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Quantum speed-limited depletion of physical resources

This is a Perspective on "Time-optimal quantum transformations with bounded bandwidth" by Dan Allan, Niklas Hörnedal, and Ole Andersson, published in Quantum 5, 462 (2021).

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Quantum speed limits: modern version of Heisenberg's uncertainty relation

Arguably among the most recognizable features of quantum mechanics are the Heisenberg uncertainty relations [1]. Whereas the mathematical underpinning of the relation for space and momentum was quickly understood [2], the similarly innocuously looking inequality for energy and time has kept researchers occupied for the better part of the last century [3]. In many introductory texts it is simply written as

$$\Delta E \Delta t \gtrsim \hbar,$$

yet a clear interpretation of ΔE and ΔT is often omitted. Decades after Heisenberg's original proposal [1], Mandelstam and Tamm [4] and even later Margolous and Levitin [5] made clear that the uncertainty relation should be interpreted as an estimate for the minimal time quantum systems require to evolve between distinct states. Nowadays, research on so-called "quantum speed limits" has become one of the most active fields [3] with a multitude of fundamental questions and potentially far reaching consequences for the development of quantum technologies.

From fundamental statements to practically relevant bounds

Until very recently, most of the work focused on understanding and generalizing the quantum speed limits to arbitrary distances and driven systems [6], or open quantum dynamics [7,8,9]. However, it has always been somewhat obvious that the fundamental bounds may be far outside the reach of experimental reality. Thus, more recently the focus shifted towards practically relevant and achievable bounds. In these analyses one can follow two fundamentally different philosophies: Either one strives to make the fundamental inequalities tighter by finding more apt ways to measure the distinguishability of quantum states [10], or one turns the problem on its head and seeks to directly estimate the rates with which physical observables change [11,12]. Rather remarkably, a recent paper published by Allan, Hörnedal, and Andersson [13] managed to do both by deriving a tight bound on the time that is required to deplete a physical resource.

Quantum speed limits for consumption of resources

To this end, Allan, Hörnedal, and Andersson [13] study dynamics governed by time-dependent Hamiltonians with bounded bandwidth. In formula this is expressed as

$$\text{tr}[H(t)^2] \leq \omega^2,$$

where ω is a fixed, positive number. The goal is then to find the special $H(t)$, for which the time-dependent expectation value of a physical observable A is minimized in the shortest time. It is actually not hard to recognize that this boils down to a brachistochrone problem, which can be solved with the usual methods from geometric control theory. For instance, one immediately has that the time-optimal path is the geodesic in the control landscape, and hence the optimal $H(t)$ is actually time-independent. Unfortunately, this is a rather formal observation, which does not simplify the issue of computing the minimal time. Therefore, the authors continue to prove a tight lower bound, which they call (in the present context) the quantum speed limit τ_{qsl} . It is defined as

$$\tau_{\text{qsl}} = \frac{\pi\sqrt{\delta}}{2\omega},$$

where δ counts how many eigenvalues of the initial quantum state have to change to minimize the expectation value of A . Remarkably, this τ_{qsl} is, indeed, the minimal time required to minimize $\langle A \rangle$ for a reasonably broad class of situations, which Allan, Hörnedal, and Andersson [13] go on to demonstrate for a variety of scenarios including catalysis and collective processes.

The last part of the analysis is dedicated to an application, whose importance can hardly be underestimated. In the development of quantum technologies so-called quantum batteries [14] take a prominent and potentially instrumental place. Allan, Hörnedal, and Andersson [13] show that their new formalism can be exploited to estimate the power, with which energy can be reversibly extracted from such a battery. In particular, it is shown that the above defined quantum speed limit τ_{qsl} is a tight bound for the duration of the complete discharge process. Thus, the rather formal analysis and mathematical findings of Ref.~[13] may have direct and immediate applications in quantum technologies.

Conclusions and outlook

The recent work by Allan, Hörnedal, and Andersson [13] constitutes an important step in the development and application of quantum speed-limited dynamics to experimentally realistic scenarios. The authors have achieved a tight and computable estimate for the time it takes to minimize a time-dependent expectation value under unitary dynamics, or in other words, the minimal time it takes to deplete a physical resource. This was demonstrated for one of the most promising examples of near-term quantum technologies, namely quantum batteries. However, it is not far-fetched to realize that similar analyses can and should be conducted, for instance, in quantum communication, quantum sensing, and quantum computation, or rather more generally for all quantum thermodynamic devices.

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► References

- [1] W. Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", Z. Phys. 43, 172 (1927) <https://doi.org/10.1007/BF01397280>.
<https://doi.org/10.1007/BF01397280>
- [2] H. P. Robertson, "The uncertainty principle", Phys. Rev. 34, 163 (1929) <https://doi.org/10.1103/PhysRev.34.163>.
<https://doi.org/10.1103/PhysRev.34.163>
- [3] S. Deffner and S. Campbell, "Quantum speed limits: from Heisenberg's uncertainty principle to optimal quantum control", J. Phys. A: Math. Theor. 50, 453001 (2017) <https://doi.org/10.1088/1751-8121/aa86c6>.
<https://doi.org/10.1088/1751-8121/aa86c6>
- [4] L. Mandelstam and I. Tamm, "The uncertainty relation between energy and time in nonrelativistic quantum mechanics", J. Phys. 9, 249 (1945) https://doi.org/10.1007/978-3-642-74626-0_8.
https://doi.org/10.1007/978-3-642-74626-0_8
- [5] N. Margolus and L. B. Levitin, "The maximum speed of dynamical evolution", Physica D 120, 188 (1998) [https://doi.org/10.1016/S0167-2789\(98\)00054-2](https://doi.org/10.1016/S0167-2789(98)00054-2).
[https://doi.org/10.1016/S0167-2789\(98\)00054-2](https://doi.org/10.1016/S0167-2789(98)00054-2)
- [6] S. Deffner and E. Lutz, "Energy–time uncertainty relation for driven quantum systems", J. Phys. A: Math. Theor. 46, 335302 (2013) <https://doi.org/10.1088/1751-8113/46/33/335302>.
<https://doi.org/10.1088/1751-8113/46/33/335302>
- [7] M. Taddei et al., "Quantum Speed Limit for Physical Processes", Phys. Rev. Lett. 110, 050402 (2013) <https://doi.org/10.1103/PhysRevLett.110.050402>.
<https://doi.org/10.1103/PhysRevLett.110.050402>
- [8] A. del Campo et al., "Quantum Speed Limits in Open System Dynamics", Phys. Rev. Lett. 110, 050403 (2013) <https://doi.org/10.1103/PhysRevLett.110.050403>.
<https://doi.org/10.1103/PhysRevLett.110.050403>
- [9] S. Deffner and E. Lutz, "Quantum Speed Limit for Non-Markovian Dynamics", Phys. Rev. Lett. 111, 010402 (2013) <https://doi.org/10.1103/PhysRevLett.111.010402>.
<https://doi.org/10.1103/PhysRevLett.111.010402>

- [10] D. Pires et al., "Generalized Geometric Quantum Speed Limits", Phys. Rev. X 6, 021031 (2016) <https://doi.org/10.1103/PhysRevX.6.021031>
- [11] E. O'Connor, G. Guarnieri, S. Campbell, "Action quantum speed limits", Phys. Rev. A 103, 022210 (2021) <https://doi.org/10.1103/PhysRevA.103.022210>
- [12] F. Campaioli, C.-S. Yu, F. A. Pollock, K. Modi, "Resource speed limits: Maximal rate of resource variation", arXiv:2004.03078 <https://arxiv.org/abs/2004.03078>
- [13] D. Allan, N. Hörnedal, and O. Andersson, "Time-optimal quantum transformations with bounded bandwidth", Quantum 5, 462 (2021) <https://doi.org/10.22331/q-2021-05-27-462>
- [14] F. C. Binder, S. Vinjanampathy, K. Modi, and J. Goold, "Quantacell: powerful charging of quantum batteries", New J. Phys. 17, 075015 (2015), <https://doi.org/10.1088/1367-2630/17/7/075015>

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