UC_V} \left\{ 1 - \left(\frac{r_0}{r} \right) \right\} \right] \quad (22)$$

We choose ϵ_0 and T_0 at 0.87 AU. T_0 can be estimated by averaging the hourly temperature data over the time period the stream was observed. The observations give $\alpha_1 = 0.07$ [Tu *et al.*, 1989]. The spectrum $E^+(k)$ at r_0 was calculated from $E_b(k)$ of Bavassano *et al.* [1982], and $E^+(k)$ was used to calculate the total dissipation rate using (13). The temperature evolution $T(r)$ in the range of $0.07 - 0.87$ AU is then obtained from (22). The same stream was encountered several times by Helios spacecraft at $r = 0.29, 0.41, 0.65$, and 0.87 AU. The average temperature of the stream at these distances is indicated by “plus” in Figure 3, which illustrates the predicted temperature evolution. The temperature gradients are listed in Table 2. The average temperature gradient in the inner heliosphere is 0.90.

Table 2. Predicted Temperature Gradients γ of the Alfvénic Streams Using the Kolmogorov-like Phenomenology

C	U , km/s	$10^{-5}T_0$, K	α_1	$10^3\epsilon$, km ² /s ³	γ		
					0.3 – 1 AU	1 – 10 AU	10 – 100 AU
8.0	640	2.10	0.07	3.0	0.89	–	–
	500	0.87	0.07	0.3	–	1.28	1.28

In the Figure 3 we see that inside of 0.27 AU, the temperature decreases with decreasing distance, becoming negative at 0.15 AU. The reason for this unphysical behavior is that in our formalism, the dissipation rate increases rapidly ($r^{-3.3}$) as we move toward the Sun. In reality the turbulent dissipation should decrease both due to the decrease in α_1 and due to the increasing value of the Parker field compared to the fluctuation level. It is possible for α_1 to become so small that dissipation virtually vanishes at some point. To compute the temperature evolution for a variable α_1 is beyond the scope of this paper.

We obtained the best agreement with the observations using $C = 8.0$. This large value acts to suppress the dissipation by more than could be accounted for by cross helicity alone. We believe the reason for this suppression is that the turbulent cascade is yet not fully developed and the energy is not cascading from the large scales to small scales as rapidly as it does in a steady state situation. The generalized Kraichnan phenomenology predicts lower dissipation than does the Kolmogorov-like and may predict the correct behavior. We will discuss this possibility in section 6.

Outer Heliosphere *Bavassano and Smith* [1986] studied the evolution of an Alfvénic stream in the outer heliosphere using data from Pioneers 10 and 11. They found that the evolution of the inertial range fluctuations was not adiabatic but rather varied as

$$E(k, r) = E_0(k) \left(\frac{r_0}{r} \right)^{1.5} \quad (23)$$

Consequently, from (13) the radial dependence of ϵ is

$$\epsilon(r) = \epsilon_0 \left(\frac{r_0}{r} \right)^{2.25} \quad (24)$$

so that (4) becomes

$$T(r, U) = \left(\frac{r}{r_0} \right)^{-4/3} \left[T_0(U) + \frac{\epsilon_0 r_0}{UC_V} \ln \left(\frac{r}{r_0} \right) \right] \quad (25)$$

We use Pioneers 10 and 11 data for this study and take the spectrum and T_0 at a 1.1 AU boundary from *Bavassano and Smith* [1986]. Again, we take $\alpha_1 = 0.07$ and $C = 8.0$. Substituting into (25), we obtain $T(r)$ for this stream between 1.1 and 10 AU. The mean temperature of this stream as encountered by Pioneers 10 and 11 at later times is shown in Figure 3 by “plus”. The scatter in the data makes it difficult to determine quantitatively whether or not $C = 8.0$ is an optimal fit.

The average temperature gradient for this stream is 1.26 (Table 2), which is consistent with the results of *Tu* [1987] and indicates that no significant heating occurs in this stream in the outer heliosphere. This contrasts with the result in the inner heliosphere. Note that if we substitute the observed density variation of $\rho \propto r^{-2.79}$, we find the temperature gradient is 1.23, which is essentially the same as the value given above.

Comparison With Tu’s Model *Tu et al.* [1984] models the dissipation using arguments similar to those of Kraichnan, while *Tu* [1988] uses a Kolmogorov-like phenomenology. Here we compare *Tu*’s [1988] model with ours.

While *Tu* assumed that α_1 was small, that is unnecessary, as shown in section 3. Because *Tu* gives both the energy density and the dissipation rates F per unit volume, we convert *Tu*’s dissipation rates to the dissipation rates per unit mass, that is, $\epsilon_{Tu} = F/(4\pi\rho)$ so that (see (11) of *Tu* [1988])

$$\epsilon_{Tu} = \alpha\alpha_1 \left[\frac{P_B(f)}{4\pi\rho} \right]^{3/2} f^{5/2} \left(\frac{2\pi}{U} \right) \quad (26)$$

where $P_B(f)$ is the spectrum of the magnetic field and α is a constant that has been determined from Helios data by *Tu et al.* [1989].

To compare the dissipation rates from the two models, we take the small α_1 limit of (13), which yields

$$\epsilon \approx C^{-3/2} \frac{\sqrt{\alpha_1}}{2} [E^+(f)]^{3/2} f^{5/2} \left(\frac{2\pi}{U} \right) \quad (27)$$

For Alfvénic streams $E^+(f) \approx 2P_B(f)/(4\pi\rho)$, so that the dissipation rate is given by

$$\epsilon = C^{-3/2} \sqrt{2\alpha_1} \left[\frac{P_B(f)}{4\pi\rho} \right]^{3/2} f^{5/2} \left(\frac{2\pi}{U} \right) \quad (28)$$

The main difference between the two models is their dependence on α_1 . In our model the dissipation rate is proportional to $\sqrt{\alpha_1}$, whereas in *Tu*’s model it is proportional to α_1 . Using the stream in the inner heliosphere studied by *Bavassano et al.* [1982] for comparison, we note that in *Tu*’s model $\alpha = 1$ so that $\alpha\alpha_1 = 1/16$. In our model (with $C = 8.0$) $C^{-3/2} \sqrt{2\alpha_1} = 1/60$ (for $\alpha_1 = 1/16$). With these parameters, our model predicts dissipation rates one quarter those of *Tu* [1988]. For $U = 640$ km/s, $\gamma = 0.89$ is higher than *Tu*’s exponent of $\gamma = 0.77$ (compare Table 1 of *Tu* [1988]) and is closer to the observations. It is impossible to tell for certain at this point which model is better for the explanation of the temperature evolution of Alfvénic streams, and an improvement on both of them is probably needed.

6. Temperature Evolution From the Generalized Kraichnan Model

In the previous section we studied the temperature variation of the solar wind using the Kolmogorov-like phenomenology. Here we use Dobrowolny et al.’s generalized Kraichnan model to investigate how well that phenomenology models the temperature evolution. A problem with the Kolmogorov model was the necessity of using very large values of C to get the correct temperature evolution of Alfvénic streams. In contrast, the generalized Kraichnan model produces lower dissipation rates, and we wish to see to what extent this translates into changes in the predicted temperature evolu-

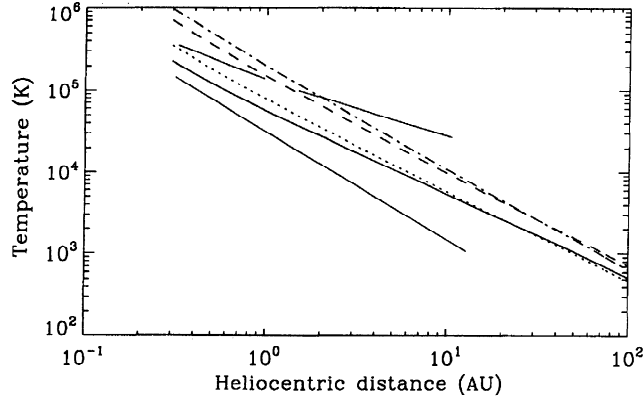


Figure 4. $T(r)$ versus r as predicted by the generalized Kraichnan's model for non-Alfvénic streams with speed of 372 (solid), 439 (dotted), 578 (dashed), and 650 km/s (chained) ($A = 1$). The temperature gradients are shown in Table 3. The straight line with slope $-4/3$ represents adiabatic cooling. The two line segments have the observed slopes of 0.9 and 0.7 in the distance intervals 0.3 – 1.0 and 1 – 10 AU, respectively.

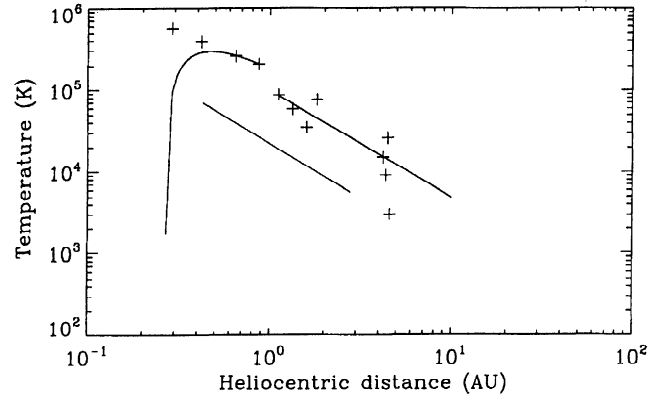


Figure 5. $T(r)$ versus r as predicted by the generalized Kraichnan's model for two Alfvénic streams ($\alpha_1 = 0.07$) with speed of 640 km/s (inner heliosphere) and 500 km/s (outer heliosphere) ($A = 1$). The observed temperature of these streams at various distances are shown by “plus”. The straight line with slope $-4/3$ represents adiabatic cooling.

tion. The derivation is essentially the same as presented above, so we only illustrate the main steps.

From section 3.3, the total dissipation rate in the generalized Kraichnan model is

$$\epsilon = A^{-2} C_A^{-1} [E^+(k)][E^-(k)] k^3 \quad (29)$$

or

$$\epsilon = \alpha_1 A^{-2} C_A^{-1} [E^+(k)]^2 k^3 \quad (30)$$

where $C_A = B_0/\sqrt{4\pi\rho}$ is the Alfvén speed, and A is a const. For simplicity we let B_0 vary as r^{-2} in the inner heliosphere and as r^{-1} in the outer heliosphere so that with $\rho \propto r^{-2}$, we have $C_A \propto r^{-1}$ in the inner heliosphere and $C_A = \text{constant}$ in the outer heliosphere.

Assuming further that the inertial range spectrum is $\propto k^{-3/2}$, the dissipation rates follow from (30). Using (4), (17), (20), (23), and (30), we find that the temperature evolution for non-Alfvénic streams is

$$T(r, U) =$$

$$\frac{\epsilon_0 r_0}{UC_V} + \left[T_0 - \frac{\epsilon_0 r_0}{UC_V} \left(\frac{r}{r_0} \right)^{-4/3} \right] \quad r < 1 \text{ AU} \quad (31)$$

$$T(r, U) = \left(\frac{r}{r_0} \right)^{-4/3} \times$$

$$\left[T_0(U) + \frac{\epsilon_0 r_0}{UC_V} 3 \left\{ \left(\frac{r}{r_0} \right)^{1/3} - 1 \right\} \right] \quad r > 1 \text{ AU} \quad (32)$$

while for Alfvénic streams the temperature evolution is

$$T(r, U) = \left(\frac{r}{r_0} \right)^{-4/3} \times$$

$$\left[T_0(U) + \frac{\epsilon_0 r_0}{UC_V} \left\{ 1 - \left(\frac{r_0}{r} \right) \right\} \right] \quad r < 1 \text{ AU} \quad (33)$$

Table 3. Predicted Temperature Gradients γ by the Generalized Kraichnan's Model for non-Alfvénic Streams

C	U , km/s	$10^{-5}T_0$, K	α_1	$10^3\epsilon$, km ² /s ³	γ		
					0.3 – 1 AU	1 – 10 AU	10 – 100 AU
1.0	372	0.56	1.0	0.52	1.16	1.02	1.01
	439	0.78	1.0	0.47	1.24	1.14	1.13
	578	1.50	1.0	0.82	1.27	1.19	1.08
	650	2.00	0.5	0.48	1.31	1.27	1.01
0.5	372	0.56	1.0	2.10	0.27	0.62	0.90
	439	0.78	1.0	1.90	0.91	0.82	0.94
	578	1.50	1.0	3.20	1.08	0.93	0.97
	650	2.00	0.5	1.90	1.23	1.12	1.07

Table 4. Predicted Temperature Gradients γ by the Generalized Kraichnan's Model for the Alfvénic Streams

C	U , km/s	$10^{-5}T_0$, K	α_1	$10^3\epsilon$, km ² /s ³	γ		
					0.3 – 1 AU	1 – 10 AU	10 – 100 AU
1.0	640	2.10	0.07	1.40	1.02	–	–
	500	0.87	0.07	0.19	–	1.30	1.30
0.5	640	2.10	0.07	5.80	0.91	–	–
	500	0.87	0.07	0.76	–	1.25	1.25

$$T(r, U) = \left(\frac{r}{r_0}\right)^{-4/3} \times \left[T_0(U) + \frac{\epsilon_0 r_0}{UC_V} 3 \left\{ 1 - \left(\frac{r}{r_0}\right)^{-2/3} \right\} \right] \quad r > 1 \text{ AU} \quad (34)$$

Taking $A = 1$, the $T(r)$ versus r plots for non-Alfvénic and Alfvénic streams are shown in Figures 4 and 5 respectively, and the calculated exponents for non-Alfvénic and Alfvénic streams are listed in Tables 3 and 4, respectively. Note that the values of γ are much higher in absolute value than those obtained previously from the Kolmogorov-like model, indicating that Kraichnan's model with $A = 1$ yields less heating. With $A = 0.5$, however, the dissipation is greater (Tables 3 and 4), but the predictions still do not generally match observations except in a few cases. In general, we have not been able to match the inner and outer heliospheric observations simultaneously for non-Alfvénic streams. This, combined with the simulation evidence discussed above, leads us to believe that the generalized Kraichnan phenomenology is not correct, at least for quasi-steady decaying turbulence, although the reason for this is not clear.

7. Discussion and Conclusions

We have estimated the dissipation rates in solar wind streams using generalized Kolmogorov and Kraichnan MHD turbulence phenomenologies. The heating rates in the two phenomenologies are dependent on values chosen for the constants C and A . These constants are often taken to be ~ 1.0 [Matthaeus and Zhou, 1989; Matthaeus and Zhou, 1990b; Marsch, 1990], but there is no rigorous derivation of their values from first principles. For a constant order unity, the temperature gradients predicted by the Kolmogorov-like phenomenology for the non-Alfvénic streams are in good agreement with the solar wind observations, while the predictions derived from the Kraichnan phenomenology, using either $A = 1$ or 0.5, do not generally match the observations.

In contrast, when dealing with Alfvénic streams, values of C of order unity do not work, and good agreement with observations was only obtained with a fairly large value of $C = 8.0$ in the generalized Kolmogorov

phenomenology, which implies that dissipation is suppressed even more than can be explained by large values of cross helicity alone. We do not understand the physics behind this suppression, although it may arise from the fact that the turbulent cascade is not yet fully developed. The nonlinear interactions may be distributing energy among large and intermediate wavenumbers, with only a small amount of energy flowing into the dissipation range. Determination of C is an important problem for understanding MHD turbulence in general and solar wind turbulence in particular. It is, of course, also possible that C is not a universal constant, but depends on the normalized cross helicity σ_c and the residual energy (the difference in kinetic and magnetic energy). Work on this problem is underway. We conclude from this study that there is no one consistent turbulence model that provides a satisfactory explanation of all aspects of the observed radial evolution of temperature in the solar wind. Nonetheless, it appears likely that sufficient energy flux is available for turbulent heating to be an important contributor throughout the heliosphere.

Acknowledgments. This work was supported by the NASA Space Physics Theory Program at the Goddard Space Flight Center.

The editor thanks B. Bavassano and Y. Zhou for their assistance in evaluating this paper.

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(Received April 26, 1994; revised April 13, 1995; accepted April 13, 1995.)