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00:00

So I'm going to present you some work that we have been doing for the last-- about two years, and some of the recent results that actually we have got in collaboration with people at NIST UMD, people at UMD in a different group, and people at UMBC for the theoretical part. And here what I'm going to show you is our work on two-dimensional frequency comb and no particular user interaction between the CW wave and the soliton.

00:26

And actually here, I tried to be fancy, showing a microchip because that's basically the only place where you're going to see a microchip. And the other thing I'm going to show you are actually totally dependent on the type of resonator you're using. It's just-- we have some, so we're using this kind of thing. So just bear that in mind. So before actually going into all the physics and all that, I'm going to actually just-- quick reminder.

00:51

I guess everybody is familiar with what's a frequency comb, what's a soliton, so this is going to be very quick how we get dissipative Kerr soliton. So first we take microring resonator, inject light on resonance. So with this we can get light into the resonator. If you inject enough power, you're going to get the modulation of the refractive index through self-phase modulation.

01:11

If you are in the correct dispersion which is anomalous for soft excitation, you're going to get modulation stability. And if you're lucky enough-- and when I speak about light here, most of the time that's actually luck-- you'll get a soliton, which is a single pulse that travels throughout the resonator. That doesn't mean you have a frequency comb yet.

01:31

Here you have a single dissipative Kerr soliton, but to get a soliton-- a frequency comb, sorry, you actually need to extract it periodically into the waveguide. And you're going to get a pulse train, and that's actually what's going to get you the frequency comb. And here it's important to note is that what you are actually measuring here is not exactly the state that you have in the resonators.

01:51

That's what you extract at every runthrough. And because of Fourier transform of the input, you get this nice CW. It's a CW light. And because you have the pulse train, you Fourier-transform that-- you get a frequency comb. The frequency comb are pretty awesome for a lot of reason, and one in particular is because it's frequency division. So I totally stole that from Scott Diddams and made a little video out of it.

02:12

The frequency comb is the frequency division between optical frequency and the repetition rate in the microwave domain. But you can go back and forth. So that means, if you have a stable optical frequency, you can get a stable repetition rate in the

microwave, and vice versa. And so that mean you can have a lot of application for this dissipative Kerr soliton frequency comb.

02:30

And I stole that from the [INAUDIBLE].. I'm seeing a lot of video at first. You'll see. Is there going to be original work? No way. In particular, at least the S stands for standards, we care about metrology application. What does that mean? That mean we focus a lot on optical frequency synthesizer and optical atomic clock.

02:50

For this kind of work we need to fully stabilize and self-stabilize the frequency comb. To do that you need an octave bandwidth, and that's why actually we work with this tiny ring-- this tiny chip that I show in the first slide-- is because for power consideration you want to minimize the number of comb teeth you have in your octave. So that's when you need to work with a large rep rate.

03:10

In our case we work with a small ring radius of 23 micron in silicon nitride, and that give you one terahertz of repetition rate. And like I said, everything that I'm going to show you is totally independent from this repetition rate, but it truly made our life much easier to have this huge repetition rate that we can resolve optically. So why is it really challenging to actually get this octave-spanning frequency comb and self-stabilization?

03:36

It's because there's not many way actually to self-stabilize a frequency comb. You need to do $f/2f$. And if you take one comb tooth, you double its [INAUDIBLE] and beat it against its harmonic-- as you can see this was a very quick math here-- you actually get the frequency of the carrier envelope offset. The issue is there's really not a lot of power at this frequency because you have the intrinsic hyperbolic square envelope of the frequency comb that's going to limit the power that you have at the frequency of interest.

04:02

And there's a trick that has been found now. It's almost like a decay where you can introduce dispersive waves using high order dispersion of your resonator while your comb teeth is actually going to be resonance, and you're going to have resonance enhancement at a given mode. If you tailor your dispersion correctly enough, you will actually have this resonance enhancement at harmonic frequency so you can do $f/2f$.

04:23

And so far to my knowledge, that's the only way you actually get a coherent octave-spanning frequency comb. That's actually not enough. So I mean, even if you get the enhancement, let's say of multiple tens of dB, you still have, like minus 30 dBm of power. If you think about second harmonic generation and you get 10 dB of losses, you try to double that beat against it or you will not measure anything.

04:46

So how we actually master the time, do stabilization, is use an auxiliary laser as shown in this red line that you actually common-lock to your frequency comb. This laser has a lot of power. It can double as much as you want, and you're going to beat it and actually retrieve your carrier envelope offset. Well the issue is you have this other laser that-- it's a degree of freedom, and you're not really doing anything else and locking it to your soliton.

05:09

So the question is, can we actually use it in a better way? And so the question is, can you actually use it in a better way, which mean, can you actually send it to your resonator? And the answer is pretty straightforward. It's yes. And at least in our group everything that's spurred all our work is really this work by Pascal Del'Haye's group, who has actually sent a second laser into the resonator.

05:32

And they have seen a spectral extension of the frequency comb up to the secondary pump. And that's like a groundbreaking work because all of a sudden you're not limited by hyperbolic second square envelope. You can have almost unlimited power wherever you want in your secondary-- in your frequency comb because of your secondary pump. What we have done-- we have done actually-- used one of the features that they show in the paper, but they don't explore it to show that you can actually create new type of dispersive wave on demand because of the secondary pump that you're sending to your resonator.

06:06

And you can obtain a record-breaking 1.5 octave of coherent frequency comb. But the issue is that even if it's a coherent frequency comb doesn't mean it's a frequency comb that you can use for metrology. Because here-- OK, we show a huge frequency comb. So that means we kind of solve the issue of stabilization, and OK, my work here is done, right? If you look closer at the frequency combs that you obtain here, you actually can zoom in and what you will see is actually two spectral component.

06:37

And what is really interesting is that both component exactly have the same repetition rate but there is this offset that we used to call the stitching back in the day that we call Δf between the translated portion and the primary portion of your frequency comb. What does that mean? That mean you actually have a degree of freedom that you still need to lock.

06:56

What is related to the stitching is actually what we have noticed. Like, there's a really obvious trend between the integrated dispersion at the secondary pump and this offset of frequency. And so we concluded that because your frequency comb is not on resonance because it's dispersion-less-- so that means the repetition rate is the same across your bandwidths-- your cavity is a dispersion.

07:18

That means your cavity resonance is going to move. So that mean when you pump, there going to be a natural offset between the cavity mode and your frequency and your-- the nearest contours, which actually is the integrated dispersion at the second item that's. Actually this value. So that's why this offset naturally occur-- because of the dispersionist nature of the frequency comb and the dispersion nature of your cavity frequency.

07:39

And like I say, it's still in those other degree of freedom. That mean for metrology purpose, we have solved absolutely nothing because it's totally not different from locking an external laser to your frequency comb or sending it into your resonator and having another frequency to lock. You still have to lock something. But as you know we have another degree of freedom, and in optics that's pretty nice because when you have another degree of freedom usually you can harness it and do nonlinear optics with it.

08:05

And so I'm going to show you that actually you can harness this frequency offset to actually do nonlinear optics and create a 2D comb. So to start first-- fancy. To start for just a recap of what you get as a frequency comb. You have this pulse train, and it's really important to recall that most of the time we only work with the envelope.

08:25

That mean that it's really nice, hyperbolic secant but there's a fast oscillation under. And if you're familiar with the work of Ted Hansch, Nathalie Picqué, you will recognize this figure because they always show this kind of phase between the first oscillation and the envelope that increases at every pulse train. And that actually is the origin of your carrier envelope offset. What happened in a resonator of this?

08:47

Pulse is actually a snapshot at every round trip when you extract your resonator-- your soliton from the resonator. So that's mean actually what's happening in the resonator-- you have a group velocity that is different from the phase velocity. And because it's different, the oscillation under the envelope are drifting by a fixed amount at every round trip. So now we can actually look at what's inside the resonator instead of what's in the waveguide.

09:12

So we can actually work in the referential that freeze the soliton. It has a fixed repetition rate, so we can introduce this file that is the angular angle. The angular-- yeah, the angle of the resonant are thinner minus the repetition rate over time, which means the envelope is effectively freezing. That means you can look at any time possible, it's always going to be there.

09:32

But because the group velocity is different from the phase velocity, the phase oscillation going to drift. So if you take a fixed point in ϕ , which is a fixed point in the azimuthal

angle, what's going to happen with time? You can see they're going to oscillate. If you record it in time, they're going to oscillate kind of differently.

09:51

But what's really important is that this period is exactly the same. It is relative to the discrepancy between group velocity and phase velocity. If you have a signal in time, you can Fourier-transform it. You get a single frequency. So effectively what does that mean? That means a single soliton in a cavity-- well, actually, a single soliton that propagate into a fiber-- can be recasted as a single frequency.

10:13

And if you want to dig deeper I invite you to look at [INAUDIBLE]---- work from [INAUDIBLE] from the '70s in action angle variable and all this stuff. If you understand it, just tell me because I still don't. And just trust our collaborator Curtis Menyuk that came up with all this stuff is a theorist. So that mean now we assume that the single soliton in the cavity is actually a single frequency. When you inject another laser, what happens is you're going to have a binding of the group velocity with the soliton.

10:42

There's very nice work that has been shown by Stuart Murdoch in New Zealand where actually they show that what you send actually has a CW wave externally, but it gripped the tail of the soliton and became essentially a dispersing wave. So what that means? That mean actually, our laser in blue here going to travel at the exact same group velocity than the soliton.

11:07

It has been shown actually recently through some other spectral extension work that it's actually exactly the case that every component travel at the same group velocity. So that means that we can still assume the little cartoons that I showed earlier about the same group velocity and actually freezing your pulse in the resonator. So now that we know that we can freeze every component into the resonator, we can start looking at the integrated dispersion.

11:31

So integrated dispersion-- if you're not familiar, it's basically a comb for the cavity resonance offset from a fixed grid of resonance. So if you have your main pump here and you inject your secondary pump here, what's going to happen is actually the same as if you have a frequency comb that is like this red line, and you're going to get an offset that is over here. So that mean you have introduced an offset of-- carrier envelope offset here.

11:55

So it's Δf here. If you recall a couple of slide above, what I show you is the carrier envelope offset is essentially a shift of phase velocity. So what you have done here-- you have actually exactly the same group velocity in your resonator-- that's what has been shown in previous work-- but you have introduced a shift of phase velocity. What does that mean?

12:13

You have one frequency that is a soliton, another frequency that is your secondary laser. We're in a nonlinear system, right? So pretty easily you can mix it. Here you have an OPO-type system where instead of a CW wave you have actually a DKS that is a single frequency that acts as a pump, you have your secondary pump that I call SP create an idler exactly at the $\delta\omega$ from the secondary pump.

12:40

So that's pretty cool. But actually these frequency are not exactly single frequencies. They do have azimuthal components, so that mean this frequency here is actually the soliton. This one is kind of like a CW wave-ish type of system, which means this one is going to become also a CW wave-ish type of stuff. Here it's like not totally random what I drew. It's actually like-- if you run the simulation that's kind of like what it looks like, which means that they will have different azimuthal component.

13:05

So that mean here, intrinsically what you have is you have two dimension. You have the first one that you're used to-- that azimuthal component. That's what we observe experimentally. But you also have the second one that we actually always play with because we tend to lock the carrier envelope offset. So actually we always use the second order-- the second dimension, just we never harnessed it from nonlinear optics.

13:27

And in this case actually by sending your secondary pump, you can actually harness it and do some mixing in this phase velocity space. So that's pretty cool. I show you everything that happened in the resonator. We don't actually see what's in the resonator. We see what's in the waveguide. So here you have three frequency in the phase velocity. So that means you have three phase velocities.

13:47

So that mean you have three kind envelope offsets. As you recall, this discrepancy of phase velocity/group velocity yield the carrier envelope offset, which means each of these single tone in the phase velocity will each yield a different carrier envelope offset. Which means this one is going to create frequency comb here, this one offsetted by the quantity $\delta\omega$, and this one offsetted by a quantity $\omega - \delta\omega$. So it's kind of nasty.

14:13

When you do experiments this way it's kind of always nasty. You have all this component everywhere. It's not very nice, so another better way actually to recast what you're seeing-- and I'm going to show that everywhere else, so it's kind of important that this thing gets through-- is that you can actually account for the fact that each azimuthal number is every component that are between $\omega - 2\pi$ and $\omega + 2\pi$.

14:34

So that mean if you take one comb tooth, every power that in between this-- in this bracket is actually the same azimuthal number. What that mean? That mean if you take

the pump here you can put it in this kind of 3D space where you have an x-axis, the azimuthal mode number and the y-axis is like minus $\text{frep over } 2$, $\text{frep over } 2$. Obviously the pump is centered in this thing because we normalize it to the frep of the frequency comb, so it's going to be 0, 0.

14:59

All the frequency-- all the tone of the DKS are actually centered in μ , so they're all going to be at 0 and ω with different μ . But this guy over there-- it has a shift of minus-- minus $\delta\omega$, right? So that mean for each of them of the μ number where the is actually power, you're going to actually shift it off my new $\delta\omega$.

15:22

If you do a very nice LLE simulation, and here that's a singularity simulation where you actually recreate the experiment-- I'm going to discuss that in more detail later-- you can actually see very well that you have one component that is at the 0 fceo-- normalize 0 fceo. And you have two other components that are spaced by this $\delta\omega$ and fceo.

15:44

And basically you show that you have this nonlinearity in this phase dimension. So here I just speak about a particular nonlinear interaction. What you can see is, well, all the four-wave mixing possible are basically at reach, right? So how to model the design? And this thing you can basically kind of trust what I'm showing because I didn't come up with.

16:04

It's Curtis Menyuk that you can trust much more than me. So on the one hand, to model the system it's very well-known. You can easily do coupled mode theory. Or on the other hand, do the Lugiato-Lefever equations. They are totally equivalent as one is in the mode domain, the other in the time domain. And the linear part, they're the same-- loss, detuning, dispersion. Same.

16:26

On the nonlinear path they are not. In one hand you have nonmixing converting-- conserving the momentum. On the other side, you actually have self-phase modulation. Which means one hand, you actually need to account for every possibility possible in the mode dimension. On the other hand, it's actually super easy. You just account for self-phase modulations that shrink or expand your pulse, and it's going to be automatic.

16:46

So in our system, we're actually in between. You do have this pulse that is very nice for the early, but you have this mode mixing that is very nice for the coupled mode theory. So the answer is coupled Lugiato-Lefever equation. So here what you will see is that instead of being just a-- as in Lugiato-Lefever equation you can have the subscript

sigma that account for all the different modes in the phase velocity when you introduce a secondary laser.

17:12

And you're going to have the sum of all the phase match component, and you're going to introduce a dispersionless term that account for the phase-- for the phase shift of each of the components. There's a lot of simplification here. It's not just like you can merge them both because this equation is actually not true all the time. It's only true when you have a soliton because the modulation instability doesn't produce a single tone in the phase velocity.

17:37

And for this equation to exist you need that. So that mean you can only add this kind of equation when a soliton already exists, which makes it more complicated. So to start first, we're going to show you that actually it's an integrated dispersion is what caused the shift of phase velocity. So we take a microring resonator-- 1 terahertz-- and we measure the dispersion across from 1,610 to 920.

18:05

And that fits very well with the simulation that we can do in COMSOL. And then we take a dissipative Kerr soliton, and here it's important-- and it was a huge pain to be honest-- it's the same exact dissipative Kerr soliton where we actually change the secondary pump across all the modes that we have access with our laser. And actually, if you just look at the dispersive wave and we zoom above it, you will see that the dispersive wave of the primary soliton line is actually not moving-- stay at this 0 normalized-- but all the different component of the secondary pumps are actually moving.

18:42

And that's followed very well the integrated dispersion that we just measured. So what essentially it is-- that mean the integrated dispersion is a measurement of the shift of frequency offset when you're in a dual tone system, which means also measure your shift of phase velocity. Great. So now I show you that if you inject a secondary pump, you know that the integrated dispersion is going to define your shift and essentially is your frequency spacing in the phase velocity stuff.

19:13

So now I can show you that we can do an OPA system. So I send the secondary pump here. Kind of forgot to put which mode number it is, so it's like 91, 92, 93, 94. And we have the single pump here-- the single pump decay is here and always the same. And we get two idler. Here you can see there is new dispersive wave that are based on phase matching.

19:32

I'm not going to describe it in detail, but if you have any questions, don't hesitate. And if we zoom across this thing you can see as expected here for the different pumped mode we're going to have a different offset frequency of that. In this case here, we're limited

by the resolution of the [INAUDIBLE],, but the same thing-- you can see the increase of the frequency offset in the other dimension.

19:54

We can recast it in this 3D plot that I presented you earlier. So that's experimental data where you can see the signal is pretty well offset from the primary comb. It's very easy to see that. And the idler is actually offset in a negative sign from the DKS. So to kind of have a better idea of what's happening actually in the phase velocity-- phase velocity mixing, we can look at the carrier envelope offset as essentially they are almost the same.

20:19

So we can integrate our comb over the mode number on μ , and so basically you just [INAUDIBLE] μ , and you get this very nice plot where you can see that you have your DKS acting as a pump. So this one is never moving. You have your signals that we tune, and we get the idler that follow exactly the frequency of that, just opposite sign from the signal. So that's a demonstration that you have an OPO in the phase dimension of your frequency comb and introduce another dimension that is not the azimuthal mode number.

20:47

We can do an LLE. So that's actually an equation that has been introduced by Hossein Taheri in his-- it's a very nice paper-- where you can just introduce another driving force based on the mode that you're pumping and the integrity dispersion where you send it. And if you reproduce exactly the experiment, which means you're not only simulating the LLE but you actually extracting your soliton at every round trip.

21:12

You recreate your pulse train then you FFT it-- that's the kind of comb that you get. So it's kind of different from the kind of easy LLEs that you can get and just get the envelope, you FFT it, and that gets your envelope of the comb. Here that's actually like the comb teeth are not just by doing a stem plot of MATLAB. That's actually the type of result that you get. And so you can zoom and actually see the offset in FCU that are all opposite sign as we see an experiment, and you reproduce the 3D plot and the OPO in the phase velocity the carrier envelope offset domain.

21:45

Great. So I show you that basically we can have an OPA in this thing. I show you also that it validates the couple-- the coupled LLE equation through this kind of full area that accounts for everything. So now can we actually cascade an OPO? So as you're all familiar with it-- how to get the frequency comb. It's pretty easy.

22:05

You always get-- almost always get a primary OPO which then cascade into all these different comb teeth. So can we do that instead of doing a μ actually in this phase velocity system? To do that we take another resonator with another dispersion, and as you can see when you single-pump it or dual-pump it in the spectra, it's quite different.

22:25

Actually if you zoom at all the different comb teeth, you can see that you do not get only one offset but you get several offset. And actually if you look at all the different components that you have, you have many more than just two or three. Can project it again in the 3D plot. As you can see you have many, many comb teeth, and that's very clear to the comb in both the mode number and this phase velocity dimension which is a frequency carrier offset.

22:50

We produce it with the LLE again, and if actually we do the integral of μ , you can see that the LLE ends the experiment much, much-- very well. So I'm kind of fooling you actually right now because I'm telling you that it's a 2D comb. It's not really a 2D comb, right? If you go back you can tell me, well, the μ s they don't really overlap.

23:08

That mean you have a nice cascading in that direction, you have a nice comb in that direction, but it's not a 2D comb because there's no overlap. For instance, the sigma of a 0 and the sigma of a 3 actually don't-- you can never get the beading between them two. So one way to do that is, you can actually try to reduce this omega repetition rate-- the repetition rate in the phase velocity dimension. Well it's pretty easy to actually get the smallest possible because that mean wherever this passive wave is, that's literally when integrated dispersion is closest to 0.

23:36

So that's what we do. We pump here and the dispersive wave-- as you can see, dispersive wave of a single pump decreases. Over here we pump it, and actually not much happen. It was really underwhelming when we got this result because basically you don't resolve anything. You don't see any fun stuff happening because we cannot resolve anything, because here omega is technically 0, right?

23:58

Omega is not 0. It's actually 1 gigahertz something, and if you send your one comb actually into a photodetector, what you will see is equispaced terms that are actually separated by 1 gigahertz. And because I show you all here that actually this cascading-- we can resolve it optically, that's not nonlinearity in our photodetector, right?

24:18

Can do the simulation, and actually the simulation show you that you have a very nice 2D comb with a very nice-- basically you can resolve every comb teeth is-- are in μ or in this omega dimension. And here I can show you in the simulation that we get about 10 comb teeth, and you can probably go much higher than only 10 comb teeth. So I do a demonstration of a 2D com. So that's the conclusion for my first talk.

24:44

So I hope in this first talk at least I convinced you that the carrier envelope offset is not just a parameter to lock. So we always play with it, and we cannot dismiss it most of the time. And we can actually do stuff with it. We can actually play with it to follow-- to do

nonlinear optics. And I discussed with you a model that accounts for the nonlinearity of both azimuthal and phase dimension.

25:04

I share something that we're working on to actually show the [INAUDIBLE] you can work. I show you multiple way to create a composite frequency comb, and that you can actually cascade nonlinearity in this phase velocity dimension to create a two-dimensional frequency comb. So what's next? So there's a lot of thing we're hoping to do with that. And in particular is that the kind of gear that I showed you in the first slide that I stole from Scott Diddams is, can we actually introduce another gear using this other frequency repetition rate?

25:36

If you remember also the very first slide, I told you that the repetition rate of our ring is 1 terahertz. The repetition rate in this frequency here is 1 gigahertz. That's meaning we have three order of magnitude difference between the different repetition rates. So can we actually introduce another wheel in this frequency division scheme to actually downconvert even further? And the last question is actually-- remember I told you that these two dimensions are orthogonal?

26:03

It's still a question. Are they actually also going to actually-- one impact the other? And actually can we disc-- have a discrepancy between them two? And that's something that we are still working on. And with that, I'd like to thank everybody that worked on it. So from [INAUDIBLE] group, Jordan, Michal, and Christy. Yann Chembo, [INAUDIBLE] and help us a lot. Avik Dutt is now at UMD.

26:25

And Curtis Menyuk and Pradyoth that are working on the theory side. And with that, I'd like to thank you. [APPLAUSE]