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# Thermal Instabilities, Oscillations, and Frequency Combs in Kerr Microresonators

Amir Leshem<sup>1</sup>, Zhen Qi<sup>2</sup>, Thomas F. Carruthers<sup>2</sup>, Curtis R. Menyuk<sup>2</sup>, and Omri Gat<sup>1</sup>

1. Racah Institute of Physics, The Hebrew University, Jerusalem, Israel 9190401. 2. University of Maryland at Baltimore County, 1000 Hilltop Circle, Baltimore, MD 21250, USA.

omrigat@mail.huji.ac.il

**Abstract:** In a theoretical study of dissipative heating in Kerr microresonators we show that thermal effects shift the coexistence wedge and modulational instability curve of continuous waves, so as to block access to frequency combs, and exhibit a path circumventing the obstruction. Thermal oscillations occur beyond a Hopf bifurcation curve. © 2022 The Author(s)

## 1. Introduction

The power dissipation of intense fields necessary for producing frequency comb in a Kerr microresonator strongly affects the wave propagation in the resonator; in particular, the sudden cooling that follows the transition from chaos to solitons by red-detuning of the pump adversely affects the soliton stability [1]. However, to-date there have been few studies that investigate systematically the role of the thermal resonance shift. Here we use a thermo-optical Lugiato-Lefever model to show that thermal effects shift the modulational instability curve to higher values of the detuning parameter, effectively blocking direct access to solitons and perfect soliton crystals. Nevertheless, we demonstrate an explicit access path of pump parameters that leads directly from continuous waves to soliton crystal waveforms that generate strong broadband frequency combs, and calculate the stability boundaries of these waves. Furthermore, we show that thermal effects can lead via a Hopf bifurcation to temporal oscillations of spatially uniform waves, making a connection with the well-studied phenomenon of thermal oscillations in microresonators [2].

## 2. Equations of motion

We modify the standard Lugiato-Lefever equation for the cavity waveform  $\psi$  by a thermal detuning term  $\Theta$  that is proportional to the difference temperature between the resonator and the ambient temperature, and supplement it by an equation for  $\Theta$ , that is assumed uniform because the thermal time scales are much longer than the optical scales, obtaining

$$\frac{\partial \psi}{\partial t} = -(1 + i(\alpha + \Theta))\psi + \frac{i}{2} \frac{\partial^2 \psi}{\partial x^2} + i|\psi|^2\psi + F, \quad \frac{d\Theta}{dt} = -AP - B\Theta. \quad (1)$$

Here  $\alpha$  and  $F$  are (respectively) the pump detuning and amplitude at ambient temperature, and  $A$  and  $B$  are the absorptive heating, and thermal relaxation coefficients (respectively.) Position  $x$ , time  $t$ , and wave amplitude  $\psi$  are measured in scales determined by the dispersion, loss, and Kerr coefficients (respectively), and  $P = \int_0^L |\psi|^2 dx / L$  is the mean power,  $L$  being the resonant mode circumference.

## 3. Thermal detuning and paths to frequency combs

The *steady state* waveforms of Eqs. (1) depend on the thermal coefficients only through the thermal sensitivity coefficient  $C = A/B$ ; in most experiments  $A$  and  $B$  are small, but  $C$  is of order one or larger, so thermal effects are significant. In a steady state  $\Theta$  is proportional to the mean power, so that all steady states are red-detuned, including the modulational instability threshold that marks the boundary between stable continuous waves (CW) and cnoidal wave (Turing roll) solutions, which is shifted accordingly to the right in the pump parameter plane (Fig. 1.)

The thermal response of CW is different: For  $\alpha > \sqrt{3}$  there is a wedge of bistability in which three CW branches coexist, of which the middle one is always unstable; when  $C$  is increased, the wedge shifts to lower  $F$  (Fig. 1.) The modulational instability gives access to cnoidal waves that sharpen into perfect soliton crystals by red-detuning, but when the bifurcation overlaps with the wedge of coexistence, the instability affects only the *upper* CW branch, which means that the wedge *blocks* the access to the soliton crystals using constant frequency pump. Notwithstanding, cnoidal waves *are* accessible using a two-step pump control in which the pump power is raised at a fixed frequency with  $\alpha < \sqrt{3}$ , and then red-detuned; the path shown in Fig. 1 leads to cnoidal waves with the smallest directly accessible free spectral range [3]. Figure 1 (right panel) shows the stability regions of cnoidal waves with thermal sensitivity  $C = 5$  [4].

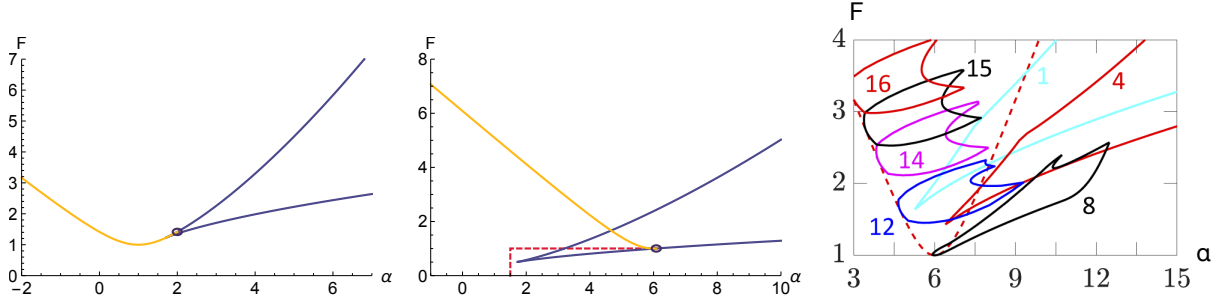


Fig. 1. Stability diagram of continuous waves without (left) and with thermal effects (right, sensitivity  $C = 5$ , relaxation rate  $B = 0.05$ ) in the plane of pump detuning  $\alpha$  and amplitude  $F$ . The blue curves are the boundaries of the wedge of coexistence, where three CW branches coexist, which moves down in  $F$  as  $C$  increases; the yellow curves are the modulational instability thresholds, that move to higher  $\alpha$  as  $C$  increases. The red dashed curve is an access path leading to cnoidal waves with the smallest accessible free spectral range, created at the point circled in black. Right: Stability regions of cnoidal waves,  $C = 5$ , mode circumference  $L = 50$ . Regions are labeled by the number of oscillations; the access path shown in the middle panel leads to the 8-oscillation region.

#### 4. Thermal oscillations

Thermal oscillations affecting spatially uniform waves in microresonators have been known for a while, and are usually attributed to competition between two effects, such as thermal detuning and thermal expansion [2]. Nevertheless, the basic thermal model (1) captures the mechanism: Without thermal effects ( $C = 0$ ) a CW branch becomes unstable only by modulational instability in space; together with the thermal degree of freedom on the other hand, Hopf bifurcations occur at large pump amplitude for some combinations of thermal coefficients (left panel of Fig. 2.) Beyond the bifurcation line, the temperature and uniform field amplitude approach a limit cycle attractor, where they perform nonlinear oscillations of the type observed in experiments (Fig. 2, middle and right panels.)

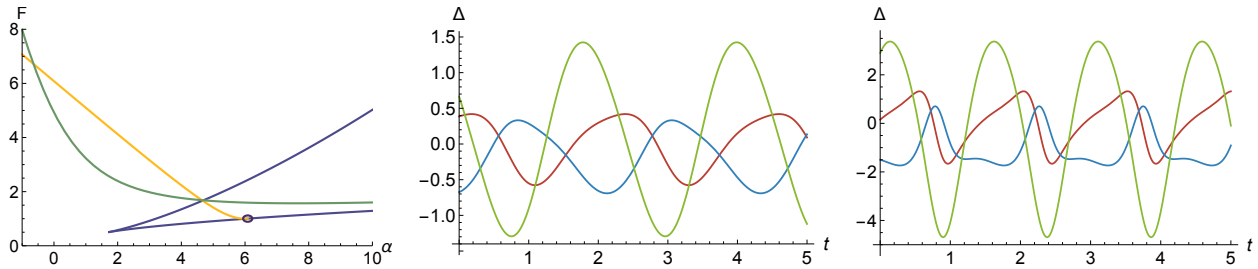


Fig. 2. Left: Stability diagrams of continuous waves with  $C = 5$  and  $B = 2$ . The blue and yellow curves have the same meaning as above; the green curve is the Hopf bifurcation curve. Middle and right: Limit cycle oscillations of the real part (red curve) and imaginary part (blue) of the field amplitude  $\psi$ , and the thermal detuning  $\Theta$  (green) with the same thermal coefficients,  $\alpha = 8$ , and  $F = F_h + 0.1$  (middle),  $F_h + 2.0$  (right), where  $F_h \approx 1.7$  is pump amplitude at the Hopf bifurcation for  $\alpha = 8$ .

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