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NONLINEAR STABILITY OF SOLAR TYPE III RADIO BURSTS. I. THEORY

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ABSTRACT

A theory of the excitation of solar type III bursts is presented. Electrons initially unstable to the linear bump-in-tail instability are shown to rapidly amplify Langmuir waves to energy densities characteristic of strong turbulence. The three-dimensional equations which describe the strong coupling (wave-wave) interactions are derived. For parameters characteristic of the interplanetary medium the equations reduce to one-dimension. In that case the oscillating two-stream instability (OTSI) is the dominant nonlinear instability. OTSI is stabilized through the production of nonlinear ion density fluctuations that efficiently scatter Langmuir waves out of resonance with the electron beam. An analytical model of the electron distribution function is also developed which is used to estimate the total energy losses suffered by the electron beam as it propagates from the solar corona to 1 AU and beyond.

Subject headings: hydromagnetics — instabilities — Sun: radio radiation — Sun: solar wind

I. INTRODUCTION

Among the various forms of active solar radio phenomena, type III bursts are unique in that they may extend throughout the solar corona and interplanetary medium, to heliocentric distances of 1 AU and beyond. These bursts are known to be excited by streams of moderately energetic electrons which are accelerated either in flares or in active storm regions and released along magnetic field lines that penetrate the high corona. The radio emission is the observable by-product of the interactions of the exciting stream with the background plasma, and the understanding of these interactions and of their effects on the propagation of the exciter presents the most fundamental problem in type III theory.

The historically accepted hypothesis is that the exciter constitutes a nonthermal feature in the plasma distribution function and is unstable against the growth of electrostatic plasma oscillations, or “plasma waves.” The plasma waves are subsequently converted to electromagnetic waves at frequencies either near the electron plasma frequency ω_e by scattering from the polarization clouds of ions, or near $2\omega_e$ by scattering from each other. This hypothesis has been questioned by Kellogg (1976) and by Gurnett and Frank (1976), who measured plasma-wave energy densities in situ in type III exciters at 1 AU and concluded that the turbulence level was several orders of magnitude lower than indicated by theory. The latter authors suggested that electromagnetic radiation might result from direct conversion of the particle energy and thus be more

efficient than the weak-turbulence mechanisms implicit in the plasma-wave hypothesis. More recent experiments and theory seem to have alleviated these difficulties. First, recent observations on board *Helios 1* and 2 (Gurnett and Anderson 1976, 1977) have actually detected wave energy levels in agreement with those predicted by Papadopoulos, Goldstein, and Smith (1974, hereafter Paper I). In addition, recent theoretical models involving strong turbulence processes and plasma wave collapse (Smith, Goldstein, and Papadopoulos 1976, hereafter Paper II) indicate the spatial localization of the electrostatic turbulence to be such that present-day experiments have marginal capability for detecting them. This aspect of the problem will be discussed in a companion paper (Goldstein, Smith, and Papadopoulos 1979).

Proceeding on the premise of the plasma wave hypothesis, several authors have noted difficulties in understanding the propagation and dynamics of the exciter. Nearly all analyses to date have been done in the context of a one-dimensional theory, in which the projection of the particle distribution function $F(v)$ onto the direction of the background magnetic field (which guides the beam) gives a reduced distribution $f(u)$ such as is schematically depicted in Figure 1. The beam-plasma instability leads to the growth of plasma waves with phase velocities V_ϕ lying in the resonant region where $\partial f/\partial u > 0$. For homogeneous systems quasi-linear theory predicts that the distribution function relaxes to a plateau (Fig. 2). This led to “Sturrock’s dilemma.” after Sturrock (1964) noted that, theoretically, the relaxation is expected to occur within a few

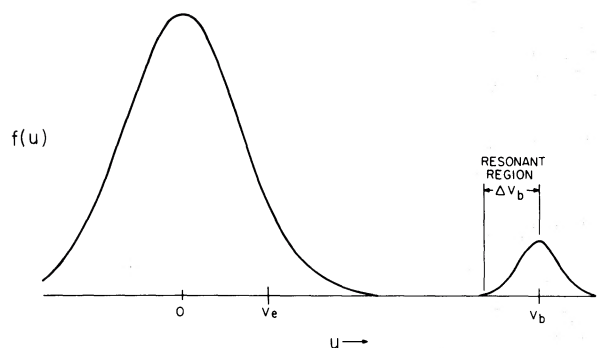


FIG. 1.—Schematic plot of a one-dimensional electron distribution function with a linearly unstable “bump” centered about v_b .

kilometers of the injection region, so that type III bursts should never be observed.

This observation led to a series of papers investigating plasma processes that could prevent quasi-linear relaxation. Two lines of approach were followed. The first one (Group I) was based on homogeneous plasma theory, neglecting wave propagation effects but including nonlinear effects within the limits of weak turbulence theory (Kadomtsev 1965). Prominent among these is nonlinear, random-phase scattering (i.e., induced scattering) on electrons, ions, or ion sound waves. Kaplan and Tsytovich (1968) considered induced scattering off thermal ions (i.e., nonlinear Landau damping on ions) as a possible mechanism that could prevent plateau formation. However, as was shown in detail (Zheleznyakov and Zaitsev 1970; Melrose 1970a; Smith and Fung 1971; Heyvaerts and de Genouillac 1974), the time scale for this process was much longer than that for plateau formation for type III bursts. As noted much earlier by Sturrock (1965), a sufficiently fast time scale for nonlinear scattering could be achieved if a sufficient level of ion-acoustic turbulence were present. This mechanism will then be similar to the presence of a high-frequency (ω_e) anomalous resistivity (Dawson and Oberman 1963) with $\nu_{\text{eff}} \gtrsim \gamma_b$ (ν_{eff} is the anomalous collision frequency and γ_b the instability growth rate). Follow-

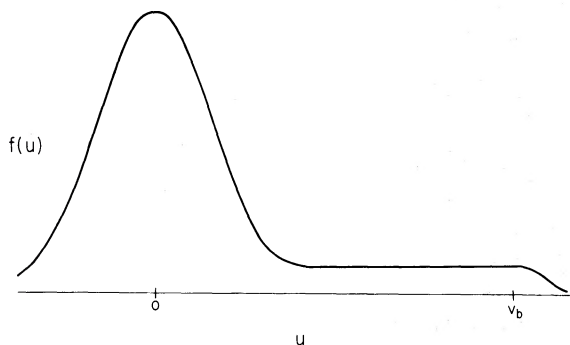


FIG. 2.—The final state of the distribution function shown in Fig. 1 as predicted by the quasi-linear kinetic theory of an infinite, homogeneous plasma.

ing this line Melrose (1970b) proposed that a large level of ion acoustic waves could be generated as a result of the return current induced by the beam if the bulk speed of the electrons $U = (n_b/n_e)v_b$ exceeds the ion sound speed $C_s = (m/M)^{1/2}v_e$. However, a threshold velocity C_s requires a density ratio $n_b/n_e > 10^{-3}$, which is unrealistically high in the corona. This requirement becomes prohibitively severe if one notes that it is erroneous to take C_s as the threshold for a $T_e/T_i \approx 0(1)$ plasma (Papadopoulos 1977). In this case the threshold is $U \approx v_e$, so that $n_b/n_e > 10^{-1}$. The second line of approach (Group II) neglected nonlinear scattering effects, but considered dispersive propagation effects of the exciter electrons and of the plasma waves. Within this approach, Zaitsev, Mityakov, and Rapoport (1972) assumed quasi-linear relaxation but attempted to take into account the finite spatial extent of the beam in a model for its dispersive propagation. Their model is not in accord with observations of the exciter evolution at 1 AU (Lin, Evans, and Fainberg 1973), and their theory has been criticized on other grounds by Smith (1973, 1974).

Harvey and Aubier (1973) and Aubier (1974) considered the effects of wave propagation into a medium of gradually decreasing density. Because the frequency of a wave is preserved during propagation, the wave-number increases in time and the phase velocity decreases; the wave thus passes gradually out of resonance with the particles that excited it. This slowing down, however, occurs over distances comparable to that on which the velocity distribution of the beam must change owing to dispersive propagation, so that the waves should continue to be in resonance with some beam particles and only the head of the beam passes out of resonance with the waves. This fact is not correctly accounted for in these papers. Harvey (1976a, b) constructed a model for the exciter evolution in both time and space, including propagation effects. Although this model is detailed and elegant, it encounters difficulties which may limit its self-consistency by neglecting to account for processes such as pitch-angle scattering and resonance broadening, which should be incorporated, at least phenomenologically, in such a model.

In Paper I it was noted that the plasma wave levels expected from all the above theories were such that nonlinear, coherent effects associated with strong turbulence theory, such as parametric instabilities, plasma wave collapse, and density cavities, will dominate, thereby invalidating their calculations. The main nonlinear effects which can be experienced by the electron plasma waves were included in the Appendix of Paper I. These equations reduce to a nonlinear Schrödinger equation when ion inertia is neglected, which was first studied by Zakharov (1972). To simplify the formalism it was noted that for type III burst parameters the nonlinear equations of Paper I would be equivalent to the parametric, oscillating, two-stream instability. Notice that as shown in Manheimer and Papadopoulos (1975) soliton formation and wave collapse are nothing more than the description in configuration space of well-known

parametric instabilities discussed in Paper I. These instabilities (or, equivalently, the spatial collapse of the waves) act to transfer waves in k -space from the linearly unstable beam-resonant region to lower, nonresonant phase velocities. The OTSI produces symmetric spectra of plasma waves at both positive and negative phase velocities in the nonresonant region, as well as purely growing (i.e., aperiodic) ion-density waves. The object of Paper I was to demonstrate that the OTSI may play an important role in the interaction of the type III exciter with the ambient plasma. With that aim, Paper I considered the temporal instability of a uniform beam, with the beam-resonant waves treated as constituting a monochromatic pump with $k_0 = 0$. The use of $k_0 = 0$ (dipole approximation) allowed us to concentrate on spectral transfers that, contrary to the weak turbulence theory, go toward shorter wavelengths similar to wave collapse.

In this paper we extend the analysis of Paper I. In § II we briefly describe the quasi-linear theory of the injection of an electron beam into a half-infinite space. The levels of Langmuir turbulence produced in that situation are found to be characteristic of strong turbulence; and thus use of the quasi-linear theory, which ignores all strong-coupling (wave-wave) interactions, is generally invalid when the dynamical evolution of the beam at a particular spatial location is considered. Thus an essential difference between this work and those of Group I is that we consider only strong turbulence processes, which are ignored in previous analyses. Because the physical situation is dominated by strong turbulence, we do not consider scattering by weak turbulence. Moreover, in contrast to the papers of Group II, we do not consider wave propagation effects; our theory is strictly local. The justification for this procedure is that the time scale Δt of the spectral evolution is such that $v_g \Delta t \ll L_n$, where $v_g \approx v_e^2/v_b$ is the group velocity of the plasma waves and L_n is the scale length of the solar-wind density gradient (Brejzman and Ryutov 1974).

In addition, our computations do not consider effects of possible density inhomogeneities created by the beam-plasma interaction itself, except insofar as the strong turbulence modifies the linear Bohm-Gross dispersion relation.

As in our previous work, the theory is one-dimensional. The latter feature has been criticized recently by Bardwell and Goldman (1976), who argue that the interaction is significantly modified by consideration of modulational instabilities in three dimensions. We show in § III that when the three-dimensional effects are properly analyzed the spectrum of pump waves excited by the electron beam is indeed predominantly one-dimensional, as we previously assumed. Furthermore, the daughter waves resulting from the OTSI are shown to propagate predominantly parallel and antiparallel to the direction of the beam. Finally, the decay instability is shown to have a higher threshold than the OTSI, at least for parameters characteristic of the interplanetary medium. Section III also contains a derivation of the corrections to the Bohm-Gross dispersion relation of linear Langmuir waves which are

of great importance in the presence of nonlinear ion-density fluctuations, such as are excited by the OTSI or collapse. This dispersion relation is shown to eventually lower the initial threshold for the OTSI, which in turn enhances the scattering of the pump waves to low phase velocities out of response with the electron beam. The final stabilization of the OTSI is accomplished by an anomalous resistivity that is also excited by the ion fluctuations in a fashion similar to Sturrock's. In the following section, we develop an analytical model that describes the total energy loss of an electron beam as it propagates from the corona to 1 AU.

In a subsequent paper (Goldstein *et al.*) the theoretical equations derived here are solved numerically, using parameters appropriate to interplanetary conditions between 0.1 and 1 AU. Comparisons between the predictions of our theory and observations, as well as calculations of the radio intensity resulting from the computed Langmuir turbulence appear in that paper and constitute the second part of our work.

II. LIMITATIONS OF QUASI-LINEAR THEORY

The fact that the type III exciter is not a uniform infinite beam, but rather evolves continuously in both coordinate and velocity space, has important consequences for the nature of the beam-plasma interaction. Regardless of the details of the beam evolution, two consequences in particular may be deduced. The first is that at any point in space, the plasma-wave energy density may exceed by a large factor the limit imposed by quasi-linear theory for the uniform case. Notwithstanding this fact, however, the head of the beam does not suffer significant quasi-linear relaxation.

To better understand these assertions, consider first the simple model of a beam such as in Figure 1 injected into a half-space (Fig. 3a). In this situation the phenomenon of "oscillation pileup" occurs (Tsytovich 1970, p. 177). The unstable waves are convected away from the injection point at the group velocity v_g , while the beam particles move at $v_b \gg v_g$. Conversely, the continuous injection of "fresh" particles into the plasma may build up the local-wave energy density beyond the limit of the uniform case, in which it obviously cannot exceed the energy density of the beam. This buildup is asymptotically limited by the competition between convection and instability. The asymptotic energy density is given (Tsytovich 1970) by

$$\tilde{W} \equiv \frac{E^2}{8\pi} \approx (v_b/v_e)^2 n_b m v_b \Delta v_b, \quad (2.1)$$

which represents an enhancement of the uniform limit by the factor $(v_b/v_e)^2$. The physical reason for this enhancement is the fact that in a steady-state injection the important quantities to be balanced are the energy fluxes [i.e., $v_g(E^2/8\pi) \approx \frac{1}{2} n_b m v_b^3 (\Delta v_b/v_b)$]. This level is reached within a distance

$$\tilde{X} \approx v_g/\gamma_b \quad (2.2)$$

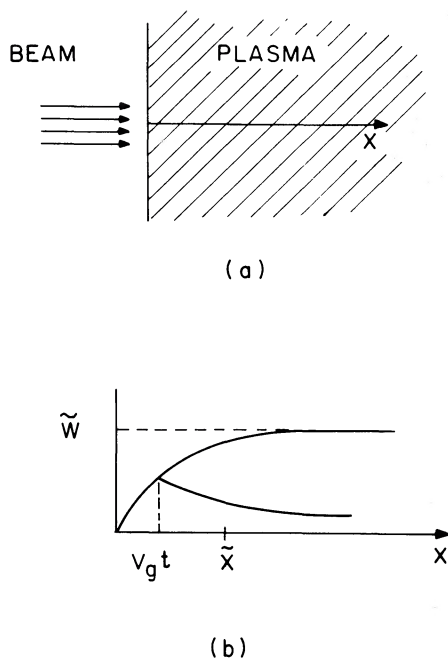


FIG. 3.—The effect of “oscillation pileup” which occurs when the electron beam is injected into a half-infinite plasma. Again, the final state (b) is that predicted by quasi-linear theory.

from the injection point, where

$$\gamma_b \approx \frac{n_b}{n_e} \left(\frac{v_b}{\Delta v_b} \right)^2 \omega_e \quad (2.3)$$

is the coarse-grained average of the beam-plasma instability growth rate.

The distance \tilde{X} is limited at any point x to

$$\tilde{X}(x) < \left(\frac{\Delta v_b}{v_b} \right)^2 \left(\frac{v_e}{v_b} \right) n_e \lambda_e^4, \quad (2.4)$$

where λ_e is the electron Debye length. For $v_b \sim \frac{1}{3}c$, $\Delta v_b/v_b \approx 1$, (2.4) gives values for \tilde{X} of less than about 60 km at the 100 MHz level. Even at 1 AU, $\tilde{X} < 10^{-2}$ AU. Because these values are much smaller than the corresponding length scales of the ambient density gradient, the injection model of oscillation pileup is at least qualitatively appropriate. Then equation (2.1) implies that values

$$W \equiv \frac{\tilde{W}}{n_e T_e} \approx \eta \left(\frac{E_b}{T_e} \right)^2 \quad (2.5)$$

may be attained, where $\eta \equiv n_b/n_e$, and E_b is the nominal beam energy. At 1 AU, the observations of Lin, Evans, and Fainberg (1973) and of Frank and Gurnett (1972) imply $\eta \approx 10^{-6}$, while equation (2.5) will give $W \gg 1$. The significance of this fact is that such high-energy densities in the resonant wave

spectrum are sufficient to drive fixed-phase instabilities even with broad-band pump spectra.

Notice that the presence of plasma waves with energy W in an isothermal plasma will produce a density depression $\delta n/n = \tilde{W}/nT$, as a result of pressure balance ($nT + \tilde{W} = \text{constant}$). Since $\omega_k = \omega_e(1 + \frac{3}{2}k^2\lambda_e^2)$, the nonlinear plasma frequency will be

$$\begin{aligned} \omega_k &= \omega_e \left(1 + \frac{3}{2} k^2 \lambda_e^2 - \frac{1}{2} \frac{\delta n}{n} \right) \\ &\approx \omega_e \left(1 + \frac{3}{2} k^2 \lambda_e^2 - \frac{1}{2} \frac{\tilde{W}}{nT} \right). \end{aligned}$$

Therefore, when $W \approx (k\lambda_e)^2$, strong turbulence phenomena such as OTSI or collapse will become important. Since for type III bursts $(k\lambda_e)^2 \approx (v_e/v_b)^2 \approx 10^{-4}$ at 1 AU, this level which constitutes the threshold for our processes is easily achieved. A more detailed derivation of the nonlinear modifications to the Bohm-Gross dispersion relation is deferred to § III.

III. INSTABILITY THEORY

In Papers I and II, we utilized a simple derivation of the OTSI growth rate which assumed a monochromatic, one-dimensional pump wave spectrum. We give here a detailed derivation of the general dispersion relation for modulation and collapse instabilities driven by an arbitrary spectrum of plasma waves. For the parameter range of interest for solar type III bursts, we show that one is justified in approximating a general three-dimensional pump spectrum with a one-dimensional one. Furthermore, the fastest-growing daughter Langmuir waves are shown to be one-dimensional also, aligned approximately parallel and antiparallel to the mean magnetic field.¹ The general OTSI dispersion relation thus reduces to the relatively simple form used in Paper II; and so we again find that the OTSI provides a mechanism for stabilizing the bursts against catastrophic energy losses that would otherwise prevent them from traveling out to 1 AU.

We conclude this theoretical development with a discussion of additional nonlinear effects that are important to the eventual stabilization of the OTSI. The first is that the Bohm-Gross dispersion relation is changed by the density fluctuations amplified through the OTSI or their subsequent decay to ion acoustic waves. Then these density fluctuations produce an anomalous resistivity that enhances the tendency of plasma waves to scatter to shorter wavelengths. This in turn hastens the decoupling of the electron beam from the Langmuir radiation.

These diverse phenomena can be described in terms of a two-fluid hydrodynamic model of the plasma in which the motions of the plasma are separated into

¹ Note that the effects described here are completely nonlinear, and our linearization corresponds to determining the time scale of the events.

fast-plasma oscillations ($\omega \approx \omega_e$) and quasi-neutral, slow sound oscillations. We take the ion density as

$$n_i = n_0 + \delta n_i, \quad \delta n_i/n_0 \ll 1, \quad (3.1)$$

and the electron density as

$$n_e = n_i + \delta n_e, \quad \delta n_e/n_i \ll 1. \quad (3.2)$$

For the high-frequency fields the linearized electron equations are

$$\frac{\partial \delta n_e}{\partial t} - n_0 \nabla \cdot \delta \mathbf{v}_e = -\nabla \cdot (\delta n_i \delta \mathbf{v}_e), \quad (3.3)$$

$$n_0 \frac{\partial}{\partial t} \delta \mathbf{v}_e + \frac{\gamma_e T_e}{m} \nabla \delta n_e = -\frac{e}{m} n_0 \boldsymbol{\epsilon} - \nu_e n_0 \delta \mathbf{v}_e, \quad (3.4)$$

with

$$\nabla \cdot \boldsymbol{\epsilon} = -4\pi e \delta n_e, \quad (3.5)$$

and where γ_e is the polytropic index and ν_e is a phenomenological damping decrement. With the right-hand sides equal to zero, these equations reduce to the usual second-order equations that describe Langmuir waves.

The equations for the ions are similar except that the ions experience a low-frequency pondermotive force \mathbf{F} caused by the gradient of the pressure of the Langmuir waves. If we define $P = P_e + P_i$ to be the total plasma pressure, then

$$\mathbf{F} = -\frac{m}{2\omega_e^2} \left(\frac{e}{m} \right)^2 \nabla \cdot \langle \boldsymbol{\epsilon} \boldsymbol{\epsilon} \rangle, \quad (3.6)$$

where $\langle \rangle$ denotes an average over the fast time scale. The linearized ion-momentum equation can now be written as

$$nM \frac{d\mathbf{v}}{dt} = -\nabla P + \mathbf{F} - \nu_i nM \mathbf{v}_i. \quad (3.7)$$

Using an equation of state of the form $P = MC_s^2 n$ with $C_s^2 = (\gamma_e T_e + \gamma_i T_i)/M$ in conjunction with the continuity equation, the ion fluctuations are found to satisfy

$$\left(\frac{\partial^2}{\partial t^2} + \nu_i \frac{\partial}{\partial t} - C_s^2 \nabla^2 \right) \delta n_i = \frac{1}{8\pi M} \nabla \cdot (\nabla \cdot \langle \boldsymbol{\epsilon} \boldsymbol{\epsilon} \rangle). \quad (3.8)$$

Under the restriction that $(k\lambda_e)^2 \ll 1$ and $\nu_e/\omega_e \ll 1$, equations (3.3) and (3.4) can be combined, yielding

$$\left(\frac{\partial^2}{\partial t^2} + \nu_e \frac{\partial}{\partial t} - \frac{\gamma_e T_e}{m} \nabla \cdot - \omega_e^2 \right) \boldsymbol{\epsilon} = -\frac{\omega_e^2}{n_0} \delta n_i \boldsymbol{\epsilon}. \quad (3.9)$$

Equations (3.8) and (3.9) are the fundamental equations that describe the coupling of high-frequency plasma waves to low-frequency density fluctuations. The equations can be simplified by introducing the slowly varying quantity $\mathbf{E}(\mathbf{x}, t)$ defined by

$$\boldsymbol{\epsilon}(\mathbf{x}, t) = \frac{1}{2} [\mathbf{E}(\mathbf{x}, t) e^{-i\omega_e t} + \mathbf{E}^*(\mathbf{x}, t) e^{i\omega_e t}]. \quad (3.10)$$

In terms of $\mathbf{E}(\mathbf{x}, t)$ equations (3.8) and (3.9) become

$$\left(i \frac{\partial}{\partial t} + \frac{\gamma_e T_e}{m\omega_e} \nabla \cdot + i\nu_e \right) \mathbf{E} = \frac{\omega_e}{2n_0} \delta n_i \mathbf{E} \quad (3.11)$$

and

$$\left(\frac{\partial^2}{\partial t^2} + \nu_i \frac{\partial}{\partial t} - C_s^2 \nabla^2 \right) \delta n_i = \frac{1}{16\pi M} \nabla^2 |\mathbf{E}|^2. \quad (3.12)$$

Equations (3.11) and (3.12) are the basic equations that describe either the OTSI or the formation of solitons and cavitons (Manheimer and Papadopoulos 1975). In this section we are primarily interested in deriving the growth rate of the OTSI and comparing it with similar growth rates for other modulation instabilities. The spatial structures are discussed in Goldstein *et al.*

Upon defining the Fourier transform as

$$\begin{aligned} & [\delta n(\mathbf{k}, t), \mathbf{E}(\mathbf{k}, t)] \\ &= \frac{1}{(2\pi)^3} \int d\mathbf{x} e^{-i\mathbf{k} \cdot \mathbf{x}} [\delta n(\mathbf{x}, t), \mathbf{E}(\mathbf{x}, t)], \end{aligned} \quad (3.13)$$

equations (3.11) and (3.12) become

$$\begin{aligned} & \left[i \frac{\partial}{\partial t} - \frac{3}{2} \omega_e (k\lambda_e)^2 \right] \mathbf{E}(\mathbf{k}, t) \\ &= -i\nu_{ek} \mathbf{E}(\mathbf{k}, t) + \frac{\omega_e}{2n_0} \int d\mathbf{k}' \delta n(\mathbf{k} - \mathbf{k}', t) \mathbf{E}(\mathbf{k}', t) \end{aligned} \quad (3.14)$$

and

$$\begin{aligned} & \left(\frac{\partial^2}{\partial t^2} + \nu_i \frac{\partial}{\partial t} + k^2 C_s^2 \right) \delta n(\mathbf{k}, t) \\ &= \frac{-k^2}{16\pi M} \int d\mathbf{k}' \mathbf{E}(\mathbf{k}') \cdot \mathbf{E}^*(\mathbf{k} - \mathbf{k}'). \end{aligned} \quad (3.15)$$

A complete solution of these equations including all the nonlinearities is beyond the scope of this discussion, for here we are most interested in studying the linear stability properties of a spectrum of Langmuir waves that is excited by an electron beam in order to determine which of the various modulational instabilities is likely to dominate under plasma conditions characteristic of type III bursts. We assume that a pump spectrum of Langmuir waves exists between $|\mathbf{k}_1| < |\mathbf{k}| < |\mathbf{k}_2|$. These waves are taken to be zero-order quantities, while waves outside that region are the small perturbations $\delta \mathbf{E}(\mathbf{k}, t)$ and $\delta n(\mathbf{k}, t)$. From equations (3.14) and (3.15) we find

$$\begin{aligned} & \left[i \left(\frac{\partial}{\partial t} - \nu_{ek} \right) - \frac{3}{2} \omega_e (k\lambda_e)^2 \right] \delta \mathbf{E}(\mathbf{k}, t) \\ &= \frac{\omega_e}{2n_0} \int d\mathbf{k}' \delta n(\mathbf{k} - \mathbf{k}', t) \mathbf{E}(\mathbf{k}', t) \end{aligned} \quad (3.16)$$

and

$$\begin{aligned} & \left(\frac{\partial^2}{\partial t^2} + \nu_i \frac{\partial}{\partial t} + k^2 C_s^2 \right) \delta n(\mathbf{k}, t) \\ &= \frac{-k^2}{16\pi M} \int d\mathbf{k}' [E(\mathbf{k}', t) \cdot \delta E^*(\mathbf{k} - \mathbf{k}', t) + \text{c.c.}] . \end{aligned} \quad (3.17)$$

We now define the propagators $L_E(\mathbf{k}, t)$ and $L_N(\mathbf{k}, t)$ through the relations

$$\begin{aligned} L_E(\mathbf{k}, t) \delta E(\mathbf{k}, t) \\ &= \frac{\omega_e}{2n_0} \int d\mathbf{k}' \delta n(\mathbf{k} - \mathbf{k}', t) E(\mathbf{k}', t) , \end{aligned} \quad (3.18a)$$

$$\begin{aligned} L_N(\mathbf{k}, t) \delta n(\mathbf{k}, t) \\ &= \frac{-k^2}{16\pi M} \int d\mathbf{k}' [E(\mathbf{k}', t) \cdot \delta E^*(\mathbf{k} - \mathbf{k}', t) + \text{c.c.}] . \end{aligned} \quad (3.18b)$$

From equations (3.16)–(3.18) we obtain the dispersion relation

$$\begin{aligned} L_N(\mathbf{k}, \omega) \\ &= \frac{k^2 \omega_e}{32\pi M n_0} \int d\mathbf{k}' |E(\mathbf{k}')|^2 \\ &\times \left\{ \frac{[\mathbf{k}' \cdot (\mathbf{k} - \mathbf{k}')]^2}{|\mathbf{k}'|^2 |\mathbf{k} - \mathbf{k}'|^2} (L_E^{-1})^*[\mathbf{k} - \mathbf{k}', \omega - \omega(\mathbf{k}')] \right. \\ &\quad \left. + \frac{[\mathbf{k}' \cdot (\mathbf{k} + \mathbf{k}')]^2}{|\mathbf{k}'|^2 |\mathbf{k} + \mathbf{k}'|^2} L_E^{-1}[\mathbf{k} + \mathbf{k}', \omega + \omega(\mathbf{k}')] \right\} , \end{aligned} \quad (3.19)$$

where L_E^{-1} is the inverse of the propagator L_E , the integration is over the volume in \mathbf{k} -space initially occupied by the unstable waves, and $\omega(\mathbf{k}) \equiv \frac{3}{2}\omega_e(k\lambda_e)^2$. From the definitions of L_E and L_N , (3.19) can be rewritten as

$$\begin{aligned} & [-\omega(\omega + i\nu_i) + k^2 C_s^2] \\ &= -\frac{k^2 \omega_e}{32\pi M n_0} \int d\mathbf{k}' |E(\mathbf{k}')|^2 \\ &\times \left\{ \frac{[\mathbf{k}' \cdot (\mathbf{k} - \mathbf{k}')]^2}{|\mathbf{k}'|^2 |\mathbf{k} - \mathbf{k}'|^2} [-(\omega + i\nu_{ek}) - \frac{3}{2}\omega_e(k'\lambda_e)^2 \right. \\ &\quad \left. + \frac{3}{2}(\mathbf{k} - \mathbf{k}')^2 \lambda_e^2 \omega_e]^{-1} \right. \\ &\quad \left. + \frac{[\mathbf{k}' \cdot (\mathbf{k} + \mathbf{k}')]^2}{|\mathbf{k}'|^2 |\mathbf{k} + \mathbf{k}'|^2} [-(\omega + i\nu_{ek}) - \frac{3}{2}\omega_e(k'\lambda_e)^2 \right. \\ &\quad \left. + \frac{3}{2}(\mathbf{k} + \mathbf{k}')^2 \lambda_e^2 \omega_e]^{-1} \right\} . \end{aligned} \quad (3.20)$$

The theory as originally presented in Papers I and II was one-dimensional. Bardwell and Goldman (1976) have maintained that a three-dimensional stability analysis is essential. In asking whether a one-dimensional treatment is allowed, there are actually three separate questions that should be addressed: (1) Is the spectrum of waves excited by the electron beam predominantly one-dimensional? (2) Are the daughter waves produced by the parametric instabilities predominantly one-dimensional? (3) What is the role of the decay instability?

We discuss each question separately below.

1. The growth rate of the beam instability is given by

$$\gamma = (\pi/2)^{1/2} (n_b/n_0) (V_b/\Delta V_b)^2 \omega_e \cos^2 \Psi , \quad (3.21)$$

where Ψ is the angle between the wave vector and the magnetic field. Notice that the maximum growth occurs along the field ($\Psi = 0$), while it rapidly decreases for $\Psi \rightarrow \pi/2$. This implies that the off-angle waves grow more slowly than the field aligned ones until they reached the nonlinear stage. To determine the dependence of the wave spectrum on k_\perp or angle Ψ , we consider the equation $W = W_0 e^{2\gamma t}$ with growth determined by (3.21). The half-width of the spectrum when the waves have reached the nonlinear level ($W \approx 10^{-3}$ – 10^{-5}) is then given by

$$\Psi_{1/2} \approx (\Delta k_\perp/k_0) \approx \left(\frac{\ln 2}{24} \frac{V_b}{V_b + \sqrt{2}\Delta V_b} \right)^{1/2} \ll 1 . \quad (3.22)$$

Note that for $\Delta V_b \approx \frac{1}{2}V_b$, $(\Delta k_\perp/k_0)^2 \approx 1/60$, an angle substantially less than 7° . Therefore, the instability spectrum is basically one-dimensional (i.e., the instability is confined to a narrow cone about the direction of the magnetic field).

2. From the general dispersion relation (3.20), we can now assume that the pump spectrum $W(k')$ is one-dimensional to a high degree of approximation. Therefore, in spherical coordinates we write the spectrum of pump waves as

$$W(k') = \frac{W_0}{2\pi^2 k'^2} \frac{W k_0 \delta(\cos \Psi' - 1)}{(k - k_0)^2 + (\Delta k_0)^2} \frac{\Delta k_0}{k_0} \ll 1 , \quad (3.23)$$

where Δk_0 is the width of the pump-wave spectrum centered at k_0 and now is assumed to be aligned along the beam direction. For convenience we will use a dimensionless notation in which $k \rightarrow (k\lambda_e)$, $\omega \rightarrow \omega/\omega_e$, $\nu \rightarrow \nu/\omega_e$, $\mu \equiv m/M$, $\int d\mathbf{k}' (|E(\mathbf{k}')|^2/8\pi n T_e) = W_0$, and $T \equiv 1 + (3T_i/2T_e)$.

From equations (3.20) and (3.23), we find on integrating over the resonance that

$$\begin{aligned} & [\omega(\omega + i\nu_i) - \mu k^2 T] \\ &+ \frac{\frac{3}{4}\mu k^4 W_0 F(k, k_0, \Psi)}{\frac{9}{4}k^4 - [(\omega + i\nu_e) + 3i|k|\Delta k_0 \cos \Psi' - 3kk_0 \cos \Psi]^2} \\ &= 0 , \end{aligned} \quad (3.24)$$

where $F(\Psi) \approx \cos^2 \Psi$ for $k \gg k_0$, and

$$F(\Psi) \approx 1 + (4k^2 \cos^2 \Psi / k_0^2) - (k^2 \sin^2 \Psi / k_0^2)$$

for $k \ll k_0$. In the limit $(\Delta k_0 / k_0) \rightarrow 0$, equation (3.24) reduces to the dispersion relation found by Papadopoulos (1975) and Bardwell and Goldman (1976). It is interesting to note that the finite width of the spectrum enters like an electron dissipation [i.e., $\omega + i\nu_e \rightarrow \omega + i(\nu_e + 3k\Delta k_0 \cos \Psi)$]. Physically, this is due to a decorrelation of the waves produced by the finite bandwidth.

In order to achieve nonlinear stabilization, the phase velocity of the daughter waves should be outside the range of the resonances of the linear instability. Because k is the wavenumber of the low-frequency wave, the phase velocity of the daughter waves is $\omega_e / (|k_0 \pm k|)$. The only way that both of the daughter waves can be outside the instability region is if $k \gg k_0$. If $k \ll k_0$, nonlinear stabilization is impossible and instead one gets a redistribution of the wave energy among the unstable modes. For the relevant case of $k_0 \ll k$, with $\Delta k_0 \lesssim \frac{1}{3}k_0$, and neglecting collisions, equation (3.24) becomes

$$\omega^2 - \mu k^2 T = \frac{1}{2} \mu^2 k^2 W_0 \frac{\delta \cos^2 \Psi}{\delta^2 - \omega^2}, \quad (3.24a)$$

where $\delta \equiv -\frac{3}{2}k^2$.

From equation (3.24a)

$$\begin{aligned} \omega^2 &= \frac{1}{2}(\delta^2 + k^2 \mu T) \\ &\pm \left[\frac{1}{4}(\delta^2 + k^2 \mu T) - 2A\delta \cos^2 \Psi - k^2 \mu T \delta^2 \right]^{1/2}, \end{aligned} \quad (3.25)$$

where $A \equiv (k^2 \mu W_0 / 4)$. The condition for instability with $\delta < 0$ is $\cos^2 \Psi > 3k^2 T / W_0$, which shows that there exists an angular cone outside of which the instability is suppressed. The behavior of the instability growth rate as a function of δ is as follows: For $|\delta| \ll A^{1/3} \cos^{2/3} \Psi$ it is $\gamma_{\max} \sim A^{1/3} \cos^{2/3} \Psi$; finally, for $|\delta| \gg A^{1/3} \cos^{2/3} \Psi$ the growth rate decreases, vanishing at $\cos^2 \Psi = (2|\delta|/W)$. From the above discussion it is clear that maximum γ occurs *always along the pump direction*. We should note that the above results were generalized recently by including a magnetic field (Krasnosel'skikh and Sotnikov 1977). The resulting dispersion relations are exactly equations (3.24a) and (3.25), if δ is replaced by

$$\delta = -\frac{3}{2}k^2 + \Omega_e^2 (\sin^2 \alpha - \sin^2 \beta),$$

where Ω_e is the electron cyclotron frequency normalized to ω_e and $\sin \alpha \equiv (k_{0\perp} / k_0)$, and $\sin \beta \equiv (k_{\perp} / k)$. The inclusion of the magnetic field further strengthens the one-dimensionality as shown by Krasnosel'skikh and Sotnikov (1977) for both the periodic and aperiodic instabilities. An important point proved there is that the shape of the pump spectrum is preserved (cf. eq. [17] of their paper).

The problem with the work of Bardwell and Goldman (1976) is that they restricted their analysis of the

aperiodic instability to the case $k \ll k_0$, which does not provide stabilization. However, even this analysis leads to deceptive conclusions because of the neglect of the finite bandwidth of the spectrum. This can be seen by considering equation (3.24) for $W_0 < \mu$, when the ion response can be neglected. The dispersion relation then gives

$$\begin{aligned} \omega &= 3kk_0 \cos \Psi - 3i|k|\Delta k_0 \cos \Psi \\ &+ (\frac{1}{2}3^{1/2})i[W_0 F(\Psi)/T - 3k^4]^{1/2}. \end{aligned} \quad (3.26)$$

The maximum growth rate as a function of angle is given by $\partial(\text{Im } \omega)/\partial \Psi = 0$, or

$$\begin{aligned} -3|k|\Delta k_0 \sin \Psi + \frac{3^{1/2}}{4} \left(\frac{W_0 F(\Psi)}{T} - 3k^4 \right)^{-1/2} \\ \times \frac{\partial F(\Psi)}{\partial \Psi} = 0, \end{aligned} \quad (3.27)$$

which can be satisfied *only* if $\Psi = 0$ for both $k < k_0$ and $k > k_0$.

We can therefore conclude that for strong pumps ($W_0 > k_0^2, \mu$), when the analysis leading to equation (3.25) is valid, as well as for weak pumps ($W_0 < \mu$), when equation (3.27) is appropriate, the one-dimensional treatment of parametric instabilities is valid. In the weak-pump case, the finite bandwidth of the spectrum suppresses the instability of the off-angle waves.

3. The final question which we want to investigate is whether the decay instability is important when $W_0 \approx 10^{-5}$. This is a three-wave interaction whose threshold is found by setting $F(\Psi) = 1$ and $\cos \Psi = 1$ in equation (3.24). The result is

$$W_{\text{decay}}^{\text{thr}} = 8 \frac{\nu_i}{\mu k T} (3|k|\Delta k_0).$$

For $T_e/T_i \approx 1$, $\nu_i/\mu k T \approx 2$ and $W_{\text{decay}}^{\text{thr}} \approx 50|k|\Delta k_0 \gg 50k_0\Delta k_0 = 50k_0^2\Delta k_0/k_0 \approx 50(1/5)k_0^2 \approx 10k_0^2 \gg 10^{-5}$.

Thus we have not exceeded threshold for the decay instability. We conclude, then, that for the case under consideration our assumptions are very well justified. This allows us to proceed as in Papers I and II without the necessary encumbrance of using the three-dimensional dispersion relation.

If we define the frequency shift between the pump wave with wave number k_0 and the daughter wave with wavenumber k' as $\delta = \omega_{ek_0} - \omega_{ek'}$, where both ω_{ek_0} and $\omega_{ek'}$ satisfy the Bohm-Gross dispersion relation, and $\delta < 0$ for the OTSI, then the growth rate for the OTSI can be found from (3.25) to be approximately given by

$$\begin{aligned} \gamma_{\text{OTSI}}^2(k_0, k') \\ = -\frac{1}{2}(\omega_A^2 + \delta^2) \\ + \frac{1}{2} \left\{ (\omega_A^2 + \delta^2)^2 - 4\delta^2 \omega_A^2 \left[1 + \frac{W(k_0)\omega_e^2}{2\delta\omega_{ek'}} \right] \right\}^{1/2}, \end{aligned} \quad (3.28)$$

where

$$\omega_A^2 = \frac{m}{M} \frac{(k'\lambda_e)^2}{1 + (k'\lambda_e)^2} \omega_e^2.$$

From (3.28) the threshold for the OTSI, in the absence of collisions, is given by

$$W_T = -2\delta\omega_{ek'}/\omega_e^2. \quad (3.29)$$

The amplification of aperiodic sound waves by the OTSI eventually changes the real part of the plasma dispersion relation of the Langmuir waves, and this will in turn modify the threshold condition (3.29). This effect has been studied in the dipole approximation by Kaw, Lin, and Dawson (1973), albeit in a different context. We present here a generalization of their analysis appropriate for a finite wavenumber pump. Starting with equation (3.9) one assumes the existence of an ion-density fluctuation given by

$$\delta n_i = n_0[1 + \xi \cos(k_i x)], \quad (3.30)$$

which, with $\nu_e = 0$, gives

$$\frac{\partial^2 \epsilon}{\partial t^2} + \omega_e^2(1 + \xi \cos k_i x)\epsilon - \frac{3T_e}{m} \frac{\partial^2 \epsilon}{\partial x^2} = 0. \quad (3.31)$$

Again we have restricted our attention to one dimension and have used an adiabatic polytropic index. Taking discrete Fourier space and time transforms of (3.31) gives

$$\left(\omega_e^2 - \omega_l^2 - \frac{3T_e k^2}{m}\right)\epsilon_l(k) + \xi \frac{\omega_e^2}{2} [\epsilon_\rho(k + k_i) + \epsilon_\rho(k - k_i)] = 0, \quad (3.32)$$

where the subscripts l and ρ are time transform variables, i.e., $\epsilon(x, t) \rightarrow \epsilon(k, \rho)$. Because $\epsilon_l(k)$ couples to $\epsilon_l(k + k_i)$, and $\epsilon_l(k \pm k_i)$ then couples to $\epsilon_l(2k \pm k_i)$, etc., equation (3.32) in fact represents an infinite system of equations. For simplicity, we follow Kaw, Lin, and Dawson (1973) and restrict ourselves to coupling only between k and $k \pm k_i$. The dispersion relation can then be written as

$$\begin{vmatrix} \frac{\omega_e^2 - \omega_l^2 - 3T_e k^2}{m} & \frac{\xi \omega_e^2}{2} & \frac{\xi \omega_e^2}{2} \\ \frac{\xi \omega_e^2}{2} & \frac{\omega_e^2 - \omega_l^2 - 3T_e k^2}{m} & 0 \\ \frac{\xi \omega_e^2}{2} & 0 & \frac{\omega_e^2 - \omega_l^2 - 3T_e k^2}{m} \end{vmatrix} = 0. \quad (3.33)$$

Note that for $k = 0$ we recover the results of Kaw, Lin, and Dawson which they derived by solving the (Mathieu) equation for $\epsilon_l(x)$. As it is not our purpose in this paper to present a detailed discussion of the

scattering of Langmuir waves from ion fluctuations, we give only an approximate solution of the cubic equation for ω_l^2 (3.33), and proceed by assuming that a wave at wavenumber k couples only to $k + k_i$ (i.e., to a wave of lower phase velocity) rather than to $k \pm k_i$. This approximation reduces (3.33) to a quadratic equation in ω_l^2 and introduces an error of order two in the nonlinear frequency shift which we are about to derive, as can be confirmed by returning to the dipole approximation, $k = 0$, and then solving (3.33) both exactly and with the approximation introduced above.

Equation (3.33) then reduces to

$$\frac{\omega_k^2}{\omega_e^2} \approx 1 + 3(k\lambda_e)^2 + \frac{3}{2}(k_i\lambda_e)(k_i + k)\lambda_e \pm \frac{3}{2} \left\{ (k_i\lambda_e)^2(k + k_i)^2 + \frac{\xi^2}{9} \right\}^{1/2}. \quad (3.34)$$

From (3.29) it is clear that if one chooses the “plus” sign in (3.34) to go with the pump wave and the “minus” sign for the daughter wave, any significant excitation of nonthermal ion fluctuations by the OTSI will tend to decrease $|\delta|$ and thus lower the threshold for the OTSI.

A further effect which we consider is that of anomalous resistivity (see Dawson 1968). In principle, this should be contained in the analysis described above. However, to exhibit that, it would be necessary to further generalize the dispersion relaxation (3.33) to include coupling to many modes ($l \gg 1$). Because that is beyond the scope of the present discussion, we have chosen to utilize equation (3.34) to illustrate only the physical phenomenon of a reduced threshold for the OTSI in the presence of a nonthermal level of density fluctuations. We describe the subsequent dissipation of the long-wavelength waves to shorter wavelengths in terms of the high-frequency anomalous resistivity (Dawson and Overman 1963), which is excited near ω_e by the ion fluctuations. In this process, the electron energy originally carried by long-wavelength Langmuir waves is conserved (because the ions are treated as fixed), but cascades to shorter wavelengths as the electrons scatter from the correlated ions. In Fourier space the spectrum of Langmuir waves will contain spectral components at high wavenumber. (Any new components appearing at small wavenumber would ultimately be rescattered, either by the OTSI or by further anomalous dissipation back to higher wavenumbers, and so we will not consider this possibility in our subsequent analysis.)

A complete discussion of plasma anomalous impedance excited near ω_e by oscillating electric fields is contained in Dawson and Oberman (1963) and Dawson (1968); here we only outline the derivation.

In a plasma of isotropically distributed ions the effective electron-ion collision time τ_c , is related to the scalar impedance by

$$Z(\omega) = \frac{4\pi i \omega}{\omega_e^2} (1 - i/\omega\tau_c). \quad (3.35)$$

For $\omega \approx \omega_e$, Dawson (1968) finds that in the presence of a three-dimensional spectrum of correlated, non-thermal ion fluctuations

$$Z(\omega \approx \omega_e) \approx \frac{\omega(k_i \lambda_e)}{9\omega_e^2 n_0 \lambda_e^3} \Delta, \quad (3.36)$$

where Δ is the enhancement of the plasma impedance due to the correlations of the ions, and is given by

$$\Delta \approx \frac{(2\pi)^3 (\delta n_i / n_0)^2 n_0}{4\pi k_i^2 \Delta k}, \quad (3.37)$$

so that

$$\begin{aligned} \gamma_{NL}/\omega_e &\equiv 1/\omega_e \tau_c \approx \frac{\pi}{2} \frac{(\delta n_i / n_0)^2}{(k_i \lambda_e)^2} \\ &= \sum \frac{[S(k_i) - S_0(k_i)]}{(k_i \lambda_e)^4}, \end{aligned} \quad (3.38)$$

where $S_0(k)$ is the thermal noise level of the ion spectrum.

A complete description of this nonstationary turbulence should include a mechanism for the damping of the aperiodic ion fluctuation in the presence of the Langmuir solitons when the electrostatic fields have fallen below threshold for the OTSI. A theory of this highly nonlinear situation has yet to be developed, though various aspects of the problem have been studied. For example, Yu and Spatschek (1976) have considered the damping of ion acoustic waves in a medium containing strong Langmuir turbulence. In their analysis, the ion waves are assumed to satisfy a normal mode dispersion relation, whereas the ion fluctuations excited by the OTSI are not normal modes, but must have frequencies $\omega < \gamma_{OTS}$. In fact, we cannot make use of the Yu and Spatschek result in the problem at hand.

However, a detailed knowledge of the ion-fluctuation damping mechanism is not really necessary in the present discussion so we have adopted the following approximations, which we feel do no violence to the basic physics. For $T_e > T_i$ we find

$$\frac{\nu_s}{\omega_e} \approx (\pi/8)^{1/2} \left(\frac{\omega}{\omega_e} \right)^2 \frac{1}{k \lambda_e} = (\pi/8)^{1/2} (k \lambda_e)^3. \quad (3.39)$$

If $T_e \approx T_i$

$$\frac{\nu_s}{\omega_e} \approx \frac{1}{2} (m/M)^{1/2} (k \lambda_e). \quad (3.40)$$

The electron oscillations are also damped by the thermal plasma. In the solar wind the electron distribution function is characterized by a thermal component at a $T_e \approx 1.15 \times 10^5$ K and a suprathermal tail ($\beta \geq 0.01$) which we have approximated by (Lin, Anderson, and Cline 1972)

$$\begin{aligned} f_w &= 1.07 \times 10^{-11} \eta_1 |\beta|^{-9.8}, \quad 0.01 \leq |\beta| \leq 0.088, \\ &= 1.02 \times 10^{-4} \eta_2 |\rho|^{-5.4}, \quad |\beta| \geq 0.088, \end{aligned} \quad (3.41)$$

where $\eta_1 = 5.5 \times 10^{-4}/n$ and $\eta_2 = 2.5 \times 10^{-6}/n$. Defining

$$\begin{aligned} f_T &= f_w(\beta) + f(\beta), \quad \beta > 0, \\ &= f_w(\beta), \quad \beta < 0, \end{aligned} \quad (3.42)$$

we can include Landau damping by solar-wind electrons along with linear growth of Langmuir waves excited by the electron beam and reabsorption caused by evolution of the beam to lower velocities, by evaluating

$$\frac{\gamma_L}{\omega_e} = \pi \frac{\beta^3}{|\beta|} \frac{\partial f_T}{\partial \beta} \Big|_{\beta = \omega/kc}. \quad (3.43)$$

The transfer rates γ_L , γ_{OTS} , γ_{NL} and ν_s enable us to describe the evolution of the turbulence spectrum by a set of rate equations similar to those used in Paper II. As in that discussion we denote by $P(k)$ and $A(k)$ the plasmon and phonon number densities which correspond to the energy densities $W(k)$ and $S(k)$:

$$\begin{aligned} W(k) &\equiv P(k) \hbar \omega_k / n_e T_e, \\ S(k) &\equiv A(k) \hbar \omega_A / n_0 T_e. \end{aligned} \quad (3.44)$$

The rate equations are then

$$\begin{aligned} \frac{dA(k)}{dt} &= \sum_{k' < k} \{ [\gamma_{OTS}(k', k) + \gamma_{OTS}(-k', k)] \\ &\quad - \nu_s(k) \} A(k), \end{aligned} \quad (3.45)$$

$$\begin{aligned} \frac{dP(k)}{dt} &= \gamma_L(k) P(k) - \sum_{k' > k} \gamma_{OTS}(k, k') A(k') \\ &\quad - P(k) \sum_{k' > k} \gamma_{NL}(k') \\ &\quad + \frac{1}{2} \gamma_{NL}(k) \sum_{k' < k} [P(k') + P(-k')] \\ &\quad + \frac{1}{2} \sum_{k' < k} [\gamma_{OTS}(k', k) + \gamma_{OTS}(-k', k)] A(k), \end{aligned} \quad (3.46)$$

$$\begin{aligned} \frac{dP}{dt}(-k) &= \gamma_L(-k) P(-k) - \sum_{k' > k} \gamma_{OTS}(-k, k') A(k') \\ &\quad - P(-k) \sum_{k' > k} \gamma_{NL}(-k') \\ &\quad + \frac{1}{2} \gamma_{NL}(-k) \sum_{k' < k} [P(-k') + P(k')] \\ &\quad + \frac{1}{2} \sum_{k' < k} [\gamma_{OTS}(k', k) + \gamma_{OTS}(-k', k)] A(k). \end{aligned} \quad (3.47)$$

In writing (3.45)–(3.47) we have normalized all the transfer rates to ω_e and have written $A(k) \equiv A(+k) + A(-k)$ so that all k and k' are assumed positive. A

detailed discussion of the results of the integration and their relationship to type III bursts observed near 1 AU is given in the companion paper (Goldstein *et al.*).

However, before concluding this description of the basic ideas which comprise our theory of type III bursts, it is necessary to discuss the analytical model which we have used to describe the temporal evolution of the electron exciter distribution function. Once this is accomplished, it is then possible to estimate the total amount of energy lost by the electron beam as it propagates to 1 AU from the Sun. These are the subjects of the next two sections.

IV. EVOLUTION OF THE EXCITER DISTRIBUTION FUNCTION

The beam distribution used in our calculations has been modeled semiempirically from in situ particle observations in type III sources at 1 AU (Lin 1973; Lin, Evans, and Fainberg 1973). Such a model cannot be developed from *a priori* arguments because the propagation of the exciter in the corona and interplanetary medium is affected by magnetic field pitch-angle scattering, wave-particle interactions, and the inherent dispersion of the injected distribution. The effects of these phenomena are manifested on disparate length and time scales. Thus any attempt to model the beam evolution must be somewhat heuristic.

Lin observed the differential energy flux of the beam electrons rather than the velocity distribution. His detector accepted only large pitch-angle particles (roughly 60° – 120°) because the axis of the acceptance cone was oriented perpendicular to the ecliptic plane, while the magnetic field was predominantly in the ecliptic plane. A second detector, with less energy resolution, provided pitch-angle distributions with resolution of about 30° . Time sequences of both energy flux distributions and pitch-angle distributions during several events were provided to us by Lin (private communications). While these distributions are difficult to model analytically owing to lack of resolution, they do indicate, however, that marked anisotropy occurs early in the event and persists for some time. Thus we have interpreted the differential energy flux distributions as one-dimensional velocity distributions. Within this context, the parameters to be deduced for the beam are η , v_b , and Δv_b all as functions of time. We sketch the method briefly here; a more detailed description, including a discussion of the interpretation of the spectra as one-dimensional distributions, has been given in Paper II.

Lin, Evans, and Fainberg (1973) showed that the first particles of any given velocity to reach their detector all appeared to have traversed the same distance L after injection at a common time t_0 . Thus at time t , the slowest particles have speeds $\beta_0 c$ given by

$$\beta_0 = \frac{L}{c(t - t_0)}. \quad (4.1)$$

The peak of the distribution is at $v_b = \beta_0 c + \Delta v_b$.

Empirically, we find that v_b can be approximated for a wide range of events by

$$v_b = \beta_0 c \Theta(\beta_0), \quad (4.2)$$

and so

$$\frac{\Delta v_b}{v_b} = 1 - \frac{1}{\Theta(\beta_0)}, \quad (4.3)$$

where

$$\Theta(\beta_0) = 1.4(1 + 0.075\beta_0 + 0.085\beta_0^2)(0.3/\beta_0)^{0.4}. \quad (4.4)$$

To model $\eta(t)$, we use the fact that the fully evolved beam is a power-law distribution in energy, $f(E) \sim E^{-\zeta/2}$. The corresponding one-dimensional distribution is

$$f_0(\beta) = A(\bar{\beta})\beta^{-\zeta}, \quad (4.5)$$

where the normalization constant $A(\bar{\beta})$ is defined through

$$\int_{\bar{\beta}}^{\infty} d\beta f_0(\beta) = \bar{\eta}_b \quad (\beta \geq \bar{\beta}) \quad (4.6)$$

$\bar{\beta}$ is defined implicitly through equation (4.6), and $\bar{\eta}_b$ is determined for various events by integration of the observed spectra over energy. This latter procedure involves assuming a form for the pitch-angle distribution, but a minimum density is attained by integrating the spectra observed by Lin with the assumption that it is isotropic. The effect of this is to tend to underestimate the growth rate of the linear instability; we shall consider the consequences of this below.

We assume that the peak of the distribution, at $\beta \equiv \beta_b(t)$, is at the fully evolved value implied by equations (4.5) and (4.6). The distribution for $\beta_0(t) \leq \beta \leq \beta_b(t)$ is modeled by a ramp, while for $\beta \geq \beta_b$ it is given by equation (4.5). Thus,

$$\begin{aligned} f(\beta, t) &= f_0(\beta) = \bar{\eta}_b \frac{(\zeta - 1)}{\beta} \left(\frac{\bar{\beta}}{\beta}\right)^{\zeta}, \quad \beta \geq \beta_b, \\ &= f_1(\beta) = f_0(\beta_b) \frac{(\beta - \beta_0)}{(\beta_b - \beta_0)}, \quad \beta_0 \leq \beta \leq \beta_b. \end{aligned} \quad (4.7)$$

The evolution model of equations (4.1)–(4.4) was developed from observations at 1 AU in order that the numerical results of our calculations may be compared to the corresponding radio observations. In this context, we may then ask to what extent the model permits such comparisons and what are the uncertainties involved? There are essentially three points to consider:

i) How valid is the direct interpretation of the differential energy flux dj/dE as a one-dimensional distribution function $f(u) \approx mdj/dE$?

ii) For calculations using the measured flux distribution parameters of a particular event, how accurate is the determination of the density normalization $\bar{\eta}_b$ in equation (4.6)?

iii) What is the validity and what are the consequences of assuming a one-dimensional ramp for $f_1(\beta)$, as in equation (4.7)?

With regard to point (i), our assumption is equivalent to assuming that the distribution is dominated by small pitch-angle particles at any given energy. Assuming that the differential energy flux distribution is separable into the form (for $E > E_b = mv_0^2/2$)

$$\frac{dj}{dEd\Omega} = AE^{-1/2}G(\mu)H(E - E_b), \quad (4.8)$$

where $\mu = \cos \alpha$, gyrotropy is assumed, and $H(x)$ is the Heaviside step function, $f(u)$ is given by

$$f(u) \propto \int_{\lambda}^{\infty} dv v^{-1} G(u/v),$$

where

$$\begin{aligned} \lambda &= u \quad (u \geq v_b), \\ &= v_b \quad (u < v_b). \end{aligned}$$

Taking $f(u) \approx mdj/dE$ implies that $G(\mu)$ is sharply peaked near $\mu = 1$. If $G(\mu)$ is expanded in Legendre polynomials, one can show that the peak of $f(u)$ is preserved at $u = v_b$, while the slope for $u < v_b$ depends on the relative weights of the various terms in the expansion. For example, if $G(\mu) = (1 + \mu)/2$, then $f(u)$ is as shown in Figure (4a), while if $G(\mu)$ is dominated by larger-order ($n > 1$) odd polynomials, $f(u)$ may be as depicted schematically in Figure 4b. The particle spectra of Lin from which equations (4.1)–(4.4) were derived apply to the portion of (4.8) in the range of solid angle

$$\Delta\Omega \approx 2\pi \int_{\sin^{-1}160^\circ}^{\sin^{-1}120^\circ} d\theta \sin \theta,$$

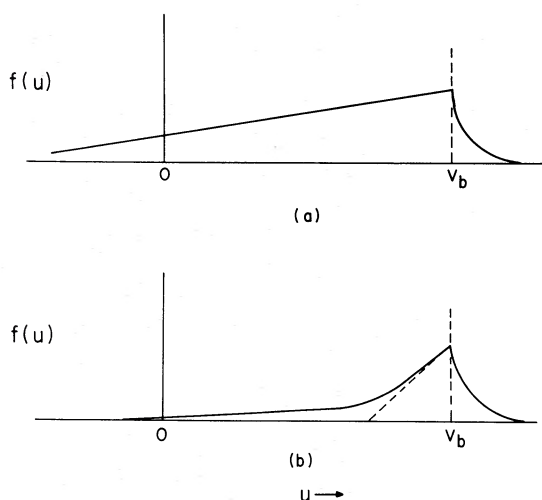


FIG. 4.—Various possibilities for reduced one-dimensional distribution functions resulting from a simple first-order pitch angle anisotropy (a) proportional to μ or (b) proportional to μ^{2n+1} , $n \gg 1$.

and so cannot be used for determining $G(\mu)$. Supplemental pitch-angle distributions indicate a degree of anisotropy that is hard to model analytically, but we feel that the distribution, at least over the rise time of the event, is closer to that depicted in Figure 4b.

With regard to point (ii), we have already noted above that an integration of the flux distributions observed by Lin, assuming them to be isotropic in the forward hemisphere $\theta < \pi/2$, tends to underestimate the density, if in fact it is dominated by particles of low pitch angle. The factor by which the density is thus underestimated is difficult to determine precisely, but an inspection of Lin's unpublished data indicates that it may be between 1 and 10. On the other hand, the assumption noted above that $f(\beta_b)$ is taken to be at its fully evolved value tends to overestimate the density, because the peaks we identify in the time-evolving spectra are in fact at slightly lower flux levels—for a given energy—than that of the asymptotic spectrum (cf. Fig. 3 of Lin, Evans, and Fainberg 1973) by a factor which is typically about 3. Thus, although we do not attempt in practice to correct for either of these errors, their combined effect is to produce an inaccurate estimate of the density $n_b(t)$ for any particular event by a factor of order unity, which is not important in the context of the type of comparison to observations that we shall make below. Moreover, because there is a wide range of densities possible in type III excitors in general, the normalizations we use are appropriate for illustrating the general physics independent of detailed comparisons to particular events. We note also that attempts to derive an overall normalization from Lin's data are also subject to uncertainties by factors of order unity.

With regard to point (iii), we note first that the schematic depiction of $f(u)$ as in Figure 4b includes only the effect of the pitch-angle distribution on particles with speed $v > v_b$. The error bars in the flux distributions for $E_0 < E < E_b$ preclude developing a model for dj/dE in this region, although the measurements allow the identification of β_0 according to equation (4.1).

Thus we have merely modeled $f(\beta)$ in the unstable region by a ramp. The confinement of the ramp to $\beta_0 \leq \beta \leq \beta_b$ is consistent with our earlier assumptions because the steeply rising portion dominates the exponential growth of the linearly unstable spectrum. In addition, it must be remembered that observations of the radio emission integrate the emission from a greatly extended exciter, which typically has lateral dimension of about 1 AU at 1 AU. Thus the effects of fine structure in $f(\beta)$ are unobservable in principle, because such structure would not be spatially uniform. Such fine structure, however, has an effect locally on our calculations, because it affects the temporal development of the pump spectrum. In addition, the nonlinear processes themselves may affect $f(\beta)$ in the resonant region. For all these reasons, it may easily be appreciated that the model of a ramp for $f(\beta)$ means that we do not treat the interaction of neighboring modes in the resonant region correctly in detail, nor can we.

A final point is that, as can be seen in § III, the Landau damping rate in the phase-velocity range $\beta < \beta_0$ contributes to the spectral dynamics. In principle, the beam contribution from small-pitch-angle particles with $\beta < \beta_0$ can modify this rate, weakening Landau damping or even contributing slow linear growth to sustain the spectrum at these phase velocities. We have assumed that this is a small effect and do not consider it further.

V. ENERGY LOSS OF THE EXCITER DURING PROPAGATION

Because of the disparity between the plasma-wave group velocity and the velocity of the exciter electrons, local comparisons of the energy densities of waves and particles are meaningless. Thus, to evaluate the adequacy of the theory from the standpoint of energetics, we must estimate the total energy lost in the exciter propagation volume and compare it to the total beam energy flux.

The total energy lost between the injection near the solar surface and the point R , say, is given by

$$\Delta E = \int_{R_0}^R dr A(r) \int_{t_1(r)}^{t_2(r)} dt \frac{d\tilde{W}(r, t)}{dt}, \quad (5.1)$$

where $A(r)$ is the source area at r , and $t_1(r)$ and $t_2(r)$ are the times at which instability at r begins and ends, respectively. For simplicity, equation (5.1) implies uniform conditions across the area, i.e., $d\tilde{W}/dt$ is an average over $A(r)$.

From our theory we may estimate $d\tilde{W}/dt$ reliably to within an order of magnitude on quite simple physical arguments. In the resonant region the beam loses the energy necessary to trigger the OTSI, after which this energy is quickly scattered into the non-resonant region and the linear instability starts again in the resonant region after the ion fluctuations have been damped. Because all the beam energy loss occurs in the resonant region, we may consider that it occurs at the steady rate

$$\frac{d\tilde{W}}{dt} \approx \frac{\tilde{W}_T}{\tau_c} = \frac{\tilde{W}_T \gamma_L}{A_c}, \quad (5.2)$$

where, as above, \tilde{W}_T is the OTSI threshold energy density, and A_c and τ_c are defined by

$$\tilde{W}_T = \tilde{W}_0 \exp(A_c) = \tilde{W}_0 \exp(\gamma_L \tau_c). \quad (5.3)$$

Using the definition $\beta_0(x, t) = x/ct$ as in § IV, we have the transformation

$$\int_{t_1(x)}^{t_2(x)} dt g(x, t) = \frac{-x}{c} \int_{\beta_0[x, t_1(x)]}^{\beta_0[x, t_2(x)]} d\beta_0 \beta^{-2} g[\beta_0(x)]. \quad (5.4)$$

The beam evolution model of § IV gives

$$\frac{v_b}{\Delta v_b} \approx \left(1 - \frac{\beta_0^{0.4}}{0.9}\right)^{-1}. \quad (5.5)$$

This is not a simple or convenient function of β_0 . As we shall see below, however, most of the beam energy loss occurs at low β_0 , where η is largest, and there $(v_b/\Delta v_b)^2$ tends to values of order 5–10. Thus we shall take this factor as simply a constant in the calculation below.

Equation (5.1) then becomes

$$\Delta E(r \leq R) \approx -\pi \left(\frac{v_b}{\Delta v_b}\right)^2 \int_{R_0}^R dr A(r) \omega_e(r) \left(\frac{r - R_0}{c}\right) \times \int_{\beta_0^{\max}(r)}^{\beta_0^{\min}(r)} d\beta_0 \frac{\tilde{W}_T(\beta_0) \eta(\beta_0)}{\beta_0^2}. \quad (5.6)$$

The limits on the integral over β_0 require discussion. The value $\beta_0^{\min}(r)$ is the velocity at which the linear instability stops owing to merging of the beam distribution into that of the solar wind, so that $\partial f/\partial u \lesssim 0$. The linear instability, of course, will be quite weak before the beam evolves down to $\beta_0 = \beta_0^{\min}$, so that use of this value for the limit of the integral, together with equation (5.4), will tend to overestimate ΔE . The limit β_0^{\max} is the highest value of β_0 for which the beam does not evolve through the velocity interval $\beta_b(\beta_0) - \beta_0$ in a time less than $\tau_c(\beta_0)$. It will turn out that the integral in equation (5.6) will not be sensitive to β_0^{\max} , but is very sensitive to β_0^{\min} , through a factor $(\beta_0^{\min})^{-\zeta}$, where ζ is again the spectral index of the beam. Thus, e.g., for $\zeta = 6$, we will overestimate ΔE by about two orders of magnitude if we underestimate β_0^{\min} by a factor of 2. We shall discuss below the estimation of β_0^{\min} in more detail.

From § IV, we write

$$\eta(\beta_0) = N(\bar{\beta}) \int_{\beta_0}^{\beta_b(\beta_0)} \beta^{-\zeta} \frac{\beta - \beta_0}{\beta_b - \beta_0} d\beta \approx \frac{N(\bar{\beta})}{\beta_0^{0.6} - \beta_0} \frac{\beta_0^{-\zeta+2}}{(\zeta - 1)(\zeta - 2)}, \quad (5.7)$$

where we have approximated $\beta_b(\beta_0) \approx \beta_0^{0.6}$ and neglected a small term, and where

$$N(\bar{\beta}) = \bar{\eta}_b(\beta \geq \bar{\beta})(\zeta - 1)(\bar{\beta})^{\zeta-1}.$$

Furthermore, we shall make the following approximations:

i) In equation (5.3), we take $A_c = \text{constant}$ (of order 10, say) and write

$$\tilde{W}_T \equiv W n(r) T_e(r), \quad (5.8)$$

with $W = \text{constant}$.

ii) In equation (5.7) we write

$$\frac{1}{\beta_0^{0.6} - \beta_0} \approx \frac{2}{\beta_0^{0.6}}, \quad (5.9)$$

which is good for $\beta_0 \lesssim 0.3$.

iii) Last, we take the source area $A(r)$ to be given by

$$A(r) = \Omega(r)(r - R_0)^2.$$

If Ω is constant, the source is a cone of solid angle Ω with vertex at the solar surface $r = R_\odot$. If Ω is, for example, piecewise constant over an interval in r , then the source volume is a portion of such a cone over this interval.

Defining $\rho = r/R_\odot$ and using these approximations, equation (5.6) becomes

$$|\Delta E(r \lesssim R)| \approx G(W, A_c, \zeta) \times \int_1^{(R/R_\odot)} d\rho (\rho - 1)^3 n_e^{3/2}(\rho) T_e(\rho) \times N(\bar{\beta}) \Omega(\rho) [\beta_0(\rho)]^{-\zeta+0.4}, \quad (5.10)$$

where

$$G(W, A_c, \zeta) = \frac{2\pi k_B R_\odot^4}{c} \left(\frac{4\pi e^2}{m} \right)^{1/2} \left(\frac{\bar{v}_b}{\Delta v_b} \right)^2 \times \frac{W}{A_c(\zeta - 1)(\zeta - 2)(\zeta - 0.4)} \approx 10^{22} \frac{W}{(\zeta - 1)(\zeta - 2)(\zeta - 0.4)}, \quad (5.11)$$

where k_B is Boltzmann's constant, the last estimate assumes $(\bar{v}_b/\Delta v_b)^2 \approx 10$, and ΔE is expressed in ergs and $T_e(\rho)$ in K.

To evaluate the integral in equation (5.10), let us assume a model of the corona in which there are J partitions such that

$$n(\rho) = n(\rho_0) \left(\frac{\rho}{\rho_0} \right)^{-\nu_j}, \\ T(\rho) = T(\rho_0) \left(\frac{\rho}{\rho_0} \right)^{-\tau_j},$$

where $\rho_{l,j} \leq \rho$, $\rho_0 \leq \rho_{u,j}$, and $j = 1, 2, \dots, J$; and take

$$\Omega(\rho) = \Omega_j, \quad \rho_{l,j} \leq \rho \leq \rho_{u,j}.$$

Then

$$|\Delta E| \approx G \sum_{j=1}^J \Omega_j (n_e^{3/2} T_e)_{\rho_{l,j}} \times \int_{\rho_{l,j}}^{\rho_{u,j}} d\rho (\rho - 1)^3 (\rho/\rho_{l,j}) \exp(-3\nu_j/2 - \tau_j) \times N(\bar{\beta}) [\beta_0^{\min}(\rho)]^{-\zeta+0.4}. \quad (5.12)$$

We remark that $N(\bar{\beta})$ is left under the integral because under this model we expect

$$n_b(\rho) \approx n_b(\rho_0) \left(\frac{\rho}{\rho_0} \right)^{-2}$$

whereas below some point $\bar{\rho}$ (i.e., $\rho \leq \bar{\rho}$) we shall have

$\nu(\rho) \gg 2$. Then the normalization of the density changes (cf. eq. [4.6]) such that

$$\eta(\rho) = \eta(\bar{\rho}) \left(\frac{\rho}{\bar{\rho}} \right)^{-2+\nu(\rho)}. \quad (5.13)$$

Now, in order to evaluate equation (5.12) we need β_0^{\min} (thus overestimating $|\Delta E|$) by noting that instability will certainly have ceased when the total one-dimensional plasma distribution function satisfies

$$\left(\frac{\partial f}{\partial \beta} \right)_{\text{total}} = \left(\frac{\partial f}{\partial \beta} \right)_{\text{background}} + \left(\frac{\partial f}{\partial \beta} \right)_{\text{beam}} = 0. \quad (5.14)$$

Because we take $(\partial f/\partial \beta)_b$ as a ramp,

$$\left(\frac{\partial f}{\partial \beta} \right)_b = \frac{f(\beta_b)}{\beta_b - \beta_0}, \quad (5.15)$$

where the subscript b denotes the beam. Equation (5.14) should be satisfied near the point where

$$f_b(\beta_b) \approx f_0(\beta_0). \quad (5.16)$$

For these considerations, we may assume that in the inner corona the suprathermal portion of the electron distribution is Maxwellian.

Thus the ambient distribution is

$$f_t = N_t \exp(-\beta^2/2\beta_t^2), \quad (5.17)$$

where the subscript t refers either to the total ambient distribution if there is no distinct suprathermal tail, or to such a tail if it exists. Using equations (5.15)–(5.17), equation (5.14) becomes

$$\frac{N(\bar{\beta})(\beta_0^{\min})^{-\zeta-1}}{(\beta_0^{\min})^{0.6}[1 - (\beta_0^{\min})^{0.4}]} = \frac{N_t}{\beta_t^2} \exp \left[- \left(\frac{\beta_0^{\min}}{2^{1/2}\beta_t} \right)^2 \right].$$

Again, approximating $[1 - (\beta_0^{\min})^{0.4}]^{-1} \approx 2$, and defining $y \equiv (\beta_0^{\min}/\beta_t)^2$, we arrive finally at the expression

$$\frac{y}{2} - \left(\frac{\zeta}{2} + 0.8 \right) \ln y = \ln \left[\frac{N_t}{N(\bar{\beta})} \beta_t^{\zeta-0.4} \right] \equiv \ln C_1. \quad (5.18)$$

Equation (5.18) may be solved numerically for β_0^{\min} in the appropriate partition region.

For the sake of definiteness and illustration, let us consider a two-partition coronal model as described in Table 1. This model is in part derived from the density model of Riddle (1974). Using this model, we may evaluate equation (5.12) straightforwardly in terms of β_0^{\min} at the lower boundary of each region. To determine β_0^{\min} , we consider that in the inner corona the ambient density is Maxwellian with the temperature dependence of Table 1. We use equation (5.13) to scale the beam density back to the inner corona, using as parameters at 1 AU the typical values

$$\eta = 10^{-6}, \quad \zeta = 5, \quad \bar{\beta} = 0.1, \quad (5.19)$$

TABLE 1
CORONAL MODEL

Region (<i>j</i>)	$\rho_{l,j}$	$\rho_{u,j}$	$n(\rho_l)$ (cm ⁻³)	$f_e(\rho_l)$ (MHz)	ν_j	τ_j	$T_e(\rho_e)$ (K)
1.....	1	2.1	8×10^8	250	7.5	0.26	2.0×10^6
2.....	2.1	215	3×10^6	15	2.6	0.26	1.6×10^6

and assuming $\Omega = 1$ sr throughout the propagation volume of the beam. It is then straightforward to solve equation (5.18) numerically to find

$$\beta_0^{\min}(\rho \approx 1) \approx 0.16, \quad \beta_0^{\min}(\rho \approx 2.1) \approx 0.15. \quad (5.20)$$

Using these values in the integration of equation (5.12) we find the energy losses $|\Delta E_{1,2}|$ in regions 1 and 2, respectively, to be

$$\begin{aligned} |\Delta E_1| &\approx 10^{30} \text{ W ergs}, \\ |\Delta E_2| &\approx 10^{29} \text{ W ergs}. \end{aligned} \quad (5.21)$$

Our calculations indicate $W \lesssim 10^{-4}$ initially (cf. eq. [3.29] and § II of Goldstein *et al.*). The total energy in the type III exciter at 1 AU has been estimated to be or order 10^{25} – 10^{26} ergs (Lin and Hudson 1971; Kane and Lin 1972; Lin 1974; Lin, private communication). Thus, with $W \lesssim 10^{-4}$ the exciter can lose anywhere from a few percent of its energy to all of it. Ninety percent of the energy loss occurs in the frequency range above 15 MHz; thus very weak exciters would give rise to bursts which would end in this range, as is frequently observed, while stronger bursts, presumably from large flares, would lose only a small fraction of the available energy. Note that because of the approximate nature of this calculation, we may have overestimated the energy loss. Furthermore, the 10^{25} – 10^{26} ergs in the exciter is the energy observed at 1 AU. We do not know the energy initially contained in it as it propagates through region 1. We may conclude, therefore, that our theory is in excellent accord with energetic requirements.

There is an even more striking conclusion to be drawn from this calculation, however, which affects the general interpretation of observations. Recall that the exciter velocity is observationally inferred from the frequency drift rate of the *peak* of the time-intensity profile at fixed frequencies. We refer to the velocity thus inferred as the “nominal velocity.” It is usually stated that at high frequencies (> 15 MHz) the nominal velocity is $\sim \frac{1}{3}c$ (cf. Wild and Smerd 1972), while Evans, Fainberg, and Stone (1973) found that at low frequencies ($\lesssim 1$ MHz) the nominal velocity decreased by a factor of about 2 between $r \sim 10 R_\odot$ and $r \sim 1$ AU.

Note first that the peak intensity at any frequency is reached at or just above the velocity at which the linear beam-plasma instability stops at that frequency; this is just the point which we have calculated above, when $\beta_0 \rightarrow \beta_0^{\min}(r)$. Recalling that our model of the

beam evolution gives the result

$$\beta_b(\beta_0) \approx \beta_0^{0.6},$$

we find $\beta_b \approx 0.32$ for $\beta_0^{\min} = 0.16$. If we determine β_0^{\min} at 1 AU, assuming that the high-velocity tails of the solar-wind electrons are described by Maxwellians with $T_e \approx 1.2 \times 10^5$ K and a density fraction of ~ 0.05 (W. C. Feldman, private communication), then we find $\beta_0^{\min} \approx 0.07$ and $\beta_b \approx 0.2$.

These calculations suggest the following important conclusions: (1) The nominal velocity $\frac{1}{3}c$ is not the characteristic velocity to which electrons are accelerated in flares or other active phenomena, but reflects the dispersive evolution of a particle spectrum which is monotone-decreasing when fully evolved. (2) The observations of Evans, Fainberg, and Stone (1973) do not indicate an actual deceleration of the exciter, but rather reflect the fact that the solar wind temperature decreases slowly with radius, so that merging of the beam and ambient distributions occurs at lower velocities.

There is one caveat to be made regarding these conclusions. Although the numerical coincidence of the derived β_b with the nominal values is striking, it is of course somewhat dependent on the beam parameters. This dependence is weak, however, being manifested only in the term $\ln C_1$ in equation (5.18). Thus it seems safe for these purposes not only to adopt the typical beam parameters (5.19) but also to neglect consideration of changes in the parameters during propagation. To the extent that the model is valid, then, the small characteristic dispersion of the observed nominal velocity is due to the weak dependence of β_b on β_0 : $\beta_b \approx \beta_0^{0.6}$. This law was derived in § IV, however, only on the basis of observations near 1 AU, for a few cases of beam evolution in which L (the only parameter in applying the beam evolution model to a particular event) varied only about 20%. In the calculation of this section, however, we have used this law all the way through the corona. That the results are so benign to our estimates of energetics, and to the correspondence with observations of the nominal velocity, indicates that such a weakly dependent connection between β_0 and β_b probably describes the integrated effects of pitch-angle scattering and reabsorption of the wave energy by the beam particles during their propagation. This finding may be of interest to future theoretical work.

VI. SUMMARY

We have presented the complete theory of modulational and collapse instabilities in three dimensions.

As in Paper I, we again find that for parameters appropriate to the interplanetary medium, these strong instabilities are essentially one-dimensional. The pump waves excited by the linearly unstable electron beam were found to be confined to a narrow propagation cone aligned along the ambient magnetic field. The daughter waves produced by the modulational instabilities were also found to be one-dimensional: parallel and antiparallel to the direction of \mathbf{B} . Furthermore, the decay instability was shown to have a higher threshold than the OTSI. Consequently, the criticisms made by Bardwell and Goldman (1976) about the inadequacy of a one-dimensional treatment of modulational instabilities are not germane to type III bursts in the interplanetary medium.

As in our earlier treatments of the initial stages of OTSI (Papers I and II), spectral transfer of Langmuir waves resonant with the electron beam to short-wavelength, nonresonant, waves takes place on a time scale that is short compared with the time scale for plateau formation and exciter energy loss associated with the quasi-linear kinetic theory. The present paper has extended those results to include a description of the processes which in turn stabilize the OTSI: enhanced scattering of Langmuir waves by nonthermal ion fluctuations that have modified the Bohm-Gross dispersion relation of the pump and daughter Langmuir

waves, and an anomalous resistivity created by these same ion fluctuations.

While the derivation of the equations of spectral transfer which include all of the above effects has been given above, the solution of these equations using parameters appropriate to solar type III bursts observed in the interplanetary medium has been deferred to the companion paper (Goldstein *et al.*). In addition to the detailed development of modulational instabilities, we have also presented an analytic model that describes the energy losses suffered by an electron exciter of type III bursts that is stabilized by the OTSI. Most of the energy loss is found to occur above the 15 MHz level. If the electrons have a total energy of some 10^{26} ergs, then we find that, as observed, many bursts will not be seen below 15 MHz, while those that are sufficiently strong to propagate out of the inner corona will not be further decelerated as the electron beam propagates out to 1 AU. Finally, the apparent deceleration of the electron exciters that has been observed by Evans, Fainberg, and Stone (1973) can be interpreted in the context of this model calculation as being due to the decrease in solar-wind temperature with heliocentric distance.

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