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THE PLASMA PHYSICS OF THE JOVIAN DECAMETER RADIATION

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ABSTRACT

We have assumed that the decameter radiation from Jupiter is produced near the local electron gyrofrequency and is amplified as it propagates out of the Jovian magnetosphere. Using the Vlasov-Maxwell equations, which describe the propagation of radiation in hot collisionless plasmas, we have derived the growth rate for radiation that propagates almost perpendicular to the direction of the magnetic field. When the electrons are described by a loss-cone distribution function, the growth rate is large enough to lead to a large amplification factor over a source region of 100–4000 km, depending on the choice of parameters. Because we expect low-energy electrons to be trapped in the Jovian dipole field regardless of the position of the satellite Io, we maintain that this model provides a plausible mechanism for the decametric radiation not associated with Io.

I. INTRODUCTION

The radio emission of Jupiter covers a broad spectrum, including intense sporadic bursts at decametric wavelengths (Burke and Franklin 1955). Our understanding of the basic physics of this emission is obscured by our unfamiliarity with the nature of the generating mechanism, its spectral characteristics, and the physical environment of the source region. In the following exposition, we assume that the decameter noise (DAM) is produced by an electron cyclotron resonance effect. The range of frequencies observed then represents the spatial variation of local electron gyrofrequencies. The duration of the DAM bursts is determined by the lifetime of the electrons as they lose energy by the coherent radiation mechanism we have considered. The alternative assumption, in which DAM is produced near the local plasma frequency (Warwick 1967; Gledhill 1967*a, b*) requires a high plasma density ($N \simeq 10^7 \text{ cm}^{-3}$) in the source region. Estimates of plasma densities in the Jovian ionosphere (Gross and Rasool 1964) predict $N \sim 10^5 \text{ cm}^{-3}$, while the computed plasma distribution in model Jovian magnetospheres (Melrose 1967; Parthasarathy, Whitten, and Sims 1970; Ioannidis and Brice 1971) falls off from the ionospheric value out to the magnetosphere boundary at $L = r/r_J \approx 80$ (Brice and Ioannidis 1970). (The radius of Jupiter, r_J , is $7.135 \times 10^9 \text{ cm}$.) In the calculation of Ioannidis and Brice (1971), the density decreases from $N \simeq 10^5 \text{ cm}^{-3}$ at the ionosphere to $N \simeq 1\text{--}10 \text{ cm}^{-3}$ at $L = 1.1$. N is of this order out to $L \simeq 80$. Because such densities are too low to produce plasma oscillations at DAM frequencies, we shall give no further consideration to that mechanism.

Chang (1963) (also see Zheleznyakov 1960*a, b*) derived the equations describing instabilities for waves propagating parallel to the ambient magnetic field ($\mathbf{k} \parallel \mathbf{B}$). Chang found that growth was possible only for frequencies $\omega_{pe}^2 \gg |\Omega_{0e}| \gg \omega^2$; and $\omega \approx |\Omega_{0e}|$, $\omega_{pe}^2 \gg \omega|\Omega_{0e}| - \omega^2 > 0$, where $\Omega_{0e} = -|eB/Mc|$ is the electron gyrofrequency, and $\omega_{pe} = (4\pi Ne^2/M)^{1/2}$ is the electron plasma density. In general when $\mathbf{k} \parallel \mathbf{B}$, radiation can escape only near the poles of a dipole field. In addition, in the frequency ranges discussed by Chang, even near the poles, radiation may encounter a stop zone on the way out of the Jovian magnetosphere (Stix 1962), and be absorbed. In the Earth's magnetosphere, the similar problem of the damping of whistlers by energetic particles has been discussed by Tidman and Jaggi (1962) and by Scarf (1962).

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For the radiation to escape the Jovian magnetosphere it must propagate at large angles to \mathbf{B} . Fung (1966*a, b, c*) discussed the problem of propagation at arbitrary angles using an electron distribution function given by $f = (1/2\pi p_\perp) \delta(p_\perp - p_\perp^0) \delta(p_\parallel - p_\parallel^0)$. His results should be compared to those of Bekefi (1966) and Goldreich and Lynden-Bell (1969) for the limit $p_\parallel^0 \rightarrow 0$. In this limit, his expression appears to be singular.

One of the most complete theories of the origin of DAM radiation is that of Goldreich and Lynden-Bell (1969). If the satellite Io has a certain minimum conductivity (Dermott 1970, Webster *et al.* 1971), this model may explain Io's modulation of DAM. Observations of DAM at Earth is most probable when Io is at 90° and 240° (measured from superior geocentric conjunction) and the northern end of Jupiter's dipole axis sweeps past Io (Duncan 1965). (For a review of the history and observations of the Io effect, see Carr and Gulkis 1969, and Warwick 1967.)

The theory of Goldreich and Lynden-Bell explains the Io-associated events, but cannot explain DAM that is not well correlated to the position of Io (in particular the main source at 210° – 270° longitude of central meridian [LCM]; see, e.g., Warwick 1967). In the following sections, we derive the growth rate for radiation propagating at 90° to \mathbf{B} . Using a loss-cone distribution function we have a model which can lead to a sufficiently large growth rate to explain DAM that is independent of Io's longitude.

II. CALCULATION OF THE GROWTH RATE

The theory of propagation of radiation at an arbitrary angle to the magnetic field in a hot plasma has been discussed by several people (e.g., Barnes 1966; Kennel 1966; Kennel and Wong 1967*a, b*; Fung 1966*a, b, c*; Andronov, Zheleznyakov, and Petelin 1964). The resulting equations are so complex that we have found it simpler to rederive the theory and take the limit of almost 90° propagation ($k_\parallel/k_\perp \ll 1$), where \mathbf{k} is the wave propagation vector, and \parallel and \perp refer to directions with respect to \mathbf{B} . We begin with the set of Vlasov-Maxwell equations which describe a hot, collisionless, proton-electron plasma; linearize them; and perform a Fourier transform in the space variables and a Laplace transform in time. One can then solve for the dispersion relation. We have assumed that the plasma is homogeneous, but that the distribution function can be gyrotropic. The derivation in the case of isotropic distribution functions can be found in Montgomery and Tidman (1964). The generalization to gyrotropic distribution functions is straightforward. The condition that a solution exists is given by the dispersion relation

$$\det R \equiv |R| = 0. \quad (1)$$

The elements of the matrix R are given by

$$\begin{aligned} R_{xx} &= s^2 + c^2 k_\parallel^2 - \Gamma_1 G_1 \left(\frac{n^2 \Omega_p^3 J_n^2}{k_\perp^2 v_\perp^2} \right), & R_{xy} &= -i \Gamma_1 G_1 \left(\frac{\Omega_p^2 n J_n J'_n}{k_\perp v_\perp} \right) = -R_{yx}, \\ R_{xz} &= -c^2 k_\perp k_\parallel - \Gamma_1 G_2 \left(\frac{\Omega_p^2 n J_n^2}{k_\perp v_\perp} \right), & R_{yy} &= s^2 + c^2 k^2 - \Gamma_1 G_1 (J'_n)^2 \Omega_p, \\ R_{yz} &= i \Gamma_1 G_2 (J_n J'_n) \Omega_p, & R_{zx} &= -c^2 k_\parallel k_\perp - \Gamma_2 G_1 \left(\frac{\Omega_p^2 n J_n^2}{k_\perp v_\perp} \right), \\ R_{zy} &= -i \Gamma_2 G_1 (J_n J'_n) \Omega_p, & R_{zz} &= s^2 + c^2 k_\perp^2 - \Gamma_2 G_2 (J'_n)^2 \Omega_p, \end{aligned} \quad (2)$$

where $\Gamma_{1,2}$ are the operators

$$\left\{ \begin{array}{l} \Gamma_1 \\ \Gamma_2 \end{array} \right\} = 2\pi s \sum_v \frac{\omega_p^2}{\Omega_p} \sum_{n=-\infty}^{\infty} \int_0^\infty dp_\parallel \int_0^\infty dp_\perp (s + ik_\parallel v_\parallel + in\Omega_p)^{-1} \left\{ \begin{array}{l} p_\perp^2 \\ p_\parallel p_\perp \end{array} \right\}. \quad (3)$$

The expressions $G_{1,2}$ are given by

$$G_1 = \frac{\partial f_0}{\partial p_\perp} + \frac{i}{s} k_\parallel \left(v_\parallel \frac{\partial f_0}{\partial p_\perp} - v_\perp \frac{\partial f_0}{\partial p_\parallel} \right), \quad G_2 = \frac{\partial f_0}{\partial p_\parallel} - \frac{in\Omega_p}{sv_\perp} \left(v_\parallel \frac{\partial f_0}{\partial p_\perp} - v_\perp \frac{\partial f_0}{\partial p_\parallel} \right). \quad (4)$$

The first summation is over species ($\nu = e, p$ for electrons and protons, respectively). The $J_n(x)$ are the ordinary Bessel functions of integral order. The argument of the Bessel functions is $k_\perp v_\perp / \Omega_\nu$; $\Omega_\nu = \Omega_{0\nu}(1 - v^2/c^2)^{1/2}$, and s is the Laplace transform variable, $-i\omega + \gamma$. (Note that Ω_p is a positive quantity while Ω_e is negative.) The function f_0 is the particle distribution function normalized so that $\int dp f_0 = 1$. In the remainder of this paper we will ignore the proton dynamics and will assume that $v^2/c^2 \ll 1$; and we ignore the variation of gyrofrequency with energy.

We now make the approximation that $k_\parallel/k_\perp \ll 1$. The nonzero elements of the matrix R can be shown to be (cf. Montgomery and Tidman 1964)

$$R_{xx} = s^2 - \Gamma_1 G_1 \left(\frac{n \Omega_e J_n}{k_\perp v_\perp} \right)^2, \quad R_{xy} = -R_{yx} = -i \Gamma_1 G_1 \left(\frac{\Omega_e n J_n J'_n}{k_\perp v_\perp} \right),$$

$$R_{yy} = s^2 + c^2 k^2 - \Gamma_1 G_1 (J'_n)^2, \quad R_{zz} = s^2 + c^2 k_\perp^2 - \Gamma_2 G_2 (J_n)^2. \quad (5)$$

Now

$$\frac{\Gamma_1}{\Gamma_2} \Big\} = 2\pi s \omega_{pe}^2 \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} dp_\perp (s + ik_\parallel v_\parallel + in\Omega_e)^{-1} \left\{ \frac{p_\perp^2}{p_\parallel p_\perp} \right\}. \quad (6)$$

The dispersion relation $|R| = 0$ can be factored. The term $R_{zz} = 0$ is the dispersion relation of the ordinary wave, which does not interest us here. For future reference we give the zero-temperature limit of the elements of R :

$$R_{xx}^{(0)} = -\omega^2 - \omega^2 \omega_{pe}^2 / (\Omega_e^2 - \omega^2), \quad R_{yy}^{(0)} = c^2 k^2 + R_{xx}^{(0)},$$

$$R_{xy}^{(0)} = -R_{yx}^{(0)} = -i \Omega_e \omega_{pe}^2 \omega / (\Omega_e^2 - \omega^2). \quad (7)$$

Following Kennel (1966) we assume that the bulk of the plasma has a low, but finite, temperature T . By this we mean that the root mean square thermal velocities parallel and perpendicular to B are small compared to $(\omega - \Omega_e)/k_\parallel$ and $|k_\perp \Omega_e|$, respectively. Consequently, the thermal particles fail to achieve resonance. In addition, we assume that the number density of the high-energy particles is much less than the number density of the low-temperature component. Both of these assumptions are meant to ensure that γ , the imaginary part of the frequency, is small compared to the real part. The resonant denominators in equation (5) can be expanded using the Plemelj formulae (cf. Montgomery and Tidman 1964, chapters 5 and 10). In the resulting expression the principal-part integration can again be expanded in powers of $v_\parallel k_\parallel (\omega - n\Omega_e)^{-1}$. The lowest-order term in this expansion leads directly to the zero-temperature matrix elements (eq. [7]). The higher-order terms lead to thermal corrections to the zero-temperature eigenfrequencies. These are unimportant for our derivation of the growth rate to lowest order.

The elements of R (eq. [5]) can now be expanded into lowest-order zero-temperature terms, which are real, plus higher-order complex terms. The first-order corrections to the zero-temperature terms (eq. [7]) are

$$R_{xx}^{(1)} = -\Gamma_1 G_1 \left(\frac{n \Omega_e J_n}{k_\perp v_\perp} \right)^2, \quad R_{yy}^{(1)} = -\Gamma_1 G_1 (J'_n)^2,$$

$$R_{xy}^{(1)} = -R_{yx}^{(1)} = -i \Gamma_1 G_1 \frac{\Omega_e n J_n J'_n}{k_\perp v_\perp},$$

$$\Gamma_1 = \frac{2\pi^2 \omega \omega_{pe}^2}{|k_\parallel|} i \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} dp_\perp^2 \delta \left(v_\parallel - \frac{\omega - n\Omega_e}{k_\parallel} \right). \quad (8)$$

The determinant of R , $|R|$, can be expanded into a zero-temperature term, $|R^{(0)}|$, plus a first-order correction, $|R^{(1)}| = R_{xx}^{(0)} R_{yy}^{(1)} + R_{yy}^{(0)} R_{xx}^{(1)} + 2R_{xy}^{(0)} R_{xy}^{(1)}$. To lowest

order, the eigenfrequencies are given by the solution to $|R^{(0)}| = 0$, which is

$$c^2 k^2 = \frac{(\omega^2 - \omega_R^2)(\omega^2 - \omega_L^2)}{(\omega^2 - \omega_{UH}^2)}, \quad \omega_R = \frac{1}{2}|\Omega_e|[1 + (1 + 4\omega_{pe}^2/\Omega_e^2)^{1/2}],$$

$$\omega_L = \frac{1}{2}|\Omega_e|[-1 + (1 + 4\omega_{pe}^2/\Omega_e^2)^{1/2}], \quad \omega_{UH}^2 = \Omega_e^2 + \omega_{pe}^2. \quad (9)$$

The thermal correction $\omega^{(1)} + i\gamma$ can be found by Taylor expansion around the cold-plasma solution (see Kennel 1966 for more detailed discussion of a similar derivation). The resulting growth rate is

$$\gamma = -\text{Im} |R^{(1)}| \bigg/ \frac{\partial}{\partial \omega} |R^{(0)}|, \quad \text{Im} |R^{(1)}| = iR_{xx}^{(0)}\Gamma_1 G_1 \theta_n,$$

$$\theta_n = \left[\frac{nJ_n}{x} \left(1 - \frac{iR_{xy}^{(0)}}{R_{xx}^{(0)}} \right) - J_{n+1} \right]^2; \quad x = k_\perp v_\perp / \Omega_e,$$

$$- \frac{R_{xx}^{(0)}}{\partial |R^{(0)}| / \partial \omega} = \frac{(\omega^2 - \omega_{UH}^2)^2}{2\omega[(\omega^2 - \omega_{UH}^2)^2 + \omega_{pe}^2 \Omega_e^2]}. \quad (10)$$

The sign of the growth rate, γ , is determined from the sign of G_1 . The quantity θ_n is a weighting function which, along with the magnitude of G_1 , determines the relative importance of the separate terms in the sum over harmonic number. The distribution of resonant particles determines the magnitude of G_1 . Because we are dealing exclusively with electrons, the $n = -1$ term predominates.

Integration over p_\parallel gives

$$\gamma = \frac{2\pi^2 \omega_{pe}^2 \omega R_{xx}^{(0)}}{|k_\parallel| \partial |R^{(0)}| / \partial \omega} \sum_{n=-\infty}^{\infty} \int_0^\infty dp_\perp p_\perp^2 \theta_n G_1|_{v_\parallel^0}, \quad (11)$$

where $v_\parallel^0 = (\omega - n\Omega_e)/k_\parallel$.

For nearly 90° propagation, the $n = 0$ harmonic is damped to this order.

III. APPLICATION TO THE JOVIAN MAGNETOSPHERE

In figure 1 we have reproduced a figure from Hasegawa (1969) showing a schematic representation of the propagation characteristics of waves propagating near 90° in a zero-temperature plasma (cf. eq. [9]). When protons are neglected, only propagation in Bands II and III is important. DAM produced deep within the Jovian magnetosphere in Band II would encounter a stop zone as it propagated out into regions of decreasing plasma density and magnetic field. Under some circumstances one might expect the radiation to tunnel through the stop zone and escape Jupiter by continuing in Band III (Stix 1962). Hasegawa (1969) discussed the possibility that electromagnetic waves produced in the terrestrial magnetosheath could tunnel into the plasmasphere, i.e., tunneling from Band III into Band II. Tunneling from Band II to III is possible when the wavelength of the wave is on the order of, or greater than, the distance over which the density and magnetic field must decrease so that $\omega \gtrsim \omega_R$. In general, one needs a situation in which there exists a discontinuity in density, as at the Earth's plasmopause. Whether or not Jupiter has a plasmopause is still unclear. A Jovian plasmopause near the orbit of Io would have important consequences for the origin of DAM. However, Brice and Ioannidis (1970) claim that the Jovian plasmopause is coincident with the magnetosphere boundary. Alternatively, if $\omega_{pe} \ll -\Omega_e$, a wave can tunnel from Band II into Band III even in the absence of a sharp discontinuity in plasma density. We discuss the implications of this below.

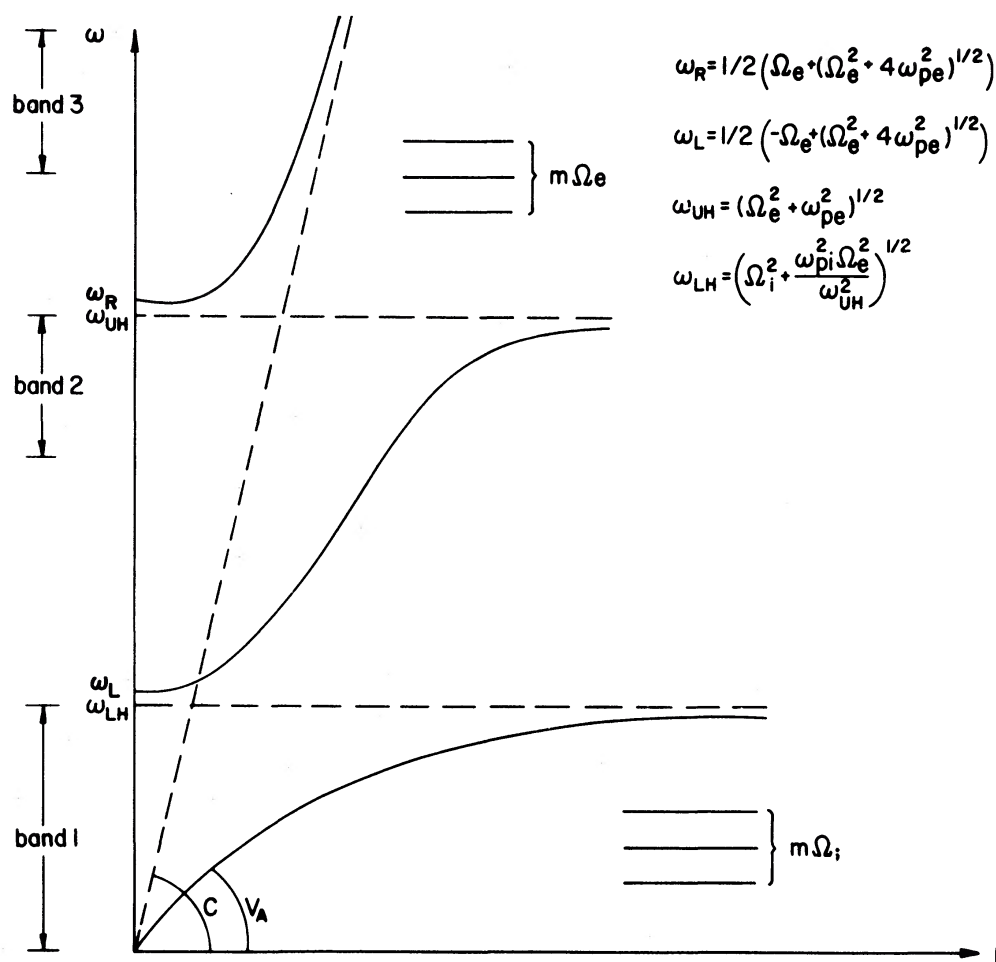


FIG. 1.—A schematic showing the allowed propagation bands for waves propagating at 90° to the direction of B . (After Hasegawa 1969.) (In this figure $\Omega_e = |\Omega_e|$.) We have ignored the ion dynamics and have set $\omega_{LH} = 0$.

Studies of the Jovian decimeter radiation have shown that high-energy electrons are trapped in the magnetosphere. The most intense concentration of these electrons is at about $L = 2$, but may extend out to $L = 6$ (Berge 1966; McAdam 1966). The DAM is produced by much lower energy particles (\sim several keV), and it is reasonable to assume that they too can be trapped. We have chosen to describe these low-energy electrons by using a loss-cone distribution function, which is appropriate for particles that mirror in a dipole magnetic field. The results are not sensitive to the details of the particular distribution we have chosen, but only to its general property of describing mirroring particles.

We let f_0 be

$$f_0 = C[\exp(-v^2/2v_E^2)] \ln \frac{\sin \alpha}{\sin \alpha_0} \quad (\alpha_0 \leq \alpha \leq \pi - \alpha_0)$$

$$= 0 \quad (0 < \alpha < \alpha_0, \pi - \alpha < \alpha < \pi), \quad (12)$$

where α is the pitch angle of the electrons, α_0 the loss-cone angle, v_E a parameter describing the hardness of the energy spectrum, and C is a normalization constant such

that $\int dp f_0(p) = 1$, and is given by

$$C = \zeta (2\pi M^2 v_E^2)^{-3/2} [\cos \alpha_0 + \ln (\cot \frac{1}{2} \alpha_0)]^{-1} \quad (13)$$

where ζ is the ratio of the concentration of energetic to thermal electrons. In the calculations that follow we have used $\zeta = 0.1$.

To calculate typical growth rates in the Jovian magnetosphere, we substituted equation (12) into equation (11) and performed the integration numerically. We have evaluated the integral for $n = -1$ at several values of L in the magnetic equatorial plane of the Jovian magnetosphere, and also for several values of the parameters α_0 and v_E . Consequently, we let $B(r) = B_s (r_{\mathcal{A}}/r)^3$. (The subscript s denotes quantities evaluated at the top of the Jovian ionosphere.) We take N between 1 and 10 cm^{-3} . This is consistent with the theoretical model of Ioannidis and Brice (1971). The magnitude of the growth rate is approximately linear in N when $\omega_{pe} \ll -\Omega_e$.

The minimum pitch angle, α_0 , is given in terms of mirroring height above the Jovian atmosphere by

$$\alpha_0 = \sin^{-1}[(\rho/r)^{3/2}(4 - 3\rho/r)^{1/4}],$$

where ρ is the mirror distance and r is the equatorial distance at which the growth rate is calculated. The calculations are not sensitive to the value of ρ that is chosen.

We computed the growth rate for waves propagating in both Bands II and III. The results are shown in figure 2. In this figure, $\gamma/|\Omega_e|$ is plotted against $\omega/2\pi$ at $L = 1.2$ and 2.0 . In the computation we used $B_s = 20$ gauss (cf. Goldreich and Lynden-Bell 1969), $v_E = 0.2c$, which corresponds to a mean electron energy of ~ 10 keV, $\rho = 1.05r_{\mathcal{A}}$, and $\zeta = 0.1$. In addition $k_{\parallel}/k_{\perp} = \tan \beta \simeq \beta = 0.1$ (i.e., $\beta \simeq 5^\circ$).

For the parameters we used in computing figure 2, one can show that radiation propagating in Band II can tunnel into Band III with almost no attenuation for $L < 2$. For $L \sim 2$, the wave would be attenuated by $\sim 10^4$ when propagating from Band II into III. If we had used $N = 1 \text{ cm}^{-3}$ (cf. Ioannidis and Brice 1971), then the growth rate would be a factor of 10 smaller. However, in this case radiation in Band II could escape the Jovian environment even if generated by electrons trapped at $L \sim 2$.

Before concluding that this instability is strong enough to produce the DAM, it is necessary to estimate the size of the source region in which coherency can be maintained. This can be obtained by insisting that the bandwidth over which the growth rate is large is comparable to the change in Ω_e as the wave propagates out of the source region. We find from figure 2 that, at $L = 1.2$, $\Omega_e/2\pi$ changes by ~ 1 MHz in a distance $d \simeq 2 \times 10^3$ km. At $L = 2$, using only Band III, $d \sim 4 \times 10^3$ km for a change in $\Omega_e/2\pi$ of ~ 0.2 MHz. The distance d was calculated on the assumption that the radiation propagated radially outward in the equatorial plane. Consequently, it is a lower limit. Recent observations of DAM using very long baseline interferometry (Carr *et al.* 1970; Dulk 1970) indicate that the size of an incoherent source must be less than $1''0$ (~ 400 km at Jupiter). The size of a coherent source, however, may be impossible to detect from Earth.

The radiation intensity will be amplified above the incoherent background by $A \equiv \exp [2\gamma d/v_g]$, where v_g is the group velocity. For these waves one finds that $v_g/c \simeq 10^{-2}$. Therefore, for $L = 1.2$, $A \sim 10^{85}$; and at $L = 2$, $A \sim 10^{200}$. In computing A we used $\zeta = 0.1$ and $N = 10 \text{ cm}^{-3}$. One should realize that long before such enormous amplifications could occur, the assumption that linear theory is valid will be grossly violated. Nonlinear effects will become important and the instability will saturate. However, the instability is sufficiently strong that radiation from Band II will escape the Jovian environment even at $L = 2$. If we had been more conservative and used $N = 1 \text{ cm}^{-3}$ and $\zeta = 0.1$, then the instability would still be sufficient to produce amplification of DAM by $A \sim 10^9$ and 10^{20} at $L = 1.2$ and 2.0 , respectively.

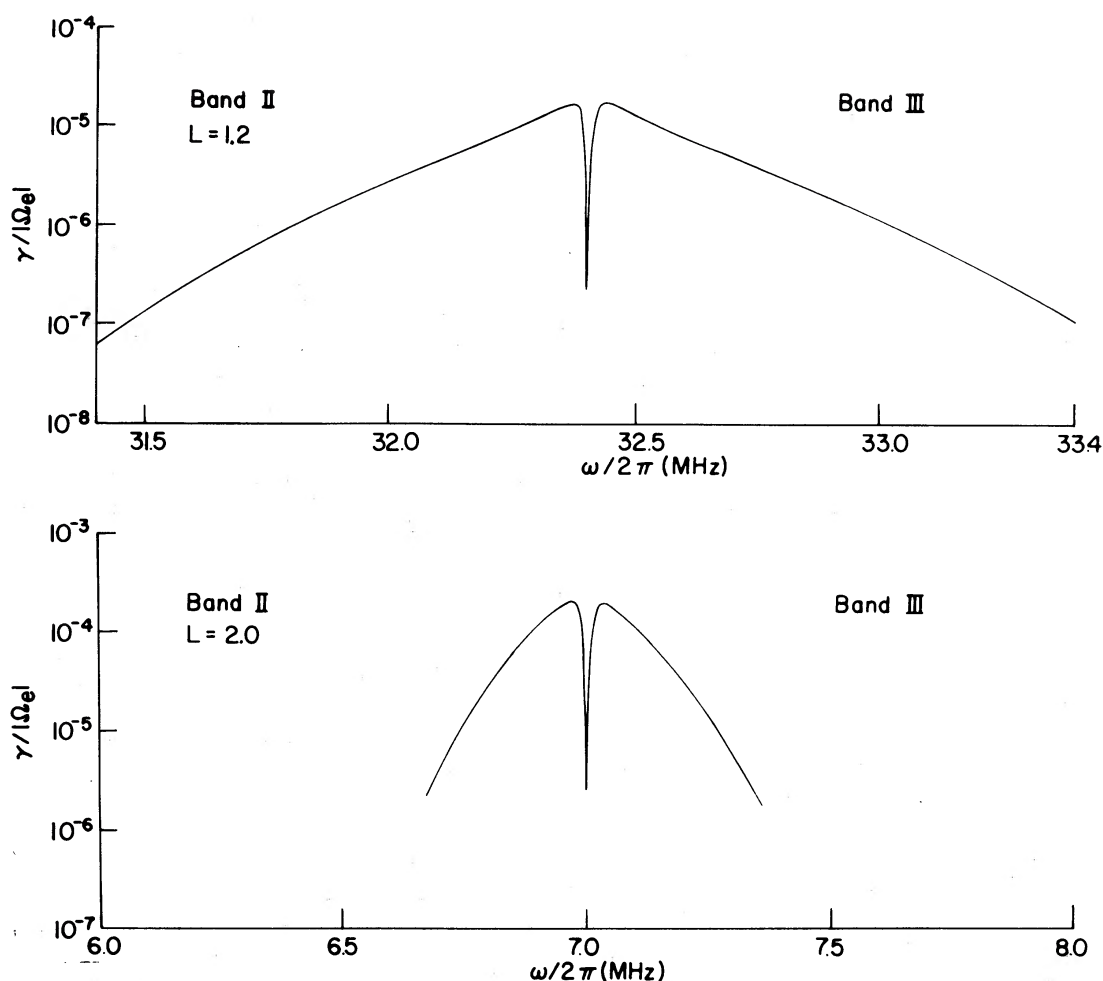


FIG. 2.— $\gamma/|\Omega_e|$ versus $\omega/2\pi$, when $L = 1.2$ and 2.0 , respectively; $\rho = 1.05$, $v_E = 0.2c$, $N = 10 \text{ cm}^{-3}$, $B_s = 20\text{G}$, $k_{\parallel}/k_{\perp} = 0.1$, and $\zeta = 0.1$. The growth rate is evaluated in the magnetic equatorial plane for wave propagation in Bands II and III. The electrons are described by a loss-cone distribution function. (See note added in proof.)

It is a general property of these results that radiation at high DAM frequencies ($\omega/2\pi \sim 20 \text{ MHz}$) grows at a smaller rate than at low frequencies ($\omega/2\pi \sim 8 \text{ MHz}$). Thus, it is reasonable to expect that DAM bursts at high frequencies would be rarer than at lower frequencies, where the growth rate is larger and the conditions necessary for producing the instability are more easily satisfied. In fact, even if the ambient plasma density is $N = 0.1 \text{ cm}^{-3}$, with a relative concentration of 10-keV electrons of $\zeta = 0.1$, one will still have coherent emission at 6 MHz (coming from $L = 2$ in Band III). But the instability will be too weak to amplify radiation at 30 MHz ($L = 1.2$). This theoretical result agrees with observations (cf. Carr and Gulkis 1969) which show that, in general, DAM bursts are more likely to occur at lower than at higher frequencies. (The observed cutoff in DAM emission at $\sim 40 \text{ MHz}$ is used as a parameter in our model as in that of Goldreich and Lynden-Bell [1969]. It has been set equal to the electron gyrofrequency at the top of the Jovian ionosphere. This in turn gives $B_s = 20 \text{ gauss}$, which is consistent with estimates of the magnetic field deduced from the decimeter flux.)

One can make a crude estimate of the amount of amplification available before nonlinear effects saturate the instability. We do this by requiring that the waves grow for time less than the lifetime of the coherently radiating electrons. The lifetime of a coherently radiating electron is approximately given by

$$\tau_c \sim \frac{Mv_E^2}{2AdE/dt}, \quad (14)$$

where dE/dt is the radiative energy loss rate of an incoherently radiating electron. With $v_E \simeq 0.2c$, $L = 1.2$, and $N\zeta = 1 \text{ cm}^{-3}$, $\tau_c \sim 10^7/2A \text{ s}$ (see Jackson 1962).

Individual DAM bursts are observed to have durations of $\sim 10^{-2}$ to 1 s for S and L bursts, respectively (Carr and Gulkis 1969). This implies that the maximum amplification is $A \sim 10^7$ – 10^9 . This can be accomplished by having a smaller source region ($d \lesssim 10^2 \text{ km}$), smaller plasma density ($N < 10 \text{ cm}^{-3}$), fewer energetic electrons ($\zeta < 0.1$), lower energy $v_E < 0.2c$), or some combination of these.

Another estimate of when nonlinear effects become important can be calculated by allowing the waves to grow until the wave amplitude is on the order of the magnitude of the zero-order field. First, we compute the energy density ϵ of incoherent emission produced by ζN electrons per cm^3 radiating for a time τ , where τ is the time it takes a wave to propagate across the source region d . We find (Jackson 1962)

$$\epsilon \simeq \frac{2\zeta N e^2 \beta^2 \Omega_0 \tau}{3c}. \quad (15)$$

The instability will amplify the incoherent radiation, given by equation (15), until $\epsilon A \approx B^2/8\pi$. At $L = 1.2$, $B \simeq 10 \text{ gauss}$. Again using $N\zeta = 1 \text{ cm}^{-3}$ and $\beta = 0.2$, we find that $\tau \lesssim 1 \text{ s}$, which is consistent with the observed duration of DAM bursts. Thus, we again have $d \lesssim 10^3 \text{ km}$.

A more precise calculation of the radiation intensity to be expected from this mechanism will require a quasi-linear or nonlinear treatment, which is beyond the scope of this article. From the results of this linear analysis we conclude that the loss-cone instability is strong enough to account for the observed DAM radiation. Because our results do not depend on I_0 , they may be more relevant to the "main source" radiation, which is often unmodulated by I_0 .

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Note added in proof.—Figure 2 is incorrect. The figure will be approximately correct if the scale for the top half of the figure ($L = 1.2$) is multiplied by 10^{-1} , and the scale for the bottom half ($L = 2.0$) is multiplied by $\frac{1}{2}$. The discussion in the text reflects these corrections.

