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NUMERICAL SIMULATION OF THE GENERATION OF TURBULENCE FROM COMETARY ION PICK-UP

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Abstract. Observations of magnetic field fluctuations near comet Halley have revealed a rapid development of a Kolmogoroff-like turbulence spectrum extending from below 10^{-2} Hz to above 0.1 Hz. Spectra obtained far from the comet have a strong peak in power near the Doppler-shifted ion-cyclotron frequency of singly ionized water. Closer to the comet, the spectrum at higher frequencies is enhanced in power level over the background solar wind spectrum by approximately an order of magnitude. We solved the equations of incompressible MHD using a two-dimensional 256×256 mode spectral method code to simulate this spectral evolution as an inertial range turbulent cascade. The initial conditions contained a constant magnetic field and a single coherent wave mode at a low wave number. The solar wind turbulence was modelled by a background noise spectrum having a Kolmogoroff spectral index. The coherent mode decayed into an inertial range spectrum with Kolmogoroff slope within a few eddy-turnover times. Both the time scale and the increase in power level of the turbulence seen in the simulation are in accord with the Giotto observations.

1. Introduction

As neutral water group molecules become ionized in the vicinity of a comet, the resulting ion distributions will form a gyrating beam distribution. This phenomenon has been studied in some detail, both for artificial and natural comets [e.g. Winske et al., 1984; Winske and Gary, 1986; Haerendel et al., 1986]. Gyrating beam distributions are unstable to the generation of low frequency magnetohydrodynamic waves with wave number $k \approx \Omega_H/v_b \approx \Omega_H/V_{sw}$ (via the resonance condition) and frequency $\omega_r < \Omega_H$ where Ω_H is the cyclotron frequency of a singly ionized ion moving with speed v_b . When Doppler shifted into the spacecraft frame of reference, the observed frequency is $\omega_{sw} \approx kV_{sw} \approx \Omega_H$ [e.g., Winske et al., 1985; Sagdeev et al., 1986; Tsurutani and Smith, 1986a, b; Gary et al., 1986; Gary and Winske, 1986; Goldstein and Wong, 1987]. The predominant ions produced near both Halley and Giacobini-Zinner have atomic masses close to that of water. Consequently, near 1 AU, the observed frequency will be ≈ 0.01 Hz.

Such waves were observed during both the ICE encounter with comet Giacobini-Zinner [Tsurutani and Smith, 1986a, b] and the Giotto encounter with comet Halley [Acuña et al., 1986]. In the

Giotto observations the power spectral peak near 0.01 Hz was accompanied by a greatly enhanced power-law spectrum extending from 0.01 Hz to above 0.1 Hz (Figure 2 of Acuña et al. [1986]). Farther from the comet, the wave spectrum was dominated by the peak near 0.01 Hz. The time series of the magnetic field data shown in Figure 1 of Acuña et al., [1986] shows this very clearly. However, the spectrum in Figure 2 of Acuña et al. [1986] includes significant low frequency power from a directional change in the interplanetary field observed between 1200 and 1220 on 13 March 1986 which obscures the dominance of the 100 s wave (Acuña, private communication, 1987).

The spectral index measured over the decade above the peak was reported by Acuña et al. [1986] to be approximately -1.8, which they noted is "typical for a turbulent cascade process" [e.g. Kraichnan and Montgomery, 1980]. Acuña et al. conjectured that a turbulent cascade was being driven by the instabilities generated by the water group ions. In this scenario, waves generated far upstream of the comet are convected back toward the comet by the solar wind, during which time nonlinear mode couplings can produce a turbulent cascade. Newly ionized water group molecules will also continue to pump the 0.01 Hz fluctuations via linear instabilities.

Several important questions need to be addressed concerning the turbulent cascade hypothesis. First, can a narrow spectral peak act as a pump for a cascade extending over an order in magnitude in wave number (or frequency)? Second, if a cascade is generated, is the resulting power spectrum a power-law with a Kolmogoroff spectral index? Third, does the process happen sufficiently rapidly to be consistent with the observations at Halley?

We have modelled the development of such a turbulent cascade in two-dimensional incompressible magnetohydrodynamics (MHD). The simulations suggest the correctness of the cascade hypothesis. In setting up this simulation we assume that a coherent low-frequency MHD wave has been generated locally by a beam or ring distribution of water-group ions. Because this class of kinetic instabilities have maximum growth along the direction of the local magnetic field, the waves are approximately incompressible. Furthermore, the frequencies are less than the ion cyclotron frequency. Therefore we adopt the idealization that the coherent wave is an incompressible Alfvén wave. Although the finite plasma beta of the solar wind would suggest that compressible effects may be important, there is substantial evidence that except in the vicinity of shocks and at scales that encompass stream compression regions, the fluctuations in the

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Paper number 7L6559.
0094-8276/87/007L-6559\$03.00

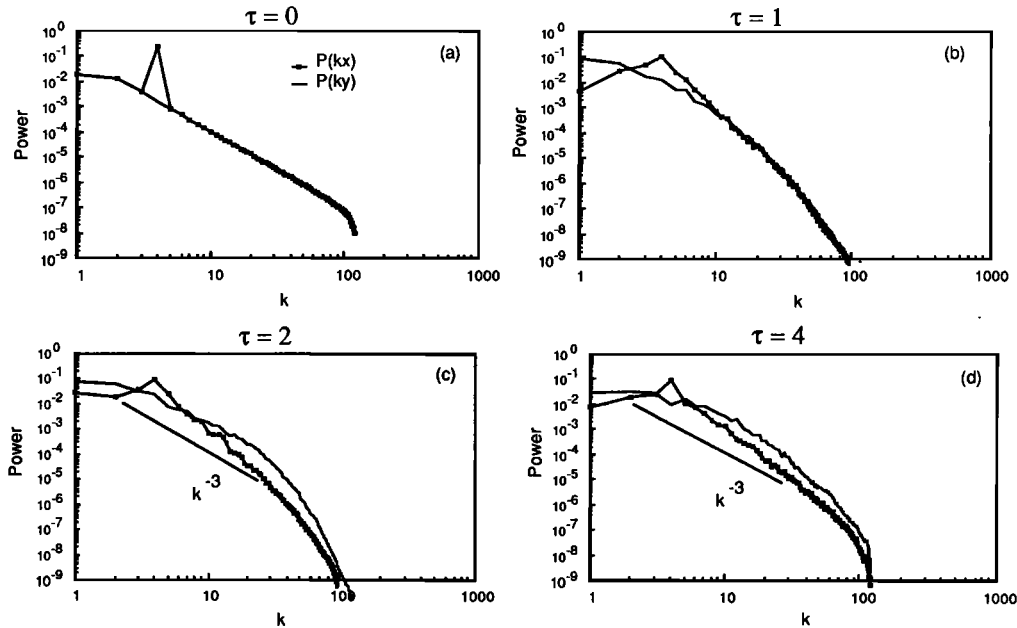


Fig. 1. Reduced spectra as functions of k_x and k_y at four times in the simulation run. The k^{-3} lines in panels (c) and (d) are drawn at the same power level as the spectrum at $\tau = 0$ in panel (a).

solar wind have only slight compressibility [cf. Montgomery et al., 1987; Roberts et al., 1987]. Consequently, the essential physics of the wave-wave interactions controlling the development of a turbulent cascade should be preserved in an incompressible MHD code.

A complete theory of the nonlinear behavior of large amplitude Alfvén waves in a compressible three-dimensional magnetofluid has yet to be developed. The early stages of evolution of large amplitude circularly polarized Alfvén waves have been studied in considerable detail using fluid and kinetic theory and it is well known that such waves are subject to modulation or decay instabilities [Hoshino and Terasawa, 1985; Longtin and Sonnerup, 1986; Wong and Goldstein, 1986; Shukla et al., 1986; and references therein]. A soliton description of large amplitude Alfvén waves in finite beta plasmas has been developed by Spangler and Sheerin [1982a,b] and Ovenden et al. [1983], among others. Fully nonlinear strong turbulence effects cannot be handled with the above techniques. Kraichnan and Montgomery [1980] reported incompressible Navier-Stokes simulations showing that, in the presence of background fluctuations, single wave number excitations such as Alfvén waves are generally nonlinearly unstable. In this letter, we use two-dimensional incompressible MHD to study the nonlinear spectral evolution of a large amplitude wave as it couples to weak background turbulence. Note that although finite amplitude Alfvén waves are generally unstable, the degree of instability depends on their amplitude, the strength of the background magnetic field, the magnitude of the cross-helicity, and the nature of any background noise, among other parameters. It is quite possible to set up conditions in which the development of turbulent cascades is inhibited.

2. Analysis

In appropriate dimensionless units the equations of incompressible MHD can be written as [e.g. Matthaeus and Montgomery, 1981]

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \mathbf{j} \times \mathbf{B} + \frac{1}{R} \nabla^2 \mathbf{v} \quad (1)$$

$$\frac{\partial \mathbf{B}}{\partial \tau} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \frac{1}{R_m} \nabla^2 \mathbf{B} \quad (2)$$

where $\mathbf{j} = \nabla \times \mathbf{B}$ is the electric current density, R is the Reynolds number, R_m is the magnetic Reynolds number, and $\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0$. Our solutions were obtained in two dimensions [Matthaeus and Montgomery, 1981] using a spectral method code with 256×256 wave numbers and $R = R_m = 1000$. Periodic boundary conditions were imposed, and the solutions were obtained inside a square box, 2π on a side.

To simulate the existence of the background interplanetary magnetic field, a mean field was included in the x -direction with magnitude $B_0 = 1$. In addition to the mean field, at $\tau = 0$ the system included a coherent mode at $k_x = 4$. The magnetic fluctuation associated with that mode was transverse to the mean field in the y -direction and had a normalized cross helicity [Matthaeus and Goldstein, 1982] of 0.98. The large value of cross helicity and the linear polarization of the coherent mode were chosen to simulate the properties of a large amplitude Alfvénic fluctuation excited by a gyrating distribution of water group ions.

The energy in modes greater than $k = 4$ contained approximately 5% of the energy in $k_x = 4$ and had an isotropic power law spectrum with spectral index -3 , which is the Kolmogoroff value expected for the turbulent inertial range of two dimensional Navier-Stokes turbulence [e.g. Kraichnan and Montgomery, 1980]. The analogous inertial range spectrum in isotropic three-dimensional turbulence has a slope of $-5/3$, which is approximately the spectral index found by Acuña et al. [1986] in the background solar wind far upstream of comet Halley (also see Matthaeus and Goldstein [1982]). Below $k = 2$, the initial noise spectrum was flat. In the noise spectrum,

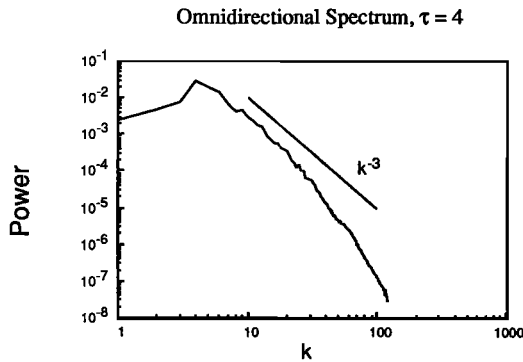


Fig. 2. The omnidirectional spectrum of magnetic energy as a function of $|k|$ at $\tau = 4$. For comparison a slope of k^{-3} is also indicated.

the energy in kinetic modes was equal to the energy in magnetic modes. The total fluctuating energy at $\tau = 0$ was normalized to unity.

An accurate simulation of the inertial range of turbulence must resolve spatial scales smaller than the Kolmogoroff dissipation scale [Matthaeus and Montgomery, 1981]. Because the magnetic and kinetic Reynolds numbers are equal, the dissipation wave number K_d is [Batchelor, 1970; Ting et al., 1986]

$$K_d = [2(\Omega + J)R^2]^{\frac{1}{2}}$$

where $\Omega = \langle \omega^2 \rangle$ is the mean square vorticity and $J = \langle j^2 \rangle$ is the mean square current. In this simulation $K_d \approx 75$ which, when compared to the maximum retained wave number of 121, indicates accurate computation of the turbulence.

In Figure 1a we have plotted one dimensional reduced power spectra as functions of k_x and k_y , at $\tau = 0$. The reduced spectrum is defined following Batchelor [1970] (also see Matthaeus and Goldstein, [1982]) as, in this case, the two-dimensional spectrum integrated over one of the orthogonal components of k . In all panels of Figure 1, the power spectrum as a function of k_x (denoted $P(k_x)$) is plotted as the line connected by circles, while the spectrum as a function of k_y (denoted $P(k_y)$) is the solid line. Above 121, all modes in the simulation are set to zero to be consistent with the dealiasing algorithm used [Patterson and Orszag, 1971].

By $\tau = 1/4$, the solutions already show that a turbulent cascade has begun. By $\tau = 1$, a cascade of energy to modes above 4 is apparent (Figure 1b). An exchange of energy between power in k_x below about 10 and power in k_y can be seen. By $\tau = 2$, the turbulent cascade has progressed out to large values of k (≈ 40). The cascade evolves more rapidly in k_y , perpendicular to B_0 , than in k_x and the spectrum becomes increasingly anisotropic in a manner described previously by Shebalin et al. [1983]. Over a fairly narrow range of k_y (≈ 10 -30), there is the suggestion of a power law with slope close to k_y^{-3} .

In the final panel of Figure 1, the power law spectrum is more clearly evident. To the extent that a power-law slope can be approximated over only one decade of wave number space, the slope of the spectrum is approximately -3 in $P(k_y)$ and somewhat steeper in $P(k_x)$. The power in both components is significantly enhanced over that

present in the inertial range at $\tau = 0$. The peak in $P(k_x)$ has diminished by almost a factor of 10 as the energy in this mode has been transferred out into the inertial range. The roll-over in the spectra in the vicinity of $k \approx 50$, which is especially evident for $\tau \geq 2$, is consistent with our estimate that $K_d \approx 75$. This is shown somewhat more clearly in Figure 2 which is a plot of the omnidirectional power spectrum (i.e. $2\pi k E(k)$; the factor of 2π is not included in Figure 2) at $\tau = 4$. The spectral slope is approximately -3 out to $k \approx 20$ where the effects of dissipation steepen the spectrum.

These numerical experiments indicate that a coherent mode in low level background turbulence can initiate a cascade that results in a Kolmogoroff-type power spectrum containing considerably more energy than was present initially in the inertial range of wave number space. Although the simulations are performed in two dimensions, it is reasonable to suppose that nonlinear processes will occur at least as quickly in three dimensions because of the larger number of nonlinear couplings available.

3. Discussion and Conclusions

It remains to show that the time scale for the development of the inertial range turbulent cascade is sufficiently fast to be consistent with the observations. To do so, we compare both the period of the coherent wave τ_w and estimates of the nonlinear (or "eddy turnover time") τ_{nl} in the simulation with the corresponding quantities estimated for the observed cometary waves. First we relate τ in the simulation to both τ_w and τ_{nl} . The dispersion relation of Alfvén waves is $f'\lambda' = V'_a = 1$ where primes denote dimensionless quantities. The wavelength of the coherent mode is $2/\pi$, so that the wave period is $\tau_w = 1.57$.

There are several ways to estimate τ_{nl} , the ratio of the characteristic scale length to the characteristic speed of the turbulent fluctuations. The most appropriate estimate for comparison with the Giotto data uses the correlation scale of the magnetofluid which can be estimated following Batchelor [1970] and Matthaeus and Goldstein [1982] from

$$L_c = \frac{\pi}{k_{\min}} \frac{P(0)}{EP(k)} \approx 1.2 \quad (3)$$

Since the characteristic speed is 1, $\tau_{nl} \approx 1.2$.

Upstream of comet Halley in the (relatively) undisturbed solar wind, the plasma density was approximately 5 cm^{-3} [Johnstone et al., 1986] and the average magnetic field was approximately 5 nT [Acuña et al., 1986] which gives an Alfvén speed of about 40 km/s. In the solar wind frame, the ion cyclotron wave period is approximately 1000s. The value of τ_{nl} in physical units can be estimated from the comet data using $\tau_{nl} = L_c/\delta B$, where L_c is the correlation scale and δB is the amplitude of the magnetic fluctuations in Alfvén speed units. We estimate L_c from T_c using $L_c = V_{sw}T_c$, eq. (3) and Figure 2 of Acuña et al. [1986]. This yields $T_c \approx 16 \text{ s}$. In Alfvén speed units with $\delta B = 10 \text{ km/s}$ and $V_{sw} = 400 \text{ km/s}$, $\tau_{nl} \approx 640 \text{ s}$ for the observed comet associated turbulence.

It is not clear whether the "correct" nonlinear time should be the eddy-diffusion time, the Alfvén transit time, or (possibly) the wave

period, or some combination of the three, but it is important that they be of the same order of magnitude because the functional dependence of the nonlinear evolution time on the three characteristic times is unknown. Given that the ratio of τ_w to τ_{nl} (as estimated above) is about 1.57 in the observations and 1.3 in the simulation, we can reasonably expect that the correspondence between the three dimensional solar wind observations and the two dimensional simulations is meaningful.

If we assume that the dimensionless wave period of $\tau_w = 1.57$ in the simulation is equivalent to the physical wave period of 1000 s in the solar wind, then $\tau = 2$ corresponds to 1300 s by which time a parcel of plasma in the solar wind will have convected a distance of approximately 5×10^5 km. The two spectra shown in Acuña et al. [1986] during the inbound trajectory (cf. the top two panels of their Figure 2) are separated in spacecraft event time by three hours during which the distance of Giotto from Halley decreased by 1.3×10^6 (cf. Figure 2 in Johnstone et al., [1986]). We conclude, therefore, that even in two-dimensional MHD there appears to be sufficient time for a turbulent cascade driven by a low frequency Alfvén wave to develop, and that the resulting spectral shape of the power spectrum is approximately a power law with index close to the Kolmogoroff value.

Acknowledgments. This work was supported in part by a NASA Solar Terrestrial Theory Program grant to the Goddard Space Flight Center. D. A. Roberts is a NAS/NRC Resident Research Associate. We thank M. H. Acuña for many stimulating discussions.

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(Received April 7, 1987;
accepted June 11, 1987.)