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The evolution of slab fluctuations in the presence of pressure-balanced magnetic structures and velocity shears

S. Ghosh¹

Applied Research Division, Space Applications Corporation, Largo, Maryland

W. H. Matthaeus

Bartol Research Institute, University of Delaware, Newark

D. A. Roberts and M. L. Goldstein

Laboratory for Extraterrestrial Physics, NASA Goddard Space Flight Center, Greenbelt, Maryland

Abstract. The traditional view that solar wind fluctuations are well-described as a spectrum of parallel-propagating Alfvén waves has been challenged many times but is still a frequently encountered perspective. Here we examine whether it remains consistent to view most of the fluctuation energy as resident in parallel-propagating Alfvén waves in situations in which there are also present either transverse pressure-balanced (PB) magnetic structures or transverse velocity shears. We address these questions through direct simulation of compressible magnetohydrodynamics, with expansion effects neglected. We show that parallel-propagating Alfvén waves are redirected to large oblique angles after refractive interactions with PB structures or advective interactions with velocity shears, reflecting the nonequilibrium nature of the initial spectral distribution. The timescale for these processes ranges from 2–8 eddy-turnover times or characteristic nonlinear times. Relatively small amounts of PB structure and/or shear energy can redirect initially parallel-propagating Alfvén waves to highly oblique angles. Velocity microstreams appear to be particularly efficient at creating highly oblique waves. Even though the excited wave vectors are eventually primarily oblique, the magnetic variance ratios show a minimum variance in the mean magnetic field direction.

1. Introduction

The nature of the spectrum of solar wind MHD scale fluctuations remains a topic of current interest in heliospheric research. Both the “geometry” of the fluctuations and the associated dynamical model have major implications in a variety of heliospheric applications, including theories of turbulent heating [Coleman, 1968; Tu, 1988; Hossain *et al.*, 1995], spatial transport of fluctuation energy [Marsch and Tu, 1993; Oughton, 1993; Matthaeus *et al.*, 1994; Oughton and Matthaeus, 1995], and cosmic ray scattering [Bieber *et al.*, 1996]. By the geometry of fluctuations we refer to the spectral distribution of energy over the direction of three-dimensional

(3-D) wave vectors. As dynamical models, we have in mind as limiting cases, on the one hand, propagating linear wave theory and, at the opposite extreme, fully developed turbulence theory of a hydrodynamic or “Kolmogoroff” nature. The contrast between a “wave” model and a pure “turbulence” model can be viewed in very stark terms when one applies a model that is purely of one kind or the other to a plasma such as the solar wind. Observations, for example, readily rule out either one-dimensional (1-D) slab wave models [Sari and Valley, 1976; Klein *et al.*, 1991; Bieber *et al.*, 1996] as well as traditional isotropic hydrodynamic turbulence models [Belcher and Davis, 1971; Matthaeus *et al.*, 1990; Bieber *et al.*, 1996]. Perhaps it is for this reason that recently a number of models have become popular in which the geometry and to some extent the dynamical model are construed to be a composite or superposition of models of the wave and turbulence types.

One simple possibility for a composite model is a two component model in which the fluctuations are represented as a superposition of parallel propagating (slab)

¹Also at Laboratory for Extraterrestrial Physics, NASA Goddard Space Flight Center, Greenbelt, Maryland.

fluctuations and an admixture of two-dimensional (2-D) fluctuations having wave vectors (\mathbf{k}) mainly perpendicular to the local mean magnetic field \mathbf{B}_0 . Here “2-D” refers to the plane orthogonal to \mathbf{B}_0 . Such models, while in large part suggested by the observation of “Maltese Cross” axisymmetric fluctuations [Matthaeus *et al.*, 1990], can actually be seen to consist of two rather distinct types.

In the first case, which is the focus of the present paper, the Fourier components of the 2-D fluctuation vectors for the magnetic field ($\delta\mathbf{B}(\mathbf{k})$) or the velocity field ($\delta\mathbf{u}(\mathbf{k})$) are themselves parallel to the local \mathbf{B}_0 . One can readily see that magnetic fluctuations of this type are, in the absence of other spectral ingredients, in a state of static pressure balance. Solar wind pressure-balanced structures have been discussed in numerous studies [e.g., Burlaga and Ogilvie, 1970; Goldstein and Siscoe, 1972; Bruno and Bavassano, 1991]. Similarly, velocity fluctuations of this type form velocity shears and velocity microstreams. Solar wind velocity shears have also been discussed in numerous studies [e.g., Roberts *et al.*, 1992; Neugebauer *et al.*, 1995; Bavassano *et al.*, 1998]. In this paper we focus on the dynamical interactions between a spectrum of field-aligned Alfvén waves and either compressive 2-D pressure-balanced (PB) structures or transverse 2-D velocity-shear (VS) structures. Of particular interest are the dynamical consequences on slab fluctuations in this type of two-component model as the fluctuation energy of the PB or VS structures is varied.

A second and equally interesting possibility, which is deferred to a subsequent paper [Ghosh *et al.*, this issue] (hereinafter paper 2), is a model that comprises a spectrum of field-aligned Alfvén waves and an admixture of 2-D fluctuations having both wave vectors and fluctuation vectors perpendicular to the mean field. Such 2-D fluctuations emerge naturally from the theory of nearly incompressible MHD [Zank and Matthaeus, 1993]. They tend to be dynamically active [Ting *et al.*, 1986], although static, constant $|\delta\mathbf{B}|$ solutions called Magnetic Field Directional Turnings (MFDTs) [Tu and Marsch, 1995] can also be found.

The two different characterizations of the 2-D fluctuations are drawn schematically in Figure 1. Figure 1a depicts PB structures with \mathbf{k} orthogonal to \mathbf{B}_0 , but $\delta\mathbf{B}$ is parallel to \mathbf{B}_0 , and Figure 1b depicts 2-D fluctuations with both \mathbf{k} and $\delta\mathbf{B}$ orthogonal to the mean magnetic field.

Here we consider a compressible MHD medium evolving from a specified initial state without external driving in the presence of a locally uniform constant magnetic field \mathbf{B}_0 , with respect to which directions may be referred to as either parallel (\parallel) or perpendicular (\perp). We use a Cartesian geometry. Initial fluctuations are of two types: (1) field-aligned Alfvén waves, having, for example, $\delta\mathbf{B}_\perp(\mathbf{k}_\parallel) = \pm\delta\mathbf{u}_\perp(\mathbf{k}_\parallel)\sqrt{4\pi\rho}$ for Fourier wave vector \mathbf{k} and mass density ρ , along with PB structures, $\delta\mathbf{B}_\parallel(\mathbf{k}_\perp)$, or (2) field-aligned Alfvén waves, as in num-

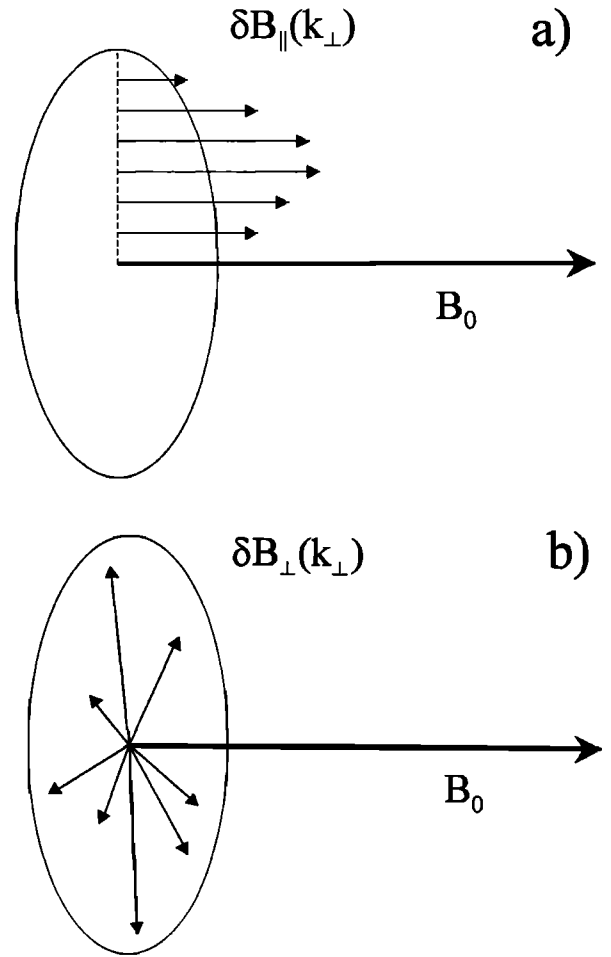


Figure 1. The two different characterizations of component 2: (a) transverse PB structures with \mathbf{k} orthogonal to \mathbf{B}_0 , but $\delta\mathbf{B}$ is parallel to \mathbf{B}_0 and (b) quasi 2-D fluctuations with both \mathbf{k} and $\delta\mathbf{B}$ orthogonal to the mean magnetic field.

ber 1, along with VS structures, $\delta\mathbf{u}_\parallel(\mathbf{k}_\perp)$. These cases are similar because the incompressible magnetic field ($\mathbf{k} \cdot \delta\mathbf{B}(\mathbf{k}) = 0$) is simply replaced by an incompressible velocity flow field ($\mathbf{k} \cdot \delta\mathbf{u}(\mathbf{k}) = 0$), while the orientation and magnitudes of the vector components remain the same. Above and throughout the paper we will sometimes employ $\delta\mathbf{B}$ and $\delta\mathbf{u}$ notation with \perp or \parallel subscripts to emphasize the character of the fluctuations. The principal result of the present study is that dynamical interaction of either PB structures or VS structures with a spectrum of field-aligned Alfvén wave results in a spectrum of Alfvénic turbulence with highly oblique wave vector directions and minimum variances parallel to the mean magnetic field.

These latter features are consistent with solar wind observations. Some of the basic physics of our conclusions were anticipated by Daryl [1973], who discussed the possibility of generation of oblique fluctuations by refraction of waves. However, from the present perspective those conclusions require clarification because they were based in part on minimum variance analysis to in-

fer wave vector directions. It is well-known that the signatures of high Alfvénicity and field aligned minimum variance are not sufficient to determine the direction of propagation [e.g., *Barnes*, 1981]. In particular, minimum variance direction cannot be used to distinguish between parallel propagating waves and purely 2-D fluctuations [*Matthaeus et al.*, 1996]. It is also possible that oblique wave vectors other than strictly 2-D may be excited. For example, recent developments on the theory of kinetic Alfvén waves [*Wong et al.*, 1997; *Leamon et al.*, 1998] suggest the presence of highly oblique Alfvén waves in the solar wind.

Another study also considered the nonlinear evolution of interplanetary Alfvén fluctuation with PB structures [*Roberts et al.*, 1996] and noted that features of the initial two component representation were lost due to refraction. However, the focus there was the lack of low cross helicity from such interactions. We will compare the present results with this earlier work, in view of the similarity of the initial data. Simulation studies of the development of turbulence from velocity shears [*Roberts et al.*, 1991; *Roberts et al.*, 1992] are also antecedents of the present work. However, unlike earlier studies, the present emphasis is a characterization of the dynamical instability of a spectrum of slab Alfvén waves. When PB or VS structures are added to slab Alfvén waves, a nonequilibrium initial state is produced, and its evolution, presumably much faster than underlying one-dimensional instability processes, rapidly changes the assumed distribution of energy in wave number space. In this sense the assumed “slab plus structures” two-component model is highly unstable.

The organization of this paper is as follows: Our numerical method and the different initial conditions for this study are discussed in section 2. The simulation results are covered in section 3. The results are discussed and the conclusions are given in section 4.

2. Numerical Method and Initial Conditions

2.1. Algorithm

Our simulations are based on a standard dimensionless representation of the compressible MHD system comprising the equations of continuity, momentum, and magnetic induction:

$$\frac{\partial}{\partial t} \rho = -\nabla \cdot (\rho \mathbf{u}), \quad (1)$$

$$\begin{aligned} \frac{\partial}{\partial t} \mathbf{u} = & -\mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{\rho} \nabla P + \frac{\mathbf{J} \times \mathbf{B}}{M_{a0}^2 \rho} \\ & + \frac{1}{\rho} \nu \nabla^2 \mathbf{u} + \frac{1}{\rho} \left(\zeta + \frac{1}{3} \nu \right) \nabla (\nabla \cdot \mathbf{u}), \end{aligned} \quad (2)$$

$$\frac{\partial}{\partial t} \mathbf{A} = -\mathbf{E} + \nabla F. \quad (3)$$

Here the fluid velocity is \mathbf{u} . The current \mathbf{J} is related to the magnetic field \mathbf{B} and the vector potential \mathbf{A} through $\mathbf{J} = \nabla \times \mathbf{B} = \nabla \times \nabla \times \mathbf{A}$. This system is closed by using an Ohm's law, $\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \mu \mathbf{J}$, and a polytropic relation between pressure and density, $P = \rho^\gamma / (\gamma M_{s0}^2)$, where $\gamma = 5/3$ in this paper. The function F is chosen to preserve the Coulomb gauge condition, $\nabla \cdot \mathbf{A} = 0$ [see *Ghosh et al.*, 1993].

We use a pseudospectral algorithm and a Cartesian geometry [see, e.g., *Ghosh et al.*, 1993]. Our notation is such that unless specified with a δ prefix or 0 subscript, all vectors represent the full quantity: constant plus fluctuations. Hence the magnetic field is $\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}$, with $\mathbf{B}_0 = B_0 \hat{x}$. Note that $\mathbf{u}_0 = 0$ since there is no mean flow, and so $\mathbf{u} = \delta \mathbf{u}$. Although $\mathbf{u}(\mathbf{k})$ obviously has spatial fluctuations for nonzero \mathbf{k} , we retain usage of $\delta \mathbf{u}$ to emphasize that we are discussing a fluctuating quantity. The two normalization coefficients are M_{s0} , representing a characteristic sonic Mach number, and M_{a0} , representing a characteristic Alfvénic Mach number. We set $M_{a0} = 1$ to run the simulations in Alfvén speed units. The average sonic Mach number is $M_s = M_{s0} u_{rms} / \rho_{rms}^{(\gamma-1)/2}$, where the rms subscript represents a (spatial) root-mean-square such that $u_{rms} = [\langle |\mathbf{u}|^2 \rangle_x]^{1/2}$ and $\langle \dots \rangle_x$ denotes a spatial (volume) average. The plasma β is defined as the ratio of squared sound speed to the squared Alfvén speed: $\beta = c_s^2 / v_a^2 = 1 / (M_{s0} B_0)^2$, where $c_s^2 = \rho_{rms}^{(\gamma-1)} / M_{s0}^2$ and $v_a^2 = B_0^2 / (M_{a0}^2 \rho_{rms})$. Several runs in this paper have $B_0 = 4$, $M_{s0} = 1/4$, and $\beta = 1$.

The last two terms in (2) represent viscous dissipation, involving viscosity coefficients ζ and ν . We let $\zeta = 0$ and adopt a simple bi-Laplacian form for the viscosity ν and the resistivity μ , which can be expressed in the transform space as $\nu_k = \nu_0 [1 + (k/k_{eq})^2]$ and $\mu_k = \mu_0 [1 + (k/k_{eq})^2]$. For the cases studied here we choose $k_{eq} = 1$, with ν_0 and μ_0 ranging between $2 \times 10^{-5} \leq \nu_0, \mu_0 \leq 5 \times 10^{-6}$ depending on the simulation's resolution. This choice is motivated purely on computational grounds and keeps the dissipation relatively low over a broad range of the spectrum while maintaining enough damping at the high wave numbers to minimize aliasing errors. (See, e.g., *Borue and Orszag* [1995] or *Siregar et al.* [1995] for discussions on the usage of nonstandard dissipation operators.)

Quantities of interest in descriptions of both waves and turbulence include the fluctuating cross helicity $\langle \delta \mathbf{u} \cdot \delta \mathbf{B} \rangle_x$ and the fluctuating magnetic helicity $\langle \delta \mathbf{A} \cdot \delta \mathbf{B} \rangle_x$ and their associated spectra. Cross helicity and, in particular, its normalized value σ_c provide a measure of “Alfvénicity,” while the normalized magnetic helicity σ_m is a measure of handedness or polarization [see, e.g., *Matthaeus and Goldstein*, 1982].

The simulation time T is in units of the transit time over unit distance of a characteristic Alfvén wave in the simulation box. The normalized length of the box is $L_0 = 2\pi$. Hence the transit time of a characteristic ($B_0 = 1$) Alfvén wave across the box is $T = L_0 / V_0 =$

2π . We will also discuss time intervals in terms of the characteristic nonlinear time: the eddy turnover time unit, $T_E = T/u_{\text{rms}}$.

We consider both three-dimensional (3-D) and two and one-half dimensional ($2\frac{1}{2}$ -D) geometries. In the 3-D case the mean field lies in the \hat{x} direction and there are two directions, y and z , perpendicular to the mean field. In the $2\frac{1}{2}$ -D geometry, fluctuations depend only upon (x, y) , while the mean field is also in the \hat{x} direction. Thus, in the $2\frac{1}{2}$ -D code, there is only one “perpendicular” coordinate, while components of vectors such as $\delta\mathbf{u}$ and $\delta\mathbf{B}$ may lie in all three spatial directions. Note that the PB structures we consider, for example $\delta\mathbf{B}_{\parallel}(\mathbf{k}_{\perp})$ have a 1-D spatial dependence in our $2\frac{1}{2}$ -D geometry, whereas they vary in two directions in the 3-D geometry.

The $2\frac{1}{2}$ -D model allows higher resolution than does a 3-D model but neglects the solenoidal nonlinear couplings in the plane perpendicular to \mathbf{B}_0 . The $2\frac{1}{2}$ -D model uses 256×256 modes, which permit wave couplings out to the maximum wave number, $|k_{i,\text{max}}| = 128$ for $i = (x, y)$. Dissipation effects are negligible for scales $|\mathbf{k}| < 35$. The 3-D model uses $64 \times 64 \times 64$ modes with a maximum wave number $|k_{i,\text{max}}| = 32$ for $i = (x, y, z)$. Here dissipation effects are negligible for scales $|\mathbf{k}| < 15$.

2.2. Initial Conditions

We consider initial conditions (ics) with magnetic PB structures or VS superposed with field-aligned Alfvén waves (slab modes).

The PB ics are similar to the ones used by *Roberts et al.* [1996], where a magnetic structure is formed from the superposition of wave vectors $(0, k_y, 0)$ and $(0, 0, k_z)$ of the δB_x component. The mean magnetic field is $\mathbf{B}_0 = B_0\hat{x}$. If $k_y = k_z = 1$, then four flux tubes are created in the quadrants of the y - z plane. These structures are pressure balanced by a nonpropagating density variation ρ_{PB} where $\nabla[\rho_{\text{PB}}/(\gamma M_{s0}^2) + \mathbf{B} \cdot \mathbf{B}/2] = 0$. Roberts et al. superposed three arbitrarily phased field-aligned Alfvén waves with $k_x = 1, 2, 3$ to these PB structures. The fluctuation amplitudes of these slab modes are equal. The total energy in slab-like fluctuations is $\sum_{k_x} (\delta\mathbf{u} \cdot \delta\mathbf{u} + \delta\mathbf{B} \cdot \delta\mathbf{B})/2 = E_v^{\text{slab}} + E_b^{\text{slab}} = E^{\text{slab}}$, where $E_v^{\text{slab}} = E_b^{\text{slab}}$, and the total energy of the structures is $\sum_{k_y, k_z} (\delta\mathbf{B} \cdot \delta\mathbf{B})/2 = E_b^{\text{PB}}$. Roberts et al. choose ics with $E_b^{\text{slab}} = 3/4$ and $E_b^{\text{PB}} = 1/4$.

In this paper we allow a finite bandwidth for the PB structures ranging up to $1 \leq k_y, k_z \leq 4$. The phases are randomly selected at each wave number, and the energy is equipartitioned between all the modes. The bandwidth of the slab modes extends to $1 \leq k_x \leq 4$. We also vary the fluctuation energy content between the PB structures and slab modes so that $0.01 \leq E_b^{\text{PB}}/E_b^{\text{slab}} \leq 0.125$. We shall describe the ics consisting of PB structures with slab fluctuations as PB+S.

The VS simulations are constructed in a similar fashion. A velocity shear is introduced from the super-

position of wave vectors $(0, k_y, 0)$ and $(0, 0, k_z)$ of the δu_x component. The VS bandwidth is $1 \leq k_y, k_z \leq k_{\text{max}}$, where in various runs k_{max} ranges from 1 to 8. The phases are randomly selected at each wave number, while the energy is equipartitioned among all the excited modes. The total energy of the VS is $\sum_{k_y, k_z} (\delta\mathbf{u}^{\text{VS}} \cdot \delta\mathbf{u}^{\text{VS}})/2 = E_v^{\text{VS}}$. Next, we add four arbitrarily phased slab modes between $1 \leq k_x \leq 4$ to the VS. Note that the VS bandwidth may exceed the slab bandwidth. This extends the VS plus slab ics to velocity microstreams. As a loose definition, we consider velocity microstreams to be any VS fluctuation whose amplitude and correlation length are the same order of magnitude or smaller than the typical Alfvénic fluctuation in a high-speed, highly Alfvénic solar wind stream. Velocity microstreams have been identified in the solar polar regions from Ulysses data by *Neugebauer et al.* [1995]. We consider ordinary velocity shears to have larger amplitudes and/or correlation lengths than the Alfvénic modes.

Similar to the $E_b^{\text{PB}}/E_b^{\text{slab}}$ ratio range, we vary the fluctuation energy content between the VS and slab modes so that $0.01 \leq E_v^{\text{VS}}/E_v^{\text{slab}} \leq 0.125$. We have modeled only cases of field-aligned stream shear ($\delta\mathbf{u}^{\text{VS}}$ parallel to \mathbf{B}_0). Our focus in this paper is the early dynamics of slab modes, corresponding to evolution out to 1–3 AU in heliocentric distance depending on the solar wind speed. The cross-field stream shear geometry, which is of interest in the outer heliosphere, is deferred to a future paper. We shall describe the ics consisting of VS with slab modes as VS+S.

We have also conducted simulations of isotropic turbulence (IT) for comparison purposes. Here the slab modes described above are replaced with an isotropic band $1 \leq |\mathbf{k}| \leq 4$ of randomly phased fluctuations. The total turbulence energy is $\sum_{\mathbf{k}} (\delta\mathbf{u}^{\text{Turb}} \cdot \delta\mathbf{u}^{\text{Turb}} + \delta\mathbf{B}^{\text{Turb}} \cdot \delta\mathbf{B}^{\text{Turb}})/2 = E^{\text{Turb}}$ and is equal to the total slab energy E^{slab} . The magnetic and cross helicities of the IT ics are the same as their equivalent slab modes.

2.3. Diagnostics

The focus of this paper is spectral anisotropies and variance ratios. We present 2-D contour plots of the magnetic energy spectrum $E_b(k_{\parallel}, k_{\perp})$ as a direct means of exploring spectral anisotropies. For 3-D simulations, $E_b(k_{\parallel}, k_{\perp})$ is defined by $E_b = 2\pi \int E_b(k_{\parallel}, k_{\perp}) dk_{\parallel} dk_{\perp}$ and represents an average of the spectral density over the two perpendicular directions to \mathbf{B}_0 .

We quantify the degree of spectral anisotropy through use of the well-known anisotropy angle θ_Q , which was suggested by *Shebalin et al.* [1983] and has been used extensively in previous studies such as *Oughton et al.* [1994], *Matthaeus et al.* [1996], and *Ghosh and Goldstein* [1997]. The anisotropy angle is defined by the relation

$$\tan^2 \theta_Q = \frac{\sum k_{\perp}^2 |\mathbf{Q}(\mathbf{k}, t)|^2}{\sum k_x^2 |\mathbf{Q}(\mathbf{k}, t)|^2},$$

where \mathbf{Q} is a selected Fourier-decomposable field such as \mathbf{B} and the summations extend over all values of \mathbf{k} . Recall that in 2 $\frac{1}{2}$ -D geometries $k_{\perp}^2 = k_y^2$ and isotropic spectra correspond to $\theta_Q = 45^\circ$ while in 3-D geometries $k_{\perp}^2 = k_y^2 + k_z^2$ and isotropic spectra correspond to $\theta_Q = \tan^{-1} \sqrt{2} \simeq 54.7^\circ$.

We consider velocity and magnetic variance ratios in this paper. Variance anisotropies such as inequalities among velocity components Δu_x^2 , Δu_y^2 , and Δu_z^2 , are independent from spectral anisotropies. A full discussion of variance versus spectral anisotropies is given by *Matthaeus et al.* [1996]. The standard definition of the (fluctuating) component variances is, for example, $\Delta u_x^2 = \sum_{\mathbf{k}} u_x^2$ with the k space summation over all wave numbers. In this paper, we consider the “small-scale” variance defined as, for example, $\Delta_5 u_x^2 = \sum_{|\mathbf{k}| \geq 5} u_x^2$, where the k space summation is $|\mathbf{k}| \geq 5$. The choice of $|\mathbf{k}| \geq 5$ is motivated by the construction of our PB and VS structures. Recalling that the initial PB or VS bandwidths are usually $1 \leq k_y, k_z \leq 4$, this definition of small-scale variance excludes the large-scale structures from the small-scale variance computation. Such a measure will be more useful for comparison with high-resolution solar wind data [e.g., *Klein et al.*, 1991] than the full variance measure computed from summation over all wave numbers.

3. Simulation Results

3.1. Magnetic Pressure-Balanced Structures

The presence of PB structures with field-aligned slab modes (PB+S) creates spectral cascades. However, the nature of the cascade is unlike that seen in MHD turbulence simulations beginning from isotropic initial conditions [*Shebalin et al.*, 1983; *Matthaeus et al.*, 1996]. The spectrum of all the PB+S runs evolves in a similar fashion. We have conducted PB+S runs over the range $1/4 \leq \beta \leq 16$, PB bandwidths $1 \leq k_y, k_z \leq 4$, and $0.01 \leq E_b^{\text{PB}}/E_b^{\text{slab}} \leq 0.125$. A representative case is shown in Figure 2, where six time snapshots of the 2-D magnetic power spectrum $E_b(k_{\parallel}, k_{\perp})$ are shown from a 3-D simulation at times $T = 0, 1, 2, 3, 4$, and 8.

Here the initial PB+S state is composed of a weak large-scale PB structure with $\delta \mathbf{B}^{\text{PB}} = B_x(k_{y,z} = 1)\hat{x}$ and $E_b^{\text{PB}} = 0.0625$. The slab mode energy is $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.5$ and is equally distributed between $1 \leq k_x \leq 3$. The fluctuation energy is large enough that one simulation time unit (T) corresponds approximately to one nonlinear time unit (T_E). The cross helicity of the slab modes is maximum ($\sigma_c = -1$), while the modal magnetic helicity is arbitrary (total $\sigma_m \approx 0$). The background magnetic field is $B_0 = 4$ with $M_{s0} = 1/4$. Hence the plasma state is $\beta = 1$ and the slab fluctuation amplitude is $\delta B/B_0 \approx 0.4$.

The early times of this simulation ($T = 0 \rightarrow 1$, see Figures 2a and 2b) are marked by a rapid energy cascade parallel to \mathbf{B}_0 . This is reminiscent of the nonlinear

field-aligned energy cascades seen in parametric instability simulations of large-amplitude circularly polarized Alfvén waves [*Ghosh and Goldstein*, 1994]. Following this rapid 1-D cascade along k_{\parallel} , the bulk of the energy lies within isocontours that sweep in an arc across the \mathbf{k} space from the k_{\parallel} direction to the k_{\perp} direction during $T = 1 \rightarrow 3$ (Figures 2b - 2d). The long-term spectral pattern is established by $T = 4$ (Figure 2e) where most of the spectral power is highly oblique, but mostly importantly, not purely perpendicular. The final time snapshot is at $T = 8$ (Figure 2f), where the highly oblique nonperpendicular character of $E_b(k_{\parallel}, k_{\perp})$ persists among the nondissipative low- k modes $|\mathbf{k}| < 25$. The Alfvénicity of the highly oblique lobe remains high ($\sigma_c \approx -1$), similar to the *Roberts et al.* [1996] study.

The migration of the spectrum from the field-aligned direction to highly oblique directions is the k space manifestation of wave refraction. This is confirmed from x space analysis as well (not shown). The initial field-aligned slab modes are refracted because of locally different Alfvén phase velocities due to the presence of the PB structures. Changing the wave number bandwidth of the PB structure or its energy content E_b^{PB} causes variations in the rate and magnitude of the refractive process. While the details are hard to quantify, the general features are easily shown using time-history plots of the (magnetic) anisotropy angle θ_b .

The time development of anisotropy for a range of PB-structure amplitudes and fixed plasma $\beta = 1$ is illustrated in Figure 3. These 3-D runs are initiated with four left circularly polarized ($\sigma_c = \sigma_m = -1$) slab modes along k_x and broadband PB structures $B_x(1 \leq k_x, k_y \leq 4)$. The kinetic and magnetic slab energies are equipartitioned with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. The PB magnetic energy is also equipartitioned within the excited band of k . Figure 3 shows the time history of the anisotropy angle θ_b for $E_b^{\text{PB}} = 0.01$ (dot-dashed line) $E_b^{\text{PB}} = 0.03125$ (dashed line), and $E_b^{\text{PB}} = 0.125$ (solid line). The time history of θ_b for an isotropic turbulence initial state starting from $E_v^{\text{Turb}} = E_b^{\text{Turb}} = 1.0$ is shown for comparison (dotted line).

The anisotropy angle is enhanced above the isotropic turbulence values for the $E_b^{\text{PB}} = 0.03125$ and $E_b^{\text{PB}} = 0.125$ cases by $T = 4$. This time corresponds to $T_E \approx 4$ in nonlinear time units. It is important to remember that the k space distribution of fluctuations in different runs can remain quite different even when their θ_b values are similar. For IT ics the maximum of the evolved energy spectrum is precisely in the perpendicular direction. This feature of decaying isotropic turbulence runs is well documented in other papers [e.g., see *Matthaeus et al.*, 1996] and is not shown here. As previously discussed, the energy spectrum is maximum along a highly oblique but nonperpendicular direction in all the PB+S runs.

The degree of anisotropy enhancement is also dependent on the strength of \mathbf{B}_0 . We show time histories of θ_b from several PB+S runs with fixed E_b^{PB} and slab

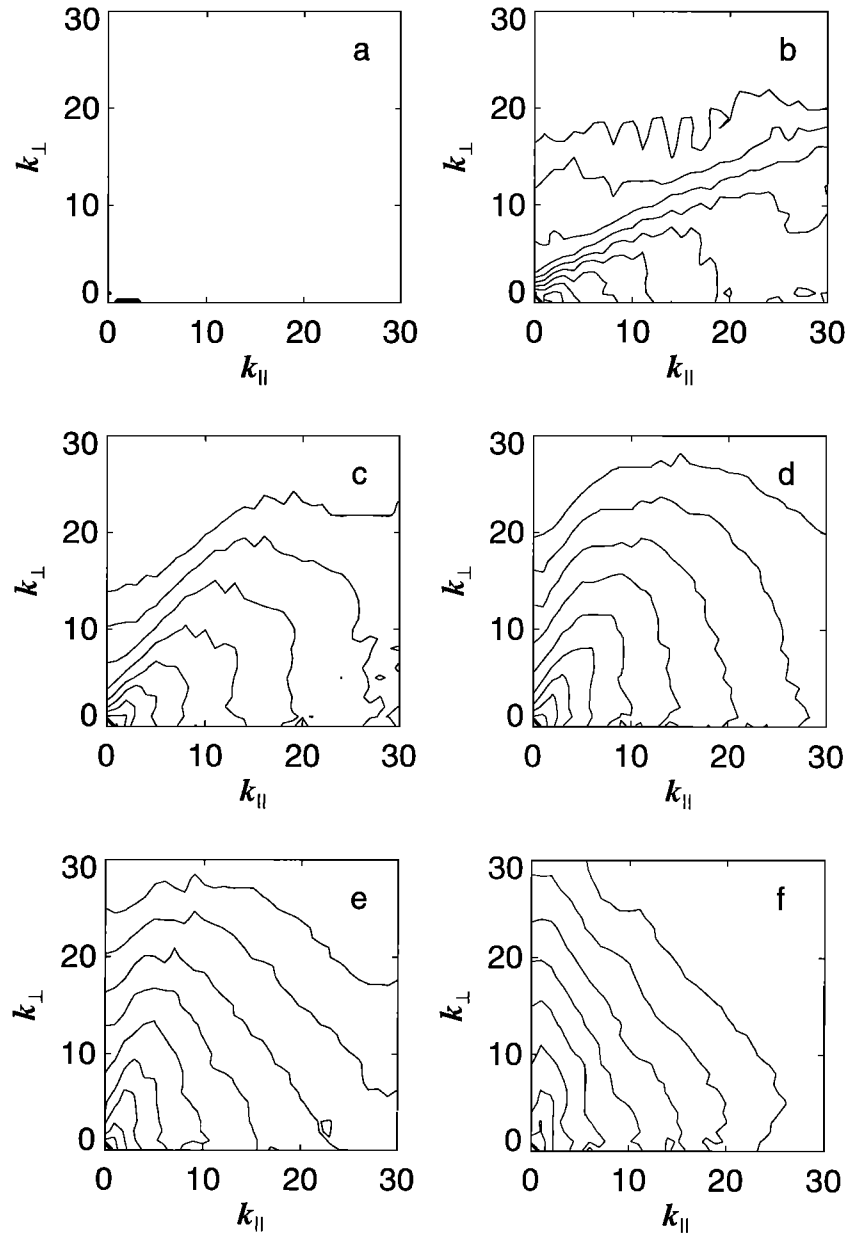


Figure 2. Time snapshots of the 2-D magnetic power spectrum $E_b(k_{\parallel}, k_{\perp})$ at several times during a 3-D PB+S run. The PB structure is a large-scale structure with $\delta \mathbf{B}^{\text{PB}} = B_x(k_{y,z} = 1)\hat{x}$ and $E_b^{\text{PB}} = 0.0625$. The slab modes are three randomly phased waves along k_x with $\sigma_c = -1$, $\sigma_m \approx 0$, and $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.5$. The plasma state is $\beta = 1$. $E_b(k_{\parallel}, k_{\perp})$ is shown at (a) $T = 0$, (b) $T = 1$, (c) $T = 2$, (d) $T = 3$, (e) $T = 4$, and (f) $T = 8$.

energies and varying \mathbf{B}_0 in Figure 4. These are $2\frac{1}{2}$ -D runs. The 3-D runs are similar. We present the $2\frac{1}{2}$ -D results to show that the process under investigation can also appear in a restricted geometry. These runs start from four high cross helicity ($\sigma_c = -1$), random magnetic helicity ($\sigma_m \approx 0$) slab modes along k_x . The kinetic and magnetic slab energies are equipartitioned with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. The PB structure is a single large-scale $k_y = 1$ mode with $E_b^{\text{PB}} = 0.03125$. The initial characteristic Mach number is $M_{s0} = 1/4$. We vary \mathbf{B}_0 between $1 \leq B_0 \leq 8$, thus sampling the plasma β

range $1/4 \leq \beta \leq 16$. Figure 4 shows the θ_b time history to $T = 8$ for $\beta = 1/4$ (dotted line), $\beta = 1/2$ (long-dashed line), $\beta = 1$ (solid line), $\beta = 2$ (short-dashed line), $\beta = 4$ (dot-dashed line), and $\beta = 16$ (triple-dot-dashed line). The nonlinear time T_E is determined from the fluctuation energy E_v^{slab} ; hence, the nonlinear time evolution of all the runs is similar while their Alfvén speeds are dissimilar.

Figure 4 shows that while θ_b anisotropies develop for all β , the development is particularly rapid in the cases where $\beta < 1$. Note that all the $\beta > 1$ runs show $\theta_b \approx 45^\circ$

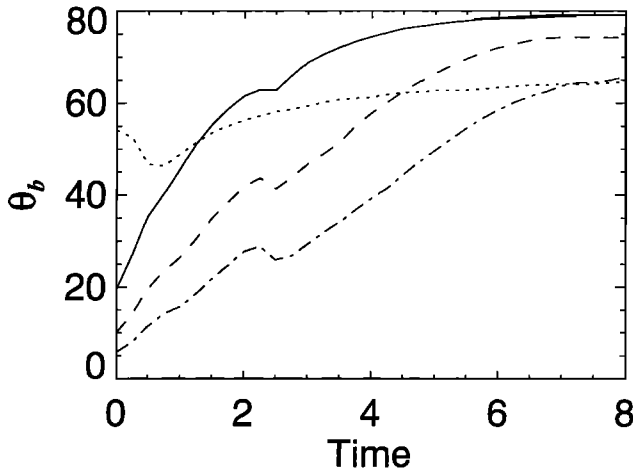


Figure 3. Time history of the magnetic anisotropy angle θ_b from three 3-D PB+S runs with different PB structures amplitudes and all other quantities held fixed. The magnetic energy of these broadband (B_x ($1 \leq k_x, k_y \leq 4$)) PB structures varies between $E_b^{\text{PB}} = 0.01$ (dot-dashed line), $E_b^{\text{PB}} = 0.03125$ (dashed line), and $E_b^{\text{PB}} = 0.125$ (solid line). The slab modes are four left circularly polarized ($\sigma_c = \sigma_m = -1$) randomly phased waves along k_x with $E_v^{\text{Slab}} = E_b^{\text{Slab}} = 1.0$. The time history of θ_b for an isotropic turbulence (IT) run with $\sigma_c = \sigma_m = -1$ and $E^{\text{Turb}} = E^{\text{Slab}}$ is shown for comparison (dotted line). The plasma state is $\beta = 1$ for all the runs.

at $T \approx 4$. This may explain why *Roberts et al.* [1996] reported nearly isotropic wave distributions for similar PB+S simulations. The *Roberts et al.* [1996] study considered $\beta = 4$ and ran their simulations to $T = 4$.

We now turn to consideration of the velocity and magnetic variances from PB+S simulations. As a typical example, we return to the PB+S run whose spectral evolution was shown in Figure 2. The small-scale velocity and magnetic variance ratios of this run are shown in Figure 5. The small-scale variances are obtained from summing all wave numbers $|\mathbf{k}| \geq 5$. Since the initial PB structure's bandwidth is $\mathbf{k} = 1$, the small-scale variances represent typical variances that one measures inside the inertial range at scales smaller than the PB structure. The time history of small-scale velocity variance ratios $\Delta_5 u_y^2 / \Delta_5 u_x^2$ (dashed line) and $\Delta_5 u_z^2 / \Delta_5 u_x^2$ (solid line) are shown in Figure 5a. Both quantities gradually rise from zero and saturate to $\Delta_5 u_{y,z}^2 / \Delta_5 u_x^2 \approx 4$ by $T = 4$. Ratios of the full velocity variance measures $\Delta u_{y,z}^2 / \Delta u_x^2$ (summation over all wave numbers) saturate at slightly lower values with $2 \leq \Delta u_{y,z}^2 / \Delta u_x^2 \leq 4$ for $T \geq 2$ and are not shown.

The time history of the small-scale magnetic variances ratios $\Delta_5 b_y^2 / \Delta_5 b_x^2$ (dashed line) and $\Delta_5 b_z^2 / \Delta_5 b_x^2$ (solid line) are shown in Figure 5b. Recalling that $\Delta_5 b_x^2 = 0$ at $T = 0$, the magnetic variance ratios drop from infinity to $\Delta_5 b_{y,z}^2 / \Delta_5 b_x^2 \approx 5$ at $T = 2$. The ratios saturate to $\Delta_5 b_{y,z}^2 / \Delta_5 b_x^2 \approx 8$ by $T = 8$. Ratios of the

full magnetic variance measures ($\Delta b_{y,z}^2 / \Delta b_x^2$) saturate at slightly lower values with $3 \leq \Delta b_{y,z}^2 / \Delta b_x^2 \leq 5$ for $T \geq 2$ and are not shown.

The other PB+S simulations show similar behavior in the small-scale/large-scale velocity and magnetic variance ratios. They do not categorize as easily as θ_b does with E_b^{PB} and β above; however, there are several cases with ratios as large as $\Delta_5 u_{y,z}^2 / \Delta_5 u_x^2 \approx 5$ and $\Delta_5 b_{y,z}^2 / \Delta_5 b_x^2 \approx 8$. The small-scale magnetic variance ratios are systematically higher than their velocity counterparts. To better understand this we separate each Fourier velocity fluctuation into longitudinal and solenoidal components $\delta \mathbf{u}(\mathbf{k}) = \delta \mathbf{v}(\mathbf{k}) + \delta \mathbf{w}(\mathbf{k})$, where the longitudinal and solenoidal directions are with respect to the Fourier wave vector \mathbf{k} . We then compute the small-scale longitudinal- and solenoidal-velocity variance ratios $\Delta_5 v_{y,z}^2 / \Delta_5 v_x^2$ and $\Delta_5 w_{y,z}^2 / \Delta_5 w_x^2$ from $\delta \mathbf{v}(\mathbf{k})$ and $\delta \mathbf{w}(\mathbf{k})$, respectively. We find that $\Delta_5 v_{y,z}^2 / \Delta_5 v_x^2 < 1$ consistently, indicating that the $\delta \mathbf{v}$ fluctuations are preferentially field-aligned. We find that $\Delta_5 w_{y,z}^2 / \Delta_5 w_x^2$ is larger than $\Delta_5 u_{y,z}^2 / \Delta_5 u_x^2$ with typical orderings where

$$\Delta_5 u_{y,z}^2 / \Delta_5 u_x^2 < \Delta_5 w_{y,z}^2 / \Delta_5 w_x^2 \approx \Delta_5 b_{y,z}^2 / \Delta_5 b_x^2.$$

Hence compressive velocity fluctuations appear to account for the discrepancy between the small-scale velocity and magnetic variance ratios in our PB+S simulations. These variance ratios are well above the variance ratios reported from compressible MHD turbulence sim-

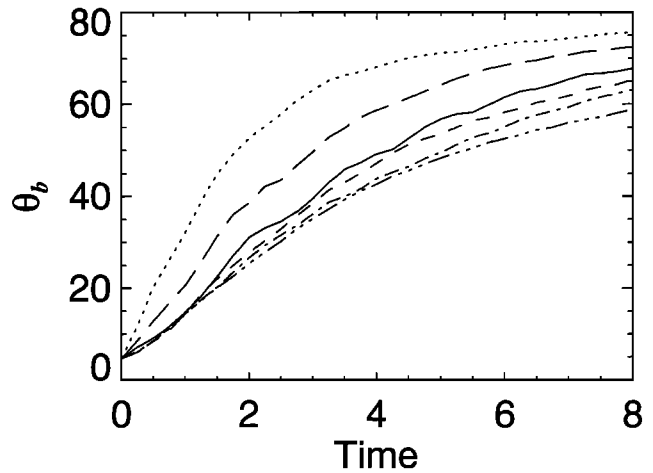


Figure 4. The time history of the magnetic anisotropy angle θ_b from several $2\frac{1}{2}$ -D PB+S runs for differing strengths of the mean magnetic field B_0 and all other quantities fixed. The PB structure has a 1-D bandwidth $k_y = 1$ with $E_b^{\text{PB}} = 0.03125$. The slab modes are four left circularly polarized ($\sigma_c = \sigma_m = -1$) randomly phased waves along k_x with $E_v^{\text{Slab}} = E_b^{\text{Slab}} = 1.0$. $M_{s0} = 1/4$ for all the runs. B_0 is varied to produce plasma states with $\beta = 1/4$ (dotted line), $\beta = 1/2$ (long-dashed line), $\beta = 1$ (solid line), $\beta = 2$ (short-dashed line), $\beta = 4$ (dot-dashed line), and $\beta = 16$ (triple-dot-dashed line).

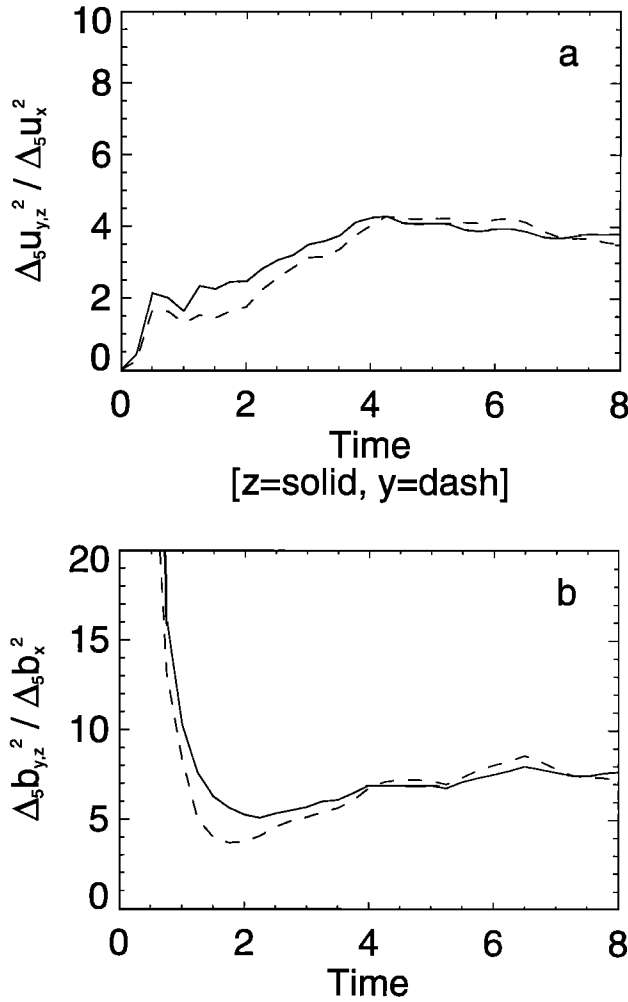


Figure 5. Time histories of the small-scale velocity and magnetic variance ratios from the same PB+S run in Figure 2. The small-scale variance is obtained from a $|\mathbf{k}| \geq 5$ k space summation. This neglects the large-scale PB structure contribution to the variance measure. (a) The velocity variance ratios $\Delta_5 u_y^2 / \Delta_5 u_x^2$ (dashed line) and $\Delta_5 u_z^2 / \Delta_5 u_x^2$ (solid line) and (b) the magnetic variance ratios $\Delta_5 b_y^2 / \Delta_5 b_x^2$ (dashed line) and $\Delta_5 b_z^2 / \Delta_5 b_x^2$ (solid line) are shown.

ulations initiated with isotropic fluctuations [Matthaeus *et al.*, 1996]. They also are in closer agreement with solar wind variance ratios [Klein *et al.*, 1991].

Note that the variance minima of both $\delta \mathbf{u}$ and $\delta \mathbf{B}$ are along \mathbf{B}_0 , while the wave vectors (\mathbf{k}) of the associated power is highly oblique. Although, it is well known in early solar wind literature [see, e.g., Barnes, 1981] that highly oblique Alfvén waves still possess a variance minimum along \mathbf{B}_0 , this aspect of Alfvénic turbulence seems to be often neglected. Studies that use the minimum variance characteristics of solar wind fluctuations to justify applications of 1-D field-aligned nonlinear Alfvén dynamics may not be focused on the major constituents of the wave dynamics.

3.2. Velocity Shears

Several evolutionary similarities exist between VS+S and PB+S simulations. The evolution of the 2-D magnetic power spectrum $E_b(k_{\parallel}, k_{\perp})$ in VS+S simulations is similar to the PB+S case (Figure 2) and is not shown. In spite of this similarity it is clear that spectral evolution has differing origins in the two cases: The PB+S case involves wave refraction to oblique directions, whereas spectral transfer in the VS+S case is driven by shear through the $\mathbf{u} \cdot \nabla \mathbf{u}$ term in the momentum equation.

Nevertheless, as with PB structures (cf. Figure 3), stronger velocity shears create larger anisotropies in θ_b . This is demonstrated in Figure 6, which shows the time history of θ_b for different VS-structure amplitudes with fixed plasma $\beta = 1$. These are 3-D runs with four left circularly polarized ($\sigma_c = \sigma_m = -1$) slab modes along k_x and a broadband VS structures $U_x (1 \leq k_x, k_y \leq 4)$. The kinetic and magnetic slab energies are equipartitioned with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. The VS fluctuation spectrum is flat in k while the total energy E_v^{VS} is varied. Figure 6 compares the time history of θ_b for $E_v^{\text{VS}} = 0.01$ (dotted line) $E_v^{\text{VS}} = 0.03125$ (dashed line), and $E_v^{\text{VS}} = 0.125$ (solid line). These are the same values in terms of fluctuation energy content E_b^{PB} as the PB+S runs shown in Figure 3. The similarities in θ_b evolution is remarkably close between the VS+S runs in Figure 6 and the PB+S runs in Figure 3.

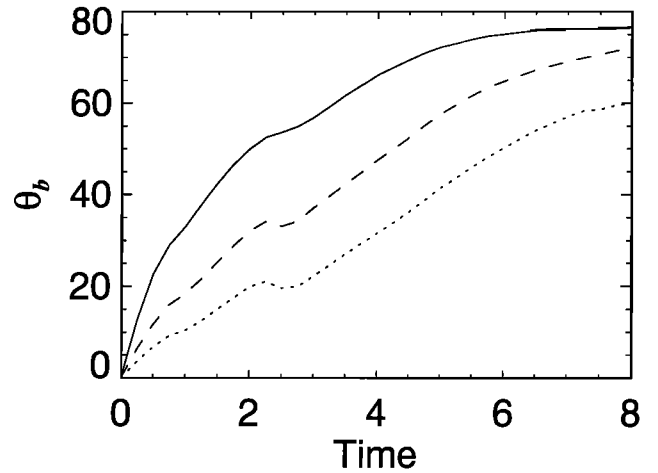


Figure 6. Time history of the magnetic anisotropy angle θ_b from three 3-D VS+S runs with different VS amplitudes and all other quantities fixed. The fluctuation energy of these broadband ($V_x (1 \leq k_x, k_y \leq 4)$) velocity shears varies between $E_v^{\text{VS}} = 0.01$ (dotted line) $E_v^{\text{VS}} = 0.03125$ (dashed line), and $E_v^{\text{VS}} = 0.125$ (solid line). The slab modes are four left circularly polarized ($\sigma_c = \sigma_m = -1$) randomly phased waves along k_x with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. The plasma state is $\beta = 1$ for all the runs. Note these are the same values in terms of energy content E_b^{PB} as the PB+S runs shown in Figure 2.

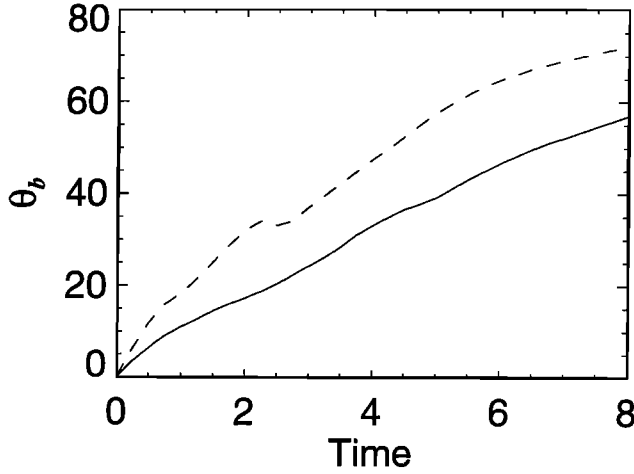


Figure 7. Time history of the magnetic anisotropy angle θ_b from two 3-D VS+S runs with different VS bandwidths and all other quantities fixed. The fluctuation energy of the VS structures is $E_v^{VS} = 0.03125$. In one case the bandwidth is $k_y, k_z = 1$ (solid line), while in the other case the bandwidth is $1 \leq k_y, k_z \leq 4$ (dashed line). The slab modes are four left circularly polarized ($\sigma_c = \sigma_m = -1$) randomly phased waves along k_x with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. The plasma state is $\beta = 1$.

There are important differences between the PB+S and VS+S cases as well. Changing the bandwidth of the PB structure while holding E_b^{PB} fixed does not influence significantly the development of anisotropy. The rate and magnitude of spectral anisotropy development of a $E_b^{\text{PB}}(k_y, k_z = 1)$ run is virtually the same as the $E_b^{\text{PB}}(1 \leq k_y, k_z \leq 4)$ run previously shown in Figure 3. On the other hand, changing the bandwidth of the VS while holding E_v^{VS} fixed brings noticeable changes to the rate of anisotropy development. This is shown in Figure 7, where the time history of θ_b is plotted for two 3-D VS+S runs. Again, four left circularly polarized ($\sigma_c = \sigma_m = -1$) Alfvén waves along k_x with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$ define the initial population of slab modes. The VS fluctuation energy is also fixed at $E_v^{VS} = 0.03125$. However, in one case the VS bandwidth is $E_v^{VS}(k_y, k_z = 1)$ (solid line), while in the other case the VS bandwidth is $E_v^{VS}(1 \leq k_y, k_z \leq 4)$ (dashed line). The broader band VS+S run develops larger and faster anisotropies. Runs incorporating a further increase of the VS bandwidth (not shown) to $1 \leq k_y, k_z \leq 8$, while the slab bandwidth remains at $1 \leq k_x \leq 4$, do not exhibit greater anisotropy development beyond the $k_{\text{max}} = 4$ case (Figure 7). This hints that the influence of velocity shears on slab turbulence extends only to the bandwidth of the initial slab modes. The physical effects illustrated in Figure 7 suggest that velocity microstreams may play a significant role in creating highly anisotropic turbulence out of slab turbulence in the solar wind [see Roberts et al., 1992].

Another significant difference between the PB+S and VS+S cases is that in the case of velocity shears the rate

of spectral anisotropy development is not influenced by plasma β . We have already shown the strong dependence on β for PB+S runs in Figure 4. Here we show the equivalent plots for VS+S simulations in Figure 8. As before, these are 2½-D simulations. The 3-D runs (not shown) are similar. The slab modes are identical to the PB+S runs in Figure 4, but the PB structures are now replaced by VS structures with a single large-scale $k_y = 1$ mode with $E_v^{VS} = 0.03125$. The characteristic Mach number is $M_{s0} = 1/4$. We vary B_0 between $2 \leq B_0 \leq 8$ to sample the plasma β range $1/4 \leq \beta \leq 4$. Figure 8 shows the θ_b time history to $T = 8$ for $\beta = 1/4$ (dotted line), $\beta = 1/2$ (long-dashed line), $\beta = 1$ (solid line), $\beta = 2$ (short-dashed line), and $\beta = 4$ (dot-dashed line). There is virtually no difference in the θ_b evolution for the range of β shown.

This lack of dependence on β and, more specifically, B_0 , should come as no surprise. The development of anisotropies in the VS+S runs is due to advective influences, which are held fixed; while the development of anisotropies in the PB+S runs is due to refractive variations in local phase speed, which depend strongly on β . Nonetheless, this absence of a θ_b - B_0 dependence indicates that such dependencies are not necessarily present in all turbulence simulations. Matthaeus et al. [1998a] report clear θ_b - B_0 dependencies for incompressible and compressible 3-D MHD decay simulations starting from isotropic turbulence initial conditions. The results presented here suggest that only the $\delta \mathbf{B} \parallel \mathbf{B}_0$ components of the isotropic ics may contribute to the θ_b - B_0 dependencies in the Matthaeus et al. study.

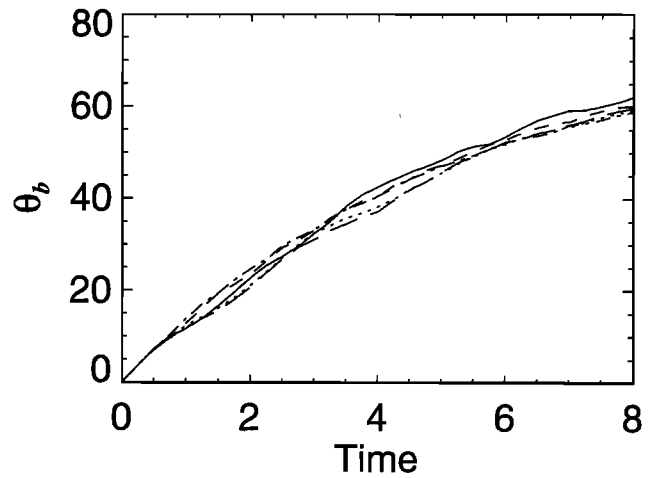


Figure 8. The time history of the magnetic anisotropy angle θ_b from several 2½-D VS+S runs for differing strengths of the mean magnetic field B_0 and all other quantities fixed. The velocity shear has a 1-D bandwidth $k_y = 1$ with $E_v^{VS} = 0.03125$. The slab modes are four left circularly polarized ($\sigma_c = \sigma_m = -1$) randomly phased waves along k_x with $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. $M_{s0} = 1/4$ for all the runs. B_0 is varied to produce plasma states with $\beta = 1/4$ (dotted line), $\beta = 1/2$ (long-dashed line), $\beta = 1$ (solid line), $\beta = 2$ (short-dashed line), and $\beta = 4$ (dot-dashed line).

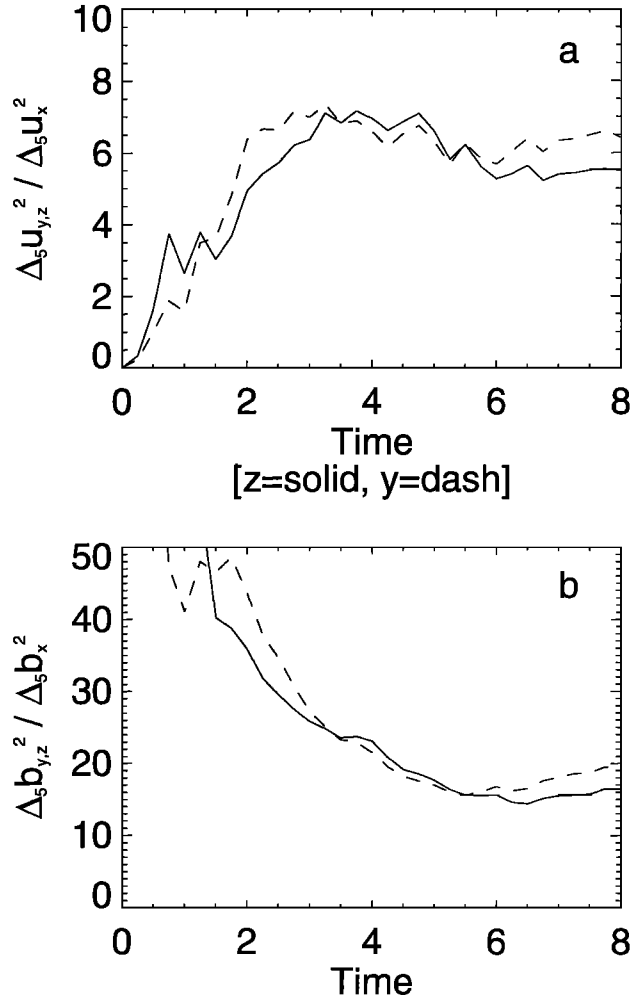


Figure 9. Time histories of the small-scale velocity and magnetic variance ratios from a 3-D VS+S run. The VS is a randomly phased, modally flat spectrum between $1 \leq k_y, k_z \leq 4$ with $E_v^{\text{VS}} = 0.01$. The slab component is a randomly phased, modally flat spectrum between $1 \leq k_x \leq 4$ with $\sigma_c = -1$, $\sigma_m \approx 0$, and $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$. The small-scale variance is obtained from a $|\mathbf{k}| \geq 5$ k space summation. This neglects the initial VS contribution to the variance measure. (a) The velocity variance ratios $\Delta_5 u_y^2 / \Delta_5 u_x^2$ (dashed line) and $\Delta_5 u_z^2 / \Delta_5 u_x^2$ (solid line) and (b) the magnetic variance ratios $\Delta_5 b_y^2 / \Delta_5 b_x^2$ (dashed line) and $\Delta_5 b_z^2 / \Delta_5 b_x^2$ (solid line) are shown.

Similar to the PB+S case, time histories of the velocity and magnetic variance ratios of all VS+S runs show a propensity for the minimum variance to lie in the direction B_0 . Figure 9 shows the small-scale velocity and magnetic variance ratios of a 3-D VS+S run with $(\sigma_c = -1, \sigma_m \approx 0)$ $E_v^{\text{slab}} = E_b^{\text{slab}} = 1.0$ slab modes between $1 \leq k_x \leq 4$ and VS structures with $E_v^{\text{VS}} = 0.01$ between $1 \leq k_y, k_z \leq 4$. Again, the small-scale variances are obtained from summing all wave numbers $|\mathbf{k}| \geq 5$, thereby avoiding the contributions of the initial VS structure. The time history of small-

scale velocity variance ratios $\Delta_5 u_y^2 / \Delta_5 u_x^2$ (dashed line) and $\Delta_5 u_z^2 / \Delta_5 u_x^2$ (solid line) are shown in Figure 9a. Both quantities gradually rise from zero and saturate to $\Delta_5 u_{y,z}^2 / \Delta_5 u_x^2 \approx 7$ by $T = 3$. The time history of the small-scale magnetic variances ratios $\Delta_5 b_y^2 / \Delta_5 b_x^2$ (dashed line) and $\Delta_5 b_z^2 / \Delta_5 b_x^2$ (solid line) are shown in Figure 9b. The ratios remain above $\Delta_5 b_{y,z}^2 / \Delta_5 b_x^2 > 15$. Ratios of the full variances (not shown) are considerably higher with $\Delta u_{y,z}^2 / \Delta u_x^2 \approx 10$ and $\Delta b_{y,z}^2 / \Delta b_x^2 > 60$.

All small-scale velocity and magnetic variance ratios of any selected VS+S run are higher than the equivalent ratios of a PB+S run for identical energy and excitation bandwidths. Although we cannot quantify this in terms of what energies and bandwidths bring equal variance ratios between VS+S and PB+S runs, this is evident from comparing Figures 5 and 9, where higher small-scale variance ratios appear in the VS+S run while the E_v^{VS} energy content is less than 1/6 of the E_b^{PB} energy content. This supports our previous assertion that velocity microstreams may create stronger anisotropies than PB structures. Varying the excitation bandwidth of the VS imparts little change in the small-scale variance ratios. We have already noted that spectral anisotropies develop faster as the excitation bandwidth is increased.

Similar to the PB+S runs, the small-scale magnetic variance ratios are systematically higher than the small-scale velocity variance ratios of all VS+S runs. Again, we separate the velocity fluctuations into longitudinal and solenoidal components $\delta \mathbf{u}(\mathbf{k}) = \delta \mathbf{v}(\mathbf{k}) + \delta \mathbf{w}(\mathbf{k})$ with respect to the Fourier wave vector \mathbf{k} and compute the corresponding small-scale longitudinal- and solenoidal-velocity variance ratios

$$\Delta_5 v_{y,z}^2 / \Delta_5 v_x^2 \text{ and } \Delta_5 w_{y,z}^2 / \Delta_5 w_x^2.$$

As before, we find that $\Delta_5 v_{y,z}^2 / \Delta_5 v_x^2 < 1$ consistently, indicating that the $\delta \mathbf{v}$ fluctuations that arise from the VS+S ics are preferentially field-aligned. The $\Delta_5 w_{y,z}^2 / \Delta_5 w_x^2$ ratios are not quite as large as the $\Delta_5 b_{y,z}^2 / \Delta_5 b_x^2$ ratios. A typical case is the VS+S run previously discussed in Figure 9. We compute $\Delta_5 w_{y,z}^2 / \Delta_5 w_x^2 \approx 10$ when, as can be measured from Figure 9b, $\Delta_5 b_{y,z}^2 / \Delta_5 b_x^2 \approx 15$. Interestingly, the small-scale longitudinal-velocity variance ratios are $\Delta_5 v_{y,z}^2 / \Delta_5 v_x^2 < 0.1$ for this case, implying extreme field-alignment of the $\delta \mathbf{v}$ fluctuations.

3.3. Isotropic Turbulence

The 2-D magnetic power spectrum $E_b(k_{\parallel}, k_{\perp})$ from MHD simulations with an ambient magnetic field and IT ics is well-documented [see, e.g., *Matthaeus et al.*, 1996] and is not shown here. The key distinguishing feature between IT power spectra and PB+S or VS+S power spectra is that spectral energy in the k_{\perp} direction is fully populated for IT simulations, while energy is depleted in the k_{\perp} direction for PB+S and VS+S simulations.

The reason for the perpendicular energy depletion in the PB+S and VS+S cases is quite simple: The refractive or advective forces that drive the spectral evolution can never remove the k_{\parallel} component of the initial slab fluctuations. These modes may refract or advect to very large oblique angles; however, a k_{\parallel} component will always be present. Hence these modes can never contribute to energy in the purely k_{\perp} direction. If the energy in PB or VS structures does not readily cascade to higher wave numbers, then a relative energy depletion will always be seen along the k_{\perp} direction.

We have already discussed how the spectral anisotropies of PB+S and VS+S ics are typically enhanced over the IT case. As shown in Figure 3, the relative anisotropy enhancements between PB+S or VS+S ics and an equivalent high cross-helicity IT state can be as much as $\Delta\theta_b \approx 20^\circ$. We have run PB+S and VS+S cases with $\sigma_c \approx -0.5$ among the slab modes to model forward- and backward-propagating Alfvén waves. The spectral anisotropies of these runs are the same as the pure $\sigma_c = -1$ runs shown in Figure 3. Consistent with previous studies, the spectral anisotropies of the equivalent low- σ_c IT cases are higher than those of the high- σ_c IT case. Hence the relative anisotropy enhancements between the low- σ_c PB+S or VS+S ics and the equivalent low- σ_c IT state is not as large. Typical values are $\Delta\theta_b \approx 5^\circ$.

In spite of the differences in the angular distribution of energy in k space, the omnidirectional $|\mathbf{k}|$ -dependence of the spectrum is remarkably similar between the IT, PB+S, and VS+S ics. We show the time-averaged omnidirectional (modal) magnetic power spectra $E_b(k)$ from the three types of initial states for the same plasma state ($\beta = 1$) and a $2\frac{1}{2}$ D geometry in Figure 10. $E_b(k)$ is defined by $E_b = \langle \delta\mathbf{B} \cdot \delta\mathbf{B}/2 \rangle_x = \int E_b(k) 2\pi k dk$, where $\langle \dots \rangle_x$ denote a spatial average and seven such spectra equally separated between $2.5 \leq T \leq 4$ have been averaged to obtain the time-averaged value.

The $2\frac{1}{2}$ D geometry is used for increased spectral resolution. The 3-D results are similar. $E_b(k)$ from a run with IT ics is shown as the dotted line in Figure 10. Here the initial bandwidth is $1 \leq |\mathbf{k}| \leq 4$ with $\sigma_c = -1$ and $\sigma_m \approx 0$. The total initial energy is equipartitioned $E_v^{\text{Turb}} = E_b^{\text{Turb}} = 1$, so the Alfvén ratio is unity for all modes. The initial density is a constant $\rho = 1$. The solid line in Figure 10 shows $E_b(k)$ from a run with PB+S ics. The total energy, σ_c , and σ_m of the slab modes are the same as the IT ics. The dashed line in Figure 10 shows $E_b(k)$ from a run with VS+S ics. The slab modes are identical to those of the PB+S run. The PB and VS structures in these two examples are due entirely to the $k_y = 1$ mode. The PB and VS fluctuation energies are identical with $E_b^{\text{PB}} = E_v^{\text{VS}} = 0.03125$. Other quantities such as the omnidirectional internal energy and the omnidirectional kinetic energy are also similar for the IT, PB+S, and VS+S ics and are not shown.

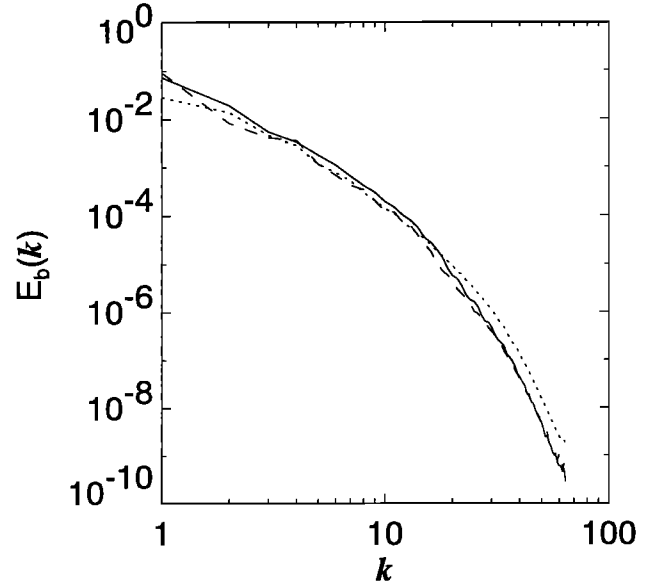


Figure 10. The time-averaged modal omnidirectional magnetic power spectra $E_b(k)$ from an IT run (dotted line), a PB+S run (solid line), and a VS+S run (dashed line) conducted using a $2\frac{1}{2}$ D geometry and $\beta = 1$. The time averaging uses seven equally separated snapshots between $2.5 \leq T \leq 4$. The initial bandwidths of the IT and slab modes are $1 \leq |\mathbf{k}| \leq 4$ with $\sigma_c = -1$, $\sigma_m \approx 0$, and $E_v^{\text{slab}} = E_b^{\text{slab}} = E_v^{\text{Turb}} = E_b^{\text{Turb}} = 1$. The PB and VS bandwidth is $k_y = 1$ with energies $E_v^{\text{VS}} = E_b^{\text{PB}} = 0.03125$.

This similarity in the omnidirectional spectra suggests that reduced spectra, as is typically measured for the solar wind, may also be indistinguishable although their sources (IT, PB+S, or VS+S) may be varied. A closer examination of spectral anisotropies, particularly in the highly oblique direction to \mathbf{B}_0 , may indicate whether the turbulence source is slab-like or isotropic.

4. Discussion and Conclusions

Early observational studies of interplanetary turbulence made few attempts to determine the rotational symmetry of fluctuations, concentrating instead on directly observable features, such as the reduced power spectrum and variances. For some time theoretical modeling proceeded by making the simplest assumptions: Slab geometry was frequently assumed, often with little specific motivation beyond Alfvénic correlations and minimum variance arguments. Both of these are known to be imprecise with regard to determination of fluctuation geometry. Over time, indications have emerged that the spectrum of MHD scale fluctuations is systematically anisotropic and that the observed fluctuations cannot easily be described by a pure slab model. *Sari and Valley [1976]* found evidence of field-aligned Alfvén waves along with oblique magnetosonic modes based on power spectra, correlation, and

coherency analyses of Pioneer 6 interplanetary magnetic field data.

Like *Matthaeus et al.* [1990], *Sari and Valley* [1976] based their analysis on comparison of intervals in which \mathbf{B}_0 subtended varying angles with respect to the solar wind flow direction. Properties of the statistics as a function of field-flow angle become interpreted in these studies as evidence for particular fluctuation geometry or, in some cases, evidence against a pure symmetry such as slab. *Carbone et al.* [1995] and *Bieber et al.* [1996] have also examined this issue using various improved techniques. In spite of differences in details, a common feature of these studies is that they rule out the pure slab model as a reasonable representation of solar wind fluctuations. One concludes, very conservatively, that an acceptable model of the fluctuation geometry must include at least some small admixture of fluctuations with a symmetry other than slab.

All of the above studies examined the kinematic properties of observed fluctuations, attempting in one way or the other to find a consistent recipe for the average geometrical properties of a fluctuation model. So far as we are aware the associated dynamical questions have not been fully addressed with regard to inherent nonequilibrium and instability of models with particular symmetry in the context of a reasonable dynamical model such as magnetohydrodynamics. Theoretical and simulation work on development of spectral anisotropy in MHD turbulence [*Shebalin et al.*, 1983; *Oughton et al.*, 1994; *Matthaeus et al.*, 1996] has generally examined the onset of anisotropy starting from an isotropic state. From these studies we know that quasi-2-D fluctuations are dynamically favored, especially at high wave number. Dynamical stability of the slab model has been less well studied.

Our main finding is that a spectrum of highly Alfvénic slab fluctuations experiences a distinctive MHD scale evolution in the presence of even small admixtures of either perpendicular PB structures or transverse velocity shears. The evolution takes place over a few large-scale eddy-turnover times and is characterized by the “refraction” of the Alfvénic lobe of the spectrum away from the direction parallel to the mean magnetic field in the case of pressure-balanced structures and a similar influence driven by advective forces in the case of velocity shears. While the most excited lobe remains somewhat identifiable (including the Alfvénic property), it does not become purely perpendicular, but the spectral distortion saturates at a highly oblique angle that appears to be related to the initial parallel wave number distribution of the Alfvénic population. The timescale for evolution to the saturated oblique Alfvénic state, about four eddy-turnover times (often less particularly for $\beta < 1$), corresponds to ~ 2 AU of solar wind flow [*Matthaeus et al.*, 1998b].

It is not clear whether the initial slab fluctuations remain wave-like, as opposed to nonlinearly dominated high cross-helicity turbulence, after their spectral dis-

tortion to a highly oblique angle. An ω - \mathbf{k} dispersion analysis is needed as well as direct measurements of the linear-versus-nonlinear driving terms in the MHD simulation algorithms. If the nonlinear forcing is relatively weak, these oblique fluctuations may develop spherically polarized Alfvén waves with imbedded rotational discontinuities as suggested by *Vasquez and Hollweg* [1998]. These issues will be addressed in a future paper.

The spectral distortion process that eliminates energy from the field-aligned slab spectral component is formally nonlinear in that it involves quadratic couplings such as $\delta\mathbf{B} \cdot \nabla\delta\mathbf{B}$. However, the basic physical couplings associated with this refractive or advective phenomenon can be understood in terms of a linear equation. For example, the Alfvén wave amplitudes can be approximated as evolving linearly with the PB (or VS) structures taken to be constant coefficients in the MHD equations. This leads to simple estimates of advective or refractive distortion of the spectrum that agree fairly well with the simulation data. For example, using the parameters for the narrow band ($k_y, k_z = 1$) VS run shown in Figure 6, one estimates from the above argument that anisotropy angles should grow initially at a rate of $\sim 10^\circ$ per simulation time unit. This agrees reasonably well with the simulation results. This geometric argument is more difficult to apply in the case of broadband VS or PB structures and, in any case, breaks down eventually due to mode-mode couplings. We believe that the bump appearing in several of the magnetic anisotropy angle time history plots at $T \sim 2$ may delineate the transition from linear to nonlinear processes.

Stability of a single parallel propagating slab wave has been studied extensively in the context of parametric processes [*Ghosh and Goldstein*, 1994; *Ghosh et al.*, 1994]. By examining the response of the Alfvén wave spectrum to transverse pressure-balanced structures, we have described a process that competes with parametric instabilities. Regardless of the formalism used to describe the evolution, it seems clear that it is difficult to maintain a spectrum of parallel propagating Alfvén waves in the presence of the types of perturbations used here. A more thorough quantitative exploration of the realm of applicability of the parametric instabilities is warranted but will not be undertaken at present.

The scenario described in this paper addresses the dynamical consistency of the “waves plus structures” model of solar wind fluctuations [*Tu and Marsch*, 1993; *Carbone et al.*, 1995]. Because of the strong dynamical interaction of the two components, it would seem unlikely that a two-component model based upon non-interacting ingredients can remain viable over the timescale of several characteristic nonlinear times. The slab and structure ingredients do not remain independent, the slab part of the Fourier space actually becomes depopulated, and what appears to be an essential element of these models — superposition — is lost.

While we expect that the present findings may be of immediate application in development and evaluation of models for solar wind fluctuations and MHD descriptions of space plasmas in general, it is also clear that additional problems along these lines remain to be tackled. We can describe in a similar way the dynamical interactions of a spectrum of slab Alfvén waves with a perturbing broadband spectrum of strictly 2-D fluctuations, having both wave vectors and amplitudes perpendicular to the mean field. Such 2-D fluctuations differ in important ways from the “structures” employed in the present paper, and such a model is another type of two-component model as envisioned by Matthaeus *et al.* [1990] and Tu and Marsch [1993]. This case is examined in paper 2. The structures in the present paper, for example, taken by themselves are in a state of classical MHD pressure balance and are therefore inert by construction. Strictly 2-D fluctuations, however, are expected to be highly active [Fyfe and Montgomery, 1976] and would exhibit their own cascade, reconnection [Matthaeus and Lamkin, 1986] and various other turbulence effects. Far from being inert, strictly 2-D fluctuations may be highly turbulent. Consequently, we may expect interesting differences in the slab plus strict 2-D case. We remark in closing that the present approach to examination of dynamical stability of a model geometry does not take into account various effects that may have considerable impact in the solar wind, including expansion, multifluid and kinetic effects.

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- S. Ghosh, Applied Research Division, Space Applications Corporation, 9315 Large Drive West, Suite 250, Largo, MD 20774. (e-mail: ron.ghosh@gsfc.nasa.gov)
- W. H. Matthaeus, Bartol Research Institute, University of Delaware, Newark, DE 19716. (e-mail: yswm@bartol.udel.edu)
- M. L. Goldstein and D. A. Roberts, Laboratory for Extraterrestrial Physics, NASA Goddard Space Flight Center, Greenbelt, MD 20771. (e-mail: melvyn.goldstein@gsfc.nasa.gov; aaron.roberts@gsfc.nasa.gov)

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