Group velocity, energy velocity, and superluminal propagation in finite photonic band-gap structures

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We have analyzed the notions of group velocity V_g and energy velocity V_E for light pulses propagating inside one-dimensional photonic band gap structures of finite length. We find that the two velocities are related through the transmission coefficient t as $V_E=|t|^2V_g$. It follows that $V_E=V_g$ only when the transmittance is unity ($|t|^2=1$). This is due to the effective dispersive properties of finite layered structures, and it allows us to better understand a wide range of phenomena, such as superluminal pulse propagation. In fact, placing the requirement that the energy velocity should remain subluminal leads directly to the condition $V_g \leqslant c/|t|^2$. This condition places a large upper limit on the allowed group velocity of the tunneling pulse at frequencies of vanishingly small transmission.

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During the past two decades electromagnetic wave propagation effects in periodic structures, usually referred to as photonic band gap (PBG) crystals [1], have been intensely investigated. Experimental studies highlighted particularly interesting linear properties of pulses propagating in structures of finite length. Examples are the measurement of superluminal group velocities at midgap frequencies [2], and the measurement of low group velocities near the band edge of a semiconductor heterostructure [3]. These effects originate with the remarkable but peculiar dispersive properties of finite multilayer stacks [4]. The concept of group velocity is particularly critical when applied to an *absorbing (or gain) homogeneous* dielectric material, because V_g may be greater than c, and it can even be negative in some circumstances. These topics were discussed at length in the seminal book by Brillouin [5], in 1970 by Loudon [6], and by Garret and McCumber [7]. The physical relevance of V_g regarding pulse propagation with superluminal or negative group velocities was experimentally studied by Chu and Wang [8], who measured the transmission time of a laser pulse tuned at a GaP:N resonance. Recently, Peatross *et al.* [9] theoretically showed that, in the context of an absorbing, homogenous material, the group velocity may still be meaningful even for broadband pulses, and when V_g is superluminal or negative. In Ref. [9], the group velocity was related to the pulse arrival time via the time expectation integral over the Poynting vector.

In the Refs. [5–9] the work dealt with pulse propagation in absorbing, homogenous dielectric materials. More recently, Wang *et al.* [10] used gain assisted linear dispersion to demonstrate superluminal light propagation in atomic cesium: the group velocity of a laser pulse under conditions of anomalous dispersion in the presence of gain can exceed c as a result of classical interference between the different frequency components. In this work, we discuss the case in which a one-dimensional (1 D) PBG structure displays anomalous effective index behavior *not as a result of gain or absorption, but as a result of scattering*. More importantly, in our case the presence of entry and exit interfaces plays a crucial role in determining the definition of the group velocity and its relationship with the energy velocity. In contrast, boundary conditions, in the sense of entry and exit interfaces, can play no role in the determination of either group or energy velocity in inhomogeneous, periodic structures of infinite length. Our simple and straightforward analysis shows that there are significant conceptual, qualitative, and quantitative differences between energy and group velocities in finite structures, in contrast to the case of infinite structures. In fact, for a periodic, infinite structure, a unique dispersion relation exists between K_β (the Bloch vector) and ω. The group velocity $V_g^{(\omega)}$ is defined as $V_g^{(\omega)}=1/[dK_\beta/d\omega]$, and it can be demonstrated that $V_g^{(\omega)}=V_E^{(\omega)}$ [11].

In order to discuss the case of 1 D, finite PBG structures, we consider a system consisting of pairs of alternating layers of high and low linear refractive indices. The thicknesses of the layers are a and b, respectively; for N periods, the length of the structure is $L=N(a+b)$. *For plane monochromatic waves*, the Helmoltz equation for the field is

$$\frac{d^2E_\omega}{dz^2}+\frac{\omega^2}{c^2}\epsilon_\omega(z)E_\omega=0. \tag{1}$$

The general boundary conditions at the input ($z=0$) and output ($z=L$) surfaces are $E_I^\omega+E_r^\omega=E_\omega(0)$, $E_t^\omega=E^\omega(L)\exp[-i(\omega/c)L]$; $i(\omega/c)(E_I^\omega-E_r^\omega)=dE_\omega(0)/dz$, and $i(\omega/c)E_t^\omega=(dE_\omega(L)/dz)\exp[-i(\omega/c)L]$. $\varepsilon_\omega(z)$ is the spatially dependent, real dielectric permittivity function, and material absorption is neglected. For simplicity, we assume the structure is surrounded by air. E_I^ω, E_r^ω, and E_t^ω are the incident, reflected, and transmitted fields, respectively. We

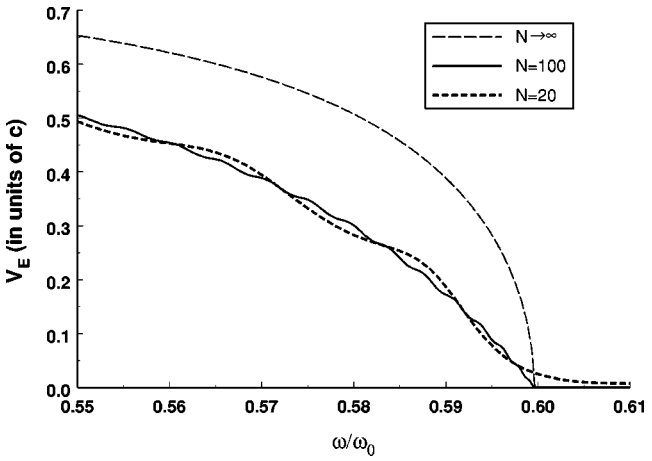

FIG. 1. $V_E^{(\omega)}$ for a 20-period structure (short dashes), a 100-period structure (solid line), and an infinite structure (long dashes) vs frequency. Increasing the number of periods, the energy velocity does not converge to the results obtained using the dispersion of the infinite structure. The elementary cell is composed of a combination of half-wave–quarter-wave layers. The indices of refraction are $n_a=1$ and $n_b=1.42857$; the respective thicknesses are $a=\lambda_0/(4n_a)$ and $b=\lambda_0/(2n_b)$, with $\lambda_0=1\mu$m and $\omega_0=2\pi c/\lambda_0$.

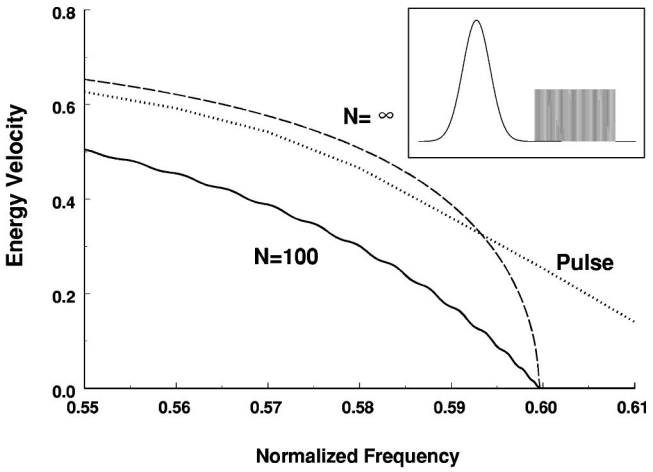

FIG. 2. $V_E^{(\omega)}$ vs frequency for a 150-fs incident pulse (dotted line) and the monochromatic regime (solid line). The monochromatic wave regime is also obtained using pulses at least several tens of picoseconds in duration. The dashed line corresponds to Bloch's velocity for the infinite structure. The structure is similar to that outlined in Fig. 1, but contains 100 periods. Inset: the 100-period structure is approximately 60 μm. in length, while the spatial extent of the full width at half maximum of the pulse is approximately 20 μm.

introduce the dimensionless quantities $\Phi_\omega(z)=E_\omega(z)/E_I^\omega$, $t_\omega=E_t^\omega\exp[i(\omega/c)L]/E_I^\omega$, and $r_\omega=E_r^\omega/E_I^\omega$, where $\Phi_\omega(z)$ is the field profile at frequency ω, and t_ω and r_ω are the transmission and reflection coefficients, respectively, all calculated using the standard matrix transfer technique. The averaged energy velocity, which measures energy flow across the sample, is defined as the ratio of the spatial average of the Poynting vector to the spatial average of the energy density within the same volume [11], which in our 1D geometry is given by

$$V_E^{(\omega)}=\frac{\frac{1}{L}\mathrm{Re}\left[\frac{ic^2}{\omega}\int_0^L\Phi_\omega\frac{d\Phi_\omega^*}{dz}dz\right]}{\frac{1}{2L}\int_0^L\left[\epsilon_\omega(z)|\Phi_\omega|^2+\frac{c^2}{\omega^2}\left|\frac{d\Phi_\omega}{dz}\right|^2\right]dz}. \qquad (2)$$

We decompose the field in the form. $\Phi_\omega=|\Phi_\omega|e^{i\vartheta_\omega}$. It follows that $|\Phi_\omega|^2 d\vartheta_\omega/dz$ is a conserved quantity admitted by Eq. (1). Substituting into Eq. (2), integrating by parts, and using the boundary conditions above, the energy velocity $V_E^{(\omega)}$ takes a form that involves both the transmittance $|t_\omega|^2$ and the imaginary part of the reflectivity r_ω of the stack:

$$V_E^{(\omega)}=\frac{c|t_\omega|^2}{\frac{1}{L}\int_0^L\epsilon_\omega(z)|\Phi_\omega|^2dz-\frac{c}{L\omega}\mathrm{Im}(r_\omega)}. \qquad (3)$$

The second term in the denominator indicates that the energy is generally not equally shared by the electric and magnetic components, which become identical only at each peak of transmittance, where r_ω vanishes. In Fig. 1 we depict $V_E^{(\omega)}$ for 20- and 100-period stacks, and compare with $V_E^{(\omega)}$ (for infinite structures $V_g^{(\omega)}$ coincides with $V_E^{(\omega)}$) of an infinite structure made with the same elementary cell. We find that the energy velocities of the infinite and finite structures do not converge to one another by increasing the number of periods. This is due to the fact that in the monochromatic approximation it is always possible to resolve each transmission resonance, and hence its curvature, even if more periods are added. In this regime, incident pulses propagate through the structure tuned at one transmission resonance, for example, with their bandwidth significantly smaller compared to resonance bandwidth. Put another way, the spatial extent of the pulse is orders of magnitude greater then the physical length of the structure. As a result, the pulse samples all internal and external interfaces simultaneously; it is delayed, and in the end completely transmitted, with minimal distortion or scattering losses [3]. Therefore, the interaction should more properly be referred to as a scattering event.

To better clarify this situation, we consider the case of a short pulse incident on a structure several pulse widths in length. If the spatial extent of the pulse is so short that it traverses the structure without simultaneously sampling both entry and exit interfaces, then we may expect that the discontinuity at the entrance and exit interfaces will not significantly affect the dynamics [12]. This is shown in Fig. 2, where we plot $V_E^{(\omega)}$ as calculated in the quasi monochromatic limit via Eq. (3), and also as numerically calculated for an incident Gaussian pulse 150 fs in duration, tuned in the passband of a 100-period structure. The energy (group) velocity of the infinite structure is also shown in the figure. The comparison between the length of the structure and the pulse

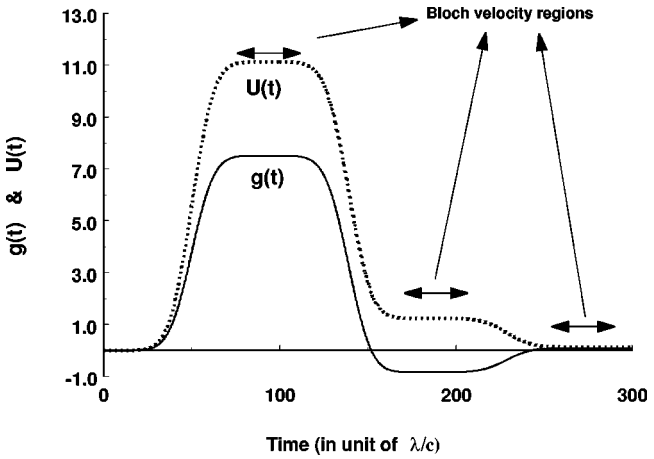

FIG. 3. Total energy (dashed) and momentum (solid) inside the structure for the pulse shown in the inset of Fig. 2. Both momentum and energy increase as the pulse traverses the entry boundary. The energy velocity for the entire process, i.e., the ratios of the areas under the curves, which is a measure of energy flow through the structure in both directions, cannot be the same as the Bloch velocity, which monitors only the velocity of the transmitted pulse. Multiple reflections inside the stack lead to ringing in reflection and transmission from the structure.

width is made in the inset. As the inset shows, the structure is several pulse widths in length. As a result, the energy velocity of the pulse propagating *inside* this structure better approximates the energy velocity of the infinite structure (Bloch's velocity). The energy velocity of the pulse does not show the same sharp cutoff near the band edge that we observe for both the infinite and the 100-period structures, because the pulse is ultrashort, and so even if the carrier frequency is tuned inside the gap a good fraction of the energy is still trasmitted. We find the same degree of convergence only if the pulse width and structure length are significantly increased simultaneously, so that the pulse can better resolve the frequencies near the band edge, but can still fit well inside the structure, as in the inset. In Fig. 3, the total momentum and energy inside the 100-period structure depicted in Fig. 2 are shown as functions of time. The total electromagnetic momentum for the pulse inside the structure can be written as follows:

$$g(t)=\frac{1}{c^2}S(t)=\frac{1}{c^2}\int_0^L E_\omega(z,t)\times H_\omega(z,t)dz. \tag{4}$$

The total energy is given by

$$U(t)=\int_0^L\left[\epsilon_\omega(z)|E_\omega(z,t)|^2+\frac{c^2}{\omega^2}|B_\omega(z,t)|^2\right]dz. \tag{5}$$

As the pulse enters the structure, there is a rapid rise in both the energy and momentum, which settle to constant values once the whole pulse travels inside the structure. When the pulse is totally inside, both group and energy velocities are equal, and given by Bloch's velocity. However, even if the structure is long, it is nevertheless finite, and so the pulse must eventually exit, leading to a reduction of energy and a reversal in sign in the total momentum. Once the momentum becomes negative, we track the first pulse reflected from the exit interface. In fact the momentum undergoes several sign reversals, until all the energy has left the structure. From Fig. 3 it should be evident that the energy velocity is equal to Bloch's velocity *only after the entire pulse has entered and remains inside the structure*, while in general the time-averaged energy velocity will be different.

With these considerations in mind, *we define the tunneling time* in a quasi monochromatic regime, consistent with our approach:

$$\tau_\omega=\frac{1}{c}\int_0^L\epsilon_\omega(z)|\Phi_\omega|^2dz-\frac{1}{\omega}\mathrm{Im}(r_\omega). \tag{6}$$

This definition of the tunneling time, derived by imposing boundary conditions on our finite structure and suggested by Eq. (3), is the electromagnetic analogue of Smith's ''dwell time'' [13], which addresses electron wave packet tunneling times through a potential barrier. According to Smith, a quantum particle spends a mean time proportional to $\int_0^L|\Psi(z)|^2dz$ in the region of space between 0 and L, which is just the probability of finding the particle within the same region of space. Following Bohm [14], we use the concept of electromagnetic energy density, instead of the quantum probability density, to define the tunneling time: Eq. (6) states that the time the field spends inside the structure is proportional to the energy density integrated over the volume. The term $-\mathrm{Im}(r_\omega)/\omega$ represents the difference in energy between the electric and magnetic components, and it has no counterpart in the quantum case. Equation (6) thus establishes a clear link between large delay times and field localization, as experimentally verified for pulse propagation near the band edge [3]. One may also define a group velocity associated with the delay of the transmitted pulse as $V_g^{(\omega)}\equiv L/\tau_\omega$. In the eyes of an observer, this definition of group velocity is an extremely useful and powerful concept. However, we remind the reader that we are not considering propagation in a uniform medium, where a true group velocity can be defined. Our system consists of a pulse whose spatial extent can be orders of magnitudes larger compared to the length of the structure, which is therefore entirely contained within the pulse [3]. As a consequence, the dynamics can only properly be described as a scattering event, with an associated tunneling time.

Once a convenient group velocity has been defined in the manner indicated, Eq. (3) can finally be recast in the following simple form:

$$V_E^{(\omega)}=|t_\omega|^2V_g^{(\omega)}. \tag{7}$$

Equation (7) is a strikingly simple result that makes it clear that for finite structures the tunneling velocity V_g and the energy velocity V_E are the same only at each transmission resonance, and can be very different from each other, espe-

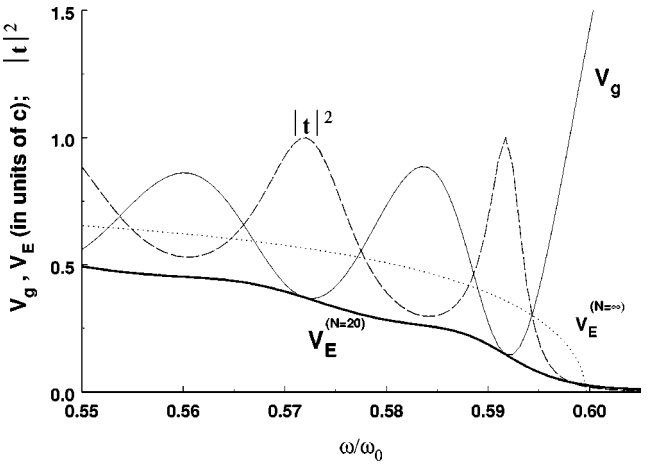

FIG. 4. $V_g^{(\omega)}$ (thin solid line), $V_E^{(\omega)}$ (thick solid line), and $|t|^2$ (dashes) for the 20-period structure described in the caption of Fig. (1). In the gap, the group velocity becomes superluminal. At resonance $V_g^{(\omega)}=V_E^{(\omega)}$, and $V_g^{(\omega)}$ is a minimum. The group velocity for the infinite structure is also depicted (dotted line) for comparison.

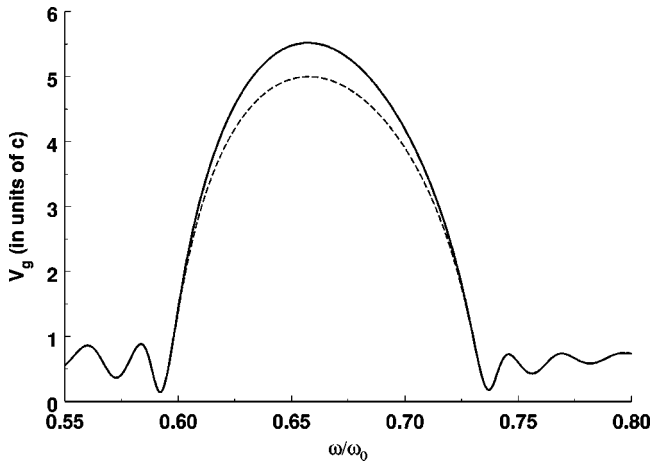

FIG. 5. $V_g^{(\omega)}$ as calculated using the definition of tunneling times given in Eq. (4) (solid line), and as calculated using the definition of phase time (dashed line) for the 20-period structure of Fig. 1.

cially at frequencies inside the gap. The implications of Eq. (7) are even more profound and far reaching if we consider superluminal tunneling behavior. We begin with the assertion that the energy velocity can never take on values greater than c, namely, $V_E^{(\omega)} \leqslant c$. This can be explicitly demonstrated for an arbitrary, nonabsorbing 1D potential barrier of finite length [15]. From Eq. (7) it immediately follows that the tunneling velocity satisfies $V_g^{(\omega)} \leqslant c/|t_\omega|^2$. That is, *the simple requirement that the energy velocity should be subluminal does not prevent superluminal tunneling times. In fact, this inequality places an unambiguous upper limit on the tunneling velocity that can be achieved without violating the requirement that the energy velocity remain subluminal*. Based on these simple considerations, statements regarding superluminal pulse propagation should always be qualified by the energy velocity and transmittance. In Fig. 4 we plot $V_g^{(\omega)}$ and $V_E^{(\omega)}$ versus frequency for the 20-period structure of Fig. 1. Inside the gap, the group velocity becomes superluminal, while the energy velocity always remains causal. In this case, minimum transmittance can be as low as one part in 10^5. We note that the maximum superluminal group velocity is approximately 5.5 times the speed of light in vacuum (see Fig. 5), far below the upper limit imposed by the condition that the energy velocity should remain subluminal, or $10^5 c$.

There is an alternative definition of the group velocity, as discussed in Ref. [16], i.e., $V_g = L/\tau_\varphi$. The associated tunneling time, also referred to in the literature as the ''phase time'' [11,14] is defined as $\tau_\varphi = d\varphi/d\omega$, where φ is the phase of the transmission function. In Fig. 5 we compare the tunneling velocity calculated using Eq. (6), and the tunneling velocity calculated using the phase time for the 20-period structure. While in the pass band the two methods yield similar results, our method gives slightly higher estimates ($\sim 10\%$) for the maximum superluminal velocity compared to the method of the phase time. This difference corresponds to a time delay of the order of 1 fs, which is small but measurable [2]. The integration of the equations of motion in the time domain yield results that are consistent with our predictions, namely, a group velocity of approximately $5.5c$ for pulses tuned inside the gap. We emphasize that although a numerical comparison shows a modest 10% quantitative change in the maximum superluminal velocity, it should be evident that the definition of tunneling time in our Eq. (6) appears more correct from a conceptual point of view because it establishes a clear nontrivial link between the energy and group velocity, as exemplified in our Eq. (7). This is link that cannot be established using the simpler phase tunneling time.

In summary, we have shown that there are nontrivial conceptual, qualitative, and quantitative differences between energy and group velocities in structures of finite length, as exemplified in our Eq. (7). These considerations have naturally led us to develop the concept of a tunneling time, which we call the electromagnetic analog of Smith's tunneling time, that can be useful in understanding the limits and meaning of what is referred to as superluminal pulse propagation under general conditions. The only requirements for the validity of our theory are that the scattering potential should be real, and the bandwidth of the incident pulses should be much narrower than a typical resonance bandwidth near the band edge; see, e.g., Ref. [3]. We note that the last requirement is also necessary for the definition of a phase time [13]. However, the tunneling time predicted by our Eq. (6) is formally and conceptually not the same as the phase tunneling time, by a measure that depends on the interplay between electric and magnetic components. These differences may be accentuated depending on the circumstances, i.e., structure length, frequency, and boundary conditions. A more challenging problem would be the extension of these results to multidimensional PBG structures.

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