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X-692-76-200

PREPRINT

NASA TM X- 71188

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(NASA-TM-X-71188) CONSEQUENCES OF USING
NONLINEAR PARTICLE TRAJECTORIES TO COMPUTE
SPATIAL DIFFUSION COEFFICIENTS (NASA) 33 p
HC \$4.00

N76-32087

CSCL 03B

G3/90

Unclas
03375

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SEPTEMBER 1976



— GODDARD SPACE FLIGHT CENTER —
GREENBELT, MARYLAND

CONSEQUENCES OF USING NONLINEAR PARTICLE TRAJECTORIES
TO COMPUTE SPATIAL DIFFUSION COEFFICIENTS

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ABSTRACT

A perturbed orbit, resonant scattering theory for pitch-angle diffusion in magnetostatic turbulence is slightly generalized and then utilized to compute the diffusion coefficient for spatial propagation parallel to the mean magnetic field, K_{\parallel} . All divergences inherent in the quasilinear formalism when the power spectrum of the fluctuation field falls off as k^{-q} ($q \geq 2$) are removed. Various methods of computing K_{\parallel} are compared and limits on the validity of the theory are discussed. For $1 < q < 2$ the various methods give roughly comparable values of K_{\parallel} , but use of perturbed orbits systematically results in a somewhat smaller K_{\parallel} than one obtains from quasilinear theory.

I. INTRODUCTION

The propagation of charged particles through interstellar and interplanetary space has often been described as a random process in which the particles are scattered by ambient electromagnetic turbulence. In general, this changes both the magnitude and direction of the particles' momentum. In this paper we confine our attention to those situations for which scattering in direction (pitch angle) is of primary interest. One can derive from this microscale phenomenon of pitch-angle scattering, a spatial diffusion coefficient, K_{\parallel} , which describes the macroscopic diffusion of the particle distribution. These diffusion coefficients have been useful for describing the solar modulation of the galactic cosmic radiation, in which the outward convection due to the solar wind is balanced by an inward diffusion [v. Jokipii, 1971 for a review]. In addition, particle observations during solar flares often show a diffusive phase in which the initial anisotropy of the particle distribution decays to isotropy. Again, knowledge of K_{\parallel} aids in calculating various characteristic times, such as the time to reach maximum flux, and the time scale for the decay of streaming anisotropies. Diffusion coefficients have customarily been computed from the quasilinear kinetic theory of pitch-angle scattering [v., e.g., Rowlands, Shapiro, and Shevchenko, 1966, Kenel and Engelmann, 1966; Jokipii, 1966, 1967, 1968, and references contained in Jokipii, 1971] along with knowledge of the power spectrum of the electromagnetic turbulence. When the turbulence is superimposed on a strong background magnetic field, as is the case in the interplanetary and interstellar media, the quasilinear theory contains some well-known difficulties if

the Larmor radius of the particles is less than the correlation length of the field [Klimas and Sandri, 1973; Jones, Birmingham and Kaiser, 1973; Kaiser, Jones, and Birmingham, 1973; and Völk, 1973]. In particular, if the magnitude of the magnetic field is constant to first order in the strength of the fluctuating field, then particle backscattering is not allowed. [For a recent discussion of this point, v. Klimas et al., 1976a.] Alternatively, when fluctuations in magnitude are present, the pitch-angle scattering coefficient, D_μ , is nearly zero near $\mu = 0$ (where μ is the cosine of the particle's pitch angle with respect to the mean field) but at $\mu = 0$ is either infinite, in the case of magnetostatic turbulence [Fisk et al., 1974 and Goldstein, Klimas and Sandri, 1975]; or, in the general electromagnetic situation, is highly peaked [Lee and Völk, 1975]. Formally one can derive expressions for K_\parallel in this situation [Jokipii, 1971], but interpreting the result in terms of a mean-free path for backscattering, which is implicitly done when discussing solar flare events, is suspect when scattering through $\mu = 0$ is strongly inhibited [Gal'perin, Toptygin, and Fradkin, 1971].

Many of the difficulties encountered in the quasilinear theory can be avoided if one modifies the trajectory followed by particles to include perturbations caused by the fluctuating fields. These perturbing forces become especially important for $\mu \sim 0$ because the duration of the wave-particle interaction is increased in that region of phase space. In recent years several nonlinear theories have been proposed that predict significant scattering through $\mu = 0$ [Jones et al., 1973; Völk, 1973; Goldstein, 1976]. [An alternative approach that also finds significant scattering through $\mu = 0$, while retaining unperturbed trajectories has

been developed by Klimas et al., 1976a, b.] In this paper we present a detailed comparison between the magnitude of K_{\parallel} computed from the perturbed orbit theory of Goldstein [1976] (Paper I) and that resulting from quasilinear theory. The discussion is restricted to the relatively simple situation of magnetostatic turbulence which is a function only of position along the mean field (the "slab" model).

Use of the perturbed trajectory can produce nearly isotropic scattering through $\mu = 0$. Then the time for backscattering through $\Delta\theta \equiv \Delta(\omega^{-1}\mu) \sim \pi$ is often about equal to the time for scattering through $\Delta\theta \sim 1$ and it becomes meaningful to interpret K_{\parallel} in terms of an approach of a particle distribution to isotropy. The value of K_{\parallel} derived from perturbed orbit theory tends to be slightly less than that computed from quasilinear orbits. In addition, for power spectra with spectral index $q \geq 2$, the perturbed orbit theory removes the divergence in K_{\parallel} that characterizes the quasilinear result when the distribution function is expanded in a perturbation expansion [v., e.g., Jokipii, 1966 and Hasselmann and Wibberenz, 1968 and 1970].

In §II we generalize the results of Paper I to include slab models with arbitrary spectral indices. We follow the basic approximation of that analysis in that we assume, to first order, that the turbulence is not modified by the particle distribution (i.e., a test particle model). In addition, the theory is restricted to the weak coupling limit in which it is assumed that the amplitude of the fluctuating field is sufficiently small so that interactions between various wave modes are negligible compared to wave-particle effects and that the particle orbits are not

grossly perturbed during a scattering time--the time necessary to propagate across several correlation lengths. In §III we evaluate K_{\parallel} for a variety of parameter values that are characteristic of the interplanetary medium and discuss the consequences of the results.

§II PITCH-ANGLE SCATTERING - A REVIEW

Ben-Israel et al. [1975] have recently developed a nonlinear kinetic theory of strong electromagnetic turbulence. In Paper I their work was adapted and modified to describe pitch-angle scattering in magnetostatic turbulence in a "slab" geometry. In order to provide a comparison with numerical experiments (Kaiser, 1975), the previous analysis was confined to exponential correlation functions to describe the statistical properties of the turbulence. In such a model, the dimensionless power spectrum is given by

$$R(k_{\parallel}) = (2\pi)^{-1/2} \epsilon (1 + \epsilon^2 k_{\parallel}^2)^{-1} \quad (1)$$

where, as in Paper I, $\epsilon = \lambda_c / r_g$ is the ratio of the correlation length of the turbulence to the Larmor radius. (Our notation follows Paper I, wherein all lengths are measured in Larmor radii and time in Larmor periods.)

In this section we first generalize the results of Paper I to include power spectra with arbitrary power law indices, q . This will enable us to compute spatial diffusion coefficients from power spectra with $7/5 \leq q \leq 2$, which is typical of interplanetary observations. For mathematical simplicity we generalize equation (1) to the following:

$$R(k_{\parallel}) = \frac{\Gamma(\nu + 1/2)}{\Gamma(\nu)} \frac{\epsilon}{\sqrt{2}} (1 + \epsilon^2 k_{\parallel}^2)^{-\nu - 1/2} \quad (2)$$

where $\Gamma(x)$ is the gamma function and $2\nu + 1 = q$. The normalization is chosen so that

$$\text{Tr } R(r=0) = 1 \quad (3)$$

where $\text{Tr } \underline{R}(\underline{r})$ denotes the trace of the correlation tensor of the field, defined by (I-23).* In the slab model, $\underline{R}(\underline{k}_{\parallel})$ is implicitly defined by

$$\underline{R}(\underline{k}) = N \delta(k_{\perp}) \underline{R}(k_{\parallel}) \quad (4)$$

where $\underline{R}(\underline{k})$ is the Fourier integral transform of $\underline{R}(\underline{r})$; and the notation follows (I-34) and (I-45). From (2) and (3) one has for the correlation tensor satisfying (2) [Erdélyi, 1954]

$$\underline{R}(\underline{r}) = \underline{R}(\lambda) = \frac{1}{\Gamma(\nu)} \left(\frac{\nu \lambda}{2\epsilon} \right)^{\nu} K_{\nu}(\nu \lambda / \epsilon) N \quad (5)$$

where λ is a dimensionless measure of length along the mean field direction, and where $K_{\nu}(x)$ is the modified Bessel function of the third kind.

The set of approximate equations that describe pitch-angle scattering in a slab model were derived in §IV of Paper I. From that analysis and equation (2) one immediately has for the pitch-angle diffusion coefficient

D_{μ} :

$$D_{\mu} \equiv \lim_{\tau \rightarrow \infty} D_{\mu}(\tau) \quad (6a)$$

$$D_{\mu}(\tau) = \frac{2\eta^2(1-\nu^2)}{\sqrt{2\pi}} \int_0^{\tau} d\lambda G(\lambda) [1 + \delta p_{\perp}(\lambda)] \quad (6b)$$

where

$$\delta p_{\perp}(\tau) = \frac{-2\eta^2}{\sqrt{2\pi}} \int_0^{\tau} d\lambda (\tau - \lambda) \cos \lambda (1 + \tan \lambda \tan \tau) [G(\lambda) - \nu H(\lambda)] \quad (7)$$

* Equation 23 of Paper I.

$$G(\tau) = \frac{\epsilon}{\sqrt{2}} \frac{\Gamma(\nu+1/2)}{\Gamma(\nu)} \cos \tau \int_0^{\infty} dk \frac{\cos[k(\nu\tau - \Delta x(\tau))]}{(1+\epsilon^2 k^2)^{\nu+1/2}} \exp[-k^2 D_1(\tau)/2] \quad (8)$$

$$H(\tau) = \frac{\epsilon}{\sqrt{2}} \frac{\Gamma(\nu+1/2)}{\Gamma(\nu)} \tau \cos \tau \int_0^{\infty} dk k \sin[k(\nu\tau - \Delta x(\tau))] \exp[-k^2 D_1(\tau)/2] \quad (9)$$

$$\Delta x(\tau) = \frac{\tau^2}{\sqrt{2\pi}} \int_0^{\tau} d\lambda (\tau-\lambda)^2 [2\nu G(\lambda) + (1-\nu^2) H(\lambda)] \quad (10)$$

$$D_1(\tau) = \frac{2\tau^2}{\sqrt{2\pi}} (1-\nu^2) \int_0^{\tau} d\lambda G(\lambda) \left(\frac{2}{3} \tau^3 - \tau^2 \lambda + \lambda^3/3 \right) \quad (11)$$

and η is the ratio of the root mean square of the fluctuation magnetic field, B' , to the mean field, B_0 ; i.e., $\eta \equiv \langle B' \cdot B' \rangle^{1/2} / |B_0|$.

For convenience we have dropped the subscript "||" on k and Δx .

For $\nu \neq \frac{1}{2}$ the integrals over k in (8) and (9) must be done numerically, but otherwise the system of equations can be solved exactly as before (cf. Appendix E of Paper I).

For comparison, the quasilinear pitch-angle diffusion coefficient is given by [Jokipii, 1971; Goldstein et al., 1975]

$$D_{\mu}^{\text{QL}} = \frac{\tau^2 \epsilon (1-\nu^2) \sqrt{\pi}}{\mu} \frac{\Gamma(\nu+1/2)}{\Gamma(\nu)} (1+\epsilon^2/\mu^2)^{-\nu-1/2} \quad (12)$$

[NOTE: The definition of D_{μ} used here and in Paper I differs slightly from that used in Goldstein et al., 1975.]

In Figure 1 we have plotted solutions of equation (6) for $\epsilon = 2$, $\eta = 0.1$, and $\nu = 1/3$. The results are qualitatively similar to those previously presented in Paper I. As before, substantial scattering through $\mu = 0$ is present.

§III SPATIAL DIFFUSION

Recently, Earl [1974] has demonstrated * t the correct expression for K_{\parallel} is (also see Hasselmann and Wibberenz, 1970)

$$K_{\parallel} = \frac{1}{2} \int_0^1 \psi^2 (1-\psi^2)^2 \left[\langle (\dot{\psi})^2 \rangle / \Delta \tau \right]^{-1} \quad (13)$$

where $\langle (\dot{\psi})^2 \rangle / \Delta \tau \equiv 2D_{\parallel}$.

In the past, K_{\parallel} has also been computed from the expression [Jokipii, 1966]

$$K_{\parallel} = \frac{2}{9} \left[\int_0^1 \psi \langle (\dot{\psi})^2 \rangle / \Delta \tau \right]^{-1} \quad (14)$$

which should be a good approximation to (13) only for nearly isotropic scattering.

For $q \geq 2$, equation (13) diverges while (14) remains finite.

Earl [1974] has previously noted that generalizations of quasilinear theory which allow scattering through $\mu = 0$, also remove the divergences in expression (13).

In the remainder of this section we discuss the results of evaluating (13) and (14) for various values of η , ϵ , ν ; and for both quasilinear and perturbed orbits. The general conclusion is that use of perturbed orbits (equations 6-11) results in approximately equal magnitudes of K_{\parallel} evaluated from either (13) or (14). This remains true even for $q = 2$. We distinguish the four possible approaches for computing K_{\parallel} by using a superscript QL or NL to denote quasilinear or nonlinear orbits, respectively; and a subscript P or L (perturbation of Legendre) to denote evaluation of K_{\parallel} using (13) or (14), respectively.

We confine our attention to values of ϵ between one and three and $\eta \leq 0.6$. At small ϵ (high energies) the slab model becomes unphysical, but in more realistic turbulence geometries the usual quasilinear treatment is adequate because the unperturbed trajectories are straight lines and consequently the difficulty for large ϵ of treating $\mu = 0$ disappears [Jokipii, 1966, 1967, 1968; Klimas and Sandri, 1971]. At low energies ($\epsilon \gg 1$) problems arise because in the interplanetary medium η can be 0.6 or larger. In this case equations (6) - (11) become inaccurate representations of the perturbed orbits for $\epsilon > 3$ and a complete derivation of the perturbed orbit equations has not been carried out for arbitrary values of μ . [See Paper I for a more detailed evaluation of nonlinear effects when $\mu = 0$.] In addition, we show in the Appendix that the weak-coupling approximation, which is essential to the derivation of (6) - (11), breaks down for $\eta \epsilon \geq 1$.

The dimensionless results can be easily transformed into more familiar dimensional units as follows: Let \tilde{K}_{\parallel} denote the dimensional diffusion coefficient, then

$$\tilde{K}_{\parallel} = K_{\parallel} \gamma^2 \omega_c = K_{\parallel} c^2 \beta^2 / \omega_c \equiv \alpha K_{\parallel} \quad (15)$$

where $\omega_c = eB_0/(\gamma mc)$, $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = v/c$ and $B_0 = \langle B \rangle$,

which we take to be 5×10^{-5} G in the interplanetary medium.

In Tables I and II we show K_{\parallel} for $\eta = 0.1$, $\epsilon = 1, 2$ and 3 ; and $v = \frac{1}{2}$ and $1/3$ ($q = 2$ and $5/3$). Use of perturbed orbits has completely removed the divergence in $K_{\parallel, P}^{QL}$ for $q = 2$ and yields $K_{\parallel, P}^{NL} \sim K_{\parallel, I}^{QL} \sim K_{\parallel, L}^{NL}$. For $q = 5/3$ the four forms of K_{\parallel} are all nearly equal. The success of

the perturbed orbit theory in removing the divergences present in quasilinear theory for steep power spectra ($q \geq 2$) is, of course, common to any approach that gives a non-zero value to D_μ at $\mu = 0$. The near equality of $K_{\parallel, P}^{NL}$ to $K_{\parallel, L}^{QL}$ arises because scattering through $\mu \approx 0$ is nearly isotropic, and either spherical harmonic or Legendre expansions of the distribution are expected to yield good approximations to K_\parallel .

In the interplanetary medium η is rarely as small as 0.1. However, it can be determined from power spectra computed from data collected from magnetometer experiments on space probes. We discuss below two examples of such spectra; the first from Jokipii and Coleman [1968] (Figure 2) and the second from Fisk and Sari [1973] (Figure 3). From Figure 2, $v \approx 0.2$ ($q \approx 7/5$) while from Figure 3, $v \sim 1/3$ ($q \approx 5/3$) and we assume, for illustrative purposes, that the spectra shown represent a slab model. $P(\tilde{f})$ is the power spectral density in γ^2/Hz ($1 \gamma = 10^{-5} \text{G}$) as a function of $\tilde{f} = V_{\text{sw}} \tilde{k}/2\pi$, where V_{sw} is the speed of the solar wind, taken to be 350 km/s. $P(\tilde{f})$ is related $R(k_\parallel)$ through the definition of $R(\lambda)$; viz.

$$\sqrt{2\pi} \gamma^2 B_0^2 \lambda_c R(k) = 2 \int_0^\infty d\lambda \cos k\lambda R(\lambda) \gamma^2 B_0^2 \lambda_c \quad (16)$$

so that

$$\gamma^2 B_0^2 / 2 = \int_{-\infty}^\infty d\tilde{f} \tilde{f} P(\tilde{f}) \quad (17)$$

where

$$P(\tilde{f}) \equiv P(\tilde{K}\lambda_c)/V_{sw} \quad (18)$$

Equation (17) also defines the total variance of the fluctuating field; which, for Figure 2 is $4.3\gamma^2$ ($\eta = .6$), (the Fisk and Sari spectrum is discussed below). For the spectrum in Figure 2, $\lambda_c \approx 2 \times 10^{11}$ cm. The resulting values of K_{\parallel} are shown in Table III, and again use of perturbed orbits does not significantly modify K_{\parallel} .

Fisk and Sari [1973] have argued that low-energy particles (≤ 100 MeV/nucleon) do not interact with tangential discontinuities that are included in computing the power spectrum shown in Figure 2. The authors then go on to remove tangential discontinuities from their data set and compute the spectra labeled P_{BETWEEN} in Figure 2. In this case $\eta \approx 0.3$ (Sari, private communication), and $\lambda_c \approx 2 \times 10^{10}$ cm. The resulting values of K_{\parallel} are shown in Table IV.

The values of K_{\parallel} in Table IV (and to some extent, also those in Table III) are consistently smaller than those deduced from the decay portion of solar flare events [v., e.g., Webb et al., 1973; and Countee and Lanzerotti, 1976]. Countee and Lanzerotti [1976] conclude that an empirical fit to the particle data gives $\tilde{K}_{\parallel} \approx 2 \times 10^{21}$ cm²/s (at proton energies of ~ 7 MeV), a factor of ~ 7 larger than shown in Table IV. Webb et al. [1973] also found that the scattering mean free path for various solar particle events was smaller than could be understood theoretically from numerical solutions of the Fokker-Planck equation which describes spatial transport. The resolution of this discrepancy is really not well understood, though there are several possibilities, some

of which we mention below: First, we have restricted the turbulence model to one in which all the k -vectors are parallel to B_0 , which tends to underestimate K_{\parallel} . Volk [1973] has argued that the radial expansion of the solar wind causes the wave vectors to refract into the radial direction, thus increasing K_{\parallel} . Recently, Morfill [1975] and Morfill et al. [1976] have investigated this possibility in more detail, with the conclusion that one can indeed increase K_{\parallel} by as much as a factor of 10 if the wave vectors of interplanetary Alfvén waves are predominately in the radial direction. Their analysis includes a discussion of effects due to an admixture of compressive modes. Unfortunately, there is little definitive information about the direction of propagation of interplanetary magnetohydrodynamic (MHD) waves and no evidence has been presented that the wave k -vectors are in the radial direction. Second, in order to fully understand interplanetary propagation it is undoubtedly necessary to have a fairly detailed knowledge of the nature of the magnetic turbulence before, during and after a solar flare event. Such a detailed study is not generally made, and consequently K_{\parallel} is not often evaluated for the actual turbulence at the time of a flare. There are, however, several analyses which have related K_{\parallel} to the magnetic power spectrum at the time of the particle events (v., e.g., Lanzerotti et al., 1973; Webb et al., 1973; and Wibberenz et al., 1973, 1976). A general conclusion of these studies is that K_{\parallel} evaluated from the weak-coupling theory is systematically smaller than implied by the observed time behavior of the particle fluxes. Third, one does not know the heliocentric dependence of K_{\parallel} . If (Sari, 1973) K_{\parallel} increases rapidly toward the Sun, then large mean free paths inside of 1 A.U. could explain the flare

observations at 1 A.U.; however, large values of N_{\parallel} are also needed to understand the small gradient in cosmic ray intensity that is observed on Pioneers 10 and 11, so again one cannot completely remove the discrepancies in this way. One could also argue that if $\lambda_c \approx 2 \times 10^{11}$ cm, then 10 MeV protons correspond to $\epsilon \approx 20$. Then, with $\eta \approx 0.6$ one has seriously violated the weak-coupling approximation; and, as there is no extant kinetic theory valid for strong coupling in strong turbulence, one might expect that a more correct theory, when it appears, would improve the situation. Even this explanation is seriously weakened, however, if Fisk and Sari [1973] are correct that low-energy particles respond to P_{BETWEEN} rather than P_{REAL} . For then, with $\eta \approx 0.3$ and $\epsilon \approx 2$ (12 MeV protons) the weak-coupling approximation is not badly violated; but from Table IV, one still finds N_{\parallel} to be too small. Yet another approximation that is always made in deriving D_{μ} is that the particle distribution behaves as a collection of test particles. While this is an excellent approximation at moderate energies it may well breakdown at the low energies (≤ 10 MeV for protons) discussed by Countee and Lanzerotti [1976]. A crude estimate of the energy density of the particles indicates that at times it equals the energy density in the fluctuation fields; a situation which in principle necessitates a full self-consistent treatment of the Vlasov-Maxwell equations for a complete theoretical treatment. [Such a theoretical framework is available only in the weak-coupling regime (Ben-Israel et al., 1975).] Finally, Nolte and Roelof [1975] have developed a mathematical treatment which examines the consequence of assuming "scatter-free" propagation at the onset.

§V CONCLUSION

We have computed the spatial diffusion coefficient K_{\parallel} using both perturbed and quasilinear orbits within the context of the weak-coupling approximation. Use of the perturbed trajectory removes divergences found in quasilinear calculations for steep power spectral indices ($q \geq 2$) when equation (13) is used to compute K_{\parallel} . For less steep spectra, $1 < q < 2$, K_{\parallel}^{NL} tends to be slightly less than K_{\parallel}^{QL} due to enhanced pitch-angle diffusion through $\mu = 0$. If the correlation length of the interplanetary turbulence is $\lambda_c \approx 2 \times 10^{11}$ cm, then for $\epsilon \leq 3$ (proton energies ≥ 400 Mev) one obtains values generally consistent with observations. At lower energies ($\epsilon \gg 3$), the weak-coupling approximation is violated and one may require a kinetic theory valid for strong coupling.

However, if Fisk and Sari [1973] are correct in their assertion that low-energy particles (≈ 30 Mev protons) traverse a stochastic field characterized by $\lambda_c \approx 2 \times 10^{10}$ cm, then one should, in principle, be able to use the present theoretical formalism to determine the spatial diffusion coefficient for ~ 30 Mev protons. The results of such a computation, using either perturbed or quasilinear orbits, yields spatial diffusion coefficients that are consistently too small by factors of 5-10. [Unless there is significant focusing of the wave k -vectors into the radial direction.] However, it is possible that for this low-energy particle population the test particle approximation has broken down. Correction of this breakdown would require significant modification to the theoretical formation that is now used, even within the context of the weak-coupling approximation.

ACKNOWLEDGMENTS

I would like to thank Drs. L. A. Fisk, A. J. Klimas, J. Sari, and G. Wibberenz for many informative and stimulating conversations.

κ_{\parallel} FOR 0.4 TO 2 GeV PROTONS ($q = 2$ and $\eta = 0.1$)

TABLE I

Proton Energy (GeV)	$\eta = 0.1$	$v = \frac{1}{2} (q = 2)$	$\lambda_c = 2 \times 10^{11} \text{ cm}$			
	ϵ	$\kappa_{\parallel, P}^{NL}$	$\kappa_{\parallel, P}^{QL}$	$\kappa_{\parallel, L}^{NL}$	$\kappa_{\parallel, L}^{QL}$	$\alpha (10^{21} \text{ cm}^2 \text{ s}^{-1})$
2	1	165	∞	114	115	5.9
1	2	219	∞	175	192	2.6
0.4	3	265	∞	230	276	1.3

κ_{\parallel} FOR 0.4 TO 2 GeV PROTONS ($q = 5/3$ and $\eta = 0.1$)

TABLE II

Proton Energy (GeV)	$\eta = 0.1$		$v = 1/3$ ($q = 5/3$)		$\lambda_c = 2 \times 10^{11}$ cm	
	ϵ	$\kappa_{\parallel, P}^{NL}$	$\kappa_{\parallel, P}^{QL}$	$\kappa_{\parallel, L}^{NL}$	$\kappa_{\parallel, L}^{QL}$	α (10^{21} cm 2 s $^{-1}$)
2	1	128	103	110	110	5.9
1	2	179	151	152	153	2.6
0.4	3	220	245	192	194	1.3

κ_{\parallel} FOR 0.4 TO 2GeV PROTONS ($q = 7/5$ and $\eta = 0.6$)
Based on the power spectrum shown in Figure 2

TABLE III

Proton Energy (GeV)	ϵ	$\nu = 1/5$ ($q = 7/5$)		$\lambda_c = 2 \times 10^{11}$ cm	
		$\kappa_{\parallel, P}^{NL}$	$\kappa_{\parallel, P}^{QL}$	$\kappa_{\parallel, L}^{NL}$	$\kappa_{\parallel, L}^{QL}$
2	1	3.4	3.0	3.1	3.4
					5.9
1	2	3.2	3.7	3.0	4.0
					2.6
0.4	3	3.0	4.2	2.7	4.6
					1.3

α (10^{21} cm² s⁻¹)

κ_{\parallel} FOR 6-50 MeV PROTONS ($q = 5/3$ and $\eta = 0.3$)
Based on the power spectrum, P_{BETWEEN} , shown in Figure 3

TABLE IV

Proton Energy (MeV)	$\eta = 0.3$		$\nu = 1/3$ ($q = 5/3$)		$\lambda_c = 2 \times 10^{10} \text{ cm}$	
	ϵ	$\kappa_{\parallel, P}^{NL}$	$\kappa_{\parallel, P}^{QL}$	$\kappa_{\parallel, L}^{NL}$	$\kappa_{\parallel, L}^{QL}$	α ($10^{20} \text{ cm}^2 \text{ s}^{-1}$)
50	1	11.3	11.5	11.0	12.2	2.0
12	2	13.0	16.8	12.7	17.0	0.45
6	3	13.5	21.7	13.1	12.6	0.23

FIGURE CAPTIONS

- Figure 1. The dimensionless pitch-angle diffusion coefficient, ϵD_{μ} , is plotted as a function of μ . The solid line is the result of using equations (6) - (11) (nonlinear orbits), while the dashed line results from use of equation (12), employing unperturbed orbits (quasilinear approximation). The spectral index is $q = 5/3$, and η and ϵ are 0.1 and 2.0, respectively.
- Figure 2. Power spectrum of the component of the interplanetary magnetic field normal to the solar equatorial plane, observed on Mariner 4. A fit to these data with an analytic function of the form of equation (2) yields $\eta \approx 0.6$ and $\lambda_c \approx 2 \times 10^{11}$ cm (after Jokipii and Coleman, 1968).
- Figure 3. Representative spectra for the total observed power density (P_{REAL}), the power density due to directional discontinuities (P_{DIS}), and the power density due to field fluctuations between the discontinuities (P_{BETWEEN}). A fit to these data with an analytic function of the form of equation (2) yields, for P_{BETWEEN} , $\eta \approx 0.3$ and $\lambda_c \approx 2 \times 10^{10}$ cm (after Fisk and Sari, 1973).

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APPENDIX

The weak-coupling approximation, as used here, is essentially the claim that (Appendix B of Ben-Israel et al. [1975] or I-13)

$$(1-A)U(\tau) \cong U_A(\tau)(1-A) \quad (A-1)$$

This is true whenever

$$\eta \int_0^\tau d\lambda U_A(\tau-\lambda)(1-A) \mathcal{L}' A U(\lambda) \ll 1 \quad (A-2)$$

All quantities in the integrand are $O(1)$, and the integral converges in after a time $\tau \geq 1$ or $\tau \geq \epsilon$ (i.e., after a cyclotron period or a correlation time, which are the two characteristic time scales of the problem. Therefore, (A-2) is true if $\eta \ll 1$ and $\eta \epsilon \ll 1$. (This is identical to the conditions of Ben-Israel et al. [1975] that $\alpha_B \ll 1$ and $\alpha_E \ll 1$.) For low energies ($\epsilon \gg 1$) one expects (A-2) to saturate after many Larmor periods ($\tau > \epsilon$) and therefore one needs $\eta \epsilon \ll 1$ for (A-1) to hold.

Similarly, the perturbed trajectories are likely to be a good approximation if (cf. equation B3 of Ben-Israel et al. [1975] or I-16.

$$U(\tau) \cong \bar{U}(\tau) \quad (A-3)$$

with

$$\eta \int_0^\tau d\lambda U(\tau-\lambda) [\mathcal{L}' \bar{U}(\lambda) - \langle \mathcal{L}' U(\lambda) \rangle] \ll 1 \quad (A-4)$$

Iteration leads to

$$\eta \int_0^1 \bar{u}(x-1) \mathcal{L}' \bar{u}(1) dx \ll 1$$

(A-5)

which is true if $\eta \ll 1$ and $\eta_\epsilon \ll 1$, as above.





