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# Adiabatic Compression of a Compact Torus

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## Abstract

A design point is presented here for a prototype fusion neutron source for waste transmutation ( $10^{17} - 10^{19}$  n/s), based on the adiabatic compression of a compact torus (spheromak). The design utilizes the CORSICA (2D equilibrium) and NIMROD (3D time-dependent MHD) codes as well as analytic modeling with target parameters  $R_{initial}=0.5\text{m}$ ,  $R_{final}=0.167\text{m}$ ,  $T_{initial}=0.4\text{keV}$ ,  $T_{final}=4\text{keV}$ ,  $n_{initial}=2\times 10^{20}\text{m}^{-3}$  and  $n_{final}=50\times 10^{20}\text{m}^{-3}$ , with radial convergence of  $C=3$ . 3D time-dependent simulations of spheromak compression agree well with analytic models for adiabatic compression, if the run-in time  $\tau_{compress} < \tau_E$ . Knowing  $\tau_{compress}$  required, we design coils and passive structure (with CORSICA) to ensure stability; then design the capacitor bank needed to both form the target plasma and drive coils. We specify target parameters for the compression in terms of plasma beta, formation efficiency and energy confinement.

**Keywords:** Compact, Neutron Source, Fusion

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## 1. Introduction

The development of compact fusion neutron sources for use in waste transmutation and fuel reprocessing has motivated an IAEA Coordinated Research Project (CRP) aiming for sources with  $P_n=1\text{-}100\text{MW}$  ( $10^{17} - 10^{19}$  n/s) [1]. As part of this study a number of different candidates are being considered, based on the Tokamak, mirror, and spherical torus (ST). Our concept is based on the adiabatic compression of a compact torus, in this case a Spheromak [2]: essentially a tokamak with reversed shear,  $q$  spanning  $q_0=0.6$  to  $q_{95}=0.3$ , and the currents that produce the toroidal field flow in the plasma rather than in external windings. We examine a low radial convergence  $C_R (=R_0/R_f) < 3$ , which differentiates our concept from magnetized target fusion concepts with  $C=10$  or more. The focus of this paper is on the design point for an experiment that will resolve matters of confinement scaling and peak pressure before building a full neutron source [3].

This paper is therefore structured as follows. In Section 2 prior work on adiabatic compression in spheromaks (S1), and tokamaks (TUMAN-3M and ATC) is briefly summarized; recent spheromak performance presented. The numerical tools

for the study are discussed in Section 3 (CORSICA, NIMROD and the analytic adiabatic scaling model). In Section 4, a 0D design point for a neutron source is defined analytically; time-dependent 3D MHD simulations results are presented (to test for adiabaticity); and a 2D equilibrium model is used to define coil positions to support equilibrium and provide compression. Given coil currents from CORSICA, a bank design is developed that allows for compression faster than  $\tau_E$ . Knowing the size of bank and coils, an engineering design point is developed. Section 5 is a discussion, contrasting the design with work ongoing in the IAEA CRP. Section 6 is a discussion of the work that follows prior to finalizing physics and engineering design points. Section 7 is the conclusion.

## 2. Background

Accessing high  $n\tau T$  adiabatically with a 1m system has been the aim of several experiments in the past. ATC [4] saw ion temperatures increase from 200eV to 600eV, TUMAN-3M [5] observed a transition to H-mode confinement and S1 (Fig. 1) was able to obtain adiabatic compression of spheromaks with  $C=1.6$ , and corresponding increase in  $T_e$  from 40 to 100eV [6]. Currently ARPA-E is supporting compression concepts, and companies are finding investment for the development of compact tori utilizing compression. Omitting the central stack (toroidal field coil, solenoid, blankets and shield) reduces cost and engineering complexity of the power core [7]. The per-

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48 performance achieved in Concept Exploration spheromaks forms  
 49 the basis for this study. Fig. 1 shows the Sustained Sphero-  
 50 mak Physics Experiment (SSPX), operated at LLNL from 1999  
 51 to 2007 [8]. In SSPX, a 1m diameter spheromak, volume-  
 52 averaged electron betas of 5% were achieved, as well as core  
 53 electron temperatures as high as 500 eV, with core thermal dif-  
 54 fusivities  $< 1 \text{ m}^2/\text{s}$  consistent with L-mode (or Ohmic) confine-  
 55 ment scaling [9].

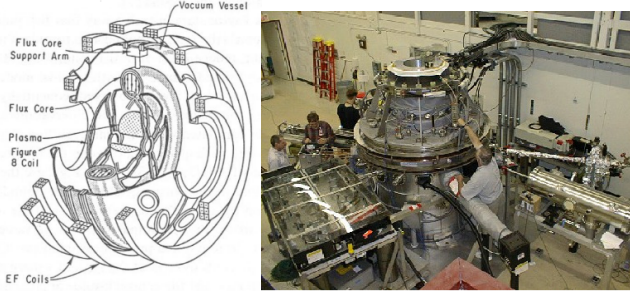


Figure 1: LEFT: S1 operated at PPPL. RIGHT: SSPX operated at LLNL.

### 3. Modeling

Analytic scaling models for the compression of plasmas generally follow the ideal gas law [10]. For processes that are adiabatic,  $(\gamma=5/3)$ :  $P_0 \cdot V_0^\gamma = P_f \cdot V_f^\gamma$ , and  $PV^{5/3} = \text{const}$ . For self-similar compression ( $C = a_0 / a_f$ )  $P_0 V_0^\gamma = P_f V_f^\gamma$  and since  $V_f = V_0 / C^3$  then:  $P_f = P_0 \cdot C^5$  and since  $P \sim n \cdot T$  and  $T_f = T_0 \cdot C^2$ . Model is implemented in Excel.

CORSICA [11] contains a 1.5-D, time-dependent plasma simulation code which solves the Grad-Hogan problem: self-consistent evolution of free-boundary plasma equilibria and internal profiles, including external conductors and magnetic diffusion, with a variety of available transport models. In addition, CORSICA provides ideal-MHD stability packages, native ballooning and vertical-stability calculations, and DCON (compiled with the code) for stability analysis.

The NIMROD code [12] uses spectral finite element discretization in two dimensions, finite Fourier series in the third dimension. It has semi-implicit and implicit temporal discretization for the range of temporal scales found in fusion experiments. The equation set includes continuity, force, temperature (two fluids), induction, and Hall terms in Ohm's law. Importantly, we use a temperature-dependent form of the thermal diffusivity, based on Braginskii's thermal transport model [13] calibrated to data from SSPX [14]. Our problem geometry is a cylinder as depicted in Fig. 2. We apply time-varying boundary conditions for the tangential electric field to emulate the effects of a magnetic compression.

Finally, we use a number of different circuit modeling tools (PSpice, 5Spice, LTSpice), and CAD (Shark, Solidworks) and FEM (COMSOL) to develop the engineering design points.

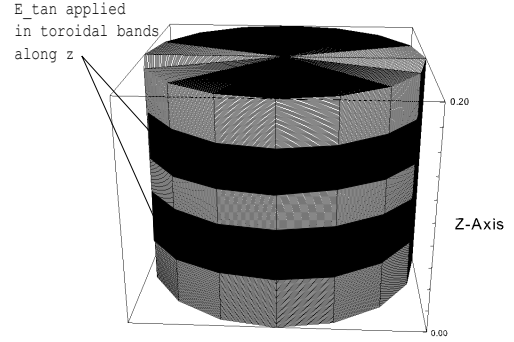


Figure 2: NIMROD mesh for runs explored in this paper.

## 4. Results

### 4.1. 0D modeling for $P_n=1\text{MW}$ design point

The point design for the compression concept is shown in Table 1. We first pick the geometrical parameters, starting with a plasma of comparable dimensions to SSPX, but compressing self-similarly to radial convergence of 3. The initial plasma beta of 15% is obtained by optimizing the shape and q-profile (see section below) to be stable to ideal pressure-driven modes. An initial density and magnetic field are then chosen consistent with what has been obtained experimentally. Next, Temperature is obtained from the initial beta, density and magnetic field strength. The power and flux are obtained from  $P_{\text{fusion}} = V \cdot E_{DT} \cdot \langle \sigma \cdot v \rangle \cdot n^2$  where  $E_{DT}$  is the energy produced in each deuterium-tritium reaction, the reaction rate,  $\langle \sigma \cdot v \rangle [\text{cm}^3/\text{s}] = 3.68 \times 10^{-12} \cdot T^{-2/3} \cdot \exp(-19.94 \cdot T^{-1/3})$ , with T in keV [15] and  $n$  is the plasma density, assuming equal amounts of deuterium and tritium. To be conservative in our 0D model, we assume that the fusion island only fills a quarter of the volume of the compressed plasma, since there will be a sharp temperature gradient in the plasma and only the core will be sufficiently hot for fusion reactions to take place (this assumption is supported by 2D modeling - see below). We calculate the Ohmic confinement time at peak compression ( $\tau_{E\text{Ohmic}} = 0.07(n/10^{20}) \cdot a \cdot R^2 \cdot q$ ) and the H-mode Ohmic confinement time  $\tau_{EH} = 5 \cdot \tau_{E\text{Ohmic}}$  - as observed in TUMAN-3M and consistent with SSPX data [9]. The steady state neutron power and rate is given, which assumes a unity duty cycle.

### 4.2. Comparison of 3D MHD simulations of compression in cylindrical geometry with 0D analytic modeling

Given the 0D design point, we set up a number of runs in a simplified geometry with a range of initial conditions (including toroidal velocity) and time-varying boundary conditions. Fig. 3 shows the solution fields at a radial convergence of 2 (in this instance the compression is only in r). Solution convergence was tested by increasing the radial and axial resolution in the mesh, and effects of increasing toroidal resolution assessed, picking a set of cases with low numerical

Parameter	Symbol	Value	Units
Radial Convergence	$C_r$	3	
Volumetric Convergence	$C_V$	27	
Initial Plasma Radius	$r_0$	0.5	m
Final Plasma Radius	$r_f$	0.166	m
Initial Volume	$V_0$	0.55	$m^3$
Converged Volume	$V_f$	0.02	$m^3$
Initial Beta	$\beta_0$	15	%
Initial Density	$n_0$	2	$\times 10^{20} m^{-3}$
Initial Magnetic Field	$B_0$	0.63	T
Initial Temperature	$T_0$	387	eV
Final Temperature	$T_f$	3487	eV
Final Density	$n_f$	54	$\times 10^{20} m^{-3}$
Final Magnetic Field	$B_f$	5.7	T
Initial plasma current	$I_0$	0.95	MA
Final plasma current	$I_f$	8.5	MA
Final Beta	$\beta_f$	45	%
Final Magnetic Energy	$U_f$	270	kJ
Ohmic Confinement time	$\tau_{E_{Ohmic}}$	2.2	ms
Ohmic H-mode confinement time	$\tau_{EH}$	10.9	ms
Steady State Neutron Power	$P_n$	1	MW
Steady State Neutron Rate	$\Gamma_n$	$4.7E+17$	n/s

Table 1: Device design point from 0D modeling.

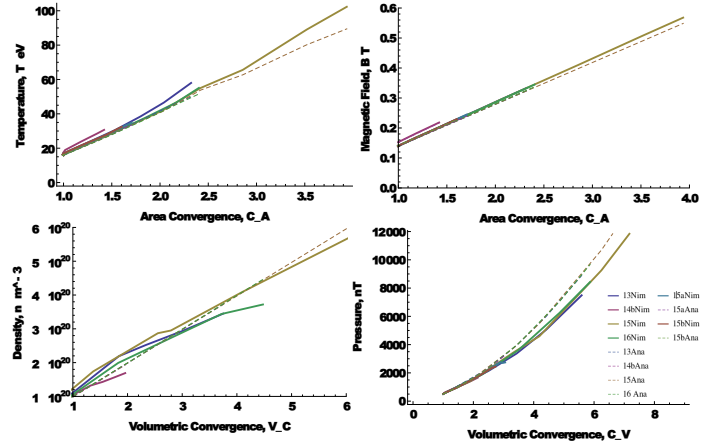


Figure 4: 3D MHD results follow 0D adiabatic scaling. Solid lines are from NIMROD simulations with a range of compression coil voltages (see Table 2) and dashed lines are from 0D analytic modeling.

Run	Ramp	Coil voltage
13	2us	80kV
14b	2us	160kV
15	2us	320kV
15a	5us	320kV
15b	6us	320kV
16	3us	80kV

Table 2: NIMROD campaigns with different coil voltages, showing ramp time, used to generate Fig. 4

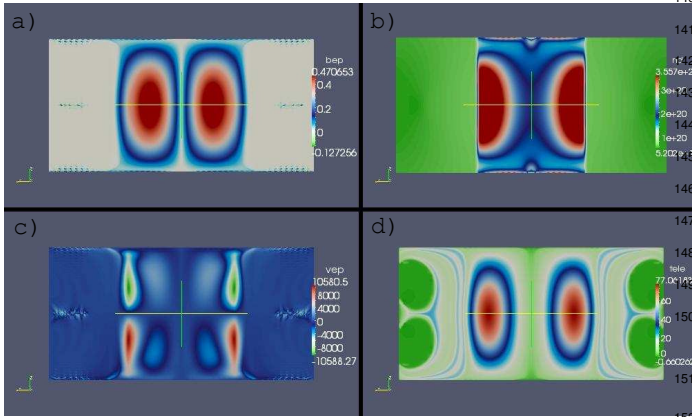


Figure 3: Adiabatic compression to  $C=2$  of a spheromak in cylindrical geometry, a) toroidal magnetic field,  $B_\phi$  (which is peaked close to the magnetic axis for a spheromak, falling to zero at the edge); b) density,  $n$ ; c) toroidal velocity  $v_\phi$ ; and d) electron temperature,  $T_e$

### 4.3. CORSICA analysis for design point

With confidence in the adiabaticity of the compression, given a fast-enough run-in, we proceeded to develop an engineering

design point with the CORSICA code, shown in Fig. 5. Here, plasma injection from outboard side using solenoid and conical annular electrodes.  $T_0 = 200\text{eV}$ , and  $n_0 = 1e20 m^{-3}$ . At peak compression, plasma current increases to 6.75MA and fusion power of 1.3MW is obtained at 5keV. Note that these parameters differ slightly from the analytic modeling, since here we are assuming profiles of temperature and density that are flux functions that are pedestal (i.e. H-mode) like.

At peak compression, the shape of the plasma dictates the attainable beta (stable to Mercier (pressure-driven) modes), so a scan of elongation was performed, and results are shown in Fig. 6, noting that for  $\kappa = 0.8$ ,  $\beta_{pol} = 50\%$ , consistent with that required at peak compression.

### 4.4. Design point for coil banks

An output from the CORSICA analysis is the coil current at peak compression, and also the mutual inductance matrix for the coil set (as well as with the plasma). Estimating the run-in time from L-mode confinement scaling allows us to define the current rise time of the coils (of the order of 1ms rise, assuming  $\tau_E$ 's of 10ms), which when coupled to capacitors of reasonable size through a switch defines a simple LRC circuit. In total there will be 4 banks for the coils (3 compression coils and one formation solenoid) and one bank to energize the formation electrodes. The outer-most compression coil bank will be 800 kJ, next smallest 200 kJ and smallest coil set will need 10

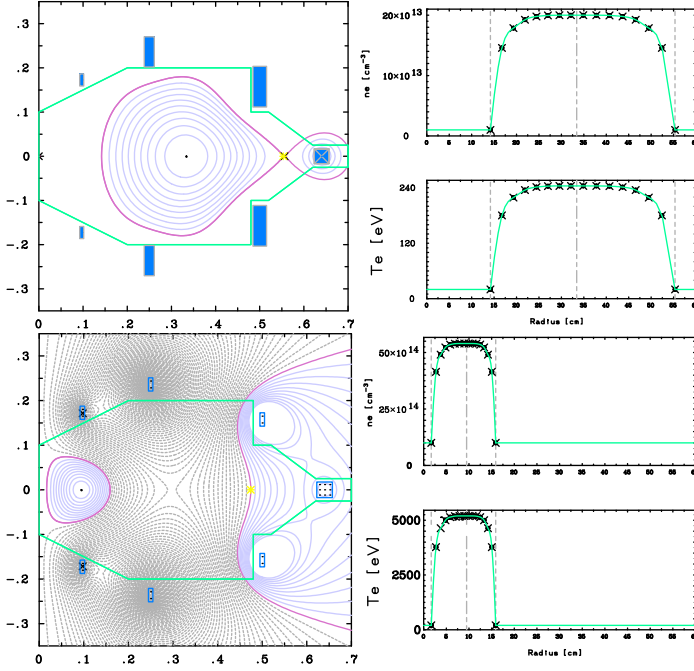


Figure 5: CORSICA design point before and after compression.

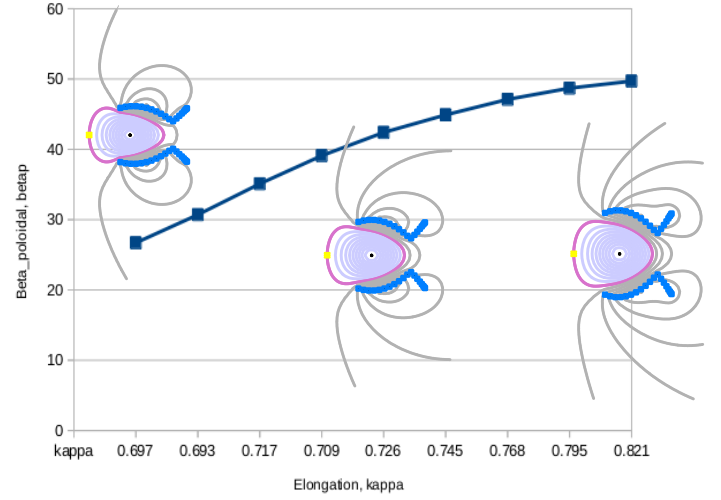


Figure 6: Effects of changing the plasma elongation on the obtainable poloidal beta.

#### 4.6. Neutron distribution

Fig. 8 shows the poloidal neutron flux distribution: peak to average ratio is 1.366 with the peak as a band centered on the equatorial plane. ( $\phi = 0, \pi$  are the poles at the axis). The flat, peak band would be a good result in terms of providing a good zone for sample exposure.

### 5. Discussion

We have seen here that the density scaling of  $n_f = C^3 \cdot n_0$  holds for the simulations, although this has not been observed experimentally in the one spheromak compression experiment previously performed (S-1), which found  $n_f = C \cdot n_0$  [6]. This is an important matter that affects the efficiency of the device, and impacts the neutron production directly. We understand this enhanced particle loss to result from pressure-driven and current-driven instabilities that occur during compressions of longer duration. Recent simulation work of longer compressions using NIMROD [18] indicates the onset of various toroidal modes consistent with q-profile evolution in a decaying spheromak, and it should be noted that the beta initially is low at 15% but increases to 45% during the cycle. This is problematic. However, when we move to larger systems and slower compressions the solution is to perform a compression that both preserves the q-profile (to prevent onset of toroidal modes) and shapes the plasma to provide better pressure stability. The first of these scenarios is discussed in the literature for in which the decay is sought in which the profiles decay self-similarly (controlled ramp-down scenarios [19][20]). Preserving pressure stability is harder, although shaping the plasma during compression to provide sufficient magnetic shear is possible [21].



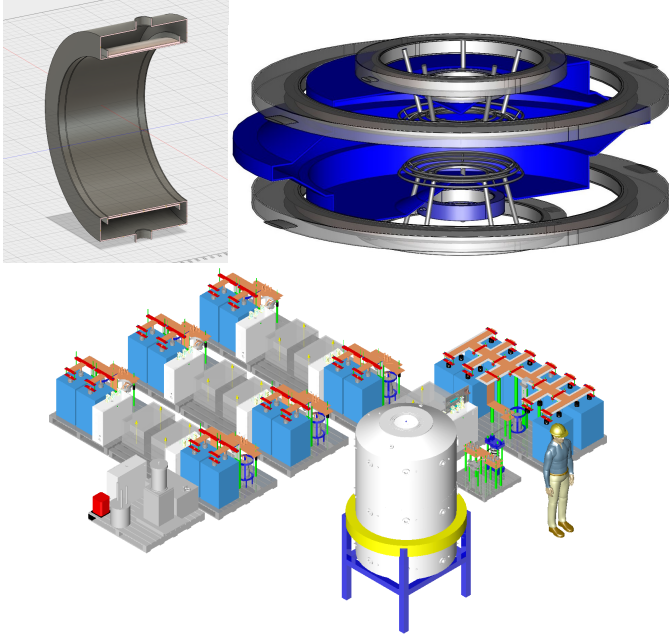


Figure 7: TOP LEFT: Coil design point - actively cooled, in-vacuum component. TOP RIGHT: all in-vacuum coils, support structure and passive stabilizers - for scale, outer coil is 1m diameter. BOTTOM: full system layout for banks, and vacuum chamber.

The design presented here is for a single pulse system, which will allow for the main scientific issues to be explored for the use of spheromak compression as a neutron source. The design point is not yet optimized and will undergo another iteration before it is finalized. The critical concerns here are that the duty cycle can be high enough for this concept to serve as a quasi-steady-state neutron source, obtaining the required flux for use in waste transmutation. Limits on the duty cycle include time of stability at peak compression, and power loading in the coils. Our design point differs from other fusion neutron sources as part of the IAEA study, in two important ways. First, the system is very compact, less than 1/10th of the linear dimensions of most other systems, and hence 1/100th of the volume. Since cost scales with volume, this represents a relatively low cost system for generating fusion neutrons (TCC estimated at \$10M). Secondly, the system is much simpler to engineer than concepts based on the ST and tokamak: the fusion island is simply connected, with no material linking it, obviating the need for shielding of the central stack (and coils).

## 6. Further Work

This work represents a pre-conceptual design stage. To follow will be a conceptual design in which all of the aspects of physics and engineering will be examined self-consistently, taking neutronics analysis with MCNP6 as a central aspect of this future work. The design point will be further optimized for efficiency, and for rep-rate. CORSICA analysis will be performed to address transport and shape optimization during run-in. NIMROD analysis (ongoing) will be performed to optimize

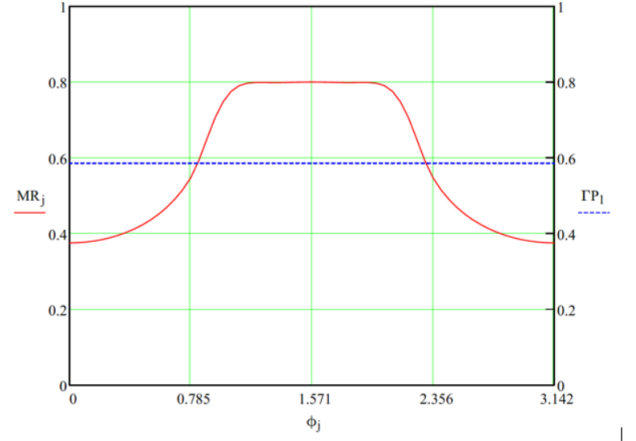


Figure 8: Poloidal neutron flux distribution.

stability at peak compression, and active feedback stabilization will be assessed. Our current study has limited itself to a narrow range of compression times ( $> \tau_E / 10$ ) and in future we may wish to extend the compression surveys to faster run-ins. Deceleration-phase instabilities that occur near peak compression will be examined as well as post-stagnation dynamics and dwell time (or period of stability at peak compression), which can strongly impact fusion yield. Finally, the electrical engineering required to increase the duty cycle will be considered.

## 7. Conclusion

The design point for a n-Source prototype has been examined analytically and numerically, using state-of-the-art tools (2D and 3D time-dependent MHD). A full engineering design point has been developed, including thermal analysis of compression coils.

## 8. Acknowledgments

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